Pion and Kaon Vector Form Factors

J. E. Palomar[1], J. A. Oller[2], E. Oset[1]

[1] Departamento de Física Teórica e IFIC, Centro Mixto Universidad de Valencia-CSIC, 46100 Burjassot (Valencia), Spain

[2] Forschungszentrum Jülich, Institut für Kernphysik (Theorie) D-52425 Jülich, Germany

Abstract

The pion and kaon coupled-channel vector form factors are described by making use of the resonance chiral Lagrangian results together with a suitable unitarization method in order to take care of the final state interactions. A very good reproduction of experimental data is accomplished for the vector form factors up to $\sqrt{s} \leq 1.2$ GeV and for the $\pi\pi$ P-wave phase shifts up to $\sqrt{s} \leq 1.5$ GeV.

PACS: 13.75.Lb, 11.55.Fv, 11.80.Et, 12.39.Fe

1 Unitarization

Using an appropriate unitarization method we take into account the final state interaction corrections to the tree level amplitudes calculated from lowest order χPT [1] and from the inclusion of explicit resonance fields in a chiral symmetry fashion as given in ref. [2]. A similar procedure has already been used in the scalar sector to describe the scalar form factor associated with the strange-change scalar current $\overline{u}s$ in ref. [3]. Starting from the unitarity of the S-matrix and the introduction of the electromagnetic meson form factor $F_{MM'}(s)$:

$$\langle \gamma(q)|T|M(p)M'(p')\rangle = e\epsilon_\mu (p-p')^\mu F_{MM'}(s) \tag{1}$$

with $q^2 = s$, e the modulus of the electron charge and ϵ_μ the photon polarization vector, we arrive at the expression:

$$\mathrm{Im} F_{MM'}(s) = \sum_\alpha F^*_\alpha(s) \frac{p_\alpha(s)}{8\pi\sqrt{s}} \theta(s-4m^2_\alpha) p_\alpha(s) \frac{T(s)_{\alpha,MM'}}{p_{MM'}(s)} \tag{2}$$

where $\theta(x)$ is the usual Heaviside function. On the other hand $p_{MM'}(s)$ and $p_\alpha(s)$ are respectively the moduli of the three momenta of the mesons in the final and intermediate meson states, and we sum over intermediate two-meson states.

We will work in the isospin limit, with $|\pi\pi\rangle$ and $|K\bar{K}\rangle$ states (and the ρ resonance) in the $I=1$ channel and only the $|K\bar{K}\rangle$ state (and the ω and ϕ resonances) in the I=0 channel. Using a matrix notation, the P-wave amplitudes $T^I(s)$ can be written as [4]:

$$T^I = [1 + K^I(s)\cdot g(s)]^{-1} \cdot K^I(s) \tag{3}$$

where $K^I_{ij}(s)$ are the tree level amplitudes derived from lowest order χPT plus s-channel vector resonance exchange contributions [2] corresponding to the transition $i \rightarrow j$. From eqs. (2) and (3) it follows, after some algebra, that $F^I(s)$ can be written as:

$$F^I(s) = \left[1 + \widetilde{Q}(s)^{-1} \cdot K^I(s) \cdot \widetilde{Q}(s) \cdot g^I(s)\right]^{-1} \cdot R^I(s) \tag{4}$$

where $\widetilde{Q}_{ij}(s) = p_i(s)\delta_{ij}$ and $K^I(s)$ is the matrix collecting the tree level amplitudes between definite $\pi\pi$ and $K\bar{K}$ isospin states. $F^I(s)$ is the column matrix $F^I(s)_i = F^I_i(s)$, $R^I(s)$ is a vector made up by functions without any cut and $g^I(s)$ is the diagonal matrix given by the loop with two meson propagators:

$$g^I_i(s) = \frac{1}{16\,\pi^2}\left[-2 + d^I_i + \sigma_i(s)\,\log\frac{\sigma_i(s)+1}{\sigma_i(s)-1}\right] \tag{5}$$

where $\sigma_i(s) = \sqrt{1 - 4m_i^2/s}$. We will label pions with 1 and kaons with 2 in the $I = 1$ case. In the $I = 0$ case we only have kaons.

In the large N_c limit loop physics is supressed and then $F^I(s) = R_{N_c leading}(s) = F^I_t(s)$, where $F^I_t(s)$ is the tree level form factor.[1]

This allows us to write:

$$F^I(s) = \left[1 + \widetilde{Q}(s)^{-1} \cdot K^I(s) \cdot \widetilde{Q}(s) \cdot g^I(s)\right]^{-1} \cdot \left[F^I_t(s) + R^I_{subleading}(s)\right] \tag{6}$$

with $R^I_{subleading}(s)$ being of $\mathcal{O}(N_c^{-1})$. If we require that the vector form factor from eq. (6) vanishes for $s \rightarrow \infty$ as is suggested by the experiments, we find that the subleading part of $R^I(s)$, which at first can be an arbitrary polynomial (the poles coming from the resonances are in the leading part $F^I_t(s)$), must be a constant. In order to fix the constants $R^I_{subleading}(s)$ and $d^I_i(s)$ of $g^I_i(s)$ we match our results with those of one loop χPT. We take $R^{I=1}_{subleading}(s) = 0$ in order to constrain further our approach. This can be done since we can match our results with one loop χPT by choosing appropriate values for $d^{I=1}_1$ and $d^{I=1}_2$. The values of the other constants given by the matching are:

$$\begin{aligned}
d^{I=1}_1 &= \frac{m_K^2}{m_K^2 - m_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + \frac{1}{2}\log\frac{m_K^2}{\mu^2} + \frac{1}{2}\right) \\
d^{I=1}_2 &= \frac{-2\;m_\pi^2}{m_K^2 - m_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + \frac{1}{2}\log\frac{m_K^2}{\mu^2} + \frac{1}{2}\right) \\
d^{I=0} &= \frac{1}{3} + \log\frac{m_K^2}{\mu^2} \\
R^{I=0}_{subleading} &= -\frac{m_K^2}{16\sqrt{2}\,\pi^2 f^2}\left(\frac{1}{3} + \log\frac{m_K^2}{\mu^2}\right)
\end{aligned} \tag{7}$$

The bare masses of the resonances (which appear in the tree level quantities) are fixed by the requirements that the moduli of the $\pi\pi$ $I = 1$ and $K\overline{K}$ I=0 P-wave amplitudes

[1] We evaluate the tree level form factors and scattering amplitudes using the lowest order χPT Lagrangian [1] plus the chiral resonance Lagrangian [2].

have a maximum for $\sqrt{s} = M_\rho^{physical}$ MeV and for $\sqrt{s} = M_\phi^{physical}$ MeV, respectively. For the mass of the ω we take directly 782 MeV since there are no experimental data in the region of the ω and its contributions to other physical regions do not depend on such fine details since the ω is very narrow. On the other hand, the coupling of the vector resonances [2] to mesons and photons are described by two real parameters G_V and F_V respectively. We use their experimental value, G_V=53 MeV (from a study of the pion electromagnetic radii [1]) and F_V=154 MeV (from the observed decay rate $\Gamma(\rho^0 \to e^+e^-)$ [2]).

2 Results and conclusions.

As can be seen in fig. 1, we can describe in a very precise way the vector pion form factor and the P-wave $\pi\pi$ phase shifts up to about s=1.44 GeV^2 (even for higher energies in the case of phase shifts). For values of $\sqrt{s}$ higher than 1.2 GeV new effects appear: 1) the presence of more massive resonances, ρ', ω', ϕ'... 2) The effect due to multiparticle states, e.g. 4π, $\omega\pi$ ... which are non negligible. In figure 2, we compare our results with those of χPT. We can see that the resummation of our scheme leads to a much better agreement with the two loop χPT pion vector form factor than with the one loop one. The resonance regions are also well reproduced.

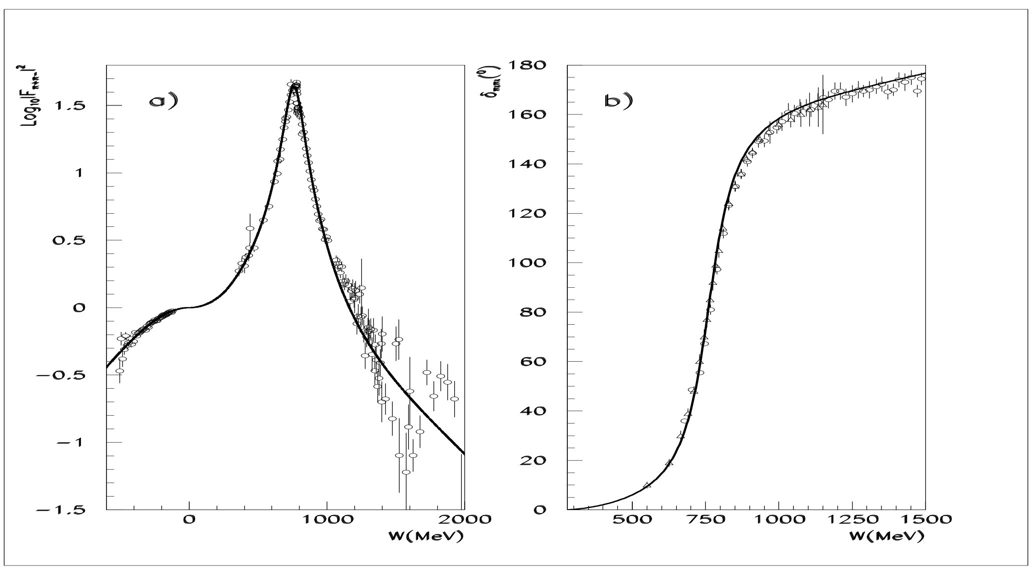

Figure 1: W is defined as $\sqrt{s}$ for $s > 0$ and as $-\sqrt{-s}$ for $s < 0$. a) $\pi^+\pi^-$ vector form factor. b) $\pi\pi$ P-wave phase shifts. Both are compared with several experimental data.

References

[1] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465, 517, 539.

[2] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. **B223** (1989) 425; G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311.

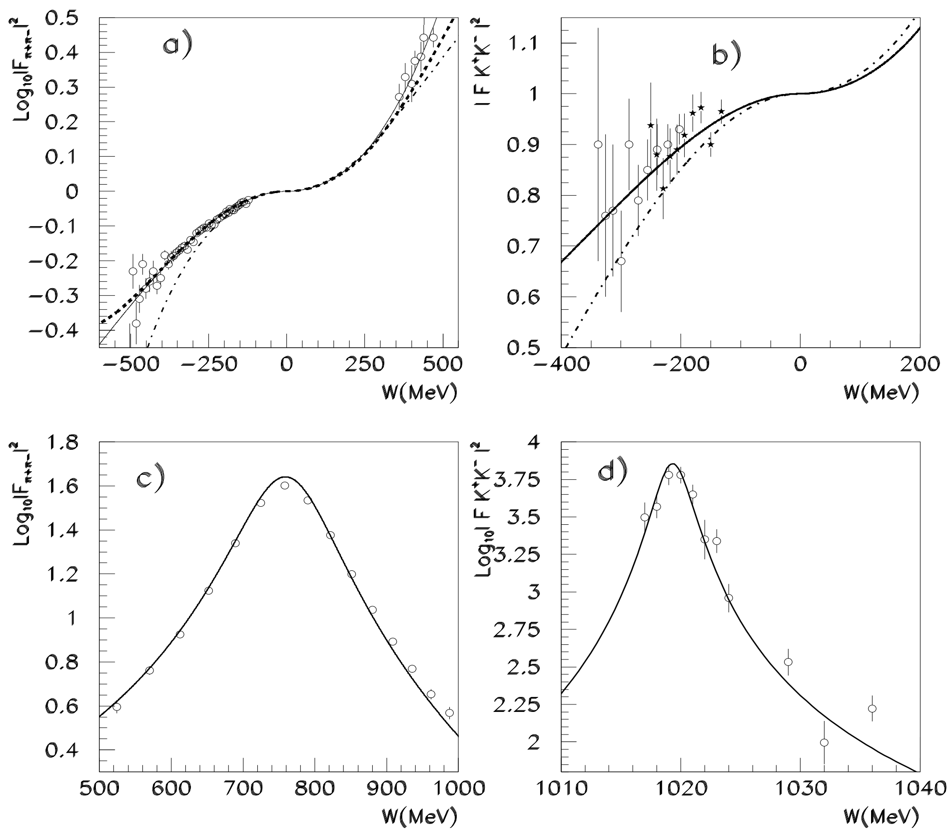

Figure 2: W is defined as $\sqrt{s}$ for $s > 0$ and as $-\sqrt{-s}$ for $s < 0$. From left to right and top to bottom: a) Vector pion form factor. The dashed-dotted line represents one loop χPT ref. [1] and the dashed one the two loop χPT result ref. [5]. b) K^+K^- form factor. The meaning of the lines is the same as before. c)Vector pion form factor in the ρ region. Data from tau decay. d) K^+K^- form factor. All the results are compared with data.

[3] M. Jamin, J. A. Oller and A. Pich, in preparation.

[4] J. A. Oller and E. Oset, Phys. Rev. **D60** (1999) 074023.

[5] J. Bijnens, G. Colangelo and P. Talavera, JHEP 9805 (1998) 014.