Life potential as a basic demographic indicator

Francisco J. Goerlich Gisbert

University of Valencia and Instituto Valenciano de Investigaciones Económicas (Ivie)
and
Ángel Soler

University of Valencia and Instituto Valenciano de Investigaciones Económicas (Ivie)

ABSTRACT

This paper proposes an indicator that integrates life expectancy with the demographic structure of the population for a given society, in this way we have a simple indicator of mortality and aging combined, and that can be very useful for well-developed societies. As is well-known, life expectancy at birth is independent of the demographic structure of the population, so it is adequate for measuring mortality; however it neglects that as life expectancy increases the society ages, so looking at life expectancy alone can produce a too optimistic view of the development process. Aging can in fact affect quality of life and sustainability in the long run. Aging indicators are usually very crude, like the share of population of 65 years old and above. We proposed a simple indicator that integrates life expectancy at different ages, not only at birth, with the demographic structure of the population at a given point in time. The indicator has an intuitive interpretation in terms of the life potential, or biological capital, of the society; and given that it is a weighted average, its changes can be easily decomposed into reductions in mortality (gains in life expectancy) and aging for different age intervals.

Key Words: Life expectancy, Life table, Aging, Demography.

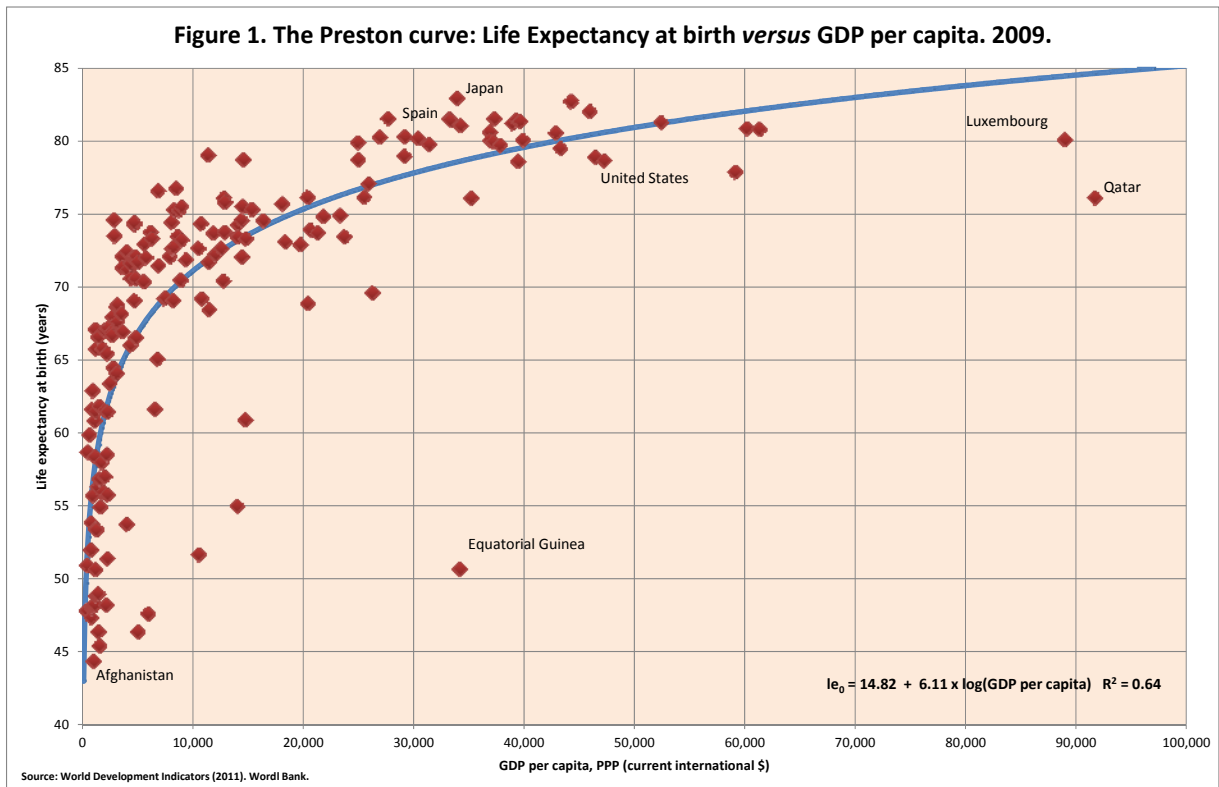**JEL Classification: J10, J11, J14.**

1.- Introduction.

Life expectancy at birth summarizes in a single number the mortality conditions of a given population, and it does it in a way that it is independent of the age structure of the underlying population. Essentially this means that the indicator is comparable, in time and across societies, with populations having very different age structures. This feature has contributed to make life expectancy one of the most widely used indicators in international comparisons about development. Thus, life expectancy at birth is one of the simplest summary measures of population health for a community (Murray, Salomon, Mathers and Lopez 2002), and as a consequence, of its degree of development (Sen 1998, 1999).

For all these reasons, life expectancy is one essential dimension in the complex and elusive concept of quality of life: without life there is no capability to enjoy consumption opportunities as represented by *per capita* income, the other well-known development indicator widely used in international comparisons. However, as has been recently recognized by the Stiglitz, Sen and Fitoussi (2009) report it is necessary to go beyond GDP in measuring the progress of actual societies. This was in fact the goal of the Human Development Index of the United Nations Development Program (http://hdr.undp.org), as well as many other proposals in including life expectancy as part of synthetic quality of life indexes (Osberg y Sharpe 2002).

It is widely recognized that there is a high correlation between life expectancy at birth and *per capita* income, in a given country and for a sufficiently long time span, as well as for a cross-section of countries at different stages of development. However, this relationship is non-linear, has no clear shape and we may find countries with relatively low *per capita* income that have a much more superior life expectancy that countries with a higher *per capita* income (Sen 1998). This relation, known as the Preston (1975) curve, can be seen in figure 1, where we can see that on average life expectancy is profoundly lower for countries with lower *per capita* income. The linear correlation coefficient between the variables represented in figure 1 is 0.62, but clearly the relationship is non-linear. The curve drawn corresponds to the regression of life expectancy at birth on the logarithm of GDP *per capita*, the correlation in this case rises to 0.80. Taken at face value, we need a bit more than a 16% of increase in *per capita* GDP for an increment in one year of life expectancy at birth, so doubling income *per capita* represents an increment of about 6 years in life expectancy at birth. The relationship

drawn shows a decreasing elasticity, which for sample values oscillates from about 0.13 to around 0.07.

Figure 1. The Preston curve: Life Expectancy at birth *versus* GDP per capita. 2009.

$le_0 = 14.82 + 6.11 \times \log(\text{GDP per capita})$ $R^2 = 0.64$

Source: World Development Indicators (2011). Wordl Bank.

An important conclusion from figure 1 is that, as income increases life expectancy at birth has lower informational content about the development of a given country. In fact we can see that the regression tends to over-fit the highest values of GDP *per capita*. At low level of income the coefficient of variation of life expectancy for the countries shown in figure 1 is 0.121, whereas for the high level income countries the coefficient of variation is just 0.020,[1] which signals the compression of life expectancy for the most developed countries.[2]

What it is not evident from figure 1 is that as life expectancy increases the society ages, a fact that results from the increase in longevity. In the first stage of the demographic transition mortality falls at early childhood (Davis 1945; Vallin 2002), so the population pyramid widens at its base, but as fertility adjusts to the new mortality conditions and mature
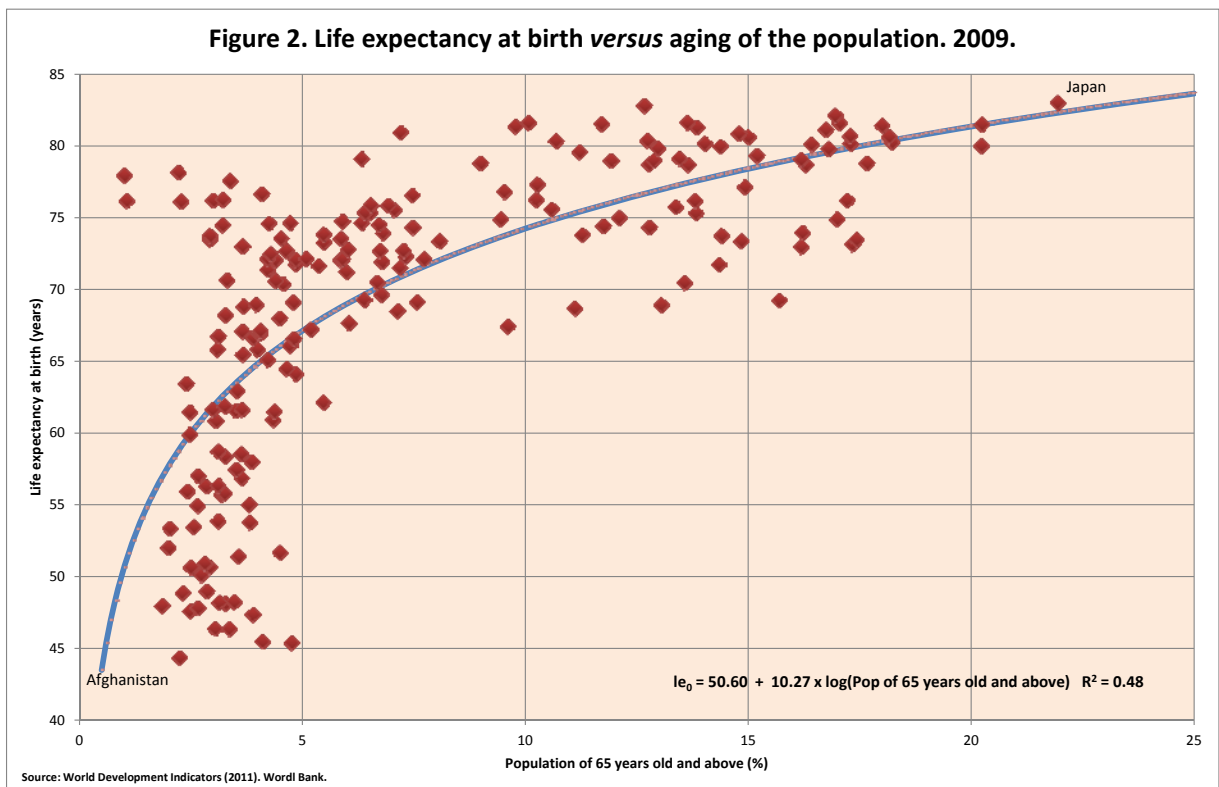
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[1] The World Bank defines for 2009 the low level income countries those with GDP *per capita*, current PPP $ lower than 1,154.04 and high level income countries those with a GDP *per capita* over 37,314.14 current PPP $. For lower and middle income countries, defined as those with a GDP *per capita* lower than 4,449.04, the coefficient of variation is 0.138.

[2] Even we don´t know the upper limit, life expectancy at birth should be bounded from above; this is not true for *per capita* income however. What it has been true historically is that the forecasted limits to life expectancy have been broken as time has elapsed (Oeppen and Vaupel 2002; Willets *et al* 2004).

societies advance in the subsequent stages of the epidemiological transition (Olshansky and Ault 1986) the base of the population pyramid begins to shrink, and the society grows older.

Eventually, the reduction of mortality at all ages, as summarized by a continuous increase in life expectancy, goes hand in hand with a reduction in fertility. Lower numbers of births are observed in highly developed countries, and this contributes to the aging of the population.

If we substitute the logarithm of *per capita* GDP in the *x*-axis of figure 1, by the logarithm of the share of people 65 years old and above, a simple index of aging, we get a very similar picture. This is done in figure 2, where again a semi-logarithmic equation is drawn. Taken at face value, an additional year of life expectancy at birth is associated with an almost 10% increase in the share of older people, so we get a high correlation between development, as measured by income *per capita*, and aging via life expectancy at birth.

Figure 2. Life expectancy at birth *versus* aging of the population. 2009.

$le_0 = 50.60 + 10.27 \times \log(\text{Pop of 65 years old and above})$ $R^2 = 0.48$

Source: World Development Indicators (2011). Wordl Bank.

Despite acknowledging that correlation does not imply causation, what figures 1 and 2 imply is that either income *per capita* or life expectancy alone can give us a too optimistic view of the potential development of the society in the future. If life expectancy increases only because longevity increases, as is the case in well advanced societies, with a very small birth rate, then sustainability and quality of life can be threatened in the long run. What we propose in the sequel is a very simple indicator that integrates life expectancy at any age with the

demographic structure of the population, because aging can be a problem beyond a certain point.

2.- Life potential: A basic demographic indicator.

We define life potential for a given individual at age x as his (uncertain) life expectancy given his age, and the **life potential** for a society, L, as the aggregate over individual life potential. Hence,

$$L = \int_0^\infty P(x)e(x)dx \qquad (1)$$

where $P(x)$ is the population at age x, and $e(x)$ is the corresponding life expectancy. From (1) it is clear that L is a weighted sum of life expectancies at different ages, and from capital theory can be understood as the **biological capital of a society**, since it is an estimate of the physical support of any other form of human capital, such as educational, job training or health capital (Shultz 1962; Becker 1962, 1964, 2007; Grosman 1972). Because L is difficult to compare among societies of different size, we may use life potential *per capita*, l. Letting P the total population, $P = \int_0^\infty P(x)dx$, we define life potential *per capita* as

$$l = \frac{L}{P} = \int_0^\infty \omega(x)e(x)dx \qquad (2)$$

where $\omega(x) = \dfrac{P(x)}{P}$, with the property that $\int_0^\infty \omega(x)dx = 1$. So, l is a weighted average of life expectancies, where the weights are given by population shares. Because life expectancy decreases with age (at least beyond a certain point), l is increasing in life expectancy at any age and decreasing in population aging. From the definition, it follows that l can be interpreted as the life expectancy of a given population, as opposed to the life expectancy of a cohort at a given age, which is the usual interpretation in demography.[3]

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[3] If we partition the population into exhaustive and mutually exclusive groups, such as by region, gender or ethnic groups, then L can be calculated as the sum of life potential over the different groups, and l is a weighted sum of life potential *per capita*, where the weights are given by the relative importance of each group in the population.

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3.- Life potential in practice.

To build an operational measure for (1) we only need population classified by age and his corresponding life expectancy. In the absence of individual (subjective) survival curves (Gan, Hurd and McFadden 2003) we don´t have individual data, so we should rely on life expectancies from standard life tables.

Published life tables are usually of the age-period type, so age-specific mortality rates for a given period, usually a calendar year, are used to construct the life experience of a fictitious generation that it is followed until it is extinguished. Life expectancies at different ages are estimated by redistributing equally all future life years lived by the survivals of the generation at a given age. In this way, period life tables represent the current mortality conditions, without taking into account future improvements in mortality, so life expectancy at birth represents the average time an individual born at a given time can expect to live on average, with the current mortality conditions. Figure 3 represents this set-up.

Figure 3: Life Tables: age-period
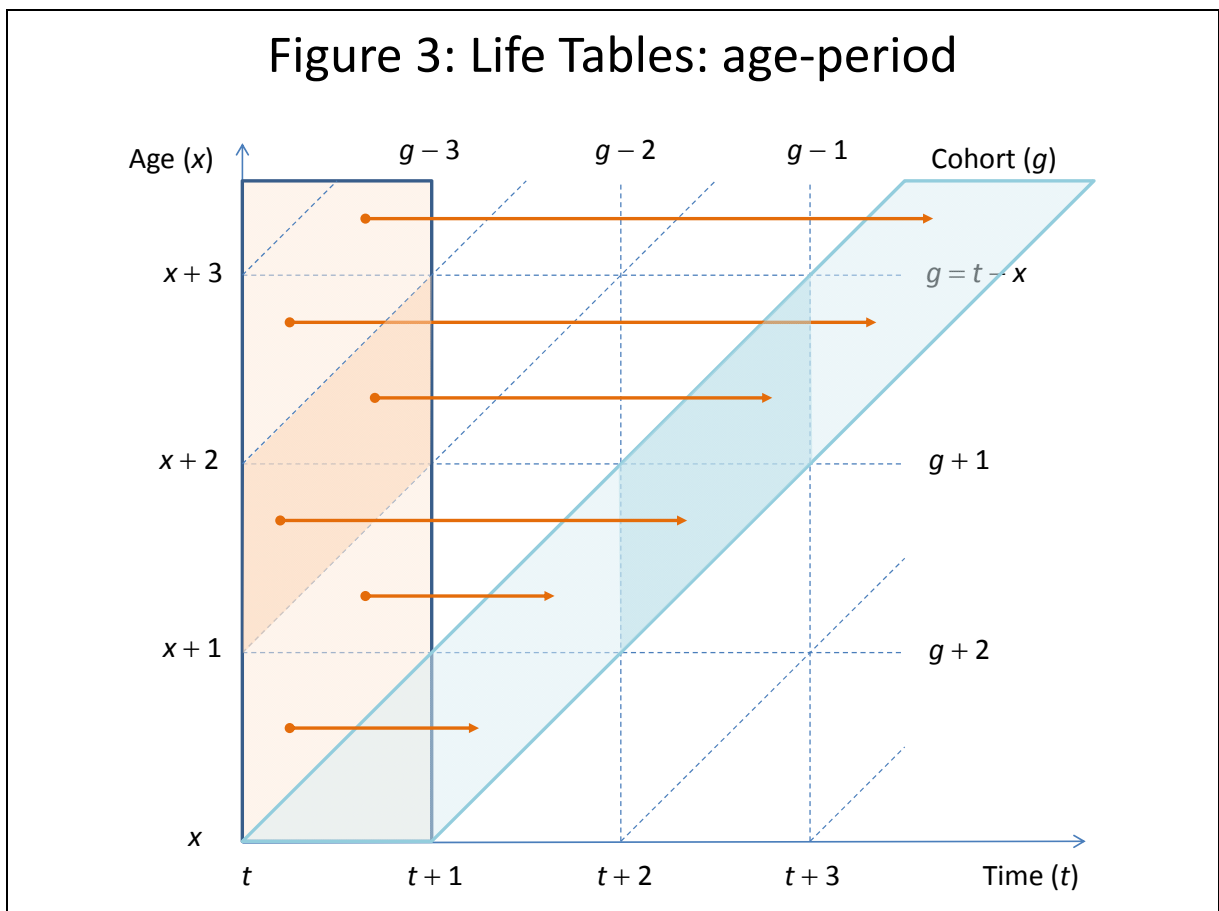
Fortunately, the Human Mortality Database (http://www.mortality.org/) builds complete life tables for a great number of countries based on a common methodology with an open

ended age interval of 110 years and above (Wilmoth *et al* 2007), and at the same time they offer population data by one year of age interval covering long periods of time, and dated 1ˢᵗ January. All the calculations in this paper use data from this database.

Period life tables estimate life expectancy at an exact age, x, e_x; this is, at the beginning of the age interval, $[x, x + 1)$ in case of single age years. On the other hand, the stock of population is dated at a given point in time, t, but it is recorded for a given age interval, $[x, x + 1)$. The empirical counterpart of (1) from discrete data with this structure is

$$L = \sum_{x=0}^{110+} P_x \overline{e}_x \qquad (3)$$

where $\overline{e}_x = \frac{1}{2}.(e_x + e_{x+1})$, P_x is the population in the age interval $[x, x + 1)$ at a given point in time and for the open ended age interval we use $\overline{e}_{110} = \frac{1}{2}.e_{110}$.

Using the weights $\omega_x = \dfrac{P_x}{P}$, where $P = \Sigma_{x \geq 0} P_x$, the empirical counterpart of life potential *per capita* is

$$l = \sum_{x=0}^{110+} \omega_x \overline{e}_x \qquad (4)$$

That it is simply a population weighted average of life expectancies.[4] Figure 4 shows life expectancy at birth and life potential *per capita* for a selection of developed countries: Spain, Japan, United Kingdom, United States of America, France and Sweden, over a long period of time; and table 1 shows the numerical values for selected years. Overall, long run tendencies in life expectancy are clear, despite short episodes related to wars or epidemics; life expectancy shows an up-ward and steadily trend. In 2006, the last common year available to all the countries considered life expectancy at birth was 82.67 years in Japan, the highest observed value, follow by Sweden, 80.95 years, and Spain, 80.94 years. The lowest value is found in United States of America with 78.09 years.

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[4] It is worth noting that the indicator (4) was used by Usher (1973) in his imputation of the value of life in the national accounts, but keeping constant the population structure and fixed to the base year in the national accounts. Maintaining the population structure or the life expectancies constant in (4) we can construct counter-factual life potential *per capita*, so we can examine the evolution of l with one of its components taken as given.

Figure 4. Life expectancy at birth and life potential *per capita*. Historical international comparisons.
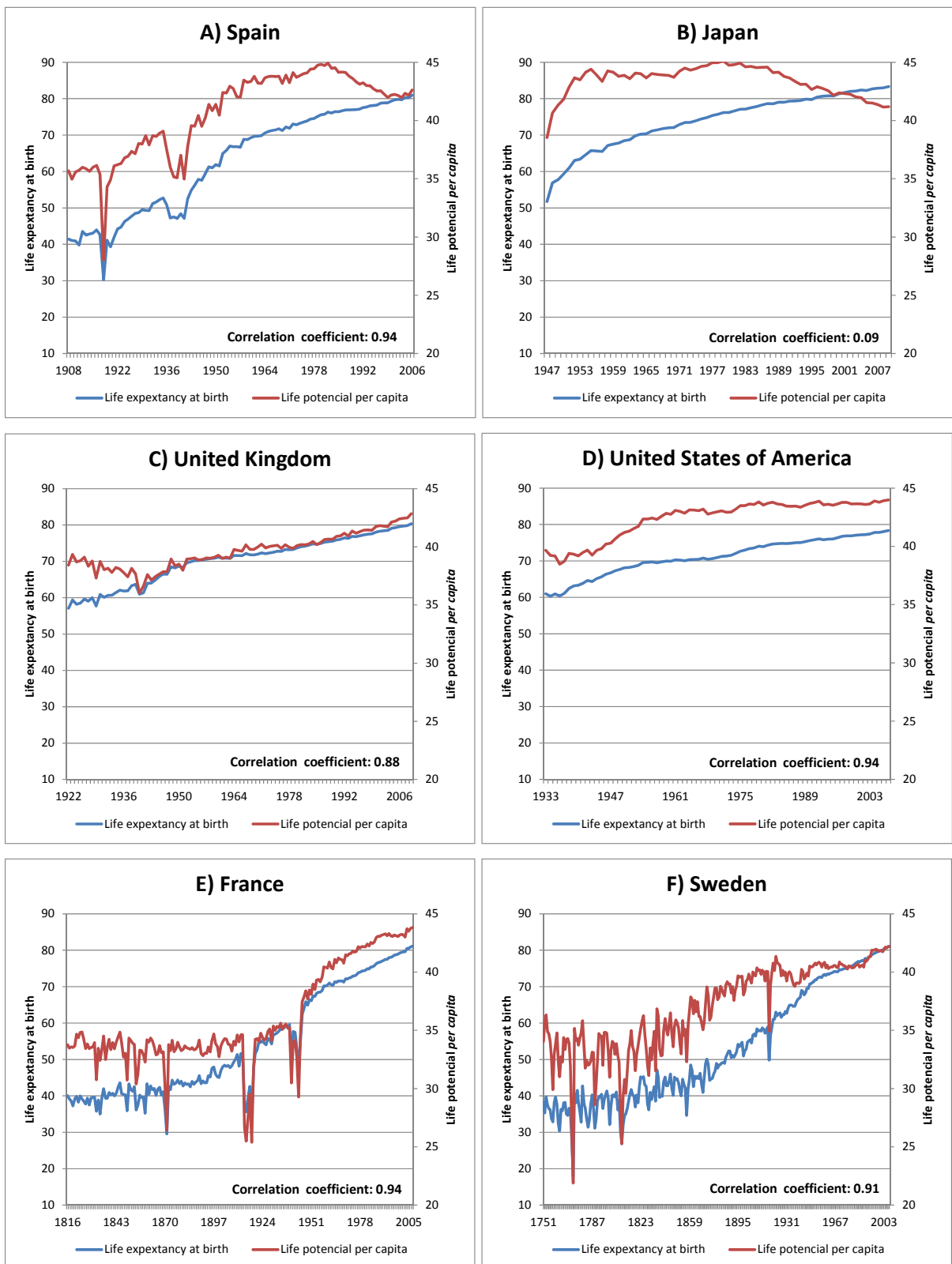
Source: Human Mortality Database and own elaboration.

Tendencies for life potential *per capita* are less clear cut, for most of the period life potential follows closely life expectancy, in fact, with the exception of Japan, the correlation between both series is very high, in excess of 0.88. However, as we will see in the sequel, this correlation changes abruptly with time, and life potential per capita appears to slow down, or even to fall in recent years in countries with the exception of United Kingdom. In this country life potential *per capita* falls at the beginning of the XX century, and this particular evolution would be worth to explore.

It is worth noting the particular evolution of life potential *per capita* in Japan and Spain. Both countries show the highest life expectancy at birth, but in both cases life potential is actually falling, since the end of the 70´s in the case of Japan, and since the beginning of the 80´s in the case of Spain. This puts a precautionary note in the optimistic signal shown by the observation of life expectancy at birth alone, given that interpreting life potential as the biological capital of the society both countries are destroying capital. From this point of view the country with the highest biological capital is United States of America, which given the observed lower life expectancies signals a younger population than the other countries considered.

Table 1. Life expectancy at birth and life potential *per capita*. Selected years from developed countries.

Spain			Japan			United Kingdom		
Year	Life expectancy	Life potential	Year	Life expectancy	Life potential	Year	Life expectancy	Life potential
1908	41.45	35.69	1947	51.75	38.54	1922	57.13	38.42
1939	47.09	35.09	1955	65.77	44.41	1940	60.97	36.04
1945	57.84	40.43	1977	75.38	44.96	1945	65.85	37.66
1980	75.60	44.82	1997	80.68	42.76	1970	72.00	39.97
2006	80.94	42.62	2009	83.31	41.19	2009	80.34	42.85

United States of America			France			Sweden		
Year	Life expectancy	Life potential	Year	Life expectancy	Life potential	Year	Life expectancy	Life potential
1933	60.96	39.67	1816	40.04	33.77	1751	38.35	34.04
1945	65.63	39.80	1871	29.59	26.45	1851	43.62	35.20
1960	69.91	42.76	1914	37.85	26.64	1914	58.26	40.03
1980	73.93	43.54	1945	54.96	34.52	1945	68.34	39.63
2007	78.32	43.99	2007	81.14	43.82	2007	81.08	42.20

Source: Human Mortality Database and own elaboration.

Given that l is a simple weighted average we can split the changes among two points in time, or even the differences between two countries at a given point in time, into the

contributions due to changes in the demographic structure, ω_x, and the contributions due to the changes in life expectancies, e_x. This is the goal of the so called *shift-share* analysis widely used in regional economics. These types of decompositions are never unique (Kitagawa 1955), but the most easy to interpret decomposition is

$$l^s - l^t = \sum_{x=0}^{110+} \left(\frac{\omega_x^s + \omega_x^t}{2} \right) . (\overline{e}_x^s - \overline{e}_x^t) + \sum_{x=0}^{110+} (\omega_x^s - \omega_x^t) . \left(\frac{\overline{e}_x^s + \overline{e}_x^t}{2} \right) \tag{5}$$

where the first term can be interpreted as the contribution of the changes in life expectancies, whereas the second term can be interpreted as the contribution of the changes in the demographic structure of the society.

Table 2 shows, for selected time periods, the changes in life expectancy at birth and the life potential *per capita*, the correlation among the two variables for the period, and eventually the decomposition (5), showing the contribution of life expectancies and demographics to the change in life potential *per capita*.

Several facts are worth mentioning. (*i*) With the exception for France for the early years, and due to the cut-off year, changes in life expectancy at birth are always positive, and they show no exhaustion symptoms, a well-known fact. (*ii*) On the other hand, changes in life potential per capita are more irregular. Spain and Japan show a negative change in recent decades, which translates into a high negative correlation for these years. (*iii*) Correlation is quite sensitive to the time period considered. The almost absent of correlation for Japan for the whole period, 1947 – 2009, is the result of a high positive correlation in the early decades and a high negative correlation in the last decades. No clear pattern emerges in this respect when we consider shorter sub-periods. (*iv*) With the exception of the first century considered for Sweden, 1751 – 1851, where the demographics contribute positively to the changes in life potential *per capita*, maybe as an indication of the early stage of the demographic transition, the contribution of the demographics to changes in life potential *per capita* is invariably negative. Moreover, this negative contribution is increasing in magnitude with time. In the two cases mentioned, Japan and Spain, the negative contribution of the demographics out-weights the positive contribution of improvements in life expectancies, resulting in the negative variation of life potential *per capita* mentioned before.

Table 2. Changes in life expectancy at birth and life potential *per capita*. Selected periods from developed countries. Shift-Share decomposition for life potential *per capita*.

Spain

Period	Changes in			Decomposition	
	Life expectancy at birth	Life potential	Correlation	Life expectancies	Demographics
1908-1939	5.64	-0.60	0.898	0.12	-0.72
1939-1980	28.51	9.74	0.980	13.11	-3.38
1980-2006	5.34	-2.20	-0.902	4.00	-6.20
1908-2006	**39.49**	**6.93**	**0.936**	**16.64**	**-9.70**

Japan

Period	Changes in			Decomposition	
	Life expectancy at birth	Life potential	Correlation	Life expectancies	Demographics
1947-1955	14.02	5.87	0.996	6.42	-0.55
1955-1977	9.61	0.56	0.598	5.32	-4.77
1977-2009	7.93	-3.78	-0.969	6.59	-10.37
1947-2009	**31.56**	**2.65**	**0.087**	**16.96**	**-14.31**

United Kingdom

Period	Changes in			Decomposition	
	Life expectancy at birth	Life potential	Correlation	Life expectancies	Demographics
1922-1940	3.84	-2.38	-0.317	0.75	-3.13
1940-1970	11.03	3.93	0.971	4.90	-0.98
1970-2009	8.34	2.89	0.975	6.33	-3.44
1922-2009	**23.21**	**4.43**	**0.878**	**12.08**	**-7.65**

United States of America

Period	Changes in			Decomposition	
	Life expectancy at birth	Life potential	Correlation	Life expectancies	Demographics
1933-1960	8.95	3.09	0.937	4.33	-1.24
1960-1980	4.02	0.78	0.897	2.71	-1.92
1980-2007	4.39	0.45	0.620	3.35	-2.90
1933-2007	**17.36**	**4.33**	**0.940**	**10.23**	**-5.90**

France

Period	Changes in			Decomposition	
	Life expectancy at birth	Life potential	Correlation	Life expectancies	Demographics
1816-1871	-10.45	-7.32	0.786	-6.10	-1.22
1871-1914	8.26	0.19	0.794	1.07	-0.88
1914-1945	17.11	7.88	0.972	9.25	-1.37
1945-2007	26.18	9.31	0.981	12.56	-3.26
1816-2007	**41.10**	**10.05**	**0.936**	**18.21**	**-8.16**

Sweden

Period	Changes in			Decomposition	
	Life expectancy at birth	Life potential	Correlation	Life expectancies	Demographics
1751-1851	5.27	1.16	0.847	0.82	0.34
1851-1914	14.64	4.83	0.938	6.53	-1.70
1914-1945	10.08	-0.40	0.388	3.50	-3.90
1945-2007	12.74	2.58	0.844	7.73	-5.15
1751-2007	**42.73**	**8.17**	**0.906**	**17.93**	**-9.77**

Note: Decomposition shows the formula (5) of the text, so it shows the contribution of the changes in life expectacies and demographics to the change in life potential *per capita* in the given period.

Source: Human Mortality Database and own elaboration.

4.- Final comments.

This short paper has introduced a simple demographic indicator that integrates life expectancy at different ages with the demographic structure of population. In this way, it tries to balance the observed increment in life expectancy with the aging of the population that characterizes advanced societies. Aging appears to be an inevitable consequence of development, so it should be incorporated in social indicators about quality of life and sustainability.

We call the indicator life potential, and it has an intuitive interpretation as the biological capital of the society at a given point in time. In this way, we can see how aging societies can suffer from a loss in biological capital that can affect sustainability and quality of life in the long run. This is the idea behind the proposal of Herrero, Martinez and Villar (2010) that in their reformulation of the Human Development Index (HDI) substitute life expectancy at birth by life potential *per capita* of a given country, in addition to other important changes in the way the HDI is calculated.

From a practical and computational point of view, life potential *per capita* is simply a population weighted life expectancy of the society; so the continuous increment in life expectancies at every age are balanced with the increment in the share of old people that enjoy shorter life expectancies. Life potential is simple to calculate, has low data requirements, not beyond the information needed to calculate life tables, and has an interesting interpretation in terms of the average life expectancy of the population at a given date.

A practical application for some developed countries in an historical context shows the clear and well-known tendency of increasing life expectancy, but a less clear cut tendency for life potential *per capita*. In general, we observe stagnation of this last variable, and in two peculiar cases, Japan and Spain, an important fall in life potential *per capita* in the recent decades, signaling an accelerated aging of the population in these two countries. Clearly, this can be a cause of concern, beyond the optimistic view we can reach by looking at life expectancy at birth alone. Aging is an important fact in developed societies, and this should be incorporated in social indicators in a more satisfactory manner than looking at shares of young or old people in society.

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