A description of the $f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$ resonances as multi-$\rho(770)$ states

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Abstract

In a previous work regarding the interaction of two $\rho(770)$ resonances, the $f_2(1270)$ ($J^{PC} = 2^{++}$) resonance was obtained dynamically as a two-ρ molecule with a very strong binding energy, 135 MeV per ρ particle. In the present work we use the $\rho\rho$ interaction in spin 2 and isospin 0 channel to show that the resonances $\rho_3(1690)$ (3^{--}), $f_4(2050)$ (4^{++}), $\rho_5(2350)$ (5^{--}) and $f_6(2510)$ (6^{++}) are basically molecules of increasing number of $\rho(770)$ particles. We use the fixed center approximation of the Faddeev equations to write the multi-body interaction in terms of the two-body scattering amplitudes. We find the masses of the states very close to the experimental values and we get an increasing value of the binding energy per ρ as the number of ρ mesons is increased.

PACS numbers:

I. INTRODUCTION

In the last decade, the chiral unitary approach has shown that many hadronic resonances can be obtained dynamically from the interaction of hadrons. This has been done through the implementation of unitarity in coupled channels using a lowest order chiral Lagrangian, to the point that these resonances can be interpreted as meson-meson or meson-baryon molecules [1–10], and it has shed new light into the issue of the nature of the scalar mesons, among others. The interaction of pseudoscalar mesons among themselves and meson-baryon interaction has given way recently to the interaction of vector mesons among themselves [11–13] and the interaction of vector mesons with baryons [14, 15], where the interaction is evaluated within the techniques of the chiral unitary approach starting from a lowest order hidden gauge symmetry Lagrangian [16–19]. In the work [11] it was found that the interaction of two $\rho(770)$ mesons in isospin $I = 0$ and spin $S = 2$ was strong enough to bind the $\rho\rho$ system into the $f_2(1270)$ ($J^{PC} = 2^{++}$) resonance. The nature of this resonance as a $\rho(770)\rho(770)$ molecule has passed the tests of radiative decay into $\gamma\gamma$ [20], the decay of J/Ψ into $\omega(\phi)$ and $f_2(1270)$ (together with other resonances generated in [12]) [21], and J/Ψ into γ and $f_2(1270)$ (and the other resonances of [12]) [22].

The $f_2(1270)$ obtained in ref. [11] as a $\rho\rho$ quasibound state or molecule implies a very large binding energy per ρ meson, about 135 MeV. This occurs only for spin S=2, where the two spins of the ρ are aligned in the same direction. In view of this strong $\rho\rho$ interaction, some natural questions arise: i) is it possible to obtain bound systems with increasing number of ρ mesons as building blocks? These systems with many ρ's with their spins aligned in the same direction would make a condensate, with features similar to a ferromagnet; ii) If so, is there a limit in the number of ρ's or, even more interesting, the mass of the multi-ρ system saturates at some number of ρ mesons?. In this latter case then it would be energetically "free" to introduce new ρ mesons into the system.

The condensates made out of mesons have been advocated some times, and concretely, the issue of pion condensates was popular for some time [23] and kaon condensation has also attracted much attention [24].

Regarding the question i), in the PDG [25] there are intriguing mesons with large spin, of the ρ and f_0 type, whose quantum numbers match systems made with 3, 4, 5 and 6 ρ mesons with their spins aligned. These are the $\rho_3(1690)$ (3^{--}), $f_4(2050)$ (4^{++}), $\rho_5(2350)$ (5^{--}) and $f_6(2510)$ (6^{++}) resonances. If these resonances were essentially multi-ρ meson molecules, they would have a binding energy per ρ of about 210, 260, 305 and 355 MeV, respectively. This increasing value as more ρ's are added to the system connects with question ii).

The main aim of the present work is to address these questions. Technically, we would have to solve the Faddeev equations [26] for a state of three ρ's to start with. Systems of three mesons, concretely $K\bar{K}\phi$, have been addressed recently with Faddeev equations [27]. States of two mesons and one baryon have also received recent attention [28–31]. In the three ρ mesons state, the fact that the two ρ meson system with S=2 is so bound, makes it advisable to use the fixed center approximation to the Faddeev equations (FCA) [32–35] in order to obtain the scattering amplitudes of one ρ with the $f_2(1270)$ state. The FCA requires the knowledge of the wave function of the bound state of the target. This information can be obtained using a recent method that connects, in an easy and practical way, the wave functions with the scattering matrices of the chiral unitary approach [36]. Proceeding by iterations we build up states with an extra ρ meson starting from the former state. In this way the multi-ρ resonances are generated, which show up as prominent bumps in the different scattering amplitudes. The iterative method suggest a way to extrapolate to many ρ states and we develop an analytical method to evaluate the mass of the n-ρ state for very large n if only the single-scattering contribution is considered in the Faddeev equations, which should be viewed only as suggestive of what might happen in the limit of a large number of ρ mesons.

II. UNITARIZED $\rho\rho$ INTERACTION

The most important ingredient in the calculation of the multi-ρ scattering is the two-ρ interaction. In this section we briefly summarize the model of ref. [11] to obtain the unitary $\rho\rho$ scattering amplitude. (We refer to [11] for further details).

The $\rho\rho$ potential is obtained from the hidden gauge symmetry Lagrangian [16–19] for vector mesons, which, up to three and four vector fields, reads:

$$\mathcal{L}^{(4V)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle \ , \tag{1}$$

$$\mathcal{L}^{(3V)} = ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle \ , \tag{2}$$

where V_ν is the $SU(3)$ matrix containing the vector-meson fields and the coupling constant g is $g = M_V/2f$ with $f = 93$ MeV, the pion decay constant. The Lagrangian of Eq. (1) gives rise to a four vector meson contact term and that of Eq. (2) to a four vector meson interaction through the exchange of an intermediate vector meson in the t and u channels (the s-channel gives rise to a p-wave that we do not consider, only the important s-wave part is studied).

From these Lagrangians, a potential V can be obtained to which the contact and ρ-exchange terms contribute. For the present work only the spin $S = 2$ and isospin $I = 0$, $I = 2$, are necessary:

$$\begin{aligned} V^{(I=0,S=2)}(s) &= -4g^2 - 8g^2\left(\frac{3s}{4m_\rho^2} - 1\right) \sim -20g^2 \\ V^{(I=2,S=2)}(s) &= 2g^2 + 4g^2\left(\frac{3s}{4m_\rho^2} - 1\right) \sim 10g^2 \end{aligned} \tag{3}$$

where the last terms are the approximate values at threshold in order to give an idea of the weight and sign of the interaction. The $\rho\rho$ $S = 2$, $I = 0$ is strongly attractive. This is the most important reason to obtain a bound $\rho\rho$ state with these quantum numbers as we explain below.

Further contributions to the previous potential where considered in ref. [11], out of which only the box diagram, which accounts for the two-pion decay mode, was relevant, and only for the imaginary part of the potential. Explicit expressions can be found in ref. [11].

With this potential the total $\rho\rho$ scattering amplitude can be obtained. In order to extend the range of applicability of the interaction to the resonance region, the implementation of exact unitarity is mandatory. In this case, we use the Bethe-Salpeter equation where the kernel is the potential V described above:

$$T = \frac{V}{1 - VG}, \tag{4}$$

for each spin-isospin channel. In Eq. 4, G is the unitary bubble or the $\rho\rho$ loop function [2, 3]

$$G(s) = i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - m_\rho^2 + i\epsilon}\frac{1}{(Q-p)^2 - m_\rho^2 + i\epsilon} \ , \tag{5}$$

with $Q = (\sqrt{s}, \vec{0})$. This loop function can be regularized by means of dimensional regularization or using a three-momentum cutoff, $p_{\max} \equiv \Lambda$:

$$G(s,m_1,m_2)=\int_0^\Lambda \frac{p^2dp}{(2\pi)^2}\frac{\omega_1+\omega_2}{\omega_1\omega_2[(Q^0)^2-(\omega_1+\omega_2)^2+i\epsilon]} \tag{6}$$

where $\omega_i=(\vec{p}_i^{\,2}+m_i^2)^{1/2}$.

In order to consider the width of the ρ particles inside the loop, a convolution with the two ρ meson spectral functions is done to Eq. (6):

$$G(s)=\frac{1}{\mathcal{N}^2}\int_{(m_\rho-2\Gamma_{ON})^2}^{(m_\rho+2\Gamma_{ON})^2} ds_1\int_{(m_\rho-2\Gamma_{ON})^2}^{(m_\rho+2\Gamma_{ON})^2} ds_2\, G(s,\sqrt{s_1},\sqrt{s_2})\, S_\rho(s_1)\, S_\rho(s_2) \tag{7}$$

where $S_\rho(s_i)$ is the ρ meson spectral function

$$S_\rho(s_i)=-\frac{1}{\pi}Im\left[\frac{1}{s_i-m_\rho^2+im_\rho\Gamma_\rho(\sqrt{s_i})}\right], \tag{8}$$

with $\Gamma_\rho(\sqrt{s})$ the ρ-meson energy dependent width

$$\Gamma_\rho(\sqrt{s})=\Gamma_{ON}\left(\frac{s-4m_\pi^2}{m_\rho^2-4m_\pi^2}\right)^{3/2} \tag{9}$$

and Γ_{ON} the on-shell ρ meson width. In Eq. (7) $\mathcal{N}$ is a normalization factor given by

$$\mathcal{N}=\int_{(m_\rho-2\Gamma_{ON})^2}^{(m_\rho+2\Gamma_{ON})^2} ds\, S_\rho(s) \tag{10}$$

The cutoff Λ is the only free parameter in the whole model and is chosen such as to produce the peak of $|T|^2$ at the experimental mass of the $f_2(1270)$. This implies $\Lambda\simeq 875$ MeV, which is of a natural size [8], about 1 GeV.

In fig. 1, the modulus squared of the $S=2$, $I=0$ scattering amplitude, $T^{(I=0,S=2)}$, is plotted. The resonance structure of the $f_2(1270)$ resonance is clearly visible.

III. MULTI-BODY INTERACTION

We are going to use the fixed center approximation of the Faddeev equations in order to obtain the interaction of a number of ρ mesons larger than two.

We will illustrate the process for the interaction of three mesons and will give the expression obtained analogously for other number of mesons. For the three ρ system, we will consider that two of the ρ mesons are clusterized forming an $f_2(1270)$ resonance, given the strong binding of the $f_2(1270)$ system. This allows us to use the FCA to the Faddeev equations.

The FCA to Faddeev equations is depicted diagrammatically in fig. 2. The external particle, the ρ in this case, interacts successively with the other two ρ mesons which form the $\rho\rho$ cluster. The FCA equations are written in terms of two partition functions T_1, T_2, which sum up to the total scattering matrix, T, and read

$$\begin{aligned} T_1 &= t_1+t_1G_0T_2\\ T_2 &= t_2+t_2G_0T_1\\ T &= T_1+T_2 \end{aligned} \tag{11}$$

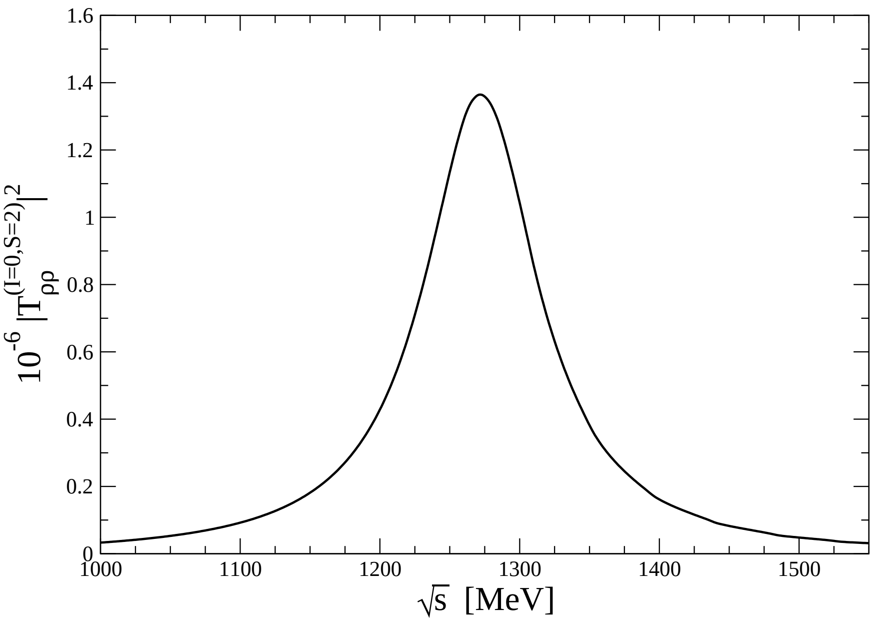

FIG. 1: Modulus squared of the $\rho\rho$ scattering amplitude with total spin $S = 2$ and isospin $I = 0$

where T is the total scattering amplitude we are looking for, T_i accounts for all the diagrams starting with the interaction of the external particle with particle i of the compound system and t_i represent the $\rho\rho$ unitarized scattering amplitude of a ρ^+ with any of the other ρ in the $I = 0$ $\rho\rho$ system. The schematic representation is depicted in fig. 2.

Fig. 2a) represents the single-scattering contribution and fig. 2b) the double-scattering. The contributions of fig. 2a and b are the two first contributions of the Faddeev equations.

In the present case, since both 1 and 2 are ρ mesons we have $T_1 = T_2$ an thus the system of equations is just reduced to a single equation

$$\begin{aligned} T_1 &= t_1 + t_1 G_0 T_1 \\ T &= 2T_1 \end{aligned} \tag{12}$$

A. Single-scattering contribution

The amplitude corresponding to the single-scattering contribution of fig. 2a comes just from the t_1 term of Eq. (12), $T = 2t_1$.

In order to write this expression in terms of the $I = 0$ and $I = 2$ unitarized amplitudes ($t_{\rho\rho}^{(I=0)}$, $t_{\rho\rho}^{(I=2)}$) of Eq. (4), let us consider a cluster of two ρ mesons in isospin $I = 0$, the constituents of which we call mesons 1 and 2. The other ρ meson will be meson number 3. The two ρ mesons forming the f_2 are in an $I = 0$ state

$$|\rho\rho\rangle_{I=0} = -\frac{1}{\sqrt{3}}|\rho^+\rho^- + \rho^-\rho^+ + \rho^0\rho^0\rangle = \frac{1}{\sqrt{3}}\Big(|(1,-1)\rangle + |(-1,1)\rangle - |(0,0)\rangle\Big) \tag{13}$$

where the kets in the last member indicate the I_z components of the 1 and 2 particles, $|(I_z^{(1)}, I_z^{(2)})\rangle$. We take the ρ meson number 3 in the state $|(I_z^{(3)})\rangle$

$$|\rho^+\rangle = -|(+1)\rangle. \tag{14}$$

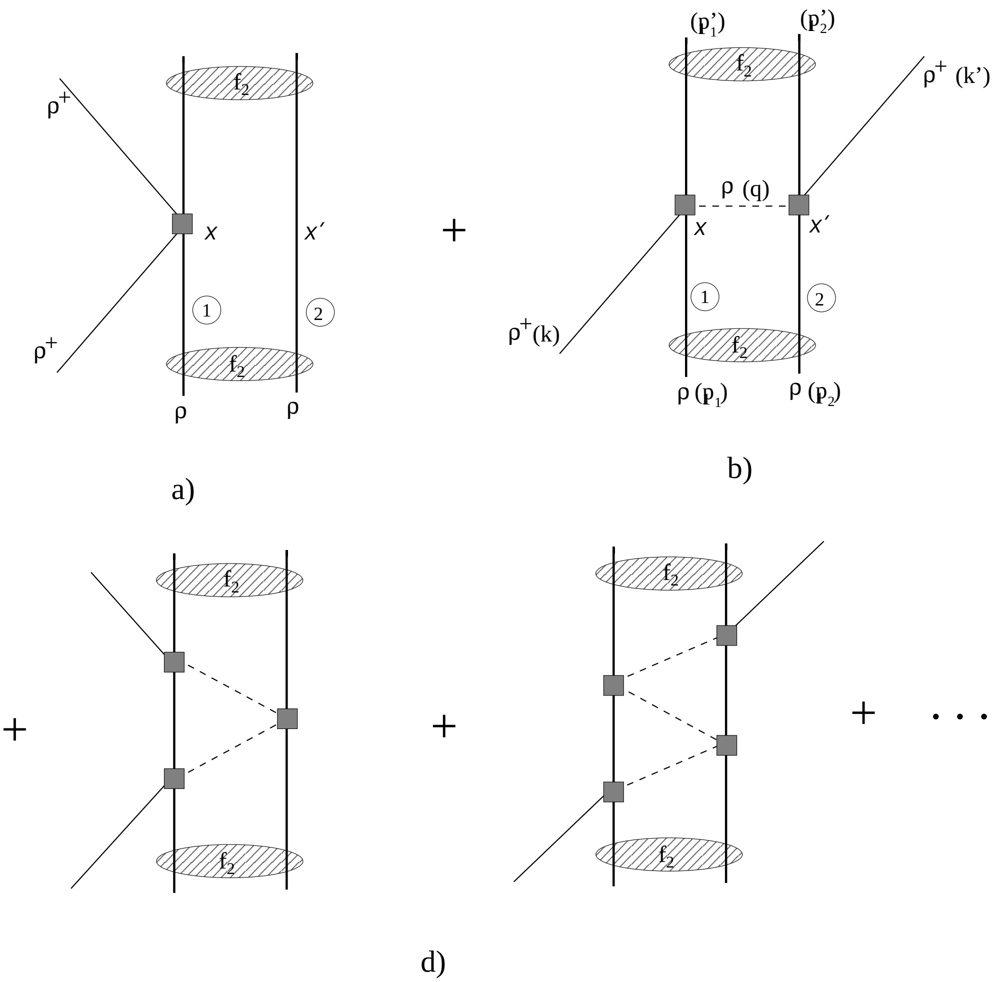

FIG. 2: Diagrammatic representation of the fixed center approximation to the Faddeev equations. Diagrams *a*) and *b*) represent the single and double scattering contributions respectively.

The scattering potential in terms of the two body potentials V_{31}, V_{32} is:

$$
\begin{aligned}
T =& \left(-\langle(+1)| \otimes \frac{1}{\sqrt{3}} \left(\langle(+1,-1) + (-1,+1) - (0,0)|\right)\right) (V_{31} + V_{32}) \\
& \left(-|(+1)\rangle \otimes \frac{1}{\sqrt{3}} \left(|(+1,-1) + (-1,+1) - (0,0)\rangle\right)\right) \\
=& \frac{1}{3}\Big\langle ((2\,{+2}),-1) + (\frac{1}{\sqrt{6}}(20) + \frac{1}{\sqrt{2}}(10) + \frac{1}{\sqrt{3}}(00),1) - (\frac{1}{\sqrt{2}}(2\,{+1}) + \frac{1}{\sqrt{2}}(1\,{+1}),0) \Big| V_{31} \\
& \Big| ((2\,{+2}),-1) + (\frac{1}{\sqrt{6}}(20) + \frac{1}{\sqrt{2}}(10) + \frac{1}{\sqrt{3}}(00),1) - (\frac{1}{\sqrt{2}}(2\,{+1}) + \frac{1}{\sqrt{2}}(1\,{+1}),0) \Big\rangle \\
&+ \frac{1}{3}\Big\langle (\frac{1}{\sqrt{6}}(20) + \frac{1}{\sqrt{2}}(10) + \frac{1}{\sqrt{3}}(00),1) + ((2\,{+2}),-1) - (\frac{1}{\sqrt{2}}(2\,{+1}) + \frac{1}{\sqrt{2}}(1\,{+1}),0) \Big| V_{32} \\
& \Big| ((\frac{1}{\sqrt{6}}(20) + \frac{1}{\sqrt{2}}(10) + \frac{1}{\sqrt{3}}(00),1) + (2\,{+2}),-1) - (\frac{1}{\sqrt{2}}(2\,{+1}) + \frac{1}{\sqrt{2}}(1\,{+1}),0) \Big\rangle . \qquad (15)
\end{aligned}
$$

where the notation followed in the last term for the states is $\langle (I^{\rm total} I_z^{\rm total}, I_z^k) | V_{ij} | \rangle$, where $I^{\rm total}$

means the total isospin of the ij system and $k \neq i, j$ (the spectator ρ).

This leads, in terms of the $I = 0$ and $I = 2$ unitarized amplitudes ($t_{\rho\rho}^{(I=0)}$, $t_{\rho\rho}^{(I=2)}$), to the following amplitude for the single scattering contribution:

$$t_1 = \frac{2}{9}\left(5t_{\rho\rho}^{(I=0)} + t_{\rho\rho}^{(I=2)}\right). \tag{16}$$

where we have added an extra 2 factor in order to match the unitary normalization of ref. [11] in $t_{\rho\rho}^{I}$.

It is worth noting that the argument of the function $T(s)$ is the total invariant mass energy s, while the argument of t_1 is s', where s' is the invariant mass of the ρ meson with momentum k and the ρ meson inside the f_2 resonance with momentum p_1 and is given by

$$s' = (k + p_1)^2 = \frac{1}{2}\left(s + 3m_\rho^2 - M_{f_2}^2\right) \tag{17}$$

For latter applications, let us write the general expression of s' for the interaction of a particle A with a molecule B with n equal building blocks b. Then, s' represents the invariant mass of the particle A and a particle b of the B molecule and is given by

$$s' = \frac{1}{n}\left(s - M_B^2 - M_A^2\right) + M_A^2 + m_b^2 \tag{18}$$

where $M_{A(B)}$ is the mass of the $A(B)$ system and m_b is the mass of every building block of the B molecule.

Let us consider the wavefunctions of the incident and outgoing ρ particles being plane waves normalized inside a box of volume $\mathcal{V}$ and let us call φ_i the wavefunctions of the ρ mesons inside the f_2 resonance. The S-matrix for the process of Fig. 2a is written as

$$S^{(1)} = \int d^4x \frac{1}{\sqrt{2\omega_{p_1}}} e^{-ip_1^0 x^0} \varphi_1(\vec{x}) \frac{1}{\sqrt{2\omega_{p_1'}}} e^{ip'^0_1 x^0} \varphi_1(\vec{x}) \frac{1}{\sqrt{2\omega_k \mathcal{V}}} e^{-ikx} \frac{1}{\sqrt{2\omega_k' \mathcal{V}}} e^{ik'x}(-it_1) \tag{19}$$

which we can multiply by the identity

$$\int d^3\vec{x}\,'\varphi_2(\vec{x}\,')\varphi_2(\vec{x}\,') = 1. \tag{20}$$

The integration of the time component x^0 provides the energy conservation at the interaction point x:

$$\int dx^0 e^{-ip_1^0 x^0} e^{ip'^0_1 x^0} e^{-ik^0 x^0} e^{ik'^0 x^0} = 2\pi\,\delta(p_1^0 + k^0 - p'^0_1 - k'^0) \equiv 2\pi\,\delta(k^0 + E_{f_2} - k'^0 - E'_{f_2}), \tag{21}$$

where in the last step we have assumed $p_2^0 = {p'_2}^0$, as corresponds to having the second particle as spectator (impulse approximation). We can take

$$\varphi_1(x)\varphi_2(x') = \frac{1}{\sqrt{\mathcal{V}}} e^{i\vec{K}_{f_2}\cdot\vec{R}} \Psi_{f_2}(\vec{r}), \tag{22}$$

with Ψ_{f_2} the wave function of the $f_2(1270)$ cluster and

$$\vec{R} = \frac{\vec{x} + \vec{x}'}{2}$$
$$\vec{r} = \vec{x} - \vec{x}'. \tag{23}$$

and then we get for the spatial integrals

$$\int d^3R\, e^{i\vec{K}_{f_2}\cdot\vec{R}} e^{-i\vec{K}'_{f_2}\cdot\vec{R}} e^{i\vec{k}\cdot\vec{R}} e^{-i\vec{k}'\cdot\vec{R}} = (2\pi)^3\delta(\vec{k}+\vec{K}_{f_2}-\vec{k}'-\vec{K}'_{f_2}) \tag{24}$$

and

$$\int d^3r\, \Psi_{f_2}(\vec{r})\Psi_{f_2}(\vec{r}) e^{i\vec{k}\cdot\frac{\vec{r}}{2}} e^{-i\vec{k}'\cdot\frac{\vec{r}}{2}} = F_{f_2}\Big(\frac{\vec{k}-\vec{k}'}{2}\Big) \simeq F_{f_2}(0) = 1 \tag{25}$$

where F_{f_2} is the $f_2(1270)$ form factor normalized to unity neglecting the $\vec{k}$, $\vec{k}'$ momenta, which we take equal.

Hence the S-matrix for the single scattering term is given by

$$S^{(1)} = -it_1 \frac{1}{\mathcal{V}^2} \frac{1}{\sqrt{2\omega_{p_1}}} \frac{1}{\sqrt{2\omega_{p'_1}}} \frac{1}{\sqrt{2\omega_k}} \frac{1}{\sqrt{2\omega'_k}} (2\pi)^4\, \delta(k+K_{f_2}-k'^0-K'_{f_2}). \tag{26}$$

and recall we must sum $t_1+t_2 \to 2t_1$.

B. Double-scattering and resummation contribution

We are going to evaluate the amplitude of the double-scattering contribution (fig. 2b) in a similar way as in the case of the kaon deuteron interaction in [35, 37].

The S-matrix can be written as

$$S^{(2)} = \int d^4x \int d^4x' \frac{1}{\sqrt{2\omega_{p_1}}} e^{-ip_1^0x^0} \varphi_1(\vec{x}) \frac{1}{\sqrt{2\omega_{p'_1}}} e^{ip'^0_1x^0} \varphi_1(\vec{x}) \frac{1}{\sqrt{2\omega_{p_2}}} e^{-ip_2^0x'^0} \varphi_2(\vec{x}\,') \frac{1}{\sqrt{2\omega_{p'_2}}} e^{ip'^0_2x'^0} \varphi_2(\vec{x}\,')$$
$$\frac{1}{\sqrt{2\omega_k\mathcal{V}}} e^{-ikx} \frac{1}{\sqrt{2\omega'_k\mathcal{V}}} e^{ik'x'}\, i \int \frac{d^4q}{(2\pi)^4} \frac{e^{iq(x-x')}}{q^2-m_\rho^2+i\epsilon} (-it_1)(-it_1) \tag{27}$$

The integrations of the time components x^0 and x'^0 provide the energy conservation at the two interaction points x and x':

$$\int dx^0 e^{-ip_1^0x^0} e^{ip'^0_1x^0} e^{-ik^0x^0} e^{iq^0x^0} = 2\pi\,\delta(p_1^0+k^0-p'^0_1-q^0)$$
$$\int dx'^0 e^{-ip_2^0x'^0} e^{ip'^0_2x'^0} e^{ik'^0x'^0} e^{-iq^0x'^0} = 2\pi\,\delta(p_2^0+q^0-p'^0_2-k'^0) \tag{28}$$

We implement now the change of variables $(\vec{x},\vec{x}') \to (\vec{R},\vec{r})$ of Eq. (23). The R integral gives the same expression as in Eq. (24), and the $\vec{r}$ integral gives rise to

$$\int d^3r\, \Psi_{f_2}(\vec{r})\Psi_{f_2}(\vec{r}) e^{i\vec{k}\cdot\frac{\vec{r}}{2}} e^{i\vec{k}'\cdot\frac{\vec{r}}{2}} e^{-i\vec{q}\cdot\vec{r}}$$
$$= \int d^3r\, e^{-i(\vec{q}-\frac{\vec{k}+\vec{k}'}{2})\cdot\vec{r}}\, \Psi_{f_2}(\vec{r})^2 \equiv F_{f_2}\Big(\vec{q}-\frac{\vec{k}+\vec{k}'}{2}\Big) \tag{29}$$

where $F_{f_2}\Big(\vec{q}-(\vec{k}+\vec{k}')/2\Big)$ is the $f_2(1270)$ form factor introduced above.

The final expression for the S-matrix for the double scattering process is

$$S^{(2)} = -i(2\pi)^4\delta(k + K_{f_2} - k' - K'_{f_2})\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_k}}\frac{1}{\sqrt{2\omega'_k}}\frac{1}{\sqrt{2\omega_{p_1}}}\frac{1}{\sqrt{2\omega_{p'_1}}}\frac{1}{\sqrt{2\omega_{p_2}}}\frac{1}{\sqrt{2\omega_{p'_2}}}$$
$$\times \int \frac{d^3q}{(2\pi)^3}F_{f_2}(q)\frac{1}{{q^0}^2 - \vec{q}^{\,2} - m_\rho^2 + i\epsilon}t_1 t_1. \quad (30)$$

and we will take q^0 at the f_2 rest frame, $q^0 = (s - m_\rho^2 - M_{f_2}^2)/(2M_{f_2})$, where we have considered ${p_1}^0 = {p'_1}^0$ and ${p_2}^0 = {p'_2}^0$ which is true in average. In Eq. (30) we have also taken into account that $(\vec{k} + \vec{k}')/2 = 0$ in average.

For the evaluation of the form factor of the f_2 resonance we follow the approach of [36]. In this work it is shown that the use of a separable potential in momentum space of the type

$$V = v\theta(\Lambda - q)\theta(\Lambda - q') \quad (31)$$

where Λ is the cutoff used in the theory for the scattering of two particles and q, q' are the modulus of the momenta, leads to the same on shell prescription for the scattering matrix as is used in the chiral unitary approach. The on shell prescription converts the coupled integral equations for the scattering matrix into algebraic equations, and similarly, the wave functions can be easily obtained in terms of an integral. The wave function in momentum space is written as

$$\langle\vec{p}|\psi\rangle = v\,\frac{\Theta(\Lambda - p)}{E - \omega_\rho(\vec{p}_1) - \omega_\rho(\vec{p}_2)}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi\rangle, \quad (32)$$

where $\omega_\rho(\vec{p}) = \sqrt{\vec{p}^{\,2} + m_\rho^2}$, and in coordinate space as

$$\langle\vec{x}|\psi\rangle = \int \frac{d^3p}{(2\pi)^{3/2}}e^{i\vec{p}.\vec{x}}\langle\vec{p}|\psi\rangle. \quad (33)$$

The final expression for the form factor of Eq. (29) is then given by

$$F_{f_2}(q) = \frac{1}{\mathcal{N}}\int_{\substack{p<\Lambda \\ |\vec{p}-\vec{q}|<\Lambda}} d^3p\,\frac{1}{M_{f_2} - 2\omega_\rho(\vec{p})}\,\frac{1}{M_{f_2} - 2\omega_\rho(\vec{p} - \vec{q})}, \quad (34)$$

where the normalization factor $\mathcal{N}$ is

$$\mathcal{N} = \int_{p<\Lambda} d^3p\frac{1}{(M_{f_2} - 2\omega_\rho(\vec{p}))^2}. \quad (35)$$

In fig. 3 we show the form factor of the f_2 resonance. The condition $|\vec{p} - \vec{q}| < \Lambda$ implies that the form factor is exactly zero for $q > 2\Lambda$. Therefore the $d|\vec{q}|$ integration in Eq. (30) has an upper limit of 2Λ.

We must now face the issue of normalization in our formalism. We use Mandl-Shaw [38] normalization for the fields and hence the S-matrix for ρf_2 scattering is written as

$$S = -iT_{\rho f_2}(s)\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_k}}\frac{1}{\sqrt{2\omega_{k'}}}\frac{1}{\sqrt{2\omega_{f_2}}}\frac{1}{\sqrt{2\omega_{f_2'}}}(2\pi)^4\,\delta(k + K_{f_2} - k'^0 - K'_{f_2}) \quad (36)$$

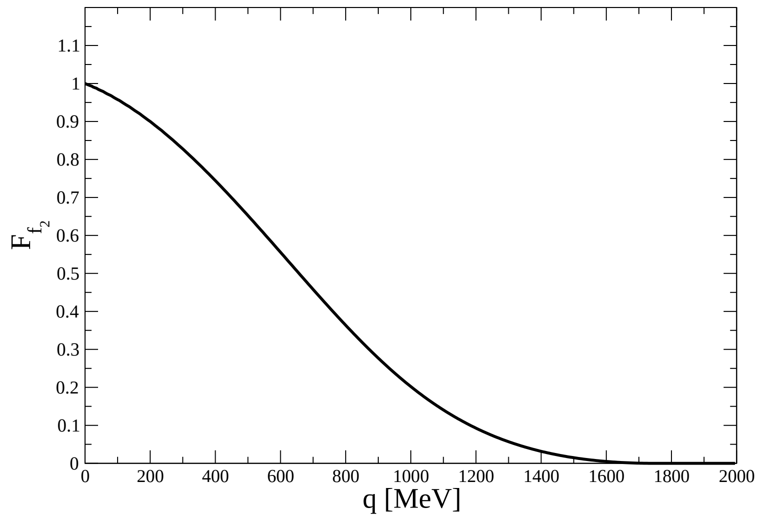

FIG. 3: Form factor of the $f_2(1270)$ resonance

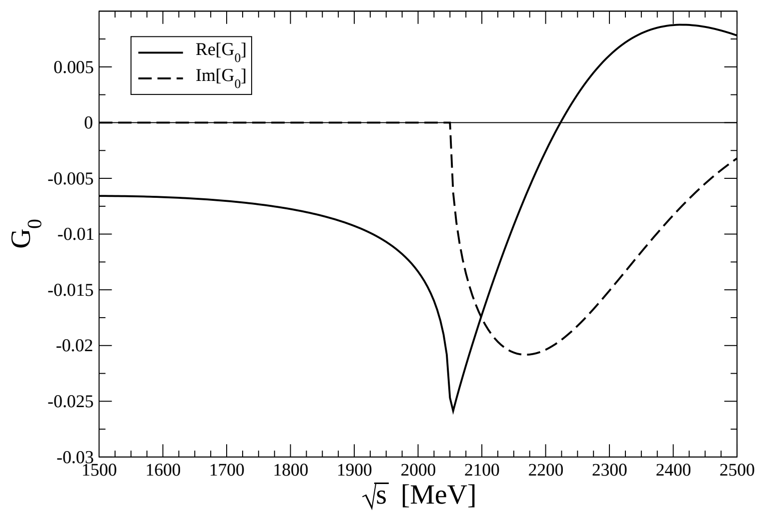

FIG. 4: Real and imaginary parts of the G_0 function, Eq. (38)

but this should be compared with expressions Eq. (26) for the single scattering and Eq. (30) for double scattering. Summing the two partitions T_1 and T_2 we find that

$$T_{\rho f_2} = 4(t_1 + t_1 t_1 G_0), \tag{37}$$

where we have made the assumption that in the f_2 rest frame, where we evaluate the amplitude, $2\omega_\rho \simeq M_{f_2}$, and G_0 is given by

$$G_0 \equiv \frac{1}{M_{f_2}} \int \frac{d^3q}{(2\pi)^3} F_{f_2}(q) \frac{1}{q^{0 2} - \vec{q}^{\,2} - m_\rho^2 + i\epsilon}. \tag{38}$$

In fig. 4 we show the real and imaginary parts of the G_0 function. Note that close to the threshold it has the typical shape of a two meson loop function, ρf_2 in this case, but it is smoothed towards zero at higher energies due to the form factor.

Equation (37) represents the two first terms of the series expansion of $4t_1/(1-t_1G_0)$. Actually, if we consider further number of scatterings in the expansion of $T_{\rho f_2}$ of the FCA, (see diagrams d in fig. 2), we get

$$\begin{aligned} T_{\rho f_2} &= 4(t_1 + t_1G_0t_1 + t_1G_0t_1G_0t_1 + t_1G_0t_1G_0t_1G_0t_1 + ...) = \frac{4t_1}{1-G_0t_1} = \frac{4}{t_1^{-1}-G_0} \\ &= 4\left[t_1^{-1}(s') - \frac{1}{M_{f_2}}\int \frac{d^3q}{(2\pi)^3}F_{f_2}(q)\frac{1}{q^{0\,2}-\vec{q}^{\,2}-m_\rho^2+i\epsilon}\right]^{-1}, \end{aligned} \tag{39}$$

where s' is given in Eq. (17).

C. Larger number of ρ mesons

For the interaction of up to 6 ρ mesons we can follow a similar procedure as in the previous subsections but considering the interaction of two different clusters. For the interaction of 4 ρ meson we can calculate the interaction of $2f_2(1270)$ resonances given the strong tendency of two ρ mesons to clusterize into an f_2. Advancing some results that we will show later, this four ρ state gives rise to the f_4 resonance. Thus, analogously, for 5ρ we can consider the interaction of one ρ meson with an f_4. And for 6ρ we can consider the interaction of an f_4 with an f_2.

Therefore the amplitude for the interaction of a cluster A with a cluster B made of two equal components b is given by

$$t(s;A,B) = 4\left[t^{-1}(s'(s;A,b);A,b) - G_0(s;A,B)\right]^{-1} \tag{40}$$

where

$$G_0(s;A,B) = \frac{1}{M_B}\int \frac{d^3q}{(2\pi)^3}F(q;B)\frac{1}{q^0(s;A,B)^2-\vec{q}^{\,2}-M_A^2+i\epsilon}, \tag{41}$$

$$F(q,B) = \frac{1}{\mathcal{N}}\int_{\substack{p<\Lambda' \\ |\vec{p}-\vec{q}|<\Lambda'}} d^3p\,\frac{1}{M_B-2\sqrt{\vec{p}^{\,2}+m_b^2}}\,\frac{1}{M_B-2\sqrt{|\vec{p}-\vec{q}|^2+m_b^2}}, \tag{42}$$

$$\mathcal{N} = \int_{p<\Lambda'} d^3p\frac{1}{\left(M_B-2\sqrt{\vec{p}^{\,2}+m_b^2}\right)^2}, \tag{43}$$

$$q^0(s;A,B) = \frac{s-M_A^2-M_B^2}{2M_B} \tag{44}$$

and, from Eq. (18),

$$s'(s;A,b) = \frac{1}{2}\left(s-M_B^2-M_A^2\right)+M_A^2+m_b^2. \tag{45}$$

Note that it is not necessary that the cutoff Λ' be the same in all the cases as the Λ used for the f_2 case. The cutoff Λ used in Eq. (6) for the $\rho\rho$ loop function, which is the same appearing in the momentum integral to get the f_2 form factor in Eq. (34) [36], can be interpreted as the typical maximum momentum that each ρ can reach inside the f_2 molecule. For the f_4 case we can argue that the maximum value would be like in the f_2 case but scaled by the typical momentum of the f_2 components inside the f_4 molecule. The typical three-momentum of the components of the f_4, γ_4, is of the order of

$$\gamma_4 \sim \sqrt{\frac{B^2}{4} + M_{f_2} B} \qquad ; \qquad B = M_{f_4} - 2M_{f_2} \tag{46}$$

where B is the binding energy of the f_4 and analogously for the f_2:

$$\gamma_2 \sim \sqrt{\frac{B^2}{4} + m_\rho B} \qquad ; \qquad B = M_{f_2} - 2m_\rho \tag{47}$$

This gives for the cutoff of the f_4

$$\Lambda'\Big|_{f_4} \sim \Lambda \sqrt{\left|\frac{\gamma_4}{\gamma_2}\right|} \simeq 1500 \text{ MeV}. \tag{48}$$

While this is just a very rough estimation, this gives us an idea of the order of Λ'. In any case this only affects the evaluation of the 5ρ and 6ρ system. In the numerical evaluation we will consider the range $\Lambda' \sim 875 - 1500$ MeV to have an idea of the uncertainties from this source. But, advancing some results, the dependence of the mass of the systems with this cutoff is small.

D. Arbitrary number of ρ mesons in single scattering approximation

For and arbitrary number of ρ mesons, it is possible to obtain a simple analytic expression for the mass of the multi-ρ system if only the single scattering mechanism is considered, which is the first order mechanism. Of course this is just a toy approximation since, as we will see in the results section, the multiple scattering is important, but it serves to make some interesting qualitative arguments.

In the single scattering approximation, Eq. (40) takes the form

$$t(s; A, B) \simeq 4\, t(s'(s; A, b); A, b) \tag{49}$$

That means that, for instance, the amplitude $t_{\rho f_2}$ is just proportional to the $t_{\rho\rho}$ amplitude but evaluated at an energy s' shifted with respect to s due to the fact that one of the ρ's involved in the $\rho\rho$ scattering is bound into an f_2 system. In general, the interaction amplitude of a number n_ρ of ρ mesons is proportional to the $\rho\rho$ amplitude with an energy obtained considering that one of the ρ mesons is bound into an $(n_\rho - 1)$ molecule. Therefore one can obtain recursively the amplitude for the n_ρ system. Because of that, the shape of $|t(s; A, B)|^2$ is the same as that of $|t_{\rho\rho}(\tilde{s})|^2$ but at a shifted energy. The $\tilde{s}$ value at which $|t_{\rho\rho}(\tilde{s})|^2$ has the maximum is precisely $M_{f_2}^2$. The value of s appearing in $t(s; A, B)$ of Eq. (49) is the value that we can assign to the mass of the n_ρ system, $M(n_\rho)$. Therefore, applying recursively the above condition one can obtain a general expression for $M(n_\rho)$ in the single scattering approximation:

$$M(n_\rho)^2 = \frac{1}{2} n_\rho\, (n_\rho - 1)\, M_{f_2}^2 - n_\rho\, (n_\rho - 2)\, m_\rho^2. \tag{50}$$

We can also define a binding energy per ρ as

$$E(n_\rho) = \frac{n_\rho m_\rho - M(n_\rho)}{n_\rho} \tag{51}$$

which will be used for later discussions.

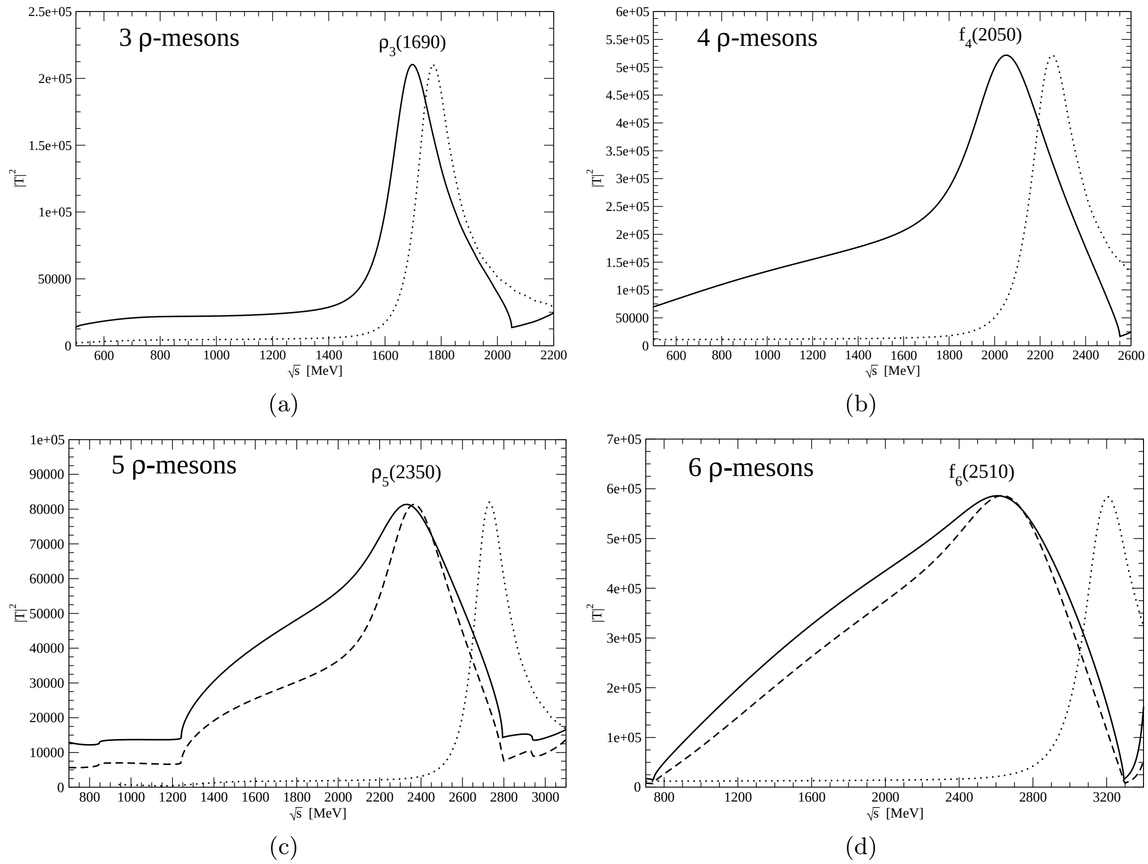

FIG. 5: Modulus squared of the unitarized multi-ρ amplitudes. Solid line: full model $\Lambda'\big|_{f_4} = 1500$ MeV; dashed line: full model $\Lambda'\big|_{f_4} = 875$ MeV; dotted line: only single-scattering contribution. (The dashed and dotted lines have been normalized to the peak of the solid line for the sake of comparison of the position of the maxima)

IV. RESULTS

In fig. 5 we show the modulus squared of the amplitudes for different number of ρ mesons considering only the single scattering mechanisms (dotted line) and the full model (solid and dashed lines). The difference between the solid and dashed lines is the value of $\Lambda'\big|_{f_4}$ of Eq. (48) needed in the evaluation of the 5ρ and 6ρ meson systems (1500 MeV in the solid line, 875 MeV in the dashed one). The dotted and dashed curves have been normalized to the peaks of the corresponding full result for the sake of comparison of the position of the maximum. The difference between the dashed and solid lines can be considered as an estimate of the error but the variation in the position of the maximum is small.

We clearly see that the amplitudes show pronounced bumps which we associate to the resonances labeled in the figures. The position of the the maxima can be associated to the masses of the corresponding resonances.

In table I the values of the masses of our generated multi-ρ systems are shown in comparison with the experimental values at the PDG [25]. The two values for the ρ_5 and f_6 masses in the full model column correspond to the different values in $\Lambda'\big|_{f_4}$ as explained above. In the last column the binding energy per ρ meson, $E(n_\rho) = (n_\rho m_\rho - M(n_\rho))/n_\rho$ is also shown.

In fig. 6 we show graphically the results for the masses of table I.

n_ρ		mass, PDG [25]	mass, only single scatt.	mass, full model	$E(n_\rho)$
2	$f_2(1270)$	1275 ± 1	1275	1285	133
3	$\rho_3(1690)$	1689 ± 2	1753	1698	209
4	$f_4(2050)$	2018 ± 11	2224	2051	263
5	$\rho_5(2350)$	2330 ± 35	2690	2330-2366	302-309
6	$f_6(2510)$	2465 ± 50	3155	2607-2633	337-341

TABLE I: Results for the masses of the dynamically generated states.

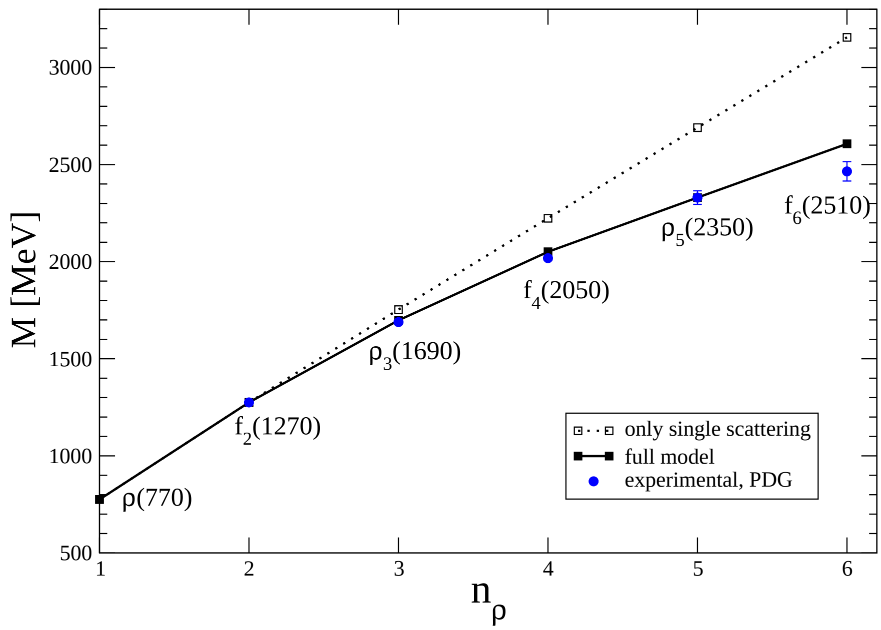

FIG. 6: Masses of the dynamically generated states as a function of the number of constituent $\rho(770)$ mesons, n_ρ. Only single scattering contribution (dotted line); full model (solid line); experimental values from the PDG[25], (circles).

We can see from the results that the single scattering mechanism produce qualitatively the resonances but the positions of the masses do not agree with the experimental values by differences ranging from about 60 MeV for the ρ_3 to 700 MeV for the f_6. The situation is drastically improved when the multiple scattering is considered. In this case, the agreement with the experimental values of the masses is remarkable. Quantitatively, the full model is essentially compatible with the experimental values within errors except for the f_6 where the discrepancy is about 150 MeV, which is still quite remarkable, given the high mass and width of the resonance. The typical discrepancy with the experimental masses is of the order of 1%, (5% for the f_6).

It is worth stressing the simplicity of our approach and the absence of parameters fitted in the model. To be more precise, only the value $\Lambda = 875$ MeV of the cutoff of the $\rho\rho$ loop function was chosen in ref. [11] to agree with the experimental f_2 pole position. No further adjustments have been done in the present work.

In principle, the widths of the bumps can be associated to the the widths of the resonances if they were Breit-Wigner like shapes, which is clearly not the case. This means that the amplitudes contain much non-resonant background which our model generates implicitly through the non-linear dynamics involved in the unitarization procedure. That means that the extraction of the widths of the resonances from our amplitudes is just very qualitative: 200, 350, 900 and 1500 MeV for ρ_3, f_4, ρ_5 and f_6 respectively. The order of magnitude agree with the experimental value of the PDG [25], 161 ± 10, 237 ± 18, 400 ± 100 and 255 ± 40 respectively, except for the two heaviest states. However, it is worth noting that, by looking at fig. 5, in these heaviest states much of the strength of the amplitude off the peak could be interpreted as a background, as would be the case in an experimental analysis of a distribution like the one obtained in fig. 5, in which case the actual width of the resonance would be significantly reduced.

Let us address again the problem of an arbitrary large number of ρ mesons. A natural question looking at fig. 6 is if the curve of the masses saturates for a large enough number of ρ mesons. That would imply that it would be energetically free to add an extra ρ meson to the system. For the single scattering case we can analyze the problem with the help of Eq. (50). The saturation would occur for

$$n_\rho\Big|_{\text{sat}} = \frac{m_\rho^2 - M_{f_2}^2/4}{m_\rho^2 - M_{f_2}^2/2} \tag{52}$$

which never happens for the actual ρ and f_2 masses, since $n_\rho\Big|_{\text{sat}}$ of Eq. (52) gives a negative value. However it is worth noting in fig. 6 that the single scattering is just a bound limit and that the multiple scatterings tend to decrease importantly the mass of an n_ρ system. If such decrease is enough to eventually reach the saturation condition cannot be answered with certainty within the present model since we do not go beyond $n_\rho = 6$. However, it is worth noting the large value of the binding energy per ρ, see last column of table I. Already for $n_\rho = 6$, it is almost half the value of the ρ meson mass. That means that the creation of the 6 ρ meson system gives back half the mass of all the particles involved which is quite a lot of energy.

The binding energy per ρ evaluated using the single scattering approximation, Eq. (51), tends asymptotically to

$$\lim_{n_\rho \to \infty} E(n_\rho) = m_\rho - \sqrt{\frac{M_{f_2}^2}{2} - m_\rho^2} \simeq 315 \text{ MeV}. \tag{53}$$

However this value is already reached at $n_\rho = 6$ if the multiple scattering mechanisms are considered.

If the $\rho\rho$ interaction were a little bit stronger, such that $M_{f_2} \sim \sqrt{2} m_\rho = 1096$ MeV, then the saturation would be reached already considering only the single scattering. And, in order to get saturation for $n_\rho = 6$ with only single scattering, the mass of the f_2 resonance should be just slightly smaller, $\sim \sqrt{20/11} m_\rho = 1056$ MeV. Of course this is just a qualitative reasoning since the width of the system would eventually increase with the number of ρ mesons, making the system fade away rapidly. The former discussion is obviously rough and speculative of what might happen for large n_ρ systems. What remains as quantitative results from the present study is the fact that the $f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$ can be essentially considered as multi-ρ molecules with increasing number of ρ mesons.

V. CONCLUSIONS

In the present work we claim for the first time that the $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$ resonances can be interpreted as multi-ρ states of 3, 4, 5 and 6 ρ mesons respectively, with their spins aligned. The main idea stems from the fact that in ref. [11] it was found that the interaction of two $\rho(770)$ mesons in isospin $I = 0$ and spin $S = 2$ is very strong, to the point to bind the two ρ mesons forming the $f_2(1270)$ resonance. This elementary $\rho\rho$ interaction is obtained implementing unitarity, using the techniques of the chiral unitary approach, with a potential obtained from a hidden gauge symmetry Lagrangian for the interaction of two vector mesons. For the multi-ρ systems we evaluate the scattering amplitudes for the interactions of two clusters made up of ρ-mesons. To this purpose we use the fixed center approximation to the Faddeev equations which considers the multiple scattering steps in addition to the single process where each ρ meson interacts with all the rest of ρ mesons within the cluster.

The position of the maximum in the modulus squared of the amplitudes can be associated with the masses of the corresponding resonances. It is worth noting that the model has no free parameters once a cutoff is chosen in ref. [11] to obtain the experimental mass of the $f_2(1270)$ resonance.

The values of the masses that we obtain are in very good agreement with the experimental values of the masses of the resonances considered in the present work, the $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$. This is a remarkable fact given the simplicity of the underlying idea.

The states obtained have an increasing binding energy per particle which induces one to speculate on the possibility that for a given number of ρ mesons it would cost no energy to produce a new ρ meson inside the meson condensate state. However, simultaneusly we observe that the width of the new systems also increases with the number of ρ mesons, to the point that for $n_\rho = 6$ the width is already very large. It might as well be that one has reached an experimental threshold and that new multi ρ states, that in principle could be created, have such a large width that they escape present detection techniques. In any case, the claims made here that the already observed states up to $J = 6$ correspond to multi ρ states is a novel idea worth consideration. New studies with different formalisms and different points of view would be most welcome, as well as possible experimental tests which could help unveil the real nature of these states.

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