# Non-perturbative chiral approach to $K^- p \to \gamma Y$ reactions

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## Abstract

A recently developed nonperturbative chiral approach to describe the $S = -1$ meson-baryon reactions has been extended to investigate the near threshold $K^- p \to \gamma\Lambda, \gamma\Sigma^0$ reactions. With the parameters governed by chiral SU(3) symmetry, we show that the predicted branching ratios $\Gamma_{K^- p \to \gamma\Lambda}/\Gamma_{K^- p \to all}$ and $\Gamma_{K^- p \to \gamma\Sigma^0}/\Gamma_{K^- p \to all}$ are close to the experimental values. The coupling with the $\eta$ channels, which was shown to be important in the $S = -1$ meson-baryon reactions, is also found to be significant here. Our results are consistent with the interpretation of the $\Lambda(1405)$ as a quasi-bound meson-baryon state as found in other similar chiral approaches.

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## I. INTRODUCTION

The near threshold $K^-p \to \gamma Y$ reaction with $Y = \Lambda, \Sigma^0$ has long attracted a lot of interest, mainly because of the possibility of using this reaction to resolve the debates [1–7] over the structure of the $\Lambda(1405)$ resonance. Most of the earlier theoretical investigations [8] neglected the initial strong $K^-p$ interactions. It was first demonstrated by Siegel and Saghai [9] that the initial $K^-p$ interactions can drastically change the predicted capture rates and thus can significantly alter the interpretation of the data. With the phenomenological separable potentials, they, however, needed an about $30 - 50\%$ deviation of the coupling constants from the SU(3) values to obtain an accurate description of the data.

Obviously, the progress can be made only when the initial strong interactions and the photoproduction amplitudes are consistently described within the same theoretical framework. For investigating the low energy processes involving strange hadrons, such as the processes considered in this work, it is now generally believed that the best starting point is the effective chiral lagrangian with SU(3) symmetry. The most developed approach in this direction is Chiral Perturbation Theory [10–14]. Such an approach, however, becomes powerless in facing the $S = -1$ meson-baryon interactions because of the formation of a resonance just below the $K^-p$ threshold. An alternative nonperturbative chiral approach was first developed by Kaiser, Siegel and Weise [15]. The essential idea is to define the meson-baryon potentials by the SU(3) chiral lagrangian and sum a subseries of the chiral expansion by solving a coupled-channel Lippmann-Schwinger equation. They consider the lowest order and next to lowest order chiral lagrangians, and implemented suitable form factors to make the resulting meson-baryon potentials of finite range such that the loops become convergent. The range parameters of form factors and some of the second order chiral lagrangians are fitted to the data. This unitary coupled-channel approach generates dynamically the $\Lambda(1405)$ resonance as a quasibound meson baryon state, and reproduces the low energy data for $K^-p$ elastic and inelastic channels. The generation of the $\Lambda(1405)$ resonance is actually not a merit of the chiral lagrangians since it can be obtained in a suitable coupled channel K matrix approach, which would implement unitarity in the amplitudes [9,16,17]. The merit of the chiral lagrangians is that they can provide an expansion for this K matrix consistent with chiral symmetry and its breaking in the different strangeness channels, $S$ = 0,1,−1. The extension of the work of [15] to the $S$ =0,1 channels is done in Ref. [18]. Particularly, when working in the S=0 sector, $\pi N$ and coupled channels, the $N^*(1535)$ resonance is also generated dynamically. Simultaneously, the $\eta$ and K photoproduction processes in the S=0 channels were also studied in a similar approach, and with a few more parameters a global reproduction of the strong and electromagnetic cross sections was obtained.

The work of [19] was inspired by Ref. [15], but followed closely the approach developed in the study of the s-wave meson-meson scattering of Ref. [20]. Starting with a coupled-channel equation based on the lowest order chiral lagrangian, it was found that the off shell effects in the vertices can be absorbed in coupling constants renormalization. Hence, only the on-shell part of the vertices needed to be considered. The loop integrals were regularized by means of a cut off, rather than using a form factor as done in [15], which allows one to keep the chiral logarithms. The resulting coupled-channel equation is similar to that of Ref. [15]. Another major step taken in Ref. [19] was to include the $\eta$ and $\Xi$ channels in order to have exact SU(3) symmetry when the mass differences among the members of each Octet, mesonic or baryonic, are neglected. Both channels are not opened at low energies and the loop integrations involving these two channels only contribute to the real part of the amplitudes. The inclusion of the $\Xi$ channels was found to have negligible effects, but the $\eta$ channels were found numerically important. The approach of [19] turns out to be more economical because with the use of only one cut off parameter and the input of the lowest order lagrangian one can reproduce fairly well all the low energy data in the $S = -1$ sector. However, the fact that the $\eta$ meson loops only provide a real part to the amplitudes, allows the effect of these channels to be effectively taken into account by means of the parameters of the second order lagrangians, as done in [15], provided one is not too close to the thresholds of these channels. Conversely, the effects of the second order lagrangian can be effectively incorporated by means of a suitable cut off in [19], much as it happened in the study of the scalar sector in the meson-meson interaction in [20,21]. The physics contained in the approaches of [15,19] is basically the same, in spite of the different inputs and treatment of the scattering equations, and the results are remarkably similar. However, it is not clear that this simple approach can be extended to the $S = 0$ and $S = 1$ sectors because higher order counterterms, which cannot be accounted for by means of just one cut off, could appear. Preliminary results for the meson-baryon interaction [22] also point in the same direction.

In this paper, we extend the unitary coupled channel chiral approach of [19] in order to investigate the $K^-p \to \gamma\Lambda, \gamma\Sigma^0$ reactions. Our main objective is to investigate whether the data for these reactions can also be well described with the parameters fixed by the SU(3) chiral symmetry and hence can provide us with an additional support to the interpretation of the $\Lambda(1405)$ as a quasi-bound meson-baryon state with $S = -1$.

In section II, we present a derivation of the approach [19] within a well-defined formulation of relativistic quantum field theory. This will allow us to clearly establish the rules by which the approximations were introduced in solving the problem. These rules then allow us to easily derive in section III the $K^-p \to \gamma Y$ amplitude that is consistent with the model of Ref. [19]. The results and discussions are presented in section IV.

## II. DERIVATION OF THE UNITARY COUPLED CHANNEL METHOD

The approaches followed in Refs. [15,19] were developed by implementing some physical considerations of chiral symmetry into a postulated coupled-channel scattering equation. To extend the model of Ref. [19] to investigate $K^-p \to \gamma Y$ reactions, it is useful to establish its derivation from a well-defined formulation of relativistic quantum field theory.

Within the relativistic quantum field theory [23], the S-matrix for the reactions we are considering can be defined as

$$S_{ij} = \delta_{ij} - (2\pi)^4 i\delta^{(4)}(P_i - P_f)\hat{T}_{ij}\,, \tag{1}$$

where $i, j$ denote either a meson-baryon($MB$) channel or a photon-hyperon($\gamma Y$) channel, and $P_i$ is the total four-momentum of the system. The scattering amplitude is defined by

$$\hat{T}_{ij} = \frac{1}{(2\pi)^6} \frac{1}{\sqrt{2\omega_i}} \sqrt{\frac{M_i}{E_i}} T_{ij} \sqrt{\frac{M_j}{E_j}} \frac{1}{\sqrt{2\omega_j}} \,, \tag{2}$$

where $M_i$ and $E_i$ denote respectively the mass and energy of the baryon , $\omega_i$ the energy of the meson. Here we have defined

$$T_{ij} = \bar{u}_i t_{ij} u_j \,, \tag{3}$$

where $u_i$ is the Dirac spinor and the invariant amplitude $t_{ij}$ is defined by a Bethe-Salpeter equation(see, for example, the derivation in Ref. [23]). In momentum-space, it takes the following form

$$\begin{aligned} t_{ij}(k_i, k_j; P) &= I_{ij}(k_i, k_j; P) \\ &+ i \sum_l \int \frac{d^4k}{(2\pi)^4} I_{il}(k_i, k; P) \frac{1}{(\not{P} - \not{k}) - M_l + i\epsilon} \frac{1}{k^2 - \mu_l^2 + i\epsilon} t_{lj}(k, k_j; P) \,. \end{aligned} \tag{4}$$

where $k$'s are the meson momenta, $P$ is the total momentum of the system, $M_l$ and $\mu_l$ are respectively the masses of the baryon and meson in the intermediate channel $l$. In the rest of the paper, we present formulae in the center of mass system with $P = (\sqrt{s}, \vec{0})$.

The driving term in Eq.(4) is the sum of all two-particle irreducible amplitudes that can be generated from the chosen SU(3) effective chiral lagrangian by using the standard perturbation theory. To lowest order these amplitudes are

$$I_{ij}(k_i, k_j) = \frac{-C_{ij}}{4f^2} (\not{k}_i + \not{k}_j) \,, \tag{5}$$

where $C_{ij}$ are SU(3) coupling constants which can be found in Ref. [19], and $f$ is the pion decay constant.

The scattering equation employed in [19] can now be obtained from Eq.(4) by using a simplification: only the positive energy component of the Fermion propagator is kept. Explicitly, the rule is to use the following substitution in evaluating any loop-integration:

$$\frac{1}{\not{p} - M + i\epsilon} \rightarrow \frac{M}{E(p)} \frac{u_{\vec{p}} \bar{u}_{\vec{p}}}{p^0 - E(p) + i\epsilon} \,. \tag{6}$$

Substituting Eq.(6) into Eq.(4) and integrating out the time component $k^0$, it is straightforward to use the definitions Eqs.(3) and (5) to obtain

$$T_{ij}(k_i, k_j, \sqrt{s}) = V_{ij}(k_i, k_j) + \sum_l \int \frac{d\vec{k}}{(2\pi)^3} \frac{M_l}{E_l(k)} \frac{1}{2\omega_l(k)} \frac{V_{il}(k_i, k) T_{lj}(k, k_j, \sqrt{s})}{\sqrt{s} - E_l(k) - \omega_l(k) + i\epsilon}\,, \tag{7}$$

where

$$V_{ij}(k_i, k_j) = \bar{u}_{-\vec{k}_i} \frac{-C_{ij}}{4f^2} (\not{k}_i + \not{k}_j) u_{-\vec{k}_j}\,. \tag{8}$$

In the near threshold region, one can use the heavy-baryon approximation to reduce the potential into a spin-independent s-wave interaction

$$V_{ij}(k', k) \to \frac{-C_{ij}}{4f^2}(k'_0 + k_0) \tag{9}$$

With the definitions Eqs.(1)-(3) for the S-matrix and the s-wave spin-independent interaction Eq.(9), it is straightforward to obtain the following expression of the total cross section

$$\sigma_{j\to i}(\sqrt{s}) = \frac{1}{4\pi} \frac{1}{s} \frac{k_i}{k_j} M_i M_j \bar{\sum} |T_{ij}(\sqrt{s})|^2 \tag{10}$$

where $\bar{\sum}$ stands for the sum and average over the final and inital spin indices. Eqs.(7), (9), and (10) define precisely the model used in [19].

In Ref. [19] Eq.(7) is solved by evaluating the potential $V_{ij}$ on-shell and hence it factors out of the integral. It was argued that the off-shell effects are in the renormalization of coupling constants and can be neglected in the calculation using physical masses and coupling constants. Eq.(7) then becomes a simple algebraic equation which can be solved easily. A cutoff $q_{max} = 630$ MeV is found to be needed to regularize the integration in Eq.(7) and give a good description of all of the existing $S = -1$ meson-nucleon reactions near threshold. This factorization technique is similar to that first introduced [20] in the study of meson-meson scattering using chiral lagrangians, where the off-shell contributions can be absorbed into the renormalization of masses and coupling constants.

## III. THE $K^-P \to \gamma Y$ AMPLITUDE

We follow here in the $S=-1$ sector similar steps as done in [9,18] in the S=0 sector, but starting from a relativistic formulation of the problem. To study the $K^-p \to \gamma Y$ reaction with $Y = \Lambda, \Sigma^0$, we return to the Bethe-Salpeter Equation (4), setting $i = \gamma Y$ and $j = K^-p$. To the leading order in the electromagnetic coupling constant $e$, we have

$$\begin{aligned} t_{\gamma Y,K^-p}(q,k';P) = A_{\gamma Y,K^-p}(q,k';P) \\ +i\sum_{MB}\int \frac{d^4k}{(2\pi)^4}\,[A_{\gamma Y,MB}(q,k;P)\frac{1}{(\not{P}-\not{k})-M_B+i\epsilon} \\ \times\frac{1}{k^2-\mu_M^2+i\epsilon}t_{MB,K^-p}(k,k';P)\,], \end{aligned} \tag{11}$$

where $MB$ denotes the allowed intermediate meson-baryon states. In general, we need to include $MB = K^-p, \pi^-\Sigma^+, \pi^+\Sigma^-, \bar{K}^0n, \pi^0\Lambda, \pi^0\Sigma^0, \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$. Eq.(11) is illustrated in Fig.1.

The photoproduction mechanism is described by the amplitude $A_{\gamma Y,MB}$ in Eq.(11). Within the SU(3) effective chiral lagrangian including the minimum electromagnetic coupling, it has the form of the standard pseudovector Born term, as illustrated in Fig.2. Explicitly, we have

$$A_{\gamma Y,MB} = A^{(a)}_{\gamma Y,MB} + A^{(b)}_{\gamma Y,MB} + A^{(c)}_{\gamma Y,MB} + A^{(d)}_{\gamma Y,MB} \tag{12}$$

with

$$A^{(a)}_{\gamma Y,MB}(q,k) = i\sum_{Y'}[\frac{-C_{Y',MB}}{2f}]\epsilon_\mu F^\mu_{\gamma YY'}\frac{1}{(\not{q}+\not{p})-M_{Y'}+i\epsilon}\gamma_5\ \not{k}\,, \tag{13}$$

$$A^{(b)}_{\gamma Y,MB}(q,k) = i\sum_{B'}[\frac{-C_{Y,MB'}}{2f}]\gamma_5\ \not{k}\frac{1}{(\not{p}'-\not{q})-M_{B'}+i\epsilon}\epsilon_\mu F^\mu_{\gamma B'B}\,, \tag{14}$$

$$A^{(c)}_{\gamma Y,MB}(q,k) = i\,[\frac{-C_{Y,MB}}{2f}]e_M\frac{\gamma_5(\not{k}-\not{q})\epsilon\cdot(2k-q)}{(k-q)^2-\mu_M^2+i\epsilon}\,, \tag{15}$$

$$A^{(d)}_{\gamma Y,MB}(q,k) = i[\frac{-C_{Y,MB}}{2f}]e_M\,[-\gamma_5\ \not{\epsilon}\,]\,, \tag{16}$$

where $e_M$ is the charge of the meson $M$. The SU(3) coupling constants $C_{B',MB}$ for the $MB \leftrightarrow B'$ transition are defined by [12]

$$C_{B',MB} = X_{B',MB}(D+F) + Z_{B',MB}(D-F) \tag{17}$$

with [12] $D + F = g_A = 1.257$ and $D - F = 0.33$. The values of X's and Z's needed for our calculations can be easily evaluated using the chiral lagrangians of Ref. [12,14] and will be given later. The photon-baryon-baryon vertices are defined by

$$\epsilon_\mu F^\mu_{\gamma BB'} = e_B \delta_{BB'} \not\epsilon - \frac{\kappa_{BB'}}{4M_p}(\not\epsilon \not q - \not q \not\epsilon)\,, \tag{18}$$

where $e_B$ is the charge of the baryon $B$ and $\kappa_{BB'}$ is the anamalous magnetic transition moment. The normalizations were chosen such that $\kappa_{\gamma pp} = \kappa_{proton} = 1.79$ for the proton.

We now apply the rule of Eq.(6) to evaluate the integration in Eq.(11). The derivation is straightforward. Here we only note that the integration over the meson-exchange term (Fig.2c) can have two meson pole contributions, and replacing the baryon propagators in Eqs.(13)-(14) by the projected propagator of Eq.(6) makes our photoproduction matrix elements different from the usual Born terms used in Refs. [8,9]. In addition, we neglect terms of order of $(k/M_B)^2$ or $k\omega/M_B^2$ and higher, typical of the heavy baryon approximation. These approximations are required for consistency with the construction of the coupled-channel potential, Eq.(9), within the approach of [19]. Accordingly, the strong amplitude $t_{MB,K^-p}$ in Eq.(11) is evaluated with the on-shell momenta and factors out of the integration in the derivation. As we show in the appendix, this factorization comes from the fact that the off-shell corrections can be absorbed in the charge renormalization. This line of argumentation is based on the work [24] for the $\gamma\gamma \to$meson-meson reaction, where it was first used.

With the above steps and the definition of the transition amplitude given in Eq.(3), we arrive at the following expression in the center of mass frame $P = (\sqrt{s}, \vec{0})$

$$T_{\gamma Y,K^-p}(q,k') = Q_{\gamma Y,K^-p}(q,k') + [QGT]_{\gamma Y,K^-p}(q,k') + \Delta_{\gamma Y,K^-p}(q,k')\,. \tag{19}$$

The second term of the above equation has a familiar form for describing the initial state interactions

$$[QGT]_{\gamma Y,K^-p}(q,k') = \sum_{MB}\{[\int \frac{d\vec{k}}{(2\pi)^3}\frac{M_B}{E_B(k)}\frac{1}{2\omega_M(k)}\frac{Q_{\gamma Y,MB}(q,k)}{\sqrt{s}-E_B(k)-\omega_M(k)+i\epsilon}] \times T_{MB,K^-p}(k_{MB},k',\sqrt{s})\}\,, \tag{20}$$

where $k_{MB}$ is the on-shell momentum for the channel $MB$. The third term in Eq.(19) is due to the second pole of meson-exchange term Fig.2c. It has the following form

$$\Delta_{\gamma Y,K^-p}(q,k') = i\sum_{MB}\{(\int \frac{d\vec{k}}{(2\pi)^3}\frac{M_B}{E_B(k)}[\frac{1}{2\omega_M(\vec{k}-\vec{q})}]\frac{1}{\sqrt{s}-E_B(k)-q^0-\omega(\vec{k}-\vec{q})+i\epsilon}$$
$$\times[\frac{-1}{2f}C_{Y,MB}]e_M\frac{-2[\vec{k}\cdot\vec{\epsilon}][\vec{\sigma}\cdot(\vec{k}-\vec{q})]}{(q^0+\omega_M(\vec{q}-\vec{k}))^2-\omega_M^2(k)})$$
$$\times T_{MB,K^-p}(k_{MB},k)\}. \qquad (21)$$

Keeping only the s-wave meson-baryon states and employing the heavy-baryon approximation described above, the photoproduction amplitude in Eqs.(19) and (20) takes the following form

$$Q_{\gamma Y,MB}(q,k) = i[\vec{\sigma}\cdot\vec{\epsilon}]F_{\gamma Y,MB}(k,q), \qquad (22)$$

where

$$F_{\gamma Y,MB}(k,q) = -e_M[\frac{-1}{2f}C_{Y,MB}][1-\frac{\omega_M(k)}{2q}+\frac{\mu_M^2}{4qk}ln\frac{\omega_M(k)+k}{\omega_M(k)-k}]. \qquad (23)$$

Note that the above expression is only from the meson-exchange term (Fig.2c) and the contact term (Fig.2d). In the heavy-baryon approximation employed here, one can show with some derivations that the baryon pole term(Fig.2a) contributes only to the meson-baryon p-wave states, while some s-wave contributions from the baryon-exchange term(Fig.2b) also vanish at threshold. This simplicity is of course due to our use of the baryon propagator shown in Eq.(6) to evaluate the matrix elements of Eqs.(13) and (14), and the s-wave nature of the meson-baryon channels within the approach followed in [19]. Consequently, the total amplitude can be calculated by using Eqs.(19)-(22). This also makes our predictions unambiguous since they do not depend on the less determined anomalous magnetic constants $\kappa_{B,B'}$ with $B, B'$ = hyperons in Eq.(18). Furthermore, the allowed intermediate states will only be the charged particle channels $MB = K^-p, \pi^+\Sigma^-, \pi^-\Sigma^+$ and $K^+\Xi$. The coupling constants $C_{B',MB}$ needed for our calculation are defined in Eq.(17) with their coefficients X's and Z's listed in Table I.

## IV. RESULTS AND DISCUSSIONS

The calculations involve two parameters: the cutoff parameter $q_{max} = 630$ MeV for regularizing the integrations in Eqs.(20) and (21) and the overall SU(3) coupling strength $f = 1.15\ F_\pi (F_\pi = 93\ \text{MeV})$, approximate average between $f_\pi$ and $f_K$. These two parameters were determined in Ref. [19] and hence the calculations based on Eqs.(19)-(22) do not have any adjustable parameters.

We have calculated the branching ratios defined by

$$B_{K^-p\to\gamma Y} = \frac{\sigma_{K^-p\to\gamma Y}(\sqrt{s}_{th})}{\sigma_{K^-p\to all}(\sqrt{s}_{th})}\,, \tag{24}$$

where $Y = \Lambda, \Sigma^0$ and the total cross sections are defined in Eq.(10). In the calculation of the denominator of Eq.(24) all $S = -1$ meson-baryon channels within the approach of [19] are included. The results presented below are obtained by evaluating the above expression at $\sqrt{s}_{th} \to \mu_{K^-} + M_p$.

Our results calculated with and without including the coupled-channel effects are listed in Table II and compared with the data [25]. We see that with no initial meson-baryon interactions (first row in Table II), the $\gamma\Sigma^0$ production is very weak and the predicted ratio between two production rates is an order of magnitude larger than the data. This is in agreement with the findings of Siegel and Saghai [9]. When the strong coupled-channel effects are included our predicted ratio(second row of Table II) is close to the experimental value. The predicted branching ratio for the $\gamma\Lambda$ production is about 50 % larger than the experimental value, but it is within the experimental uncertainty for the $\gamma\Sigma^0$ production.

To understand the dynamical origins of our results listed in Table II, we examine the coupled-channel effects in some detail. For this purpose it is sufficient to just consider the first two terms of Eq.(19). They can be cast into a form employed by Siegel and Saghai [9], if we factor out the photoproduction amplitude $Q_{\gamma Y,MB}$ and evaluate it at the on-shell momenta. With this approximation, we have( obvious variables are omitted here)

$$T_{\gamma Y,K^-p} \sim \sum_{MB} F_{\gamma Y,MB}[1 + GT]_{MB,K^-p}\,. \tag{25}$$

All quantites in the above equation are evaluated at on-shell momenta. Obviously the term $[1+GT]_{MB,K^-p}$ measures the strength of the $K^-p \rightarrow MB$ transition. In Table III, we list these matrix elements as well as the values of the on-shell photoproduction amplitudes $F_{\gamma Y,MB}$. We see that the total amplitude, defined by Eq.(25) in this estimate, involves a nontrival interplay between the strong transitions and electromagnetic transitions. In particular, the large enhancement of the $\gamma\Sigma^0$ production by the coupled-channel effects can be understood from the second column of Table III. We see that $F_{\gamma\Sigma^0,\pi^+\Sigma^-}$ and $F_{\gamma\Sigma^0,\pi^-\Sigma^+}$ are a factor of about 3 larger than $F_{\gamma\Sigma^0,K^-p}$, while their strong transition strengths in column 3 are comparable in magnitude. Consequently, the branching ratio for $\gamma\Sigma^0$ is greatly enhanced when the $\pi$ channels are included. This is verified in our exact calculations based on Eqs.(19)-(20), as can be seen in the second row of Table IV. We see that the branching ratio for $\gamma\Sigma^0$ production is increased from 0.14 to 1.28 when the $\pi^+$ and $\pi^-$ channels are included. From column 1 of Table III, we also expect that the effect of $\pi$ channels on the $\gamma\Lambda$ production is much less, mainly because the photoproduction amplitudes for the first three channels are comparable and the contributions from $\pi^+$ and $\pi^-$ channels have opposite signs. As shown in the first row of Table IV, the coupling with the $\pi$ channels can lower the production rate only from 2.47 to 1.56. Neverthelesss, it is instrumental in bringing our prediction closer to the experimental value $0.86 \pm 0.16$. We further notice that the reduction from 2.47 to 1.33 when the $\pi^+\Sigma^-$ channel is included (first row of Table IV) is largely due to the cancellation caused by the opposite signs of the imaginary parts of $[1+GT]$, as seen in the third column of Table III. Note that the relative signs between the contributions from different channels are determined not only by the meson charges($e_M$) but also by the chiral SU(3) parameters listed in Table I. Thus, the coupled-channel effects are crucial in testing the chiral SU(3) symmetry. The effect due to the $K^+\Xi^-$ channel is very weak because their strong transition is an order of magnitude smaller than others, as can be seen in the third column of Table III.

We now turn to illustrating several important features of our approach. The exact treatment of meson propagators in Fig.2c leads to a contribution($\Delta_{\gamma Y,K^-p}$ in Eq.(19)) from a second pole. By comparing the first two rows in Table V, we see that this second pole term can change the $\gamma\Sigma^0$ branching ratio by about 40%, but much less for $\gamma\Lambda$. Consequently, the the predicted ratio becomes closer to the experimental value.

An important finding of Ref. [19] was that the coupling with the $\eta$ channels is essential in obtaining a good agreement with all of the existing $S = -1$ meson-baryon reaction data when using only the lowest order lagrangian as input. The influence of this coupling on our predictions is also significant. This can be seen by comparing the second and third rows in Table V. We see that the predicted branching ratio for $\gamma\Lambda$ production is increased by about 60 % if the $\eta$ channels are omitted in the calculation of the strong amplitudes $T_{MB,K^-p}$ appearing in Eq.(19). It is clear that including the $\eta$ channels is also crucial in using this reaction to test the chiral SU(3) symmetry. We note that the $\eta$ channels were omitted in the model of Ref. [9]. At the same time the couplings in that work had to be substantially changed with respect to their SU(3) values in order to obtain a good fit to the data. In retrospective we can say that what the explicit breaking of SU(3) symmetry does in reality is to restore it after it has been broken by the omission of the $\eta$ channels.

Finally, we note that the strong meson-baryon-baryon vertex in each of the photoproduction amplitudes should in principle have a form factor because hadrons are composite particles. To see how our results will be changed when this is taken into account, we perform a calculation including a monopole form factor with typical cut off, $\Lambda_\pi$, values of 1.2 and 1 GeV. The introduction of this cut off reduces the ratio $R$ to values in better agreement with the data. Changes of $\Lambda_\pi$ from 1 to 1.2 GeV introduce only corrections of the order of 10%. The results obtained with $\Lambda_\pi = 1$ GeV, a value which is commonly accepted, roughly agree with the data within experimental errors, which are of order of 20%. If one compares with the central values of the experimental branching ratios, our results are on the upper edge of the $B_{K^-p\to\gamma\Lambda}$ ratio while those for $B_{K^-p\to\gamma\Sigma^0}$ are on the lower edge. Looked at it in the context that the coupled channels and unitarization have reduced the ratio $R$ by a factor 14, differences of the order of 10–20 % are not so significative. Yet, the fact that a better agreement with the central values of the data is obtained in [9], at the price of fitting parameters, indicates that there is probably still room in the model used here for moderate breakings of SU(3) beyond those implemented by the different masses of the particles.

In conclusion, we have extended the approach of [19] to make predictions for the branching ratios for the $K^- p \to \gamma\Lambda, \gamma\Sigma^0$ reactions near the threshold. All coupling constants are consistent with the chiral SU(3) symmetry. With only two parameters, which were fixed in the study of $S = -1$ meson-baryon reactions, our predictions are close to the data, in particular the ratio between two branching ratios. In our approach, neither the meson-baryon nor the photoproduction mechanisms involve the explicit consideration of excited hyperon states since the $\Lambda(1405)$ resonance, which plays a key role in these reactions, is generated dynamically. Our results therefore strengthen the interpretation of the $\Lambda(1405)$ as a quasibound meson-baryon system with $S = -1$, as already supported by the study of the strong interactions in [15,19]. This does not exclude a seed of a more complicated intrinsic quark substructure, but the strong coupling to the meson baryon channels imposed by the unitarization reinforces the meson baryon component and allows this resonance to approximately qualify as a meson baryon quasibound state much as it happens for the resonances in the scalar meson-meson sector [26,27]

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## APPENDIX A: RENORMALIZABILITY CHARACTER OF THE OFF SHELL PART OF THE STRONG AMPLITUDE.

We start from Eq. (11) and take just one loop in the Bethe Salpeter equation. This means that we substitute $t_{MB,K^-p}(k,k',P)$ by the $I_{ij}(k,k',P)$ function of Eq. (5). Let us take the contact term for the electromagnetic amplitude $A_{\gamma Y,MB}$ (the procedure and the conclusion for the other terms are the same). By taking the nonrelativistic reduction of the contact term and the positive energy part of the nucleon propagator, Eq. (6), as done in the evaluation of the meson nucleon strong amplitude in [19], we obtain from the loop a contribution proportional to

$$i\int \frac{d^4k}{(2\pi)^4}\frac{M}{E_B(k)}\frac{1}{p^0+k'^0-k^0-E_B(\vec{k})+i\epsilon}(k'^0+k^0)\frac{1}{k^2-\mu_M^2+i\epsilon}e\vec{\sigma}\vec{\epsilon} \tag{A1}$$

We now separate the strong amplitude, represented by the factor $k'^0+k^0$ into its on shell and off shell parts as

$$k'^0+k^0 = 2k'^0 + (k^0-k'^0) \tag{A2}$$

The part of the integral coming from the term $2k'^0$ in the former equation corresponds to factorizing the strong amplitude with its on shell value, which is the procedure that we have followed. The contribution from the off shell part will be given by

$$i\int \frac{d^4k}{(2\pi)^4}\frac{M}{E_B(k)}\frac{1}{p^0+k'^0-k^0-E_B(\vec{k})+i\epsilon}(k^0-k'^0)\frac{1}{k^2-\mu_M^2+i\epsilon}e\vec{\sigma}\vec{\epsilon} \tag{A3}$$

Here we follow the same steps that led to include the off shell contribution in the strong amplitude into a renormalization of the $f$ coupling in [19]. We use typical approximations of the heavy baryon formalism and set $M/E_B(\vec{k})=1$, $p^0-E_B(\vec{k})=0$. In this case the off shell term $(k^0-k'^0)$ and the nucleon propagator $(k'^0-k^0)^{-1}$ cancel and we get a contribution

$$\begin{aligned} &-i\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu_M^2+i\epsilon}e\vec{\sigma}\vec{\epsilon} \\ &= -\int \frac{d^3k}{(2\pi)^3}\frac{1}{2w(\vec{k})}e\vec{\sigma}\vec{\epsilon} \sim q_{max}^2 e\vec{\sigma}\vec{\epsilon} \end{aligned} \tag{A4}$$

Hence, we obtain a contribution proportional to the cut off momentum squared, independent of energy and with the same structure as the contact term in the Born amplitude, which goes into renormalizing the contact term and is taken into account when using this term with the physical electromagnetic coupling $e$.

The generalization to other terms of the electromagnetic amplitude and higher order loops proceeds analogously with the conclusion that only the on shell part of the strong amplitude must be used.

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# TABLES

TABLE I. SU(3) coupling constants defined in Eq.(17)

| $X_{B'MB}$ | $MB =$ | $K^-p$ | $\pi^+\Sigma^-$ | $\pi^-\Sigma^+$ | $K^+\Xi^-$ |
|---|---|---|---|---|---|
| $B' = \Lambda$ | – | $\frac{-2}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |
| $B' = \Sigma^0$ | – | 0 | 1 | $-1$ | 1 |

| $Z_{B'MB}$ | $MB =$ | $K^-p$ | $\pi^+\Sigma^-$ | $\pi^-\Sigma^+$ | $K^+\Xi^-$ |
|---|---|---|---|---|---|
| $B' = \Lambda$ | – | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{-2}{\sqrt{3}}$ |
| $B' = \Sigma^0$ | – | 1 | $-1$ | 1 | 0 |

TABLE II. Comparisions of the predicted $K^-p \to \gamma\Lambda, \gamma\Sigma^0$ branching ratios defined in Eq.(24)(in unit of $10^{-3}$) with the data[20]. The amplitudes are defined in Eqs.(19)-(22).

| *Amplitude* | $B_{K^-p\to\gamma\Lambda}$ | $B_{K^-p\to\gamma\Sigma^0}$ | $R = B_{K^-p\to\gamma\Lambda}/B_{K^-p\to\gamma\Sigma^0}$ |
|---|---|---|---|
| $Q$ | 1.12 | 0.073 | 16.4 |
| $Q + GQT + \Delta$ | 1.58 | 1.33 | 1.19 |
| Data [25] | $0.86 \pm 0.16$ | $1.44 \pm 0.31$ | 0.4 - 0.9 |

TABLE III. The quantities defined in Eq.(25) are compared. The coefficient $F_{\gamma Y,MB}$ is defined in Eq.(23) and is in unit of $10^{-2}$/MeV.

| $MB$ | $F_{\gamma\Lambda,MB}$ | $F_{\gamma\Sigma^0,MB}$ | $[1 + GT]_{MB,K^-p}$ |
|---|---|---|---|
| $K^-p$ | 0.588 | -0.159 | -0.68+i1.63 |
| $\pi^+\Sigma^-$ | 0.431 | 0.430 | -0.66-i1.01 |
| $\pi^-\Sigma^+$ | -0.431 | 0.430 | -0.61-i0.40 |
| $K^+\Xi^-$ | 0.157 | 0.589 | -0.087-i0.002 |

TABLE IV. Branching ratios(in unit of $10^{-3}$) predicted from the calculations with different numbers of coupled-channels included. Here we define $Ratio = B_{K^-p\to\gamma\Lambda}/B_{K^-p\to\gamma\Sigma^0}$

| | Channels included | | | | |
|---|---|---|---|---|---|
| | 0 | $K^-p$ | $K^-p+\pi^+\Sigma^-$ | $K^-p+\pi^+\Sigma^-+\pi^-\Sigma^+$ | $K^-p+\pi^+\Sigma^-+\pi^-\Sigma^++K^+\Sigma^-$ |
| $B_{K^-p\to\gamma\Lambda}$ | 1.12 | 2.47 | 1.33 | 1.56 | 1.58 |
| $B_{K^-p\to\gamma\Sigma^0}$ | 0.073 | 0.14 | 0.74 | 1.28 | 1.33 |
| *Ratio* | 16.4 | 17.5 | 1.81 | 1.22 | 1.19 |

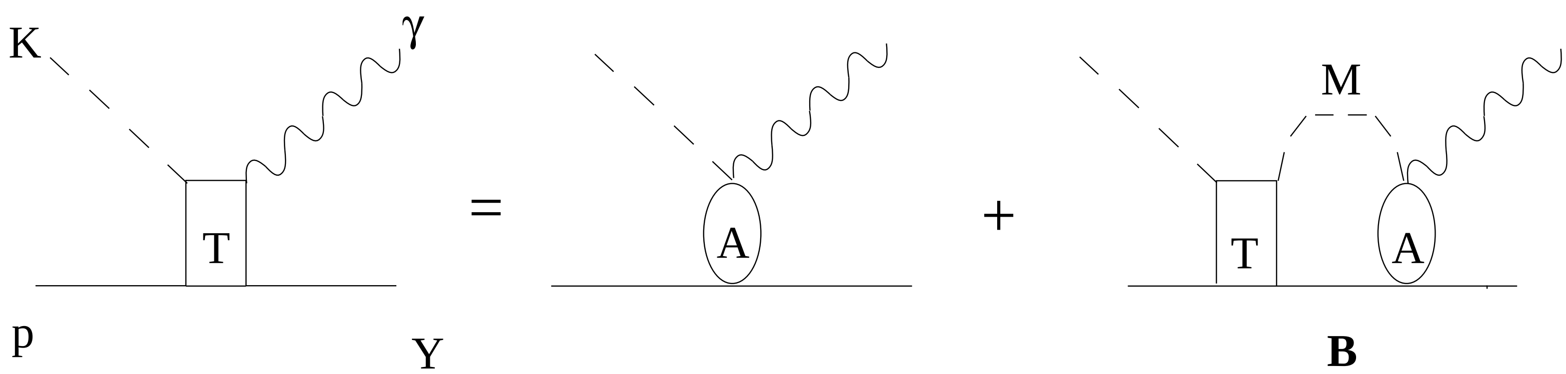
TABLE V. Same as Table II. The subindex $no-\eta$ indicates that the strong amplitude $T_{K^-p,MB}$ is calculated with the $\eta$ channels turned off. The subindex $with-\Lambda_\pi$ indicates that a monopole form factor with cutoff $\Lambda_\pi = 1$ GeV (1.2 GeV in brackets) is included in the meson-baryon-baryon vertices of photoproduction amplitudes.

| *Amplitude* | $B_{K^-p\to\gamma\Lambda}$ | $B_{K^-p\to\gamma\Sigma^0}$ | $R = B_{K^-p\to\gamma\Lambda}/B_{K^-p\to\gamma\Sigma^0}$ |
|---|---|---|---|
| $Q+QGT$ | 1.31 | 0.95 | 1.38 |
| $Q+QGT+\Delta$ | 1.58 | 1.33 | 1.19 |
| $[Q+QGT+\Delta]_{no-\eta}$ | 2.47 | 1.27 | 1.94 |
| $[Q+QGT+\Delta]_{with-\Lambda_\pi}$ | 1.10 (1.22) | 1.05 (1.13) | 1.04 (1.08) |
| Data [25] | $0.86\pm0.16$ | $1.44\pm0.31$ | $0.4-0.9$ |

## FIGURES

FIG. 1. Graphical representation of Eq.(4)

FIG. 2. Photoproduction mechanisms of Eq.(13)-(16)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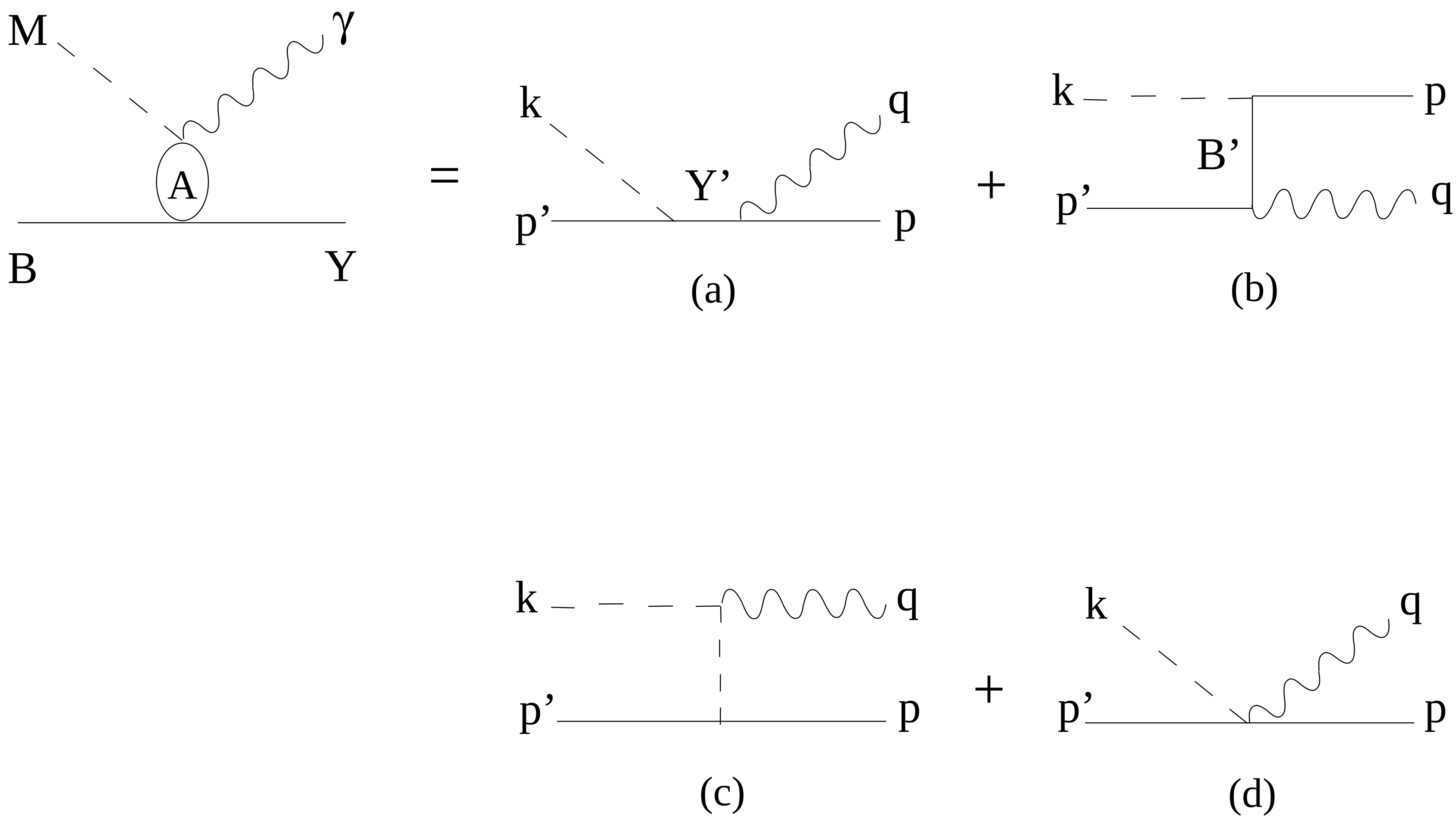
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