Unitarized chiral perturbation theory of hadrons

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An exposition is made of recent developments using techniques of unitary chiral perturbation theory, $U\chi PT$, which allows one to extend predictions using chiral Lagrangians to higher energies than ordinary chiral perturbation theory, including the region of low lying mesonic and baryonic resonances, some of which are dynamically generated in the approach. Results for meson meson scattering, pion and kaon form factors and meson baryon scattering are shown. Applications are done for nuclear problems showing the results for the kaon and eta selfenergies, phi renormalization in a nuclear medium and σ renormalization in the medium, comparing results with recent experiments.

1. Introduction

Chiral perturbation theory, incorporating the basic symmetries of the original QCD Lagrangian into an effective Lagrangian which uses ordinary mesons and baryon fields as effective degrees of freedom, has had a tremendous impact in hadronic physics at low and intermediate energies. The theory organizes the Lagrangians in powers of the momentum of the hadrons and performs ordinary field theoretical perturbation theory where the higher order Lagrangians provide counterterms which regularize the theory [1, 2]. With these Lagrangians one can make predictions for meson meson interaction and meson baryon interaction at lowest order, which reproduce the results obtained with current algebra techniques, yet in a more elegant, systematic and technically simpler way. The novelties of χPT stem from the perturbative calculations which ones performs with these Lagrangians. The infinities appearing from the loops in second order are canceled by the Lagrangians of next order which leave some finite counterterms. The obvious problem with χPT is its limited range of convergence. If one studies for instance s-wave meson meson scattering the absolute limit for convergence is in the first pole, the one of the σ meson around 500 MeV, but the lack of convergence shows already below this energy. More problematic is the case of the meson baryon interaction. For instance, in the study of the s-wave meson baryon scattering in the strangeness $S = -1$ sector, the low energy K^-p scattering amplitude is already dominated by the $\Lambda(1405)$ resonance below threshold. Ordinary χPT simply cannot be applied there. The region of resonances is inaccessible with χPT, thus putting strong limitations to the range of applicability. Yet, the question arises whether by using some suitable resummation technique one could extend this range of applicability, using still the content of the chiral Lagrangians. In the last years there has been much progress in this direction, where the consideration of the

constraints of unitarity have played a key role in the answer to the problem. In the next section we sketch the basic ideas about these developments.

2. Unitarized chiral perturbation theory

The first steps to combine chiral perturbation theory and unitarity were done in [3]. There the Lippmann Schwinger equation was used with a potential obtained from the lowest and higher order chiral Lagrangians. The approach was used to investigate πN scattering and $\bar{K}N$ scattering around the regions of the $N^*(1525)$ and the $\Lambda(1405)$ resonances, respectively, and a good agreement with data was obtained using only a few free low energies parameters. Similarly, it was also found in [4] that the meson meson scattering up to 1.2 GeV was well reproduced for s-waves using only the Bethe Salpeter equation, with the lowest order amplitudes as kernel and a cut off of the order of 1 GeV to regularize the loops. The s-waves for meson baryon scattering in the $S = -1$ sector were also studied in [5] by means of the Bethe Salpeter equation, the lowest order chiral Lagrangian and a cut off, reproducing fairly well the low energy scattering properties of $\bar{K}N$ scattering and generating the $\Lambda(1405)$ resonance dynamically. A further clarification of the issue and its extension to also p-waves in the meson meson scattering was done in [6, 7], using the Inverse Amplitude Method and the N/D method, respectively. In both cases one could obtain a good reproduction of the data and all the resonances up to 1.2 GeV, the $\sigma(500)$, the $f_0(980)$, the $a_0(980)$, the $\kappa(900)$, the ρ and the K^*.

One can find a systematic and easily comprehensible derivation of the ideas of the N/D method applied for the first time to the meson baryon system in [8], which we reproduce here below. One defines the transition T–matrix as $T_{i,j}$ between the coupled channels which couple to certain quantum numbers. For instance in the case of $\bar{K}N$ scattering studied in [8] the channels with zero charge are K^-p, $\bar{K}^0n$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\pi^0\Lambda$, $\eta\Lambda$, $\eta\Sigma^0$, $K^+\Xi^-$, $K^0\Xi^0$. Unitarity in coupled channels is written as

$$ImT_{i,j} = T_{i,l}\rho_l T_{l,j}^* \tag{1}$$

where $\rho_i \equiv q_i/(8\pi W)$, with q_i the modulus of the c.m. three–momentum, and the subscripts i and j refer to the physical channels. This equation is most efficiently written in terms of the inverse amplitude as

$$\text{Im } T^{-1}(W)_{ij} = -\rho(W)_i\delta_{ij} , \tag{2}$$

The unitarity relation in eq. (2) gives rise to a cut in the T–matrix of partial wave amplitudes, which is usually called the unitarity or right–hand cut. Hence one can write down a dispersion relation for $T^{-1}(W)$

$$T^{-1}(W)_{ij} = -\delta_{ij}\left\{\tilde{a}_i(s_0) + \frac{s - s_0}{\pi}\int_{s_i}^{\infty} ds'\frac{\rho(s')_i}{(s' - s)(s' - s_0)}\right\} + \mathcal{T}^{-1}(W)_{ij} , \tag{3}$$

where s_i is the value of the s variable at the threshold of channel i and $\mathcal{T}^{-1}(W)_{ij}$ indicates other contributions coming from local and pole terms, as well as crossed channel dynamics but *without* right–hand cut. These extra terms are taken directly from χPT after requiring the *matching* of the general result to the χPT expressions. Notice also that

$$g(s)_i = \tilde{a}_i(s_0) + \frac{s - s_0}{\pi}\int_{s_i}^{\infty} ds'\frac{\rho(s')_i}{(s' - s)(s' - s_0)} \tag{4}$$

is the familiar scalar loop integral

$$
\begin{aligned}
g(s)_i &= \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - M_i^2 + i\epsilon)((P-q)^2 - m_i^2 + i\epsilon)} \\
&= \frac{1}{16\pi^2} \left\{ a_i(\mu) + \log \frac{m_i^2}{\mu^2} + \frac{M_i^2 - m_i^2 + s}{2s} \log \frac{M_i^2}{m_i^2} + \frac{q_i}{\sqrt{s}} \log \frac{m_i^2 + M_i^2 - s - 2\sqrt{s}q_i}{m_i^2 + M_i^2 - s + 2\sqrt{s}q_i} \right\},
\end{aligned}
\tag{5}
$$

where M_i and m_i are, respectively, the meson and baryon masses in the state i. In order to calculate $g(s)_i$ one uses the physical masses both for mesons and baryons and, hence, eq.(2) holds.

One can further simplify the notation by employing a matrix formalism. Introducing the matrices $g(s) = \mathrm{diag}\,(g(s)_i)$, T and $\mathcal{T}$, the latter defined in terms of the matrix elements T_{ij} and $\mathcal{T}_{ij}$, the T-matrix can be written as:

$$
T(W) = [I - \mathcal{T}(W) \cdot g(s)]^{-1} \cdot \mathcal{T}(W) .
\tag{6}
$$

which can be recast in a more familiar form as

$$
T(W) = \mathcal{T}(W) + \mathcal{T}(W)g(s)T(W)
\tag{7}
$$

Now imagine one is taking the lowest order chiral amplitude for the kernel as done in [8]. Then the former equation is nothing but the Bethe Salpeter equation with the kernel taken from the lowest order Lagrangian and factorized on shell, the same approach followed in [5] where different arguments were used to justify the on shell factorization of the kernel. Furthermore in [8] a simple relationship is found between the cut off used in [5] and the subtraction constants used in [8]

$$
a_i(\mu) = -2\log\left(1 + \sqrt{1 + \frac{m_i^2}{\mu^2}}\right) + \dots ,
\tag{8}
$$

where μ plays the role of the cut off. Then taking values of μ around 650 MeV to 1.GeV one would find subtraction constants of the order of –2, which we call of natural size.

Eq. (7) also serves to clarify the issue of the dynamical generation of the resonances. According to the findings of [9], the second order Lagrangian for mesons represents the low energy limit of the exchange of vector mesons, essentially. In that sense for the p-waves in [7] the function $\mathcal{T}(W)$ contains the lowest order amplitude plus the exchange of a bare vector meson, and upon unitarization one can obtain a good reproduction of the data with the right properties of mass and width of the ρ and the K^* [7]. We call these resonances genuine, since one has to put them explicitly in the formalism in order to obtain them, although the unitarization provides the adequate width and some extra dressing of the resonance. On the other hand, in the scalar sector the introduction of genuine resonance exchange in the kernel was not needed and the unitarization of the lowest order amplitudes gave rise to the scalar resonances [7]. In the IAM one is however making explicit use of the second order chiral Lagrangians and expands ReT^{-1} in powers of the momenta. In this case the differentiation between genuine and dynamically generated resonances is not so clear. The IAM method has also been recently used to study meson baryon interactions in [10]. A variant of the Bethe Salpeter equation is done in [11, 12],

where rather than using the fact that one can factorize the kernel on shell introducing explicitly subtraction constants in $g(s)$, one assumes a certain general form for the off shell extrapolation which involves some free parameters and solves the Bethe Salpeter equation selfconsistently.

The formalism exposed here can be easily applied to the calculation of form factors, matching the results of the unitarized form factor to the results of one loop chiral perturbation theory [13]. This procedure, together with the requirement that the electromagnetic pion form factor has a peak at the ρ position, and the kaon form factor at the ϕ and ω masses, determines uniquely the form factors in this approach, with good agreement with the data up to 1.2 GeV [14].

The procedure of [8] has been recently used to find out more dynamical resonances [15]. The simple extrapolation of the approach to higher energies provides a resonance in $S = -1$ and isospin, $I = 0$, corresponding to the $\Lambda(1670)$ resonance and another resonance, corresponding to $\Sigma(1620)$, which is not visible in the amplitudes but is found as a pole in the second Riemann sheet of the complex plane, although with a large width. The scheme has also been applied to the study of resonances in the $S = -2$ sector in [16], where a Ξ resonance around 1620 MeV is found. When it comes to compare this resonance with empirical ones one is left at the beginning with the uncertainty to associate this resonance to the two, $I = 1/2$, Ξ resonances of the PDG, the $\Xi(1620)$ and the $\Xi(1690)$. The study of the residues at the poles in the second Riemann sheet of the complex plane for the different transition amplitudes provides the couplings of the resonance to the different channels. What one observes is that the couplings of the found resonance are such that the partial decay widths are totally incompatible with those measured for the $\Xi(1690)$ resonance, with discrepancies of the order of a factor 20 to 30 in all channels. This rules out completely the identification of the resonance found with the $\Xi(1690)$ resonance, leaving room for only the $\Xi(1620)$. With this identification, the $\Xi(1620)$ resonance has spin and parity $1/2^-$, two magnitudes which were still not determined in the PDG.

An interesting related work has been recently done by the Osaka-Spanish collaboration [17], showing that one is actually obtaining two octets and a singlet of dynamically generated resonances, as one might expect from the SU(3) decomposition

$$8 \times 8 = 1 + 8^s + 8^a + 10 + \bar{10} + 27 \qquad (9)$$

Indeed, if one takes the SU(3) symmetric, case when all masses of mesons on one side and the masses of the baryons on the other are made equal, one obtains a singlet state and an octet of states with zero width (since they are below the threshold for the average SU(3) masses taken). When the symmetry is gradually broken to account for the different masses, one can see that the poles move in the complex plane and two octets appear. One finds two states with $I = 0$ which move apart and also two states with $I = 1$ which also move apart when the symmetry is broken. One of the $I = 1$ states becomes the $\Sigma(1620)$ and the other one moves to lower masses close to the $\bar{K}N$ threshold and is too wide to have a clear repercussion in the amplitudes, but the $I = 0$ states are narrow enough to be clearly visible in the amplitudes. In particular one sees that one of the $I = 0$ states moves to the $\Lambda(1670)$ position while the other one moves to lower energies and mixes with the singlet state to give two resonances close to the $\Lambda(1405)$ position. Hence what has been so far identified as the $\Lambda(1405)$ resonance actually corresponds to two poles

which are close by, but which have different widths and different couplings to the states. It might be possible to think of new reactions which give different weight to these two resonances. One of these reactions could very well be the $K^- p \to \Lambda(1405)\gamma$ reaction, which was studied theoretically in [18], where a narrower $\Lambda(1405)$ than the standard one was produced.

In the strangeness $S = 0$ sector the approach also generates the $N^*(1535)$ resonance. This was also found in [19], but it has been reviewed recently with the formalism of the dispersion relation with subtraction constants in [20].

3. Application to nuclear problems

One of the important applications to nuclear physics problems has been the determination of the $\bar{K}$ selfenergy in a nuclear medium. Pauli blocking corrections in the intermediate nucleon states were taken into account in [21], which led to a shift of the resonance and a large attractive selfenergy of the kaon. Subsequently a selfconsistent calculation was done in [22], where the obtained $\bar{K}$ selfenergy was used in the calculation, as a consequence of which the resonance moves back to the original position and a weaker attraction for the kaon is obtained. Further developments are done in [23] where, in addition, the intermediate pions and baryons are also dressed, leading to a wider kaon spectral function and still a moderate attraction of the order of 40 MeV at normal nuclear matter density. This results seemed contradictory with earlier expectations and fits to kaonic atoms, which demanded around 200 MeV attraction. Yet, as shown in [24, 25, 26], one can get a reasonable description of the K^- atoms with just this moderately attractive potential, which would make kaon condensation in neutron stars unlikely.

Once one has the $\bar{K}$ potential and the one for K (quite reliably given by the $t\rho$ approximation), one can then evaluate the ϕ selfenergy in the nuclear medium by renormalizing the two kaons which come from the ϕ decay into $K\bar{K}$. This was done in [27], subsequently in [28] using the improved $\bar{K}$ selfenergy of [23], and more recently in [29] where also the real part was calculated. The results from these calculations show a very small shift of the mass and a substantial increase of the width which ranges from 5 to 10 times the free ϕ width. The calculations are done for a ϕ at rest but one expects a similar renormalization for a ϕ moving inside the nucleus. Although an experiment is devised in [30] to measure the changes of a slow ϕ in nuclear ϕ photoproduction, we might have sooner results for fast moving ϕ in the experiment of [31] at Spring8/Osaka.

The method of [23] has also been used recently to determine the η selfenergy in the nuclear medium [32]. One obtains a potential at threshold of the order of (54 -i29) MeV at normal nuclear matter, but it also has a strong energy dependence due to the proximity of the $N^*(1535)$ resonance and its appreciable modification in the nuclear medium. One can solve the Klein Gordon equation with this energy dependent potential and one finds bound states in medium and heavy nuclei [33], with binding energies ranging from 21 MeV to threshold, and half widths of the order 18 MeV, such that the sum of two half widths is bigger than the separation between the levels. This would make the detection of peaks unlikely, although one could measure strength in the bound region which would spread up to the lowest energy plus half the width of this state, hence around 40 MeV in the bound region. The best place to eventually see these states would be in the region of
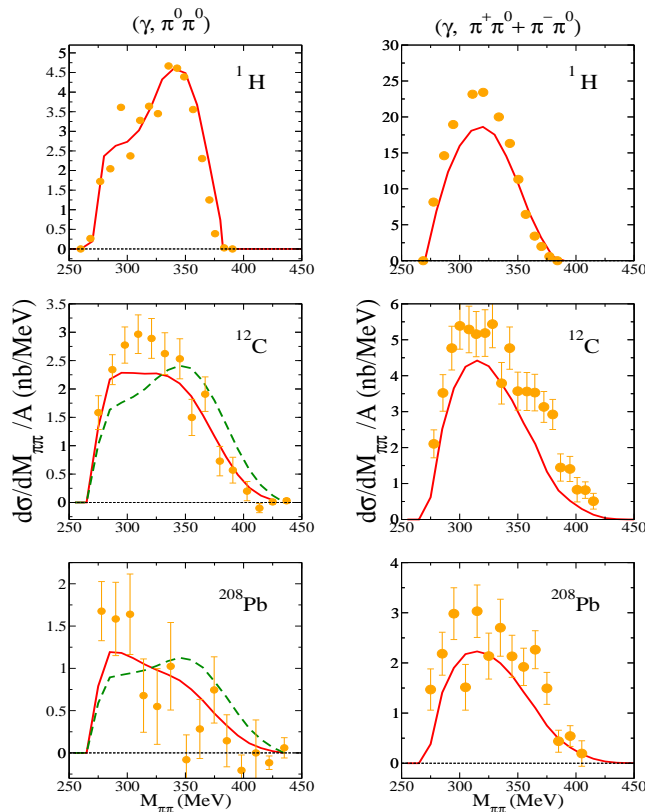
Figure 1. Two pion invariant mass distribution for $2\pi^0$ photoproduction in ^{12}C and ^{208}Pb. Continuous lines: theory with $\pi\pi$ interaction in the medium. Dashed lines: theory with $\pi\pi$ interaction in free space. Experimental points from [42].

^{24}Mg, where only one bound state appears with 13 MeV binding energy and a half width of the order of 16 MeV.

Finally, let me tackle another very recent problem concerning the renormalization of the σ meson in the nucleus. The $\pi\pi$ interaction in a nuclear medium has got much attention in the last 10 years, motivated by the original suggestion of [34], where a peak appeared at nuclear matter below the two pion threshold suggesting the creation of pion Cooper pairs. More refined calculations, dressing the pions in the series of Bethe Salpeter terms done in [35, 36], indicate that the peaks do not appear but much strength is moved to lower energies. These results could be interpreted as the σ mass getting reduced in the nuclear medium, as has been suggested in [37, 38, 39, 40], and this is also the case as has been recently shown in [41] by looking at the σ poles in the medium in the complex plane. It would be most desirable to have an experiment to test that and fortunately the experiment has been done recently at Mainz [42]. It is the photoproduction of $\pi^0\pi^0$ in nuclei at small energies. The invariant mass of the two pions has been measured and one observes an appreciable shift of strength of the invariant mass distribution to low invariant masses as seen in fig.1.

A theoretical calculation [43] was done prior to the experiment, predicting this shift which has been confirmed by the data. Since in the chiral unitary approach the σ is dynamically generated one does not have to introduce it explicitly, it simply comes from the consideration of the final state interaction of the two pions in s-wave and I=0. Consequently with this idea, in [43] one takes the model for $(\gamma, \pi\pi)$ of [44], which produces the two pions at tree level, and then allow the pions to interact in the medium as done in [36]. After this is done and the pion absorption of the final pions from the point of production till they leave the nucleus is considered, following the lines of [45], then the results of fig. 1 are obtained which show a clear shift of the pion invariant mass strength to lower invariant masses from the proton to nuclei. This shift is not present when the final state of the pions is replaced by the free one, and is also absent in the I=1 channel, both in the calculations and the experiment of [42], as one can see in fig. 1. The experiment and theoretical results represent the first solid proof of the σ meson renormalization in the nucleus, which should be corroborated with further calculations and experiments for other reactions. It also stresses the point in favor of the σ as a dynamically generated meson by contrasting consequences of this hypothesis with experiment.

4. Conclusions

In the talk I have made a brief survey of the ideas about the chiral unitary approach to study hadron dynamics at low and intermediate energies. Then several examples of successful application of these ideas have been shown in elementary and nuclear reactions. The approach is powerful and represents a natural extrapolation of χPT at higher energies. Since it allows one to enter the regime of low lying mesonic and baryonic resonances, it opens a broad field of possible applications, many of them already done which can not be reported in this limited time. A review of some of the applications is done in [46]. Applications to higher energies are possible with the likely introduction of extra degrees of freedom or resonances. But the examples shown here clearly indicate that the skillful combination of the dynamics contained in the chiral Lagrangians and the powerful constraints imposed by unitarity provide and ideal tool to face elementary and nuclear reactions at intermediate energies, using a dynamics consistent with the original one of the QCD Lagrangian and mesons and baryons as degrees of freedom, which allow an immediate comparison with experiments measuring directly these particles.

REFERENCES

1. J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.
2. U. G. Meissner, Rept. Prog. Phys. **56** (1993) 903
3. N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A **594** (1995) 325
4. J. A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438 [Erratum-ibid. A **652** (1999) 407]

5. E. Oset and A. Ramos, Nucl. Phys. A **635** (1998) 99
6. J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. **80** (1998) 3452
7. J. A. Oller and E. Oset, Phys. Rev. D **60** (1999) 074023
8. J. A. Oller and U. G. Meissner, Phys. Lett. B **500** (2001) 263
9. G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311.
10. A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D **62** (2000) 017502
11. J. Nieves and E. Ruiz Arriola, Phys. Rev. D **63** (2001) 076001
12. J. Nieves and E. Ruiz Arriola, Phys. Rev. D **64** (2001) 116008
13. J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 517.
14. J. A. Oller, E. Oset and J. E. Palomar, Phys. Rev. D **63** (2001) 114009
15. E. Oset, A. Ramos and C. Bennhold, Phys. Lett. B **527** (2002) 99 [Erratum-ibid. B **530** (2002) 260]
16. A. Ramos, E. Oset and C. Bennhold, arXiv:nucl-th/0204044, Phys. Rev. Lett., in print.
17. D. Jido et al., to be published.
18. J. C. Nacher, E. Oset, H. Toki and A. Ramos, Phys. Lett. B **461** (1999) 299
19. N. Kaiser, P. B. Siegel and W. Weise, Phys. Lett. B **362** (1995) 23
20. T. Inoue, E. Oset and M. J. Vicente Vacas, Phys. Rev. C **65** (2002) 035204
21. T. Waas and W. Weise, Nucl. Phys. A **625** (1997) 287.
22. M. Lutz, Phys. Lett. B **426** (1998) 12
23. A. Ramos and E. Oset, Nucl. Phys. A **671** (2000) 481
24. S. Hirenzaki, Y. Okumura, H. Toki, E. Oset and A. Ramos, Phys. Rev. C **61** (2000) 055205.
25. A. Baca, C. Garcia-Recio and J. Nieves, Nucl. Phys. A **673** (2000) 335
26. A. Cieply, E. Friedman, A. Gal and J. Mares, Nucl. Phys. A **696** (2001) 173
27. F. Klingl, N. Kaiser and W. Weise, Nucl. Phys. A **624** (1997) 527
28. E. Oset and A. Ramos, Nucl. Phys. A **679** (2001) 616
29. D. Cabrera and M. J. Vicente Vacas, arXiv:nucl-th/0205075.
30. E. Oset, M. J. Vicente Vacas, H. Toki and A. Ramos, Phys. Lett. B **508** (2001) 237
31. K. Imai, private communication.
32. T. Inoue and E. Oset, Nucl. Phys. A **710** (2002) 354
33. C. Garcia-Recio, J. Nieves, T. Inoue and E. Oset, arXiv:nucl-th/0206024, Phys. Lett. B, in print.
34. P. Schuck, W. Norenberg and G. Chanfray, Z. Phys. A **330** (1988) 119.
35. G. Chanfray and D. Davesne, Nucl. Phys. A **646** (1999) 125.
36. H. C. Chiang, E. Oset and M. J. Vicente-Vacas, Nucl. Phys. A **644** (1998) 77
37. V. Bernard, U. G. Meissner and I. Zahed, Phys. Rev. Lett. **59** (1987) 966.
38. V. Bernard and U. G. Meissner, Phys. Rev. D **38** (1988) 1551.
39. T. Hatsuda, T. Kunihiro and H. Shimizu, Phys. Rev. Lett. **82**, 2840 (1999).
40. D. Jido, T. Hatsuda and T. Kunihiro, Phys. Rev. D **63** (2001) 011901
41. M. J. Vicente Vacas and E. Oset, arXiv:nucl-th/0204055.
42. J. G. Messchendorp *et al.*, arXiv:nucl-ex/0205009.
43. L. Roca, E. Oset and M. J. Vicente Vacas, Phys. Lett. B **541** (2002) 77
44. J. C. Nacher, E. Oset, M. J. Vicente and L. Roca, Nucl. Phys. A **695** (2001) 295
45. J. Nieves, E. Oset and C. Garcia-Recio, Nucl. Phys. A **554** (1993) 554.

46. J. A. Oller, E. Oset and A. Ramos, Prog. Part. Nucl. Phys. **45** (2000) 157