# Heavy quark mass effects in $e^+e^-$ into three jets [*]

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## Abstract

Next-to-leading order calculation for three jet heavy quark production in $e^+e^-$ collisions, including complete quark mass effects, is reviewed. Its applications at LEP/SLC are also discussed.

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Next-to-leading order calculation for three-jet heavy quark production in $e^+e^-$-collisions, including complete quark mass effects, is reviewed. Its applications at LEP/SLC are also discussed.

The importance of the corrections due to the mass of the heavy quark in the jet-production in $e^+e^-$-collisions has been already seen in the early tests of the flavour independence of the strong coupling constant [1,2]. The final high precision of the LEP/SLC experiments required accurate account for the bottom-quark mass in the theoretical predictions. If quark mass effects are neglected, the ratio $\alpha_s^b/\alpha_s^{uds}$ measured from the analysis of different three-jet event-shape observables is shifted away from unity up to 8% [3] (see also [4]).

Sensitivity of the three-jet observables to the value of the heavy quark mass allowed to consider the possibility [5,6] of the determination of the b-quark mass from LEP data, assuming the universality of $\alpha_s$. In a recent analysis of three-jet events, DELPHI measured the mass of the b-quark, $m_b$, for the first time far above the production threshold [4]. This result is in a good agreement with low energy determinations of $m_b$ using QCD sum rules and lattice QCD from $\Upsilon$ and $B$-mesons spectra (for recent results see e.g. [7]) The agreement between high and low energy determinations of the quark mass is rather impressive as non-perturbative parts are very different in the two cases.

In this contribution we will discuss some aspects of the next-to-leading order (NLO) calculation of the decay $Z \to 3jets$ with massive quarks, necessary for the measurements of the bottom-quark mass at the $Z$-peak. Recently such calculations were performed independently by three groups [8–10].

The first question we would like to answer whether it is not at all surprising that LEP/SLC observables are sensitive to $m_b$ as the main scale involved is the mass of the $Z$-boson, $M_Z \gg m_b$. Indeed, the quark-mass effects for an inclusive observable such as the total width $Z \to b\bar{b}$ are negligible. Due to Kinoshita-Lee-Nauenberg theorem such observable does not contain mass singularities and a quark-mass appears in the ratio $\overline{m}_q^2(M_Z)/M_Z^2 \approx 10^{-3}$, where using $\overline{MS}$ running mass at the $M_Z$-scale takes into account the bulk of the NLO QCD corrections [11,6].

However, the situation with more exclusive observables is different. Let's consider the simplest process, $Z \to \bar{b}bg$, which contributes to three-jet final state at the leading order (LO). When the energy of the radiated gluon approaches zero, the process has an infrared (IR) divergence and in order to have an IR-finite prediction, some kinematical restriction should be introduced in the phase-space integration to cut out the troublesome region. In $e^+e^-$-annihilation that is usually done by applying the so-called jet-clustering algorithm with a jet-resolution parameter, $y_c$ (see [12] for recent discussion of jet-algorithms in $e^+e^-$). Then the transition probability in the three-jet

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part of the phase-space will have contributions as large as $1/y_c \cdot (m_b^2/M_Z^2)$, where $y_c$ can be rather small, in the range $10^{-2} - 10^{-3}$. Then one can expect a significant enhancement of the quark-mass effects, which can reach several percents.

The convenient observable for studies of the mass effects in the three-jet final state, proposed some time ago [1,6], is defined as follows

$$R_3^{bd} = \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^d(y_c)/\Gamma^d} \qquad (1)$$

$$= 1 + r_b \left( b_0(y_c, r_b) + \frac{\alpha_s}{\pi} b_1(y_c, r_b) \right)$$

where $\Gamma_{3j}^q$ and $\Gamma^q$ are three-jet and total decay widths of the $Z$-boson into quark pair of flavour $q$, $r_b = m_b^2/M_Z^2$. Note that above expression is not an expansion in $r_b$.

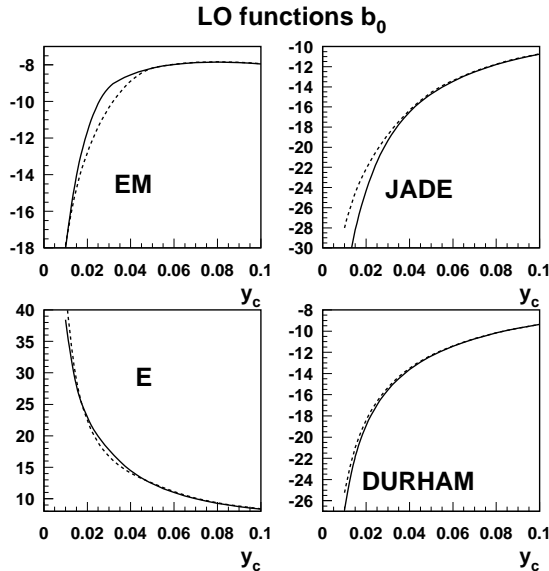The LO function, $b_0$, is plotted in fig. 1 for four different jet algorithms.

**LO functions b$_0$**



Figure 1. *LO contribution to the ratio $R_3^{bd}$ as a function of $y_c$ (see eq.(1) for the definition) for $m_b = 3GeV$ (dashed curve) and $m_b = 5GeV$ (solid curve).*

Together with well known JADE, E and DURHAM schemes we consider the so-called EM algorithm [6] with a resolution parameter $y_{ij} = 2p_ip_j/s$ and which was used for analytical calcu-

lations in the massive case [6]. The main observation from fig. 1 is that for $y_c > 0.05$, $b_0$ is almost independent of the value of $m_b$ for all schemes. Although, this remains true also for smaller $y_c$ in DURHAM and E schemes, there is a noticeable mass dependence in JADE and EM schemes.

Note that $b_0$ is positive for E-scheme. That contradicts the intuitive expectations that a heavy quark should radiate less than a light one. This unusual behavior is due to the definition of the resolution parameter in E-scheme, $y_{ij} = (p_i + p_j)^2/s$, which has significantly different values for partons with the same momenta in the massive and massless cases, and it can be used as a consistency check of the data.

In what follows we restrict ourselves to DURHAM scheme, the one used in the experimental analysis [4], and $b_0$ can be interpolated as: $b_0 = b_0^{(0)} + b_0^{(1)} \ln y_c + b_0^{(2)} \ln^2 y_c$. In the LO calculations we can not specify what value of the b-quark mass should be taken in the calculations: all quark-mass definitions are equivalent (the difference is due to the higher orders in $\alpha_s$). One can use, for example, the pole mass $M_b \approx 4.6GeV$ or the $\overline{MS}$-running mass $\overline{m}_b(\mu)$ at any scale relevant to the problem, $m_b \leq \mu \leq M_Z$, with $\overline{m}_b(m_b) \approx 4.13GeV$ and $\overline{m}_b(M_Z) \approx 2.83GeV$. As a result, the spread in LO predictions for different values of b-quark mass is significant, the LO prediction is not accurate enough and the NLO calculation should be done.

At the NLO there are two different contributions: from one-loop corrections to the three-parton decay, $Z \to b\bar{b}g$ and tree-level four-parton decay, $Z \to b\bar{b}gg$ and $Z \to b\bar{b}q\bar{q}$, $q = u, d, s, c, b$ integrated over the three-jet region of the four-parton phase-space. In the NLO calculation one has to deal with divergences, both ultraviolet [3] and infrared which appear at the intermediate stages. The sum of the one-loop and tree-level contributions is, however, IR finite. We would like to stress that the structure of the NLO corrections in the massive case is completely different from the ones in the massless case [13]. That is

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[3]The ultraviolet divergences in the loop-contribution are cancelled after the renormalization of the parameters of the QCD Lagrangian.

due to the fact that in the massive case, part of the collinear divergences, those associated with the gluon radiation from the quarks, are softened into $\ln r_b$ and only collinear divergences associated with gluon-gluon splitting remain.

In the NLO calculations one should specify the quark mass definition. It turned out technically simpler to use a mixed renormalization scheme which uses on-shell definition for the quark mass and $\overline{MS}$ definition for the strong coupling. Therefore, physical quantities are originally expressed in terms of the pole mass. It can be perfectly used in perturbation theory, however, in contrast to the pole mass in QED, the quark pole mass is not a physical parameter. The non-perturbative corrections to the quark self-energy bring an ambiguity of order $\approx 300 MeV$ (hadron size) to the physical position of the pole of the quark propagator. Above the quark production threshold, it is natural to use the running mass definition (we use $\overline{MS}$). The advantage of this definition is that $\overline{m}_b(\mu)$ can be used for $\mu \gg m_b$.
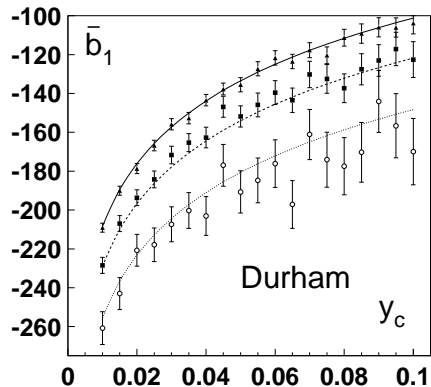


Figure 2. *NLO function $\overline{b}_1$ for different $m_b$ (see eqs.(1),(3) for the definition and text for details). The errors are due to numerical integrations.*

The pole, $M_b$, and the running masses of the quark are perturbatively related

$$M_b = \overline{m}_b(\mu)\left[1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} - \ln\frac{m_b^2}{\mu^2}\right)\right] \ . \qquad (2)$$

We use this one-loop relation to pass from the pole mass to the running one, which is consistent with our NLO calculations. To match needed precision we have to use this equation for values of $\mu$

about $m_b$. Then we can use one-loop renormalization group improved equation in order to define the quark mass at the higher scales. Substituting eq.(2) into definition eq.(1) we have

$$R_3^{bd}(y_c,\overline{m}_b(\mu),\mu) =$$
$$1 + \overline{r}_b(\mu)\left[b_0 + \frac{\alpha_s(\mu)}{\pi}\left(\overline{b}_1 - 2b_0\ln\frac{M_Z^2}{\mu^2}\right)\right](3)$$

with $\overline{b}_1 = b_1 + b_0(8/3 - 2\ln r_b)$ and $\overline{r}_b = \overline{m}_b^2/M_Z^2$.

In fig. 2 we show the NLO function $\overline{b}_1(y_c,r_b)$ calculated for three different values of the quark mass : $3 GeV$ (open circles), $4 GeV$ (squares) and $5 GeV$ (triangles).
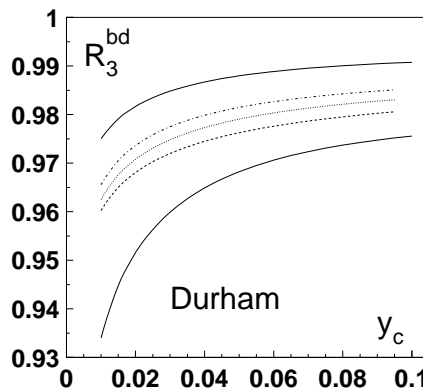


Figure 3. *The ratio $R_3^{bd}$ (eq. (1)). Solid curves - LO predictions, dashed curves give the NLO results (see text for details).*

In contrast to the $b_0$, one sees a significant residual mass dependence in $\overline{b}_1$, which can not be neglected. The solid lines in fig. 2 represent a fit by the function: $\overline{b}_1 = \overline{b}_1^{(0)} + \overline{b}_1^{(1)}\ln y_c + \overline{b}_1^{(2)}\ln r_b$ performed in the range $0.01 \leq y_c \leq 0.1$ The quality of this interpolation is very good and the main residual $m_b$ dependence in $\overline{b}_1$ is taken into account by $\ln r_b$ term. Inclusion of higher powers of $\ln r_b$ does not improve the fit.

Fig. 3 presents theoretical predictions in the DURHAM scheme for the $R_3^{bd}$ observable measured by DELPHI[4]. The solid lines are LO predictions for the b-quark mass, $m_b = \overline{m}_b(M_Z) = 2.83 GeV$ (upper curve) and $m_b = M_b = 4.6 GeV$ (lower curve). The dashed curves give NLO results for different values of scale $\mu$ : 10, 30, 91$GeV$. One sees that NLO curve

for large scale is naturally closer to LO curve for $\overline{m}_b(M_Z)$ and for smaller scale is closer to the LO one with $m_b = M_b$.

Fig. 4 illustrates the scale dependence of $R_3^{bd}$ for $y_c = 0.02$. By studying the scale dependence, which is a reflection of the fixed order calculation, we can estimate the uncertainty of the predictions. The dashed-dotted curve gives $\mu$-dependence when eq. (1) was used, so it is $\mu$-dependence due to renormalization of the strong coupling constant, $\alpha_s$.
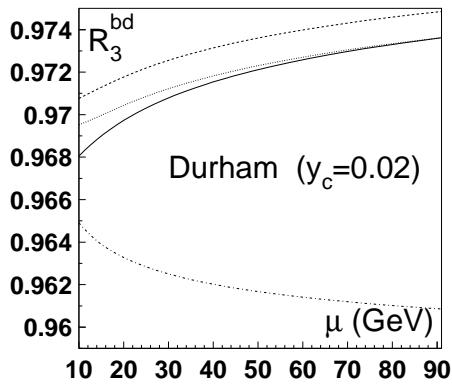
Figure 4. *The ratio $R_3^{bd}$ as a function of the scale $\mu$ for $y_c = 0.02$.*

Other curves show $\mu$-dependence when $R_3^{bd}$ is parameterized in terms of the running mass, $\overline{m}_b(M_Z)$, eq.(3), but different mass definitions have been used in the logarithms. The conservative estimate of the theoretical error for the $R_3^{bd}$ is to take the whole spread given by the curves. The uncertainty in $R_3^{bd}$ induces an error in the measured mass of the b-quark, $\Delta R_3^{bd} = 0.004 \rightarrow \Delta m_b \simeq 0.23 GeV$. This theoretical uncertainty is, however, below current experimental errors, which are dominated by fragmentation.

To conclude, the NLO calculation is necessary for accurate description of the three-jet final state with massive quarks in $e^+e^-$-annihilation. Further studies of different observables and different jet-algorithms could be very useful for the reduction the uncertainty of such calculation.

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