

The $(\pi^-, \gamma\gamma)$ reaction in nuclei and the σ meson in the medium

L. Roca and E. Oset

*Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC
Institutos de Investigación de Paterna, Apdo. correos 22085, 46071, Valencia, Spain*

Abstract

A theoretical analysis of the $(\pi^-, \gamma\gamma)$ reaction in nuclei is made in order to find the viability of this reaction to test modifications of the σ meson mass in nuclear matter. The $\pi\pi$ correlation in the scalar-isoscalar channel in nuclear matter could, in principle, manifest itself in this reaction since it plays an important role in the $\pi\pi \rightarrow \gamma\gamma$ mechanism. But we conclude that this effect is hardly visible in this reaction due to the strong background of the pion-Bremsstrahlung terms. Only with some special cuts and for some polarization states are the effect visible at the cost of a strong reduction in the cross section.

1 Introduction

The study of the π^- -induced two photon emission on nuclei has attracted much attention in the last decades. But most of the efforts have been aimed at its application in pionic atoms [1, 2, 3, 4, 5]. In this work we will aim in another direction, focusing in two main objectives: First of all we will explain how to extend the free $\pi^-p \rightarrow \gamma\gamma n$ reaction to the process in nuclei using many body techniques successfully used to describe different pionic reactions [6, 7], paying special attention to the distortion of the initial pion in the nucleus. Our second aim will be to study the viability of using the $(\pi^-, \gamma\gamma)$ reaction in nuclei as a way to test the modification of the properties of the scalar-isoscalar σ meson in nuclear matter.

In the recent past there has been a very lively discussion about the existence, nature and properties of the scalar meson $\sigma(500)$ which lasts till today [8, 9]. The most important source of controversy comes from the confrontation between the interpretation of the σ meson as an ordinary $q\bar{q}$ meson or as a $\pi\pi$ resonance. The advent of χPT has brought new light into this problem and soon it was suggested [10, 11] that the σ could not qualify as a genuine meson which would survive in the limit of large N_c . The reason is that the $\pi\pi$ interaction in s-wave in the isoscalar sector is strong enough to generate a resonance through multiple scattering of the pions. This seems to be the case, and even in models starting with a seed of $q\bar{q}$ states, the incorporation of the $\pi\pi$ channels in a unitary approach leads to a large dressing by a pion cloud which makes negligible the effects of the original $q\bar{q}$ seed [12]. This idea has been made more quantitative through the introduction of the unitary extensions of χPT ($U\chi PT$) [13, 14, 15, 16]. These works implement unitarity in coupled channels in an exact form and use the input of the lowest and second order chiral Lagrangians of [17].

Another point of interest which can help us understand the nature of the σ meson is the modification of its properties at finite nuclear density. The importance of the medium modification of the $\pi\pi$ interaction in the scalar sector was suggested in [18] where the $\pi\pi$ amplitude in the medium developed large peaks below the two pion threshold, somehow indicating that the σ pole had moved to much lower energies. The issue has been revised and the models have been polished incorporating chiral constraints [19, 20, 21] with the result that the peaks disappear at normal density, but still much strength is shifted to low energies.

Experimental tests of the renormalization of the σ properties in the medium have been performed using two pion production reactions induced by photons ($\gamma, \pi\pi$) [22] and ($\pi, \pi\pi$) [23, 24, 25]. In the latter reaction the absorption of the pions in the nucleus caused the process to be too much peripheral to manifest the in-medium σ effects, to the point that the large changes seen in the experimental $\pi\pi$ mass distribution from deuterium to heavier nuclei [23, 24] could not be explained [26]. It was argued there that the abnormal feature was not the size of the $\pi^+\pi^-$ invariant mass distribution close to threshold in nuclei, but the very small size of this magnitude in deuterium, coming from a subtle cancellation of different terms in the amplitude. The offset of this cancellation in nuclei could bring the strength of the $\pi^+\pi^-$ distribution in nuclei to its "normal size" (given by the $\pi^+\pi^+$ distribution).

So far the most successful reaction has been the ($\gamma, \pi^0\pi^0$) reaction where the $\pi^0\pi^0$ invariant mass distribution in nuclei, evaluated in [27], shows a shift of strength towards lower invariant masses with respect to the reaction on the proton. This shift has been corroborated in a recent experiment in [22].

Because of the importance and controversy on this topic, more reactions testing the modification of the σ meson in the medium would be welcome. One of the possible candidates is the ($\pi^-, \gamma\gamma$) reaction in nuclei. This process is interesting because there is no absorption of the final state particles by the nucleus and, hence, allows one to test bigger nuclear densities. In this sense, the reaction should in principle be preferable to other reactions which have been theoretically used as a test of the modification of the $\pi\pi$ interaction in the nuclear medium like ($\gamma, \pi\pi$) and ($\pi, \pi\pi$) in nuclei.

The use of the ($\pi^-, \gamma\gamma$) reaction to test experimentally the in-medium $\sigma \rightarrow \gamma\gamma$ modification has been preliminary study in [25]. In this latter work, comparison of data on ($\pi^-, \gamma\gamma$) for proton and ^{12}C targets, using simulations of the theoretical model of [28], hinted, as a preliminary result, to a small modification of the $\sigma \rightarrow \gamma\gamma$ decay in the nuclear medium.

From the point of view of $U\chi PT$ the σ meson can appear in the ($\pi^-, \gamma\gamma$) reaction via the $\pi\pi$ rescattering involved in the $\pi^+\pi^- \rightarrow \gamma\gamma$ mechanism [29]. But this mechanism is only one among all the mechanisms involved in the ($\pi^-, \gamma\gamma$) reaction [30]. The strength of the terms in the reaction proceeding through σ excitation, relative to that of other mechanisms, will determine the chances to see σ effects in the medium. Hence a quantitative study of this reaction is needed to provide a proper answer to this question.

2 Free reaction

For the evaluation of the $\pi^-p \rightarrow \gamma\gamma n$ amplitude we will first consider the mechanisms shown in Fig. 1.

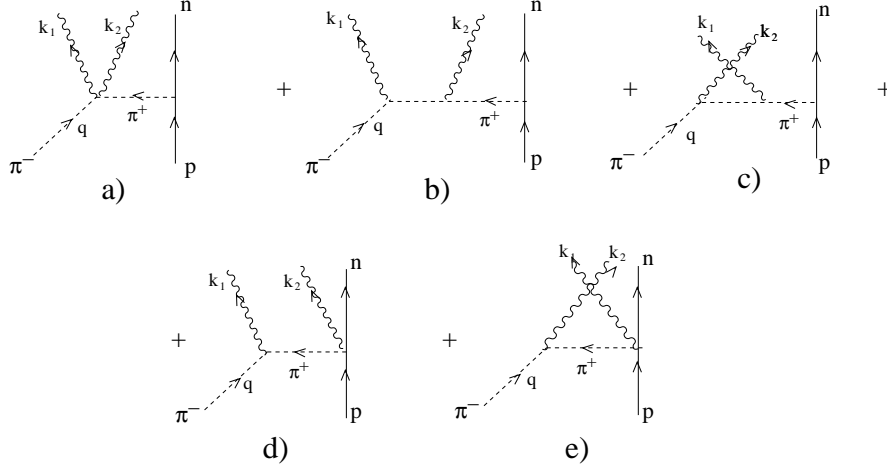


Figure 1: Tree level diagrams for the $\pi^- p \rightarrow \gamma\gamma p$ reaction

The d and e diagrams, which are negligible at small momenta, were not considered in [1] since the authors were only interested in pionic atoms. But now, since we are also interested in higher momenta of the initial pion, one must keep these terms. Other possible diagrams like those where the two photons come from one π^0 decay or the Bremsstrahlung of one photon on the nucleon lines were considered in [30]. But we will not consider them because: In the first case the diagrams with the $\pi^0 \rightarrow \gamma\gamma$ mechanism are only relevant near the region where $M_{\gamma\gamma} \sim m_\pi$ and can be easily filtered experimentally. In the second case, the diagrams with Bremsstrahlung on the nucleon lines can be neglected because their contribution to the cross section are of the order of $\mathcal{O}(p/2M)$, with p the momenta of the nucleons and the photon involved and M the mass of the nucleon. It is also possible to consider the effect of intermediate $\Delta(1232)$ states which in [31] were estimated to be around 7% of the diagram a of Fig. 1 near threshold. The smallness of these terms was also corroborated by the fair agreement of the model ignoring them with the data of $(\pi^-, \gamma\gamma)$ in pionic atoms [1]. At the higher energies where we will work, the Δ contribution should be a little bit bigger. Yet, for the purpose of the present paper, which is to build up the framework for the study of the $(\pi, \gamma\gamma)$ reaction in nuclei and its viability to see σ medium modification, we can safely ignore the nucleon-Bremsstrahlung and Δ terms.

The amplitude corresponding to the diagrams of Fig. 1, in the Coulomb gauge, takes the form

$$\begin{aligned}
T = & -i2\sqrt{2}e^2 \frac{f}{m_\pi} \vec{\sigma} \cdot \vec{q}' \frac{1}{q'^2 - m_\pi^2} \left[\vec{\epsilon}_1 \cdot \vec{\epsilon}_1 + \frac{\vec{q} \cdot \vec{\epsilon}_1 \vec{q}' \cdot \vec{\epsilon}_2}{q \cdot k_1} + \frac{\vec{q} \cdot \vec{\epsilon}_2 \vec{q}' \cdot \vec{\epsilon}_1}{q \cdot k_2} \right] - \\
& -i\sqrt{2}e^2 \frac{f}{m_\pi} \left[\frac{\vec{q} \cdot \vec{\epsilon}_1 \vec{\sigma} \cdot \vec{\epsilon}_2}{q \cdot k_1} + \frac{\vec{q} \cdot \vec{\epsilon}_2 \vec{\sigma} \cdot \vec{\epsilon}_1}{q \cdot k_2} \right]
\end{aligned} \tag{1}$$

The terms represent the diagrams a , b , c , d and e of Fig. 1 respectively, $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ are the polarization vectors of the two photons, $q' = k_1 + k_2 - q$ and $f \approx 1$. The first three terms relate to the Born terms in $\gamma\gamma \rightarrow \pi\pi$ and we shall call them also Born terms in the $(\pi^-, \gamma\gamma)$ reaction.

The cross section for the $\pi^- p \rightarrow \gamma\gamma n$ reaction is given by

$$\begin{aligned}
\sigma &= \frac{M}{\lambda^{1/2}(s, m_\pi^2, M^2)} \frac{1}{2(2\pi)^5} \int \frac{d^3k_1}{2\omega_1} \int \frac{d^3k_2}{2\omega_2} \int d^3p_2 \frac{M}{E_2} \cdot \\
&\cdot \delta^4(q + p_1 - p_2 - k_1 - k_2) \overline{\sum_{s_i} \sum_{s_f}} |T|^2 \\
&= \frac{M^2}{\lambda^{1/2}(s, m_\pi^2, M^2)} \frac{1}{8(2\pi)^4} \int d\omega_2 d\omega_1 d\cos\theta_2 d\phi_{12} \theta(1 - \cos^2\theta_{12}) \overline{\sum_{s_i} \sum_{s_f, \lambda}} |T|^2
\end{aligned} \tag{2}$$

where $q = (\omega, \vec{q})$, $p_1 = (E_1, \vec{p}_1)$, $p_2 = (E_2, \vec{p}_2)$, $k_1 = (\omega_1, \vec{k}_1)$, $k_2 = (\omega_2, \vec{k}_2)$ are the momenta of the pion, initial proton, outgoing neutron and the outgoing photons respectively. The label λ indicates that the sum is done over all the polarizations of the photons. In Eq. (2) ϕ_{12} , θ_{12} are the azimuthal and polar angles of \vec{k}_1 with respect to \vec{k}_2 and θ_2 is the angle of \vec{k}_2 with the z direction defined by the incident pion momentum \vec{q} . While ϕ_{12} is an integration variable, θ_{12} is given by energy-momentum conservation in terms of the other variables. T is the invariant matrix element for the reaction.

The pion-Bremsstrahlung mechanisms, b , c , d and e diagrams, have a typical infrared divergence when the momentum of the pion is not zero, thus some cut in the photon energy is needed.

In the upper row of Fig. 2 we can see the invariant mass distributions for the two photons at different kinetic energies of the pion, calculated removing those events where the energy of some of the photons is less than 25 MeV .

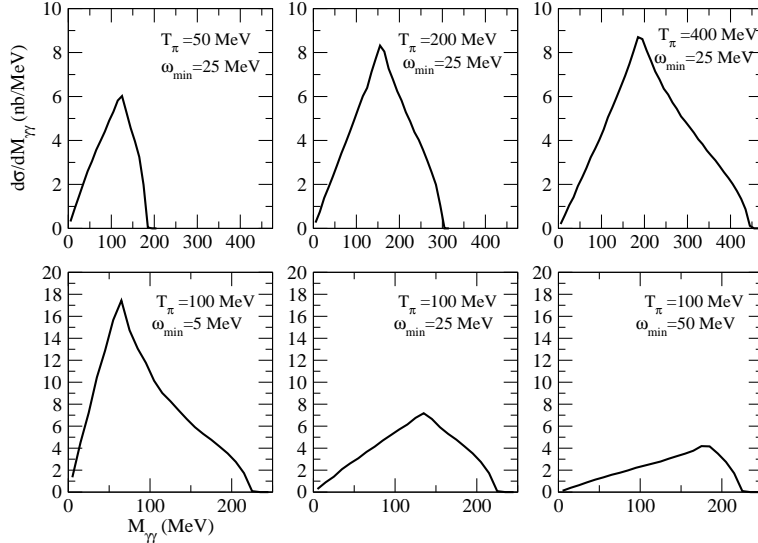


Figure 2: Invariant mass distribution of the two photons: Upper row: varying the pion kinetic energy (T_π) at a fixed cut in the photon energy (ω_{min}). Lower row: varying ω_{min} at a fixed T_π .

In the lower row of Fig. 2 we can also see the invariant mass distributions of the two photons at a fixed kinetic energy of the pion (100 MeV) but varying the cut in the photon energy. We note the trend of the cross section blowing up as the energy of the photons

goes to zero, because of the infrared divergence of the pion-Bremsstrahlung terms. Both the shape and strength of the invariant mass distribution depend crucially on the cut in the energy of the photons.

3 The $(\pi^-, \gamma\gamma)$ reaction in nuclei

The cross section for the process in nuclei can be calculated using the accurate and simple many body techniques summarized in [32], successfully used in many pion and lepton interactions with nuclei [27, 33, 34, 7, 6, 35].

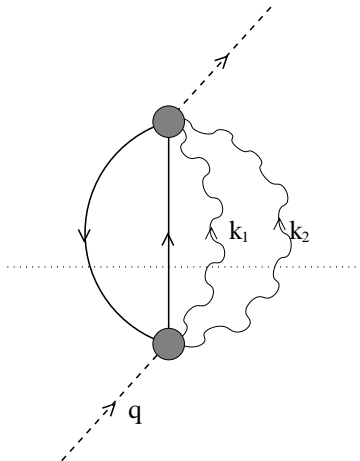


Figure 3: Diagram for the π selfenergy having as a source of imaginary part the cut (dotted line) of the two photon lines and the particle-hole excitation.

The total cross section can be related to the imaginary part of the pion selfenergy of Fig. 3 through the expression

$$\sigma = -\frac{1}{q} \int d^3\vec{r} \text{Im}\Pi(q, \rho(r)) \quad (3)$$

where $\Pi(q, \rho(r))$ is the pion selfenergy of the diagram of Fig.3 and q is the pion momentum. Equation (3) is making an implicit use of the local density approximation, since the photon selfenergy is evaluated at the nuclear density at the point \vec{r} in the integral.

The pion selfenergy corresponding to the diagram of Fig. 3 is given by

$$-i\Pi(q, \rho(r)) = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{i}{k_1^2 + i\epsilon} \frac{i}{k_2^2 + i\epsilon} \frac{1}{2} i\bar{U}(q - k_1 - k_2, \rho(r)) \frac{1}{2} \sum_{s_i, s_f, \lambda} (-i)^2 T^2 \quad (4)$$

where \bar{U} is the Lindhard function which accounts for the particle-hole excitation

$$\bar{U}(q, \rho) = 2 \int \frac{d^3k}{(2\pi)^3} \left[\frac{n(k, \rho)[1 - n(k - q, \rho)]}{q^0 + E(k) - E(k + q) + i\epsilon} + \frac{n(k, \rho)[1 - n(k + q, \rho)]}{-q^0 + E(k) - E(k - q) + i\epsilon} \right] \quad (5)$$

and the last 1/2 factor is the symmetry factor for the two photons. In this expression $E(p) = \sqrt{|\vec{p}|^2 + M^2}$, $n(p, \rho(\vec{r}))$ is the occupation number of the local Fermi sea, which is

unity for $|\vec{p}| \leq k_F(\vec{r})$ and zero for $|\vec{p}| > k_F(\vec{r})$ and $k_F(\vec{r}) = [\frac{3}{2}\pi^2\rho(\vec{r})]^{1/3}$ is the Fermi momentum. One can see in Eq. (5) the effect of the Fermi motion of the initial nucleon, $d^3k n(k, \rho)$, and the Pauli blocking of the final nucleon, $1 - n(k + q, \rho)$.

The imaginary part of the pion selfenergy is obtained when the intermediate states (a particle-hole and two photons) are placed on shell in the integrations over the momenta of the intermediate states. This can actually be implemented using Cutkosky rules, applying the following substitutions to the lines cut by a straight line drawn between the two external pions as shown in Fig. 3

$$\begin{aligned} \Pi(k) &\rightarrow 2i \text{Im}\Pi(k) \\ \bar{U}(k) &\rightarrow 2i \theta(k^0) \text{Im}\bar{U}(k) \\ D(k) &\rightarrow 2i \theta(k^0) \text{Im}D(k) \end{aligned} \quad (6)$$

with $D(k)$ the photon propagator $\frac{1}{k^2+i\epsilon}$ and conjugating the T matrix in the upper vertex.

Applying these rules to Eq. (4) and using Eq. (3) we obtain the following expression for the $(\pi^-, \gamma\gamma)$ reaction in nuclei:

$$\sigma = -\frac{1}{16(2\pi)^6} \frac{1}{q} \int d^3r \int d^3k_1 \int d^3k_2 \frac{1}{k_1 k_2} \text{Im}\bar{U}(q - k_1 - k_2, \rho(r)) \sum_{s_i, s_f, \lambda} |T|^2 \quad (7)$$

So far the distortion of the incoming pion inside the nucleus before it reaches the production point \vec{r} has not been taken into account. Since pions interact strongly with nucleons, the loss of pion flux makes the reactions in nuclei involving pions in initial or final states to be more peripheral and to produce less desired events. This strong distortion of the pions inside the nucleus has turned out to be crucial in the evaluation of cross sections and nuclear effects in reactions involving pions [26, 27].

Pions inside a nucleus can be distorted in many ways: Can be absorbed; can undergo quasielastic collisions and change direction, energy, charge or even have inelastic collisions and produce more pions. In order to quantify the distortion of the pions we make an eikonal approximation and remove from the pion flux those pions which undergo absorption, which indeed disappear. We also remove the pions which undergo quasielastic collisions because, even if they do not disappear, they lose much energy and the cross section for production of two photons in the upper part of the 2γ spectrum, where we will be interested, is considerably reduced, (see Fig. 2). Thus we have to include in the integrand of Eq. (7) an eikonal factor for this initial state interaction (ISI) of the pion which is given by

$$F_{ISI}(\vec{r}, \vec{q}) = \exp \left[- \int_{-\infty}^0 dl \mathcal{P}(\vec{r}', \vec{q}) \right] \quad (8)$$

\vec{q} : momentum of the pion

\vec{r} : production point inside the nucleus

$\vec{r}' = \vec{r} + l \vec{q} / |\vec{q}|$: integration point in the π^- trajectory

$\mathcal{P}(\vec{r}', \vec{q})$: reaction probability per unit length of the π^- in the nucleus

The interpretation of the eikonal factor (8) is very intuitive since it is nothing but an exponential decay law which represents the probability for a π^- to reach the production point \vec{r} without interacting with the nucleus. The probability of absorption or quasielastic collisions per unit length depends crucially on the energy of the pion, and there are accurate models for different energy regions to account for it: From zero to ~ 300 MeV we will use

$$\mathcal{P}(\vec{r}', \vec{q}) = -\frac{1}{|\vec{q}|} \text{Im}\Pi(\vec{r}', \vec{q}) \quad (9)$$

where Π is the pion selfenergy in the nuclear medium. For the region of very low energy pions (from 0 to ~ 85 MeV) we will use for Π the model of [36], based on an extrapolation for low energy pions of the pion-nucleus optical potential developed for pionic atoms using many body techniques. In this work the imaginary part of the potential is split into a part that accounts for the probability of quasielastic collisions and another one which accounts for the pion absorption probability. In the Δ resonance region (from ~ 85 to ~ 300 MeV) we use for Π the model of [6, 7] which considers absorption by two and three body mechanisms and quasielastic collisions at the level of $1p1h$ and $2p2h$ excitation. For kinetic energies of the pion beyond ~ 300 MeV we will use for the probability of distortion per unit length

$$\mathcal{P}(\vec{r}', \vec{q}) = \sigma_{\pi^-p}\rho_p(\vec{r}') + \sigma_{\pi^-n}\rho_n(\vec{r}') + C^{abs.(2)}\rho^2(\vec{r}') + C^{abs.(3)}\rho^3(\vec{r}') \quad (10)$$

σ_{π^-N} : quasielastic π^- -Nucleon cross section

$C^{abs.}$: absorption coefficients by 2 and 3 body mechanisms

where σ_{π^-N} are taken from experiment while C^{abs} are calculated theoretically from the model of [37]. The superindex (2) and (3) in the absorption coefficients indicates absorption by two and three nucleons respectively.

In principle we should implement other medium modifications, like the addition of a selfenergy in the internal pion line, but the virtual pion is far off-shell and the pion selfenergy in the denominator of the propagator is small in comparison with the quadri-momentum of the pion. On the other hand, we should expect some medium corrections in the vertices such as $NN\pi$ and others but, based on calculations of [38], we expect these corrections to be very small.

In Fig. 4 we present results for the invariant mass distribution of the two photons for the $(\pi^-, \gamma\gamma)$ reaction in ^{208}Pb in the same cases as in Fig. 2. We can observe there the strong π^- absorption at $T_\pi = 200$ MeV where the initial pion is in the Δ resonance region.

4 Looking for in medium modifications of the σ meson

In the work of [27] the authors observed a shift in the two pion invariant mass distribution in the reaction $(\gamma, \pi^0\pi^0)$ in nuclei due to the shift of the pole position of the $\pi\pi \rightarrow \pi\pi$

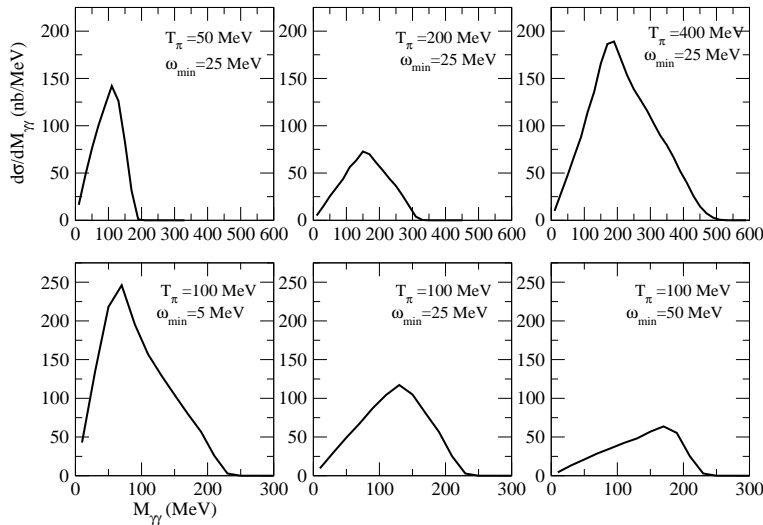


Figure 4: Same as Fig. 2 but in ^{208}Pb .

amplitude in the scalar-isoscalar channel in nuclear matter. When contrasted with the experimental data of [22], this work represented the first clear manifestation of the dropping of the σ meson mass in nuclear matter. In [27] the model of [21] for the $\pi\pi$ interaction in the nuclear medium was used. In [21] the $\pi\pi$ rescattering in nuclear matter was done renormalizing the pion propagators in the medium and introducing vertex corrections for consistency. The results obtained with the model of [21] for the imaginary part of the $\pi\pi \rightarrow \pi\pi$ in $I = 0$ as a function of the nuclear density can be seen in Fig. 5.

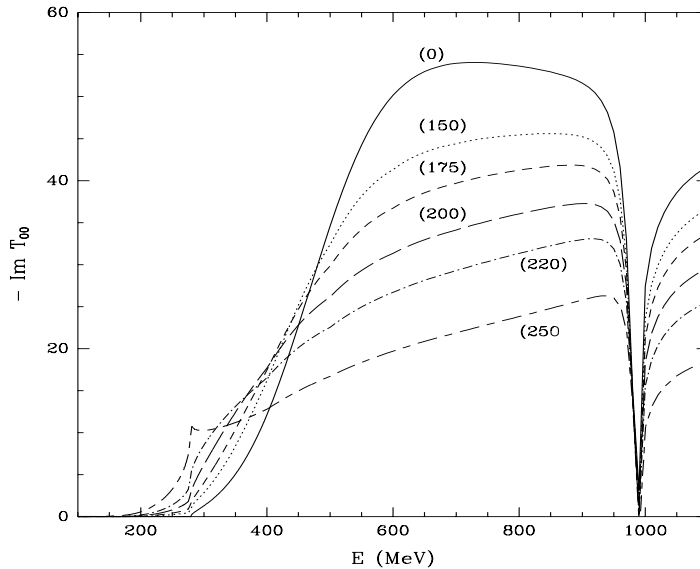


Figure 5: Imaginary part of the $\pi\pi \rightarrow \pi\pi$ in $J=I=0$ in the nuclear medium for different values of k_F versus the CM energy of the pion pair. The labels correspond to the values of k_F in MeV.

One can see in Fig. 5 that, as the nuclear density increases, there is a shift of strength to low $\pi\pi$ masses. This was the effect shown in [27] and [22].

In view of the results of these works one may wonder if the $(\pi^-, \gamma\gamma)$ on nuclei is also a suitable reaction to test this dropping of the σ mass in nuclear matter, because the $\pi\pi \rightarrow \gamma\gamma$ reaction is a very important mechanism in this reaction, see Fig 1, and the σ channel is a relevant part in the $\gamma\gamma \rightarrow \pi^+\pi^-$ amplitude as shown in [29].

In order to answer this question we have to implement the rescattering of the pions in $(I=0, L=0)$ (σ channel) in the diagrams where they lead to two photons, in the way of Fig. 6. Then we have to compare the invariant mass distribution of the two photons when the $(\pi^-, \gamma\gamma)$ reaction is done in nuclei with that where the reaction is done on hydrogen, and then see if there is a move of strength, manifesting the effect shown close to $2m_\pi$ in Fig. 5.

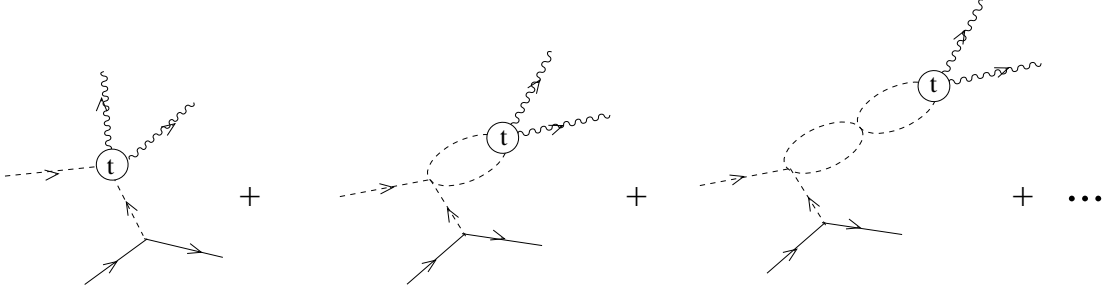


Figure 6: Rescattering of the pions for the dynamical generation of the σ meson in the $(\pi^-, \gamma\gamma)$ reaction.

For the $\pi^+\pi^- \rightarrow \gamma\gamma$ reaction in free space we will use the model of [29]. In this work the authors considered the resummation of the Born terms, those obtained from chiral Lagrangians up to order $\mathcal{O}(p^4)$ and the terms with exchange of one axial or vector meson. The diagrammatic representation is shown in Fig. 7 (see Ref. [29] for details and meaning of these terms).

The amplitude for the $\pi^+\pi^- \rightarrow \gamma\gamma$ reaction corresponding to the diagrams of Fig. 7 reads

$$\begin{aligned}
t &= t_{Born} + t_A + t_\rho + (\tilde{t}_{A\pi^+\pi^-} + \tilde{t}_{\rho\pi^+\pi^-} + \tilde{t}_{\chi\pi})t_{\pi^+\pi^-, \pi^+\pi^-} + \\
&+ (\tilde{t}_{AK^+K^-} + \tilde{t}_{\chi K})t_{K^+K^-, \pi^+\pi^-} + (\tilde{t}_{\rho\pi^0\pi^0} + \tilde{t}_{\omega\pi^0\pi^0})t_{\pi^0\pi^0, \pi^+\pi^-}
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
t_{\pi^+\pi^-, \pi^+\pi^-} &= \frac{1}{3}t^{I=0} + \frac{1}{6}t^{I=2} \\
t_{\pi^0\pi^0, \pi^+\pi^-} &= \frac{1}{3}t^{I=0} - \frac{1}{3}t^{I=2} \\
t_{K^+K^-, \pi^+\pi^-} &= \frac{1}{\sqrt{6}}t^{I=0}
\end{aligned} \tag{12}$$

The analytical expressions for $t_{A,\rho}$ and \tilde{t}_i can be found in [29]. The meson scattering amplitudes of Eqs. (12) were evaluated in [39] by solving the Bethe-Salpeter equation in coupled channels with the kernels formed from the lowest order meson-meson chiral

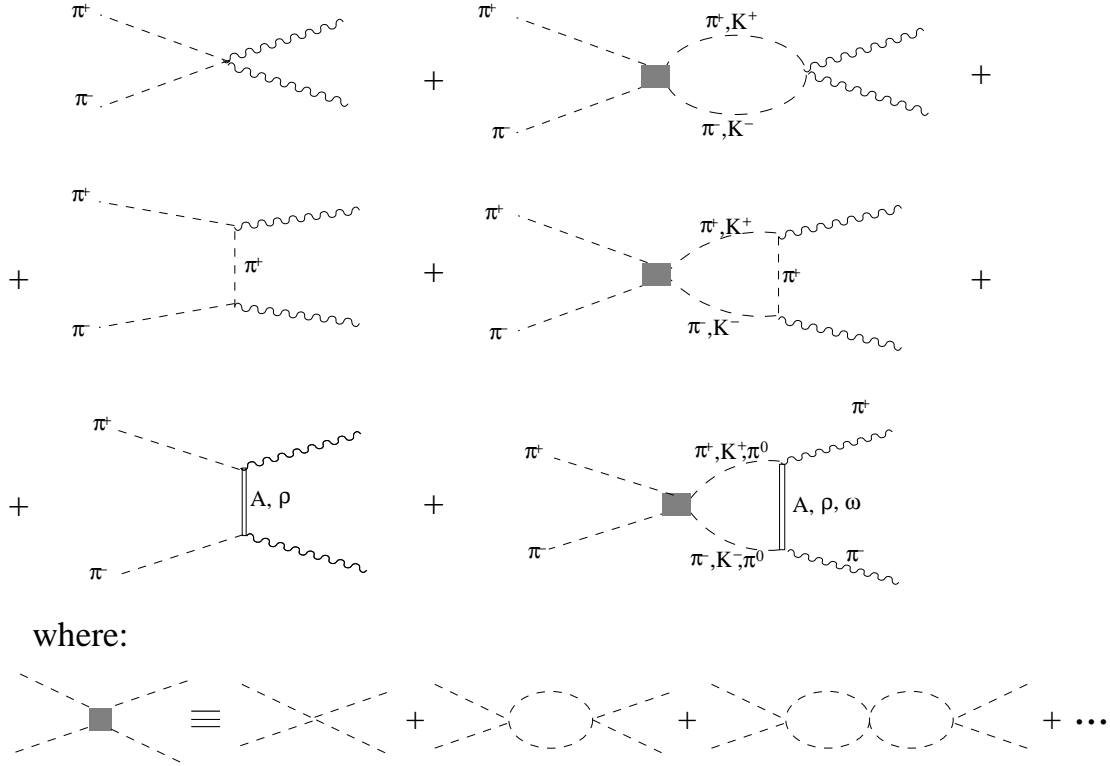


Figure 7: Diagrams for $\pi^+\pi^- \rightarrow \gamma\gamma$ included in the model of [29].

Lagrangian amplitude. This is the way the σ meson is dynamically generated. From now on we will call, for brief, "chiral terms" those of Eq. (11) without t_{Born} .

When we go to the nuclear medium we have to replace the free $\pi\pi \rightarrow \pi\pi$ in $I = 0$ and the two pion loop function, $G_{\pi\pi}$, by their corresponding values in nuclear matter, obtained in [21], calculated at the local density corresponding to the integration point in Eq. (7). When replacing the free $G_{\pi\pi}$ by its in-medium expression one automatically takes into account the terms involving loops in the third row in Fig. 7, because in [29] it was shown that these diagrams can be expressed in terms of $G_{\pi\pi}$, factorizing on-shell the rest of the amplitude. For the loops implicit in \tilde{t}_χ , the loops in the first two rows of Fig. 7, we have just multiplied the expression given in [29] by $G_{\pi\pi}(\rho)/G_{\pi\pi}(\rho = 0)$.

With all these considerations we can pass now to study if these modifications have appreciable consequences when they are introduced in the full $(\pi^-, \gamma\gamma)$ model in nuclei. In order to look for the effect shown in Fig. 5 we have to choose a kinematical region where this effect is maximized. We have chosen a kinetic energy of the pion of 380 MeV because the phase space distribution of the two photons makes more relevant the region of 200 to 400 MeV where the changes of the $\pi\pi$ amplitude show up. The cut in the energy of the photons has been chosen as 50 MeV in what follows in order to reduce the strength far away from the region of interest ($2m_\pi$).

In Fig. 8.a we can see the invariant mass distribution of the two photons for the $\pi^- p \rightarrow \gamma\gamma n$ reaction with and without the explicit inclusion of the chiral terms, dashed and continuous lines respectively, but we can see that the effect is hardly visible, around 1-2%. In Fig. 8.b we have plotted the equivalent curves to Fig. 8.a but for $\gamma\gamma \rightarrow \pi^+\pi^-$. Looking at Fig. 8.b one can see that the effect of the chiral terms, where the σ meson

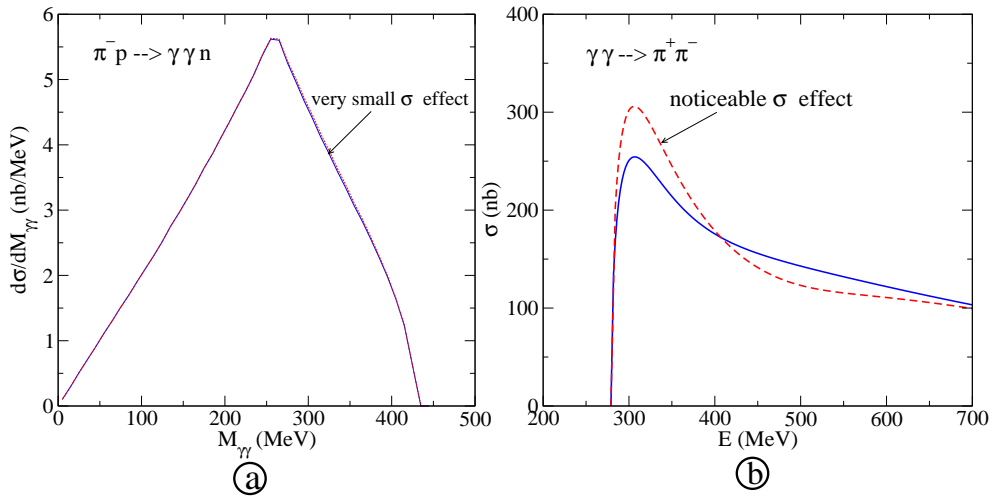


Figure 8: Invariant mass distribution and total cross section for $\pi^- p \rightarrow \gamma\gamma n$ and $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction respectively. Continuous line: full model without the inclusion of the chiral terms. Dashed line: full model + chiral terms.

plays a role, is to increase the cross section in around 20% at ~ 320 MeV of energy. Therefore one could expect, in principle, a similar effect in the $(\pi^-, \gamma\gamma)$ reaction. But in the $(\pi^-, \gamma\gamma)$ reaction we only obtain a tiny effect. This is due to a subtle combination of interferences and the allowed phase space. In order to clarify this interference we show in Fig. 9 different contributions separately, and we refer to this figure in what follow.

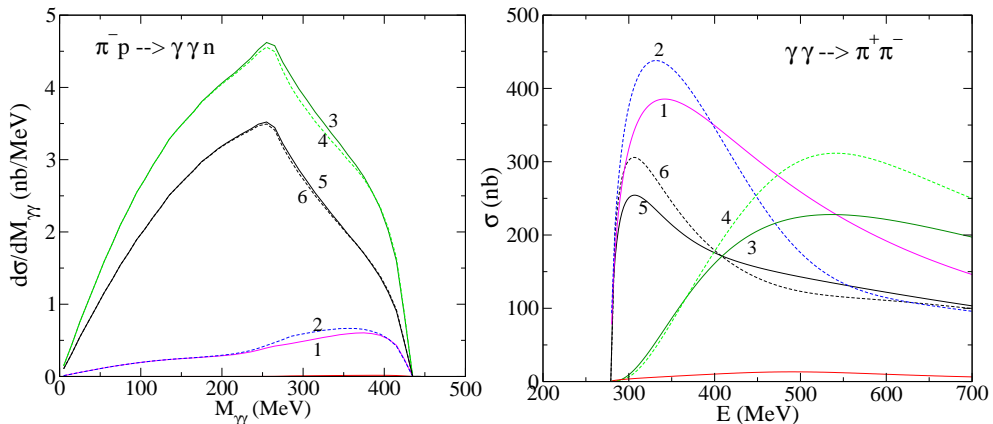


Figure 9: Effect of the chiral terms on the different contributions to the $(\pi^-, \gamma\gamma)$ and $\gamma\gamma \rightarrow \pi^+\pi^-$ reactions. See text for explanation.

We have checked that the effect of adding the chiral terms to the diagram *a* of Fig. 1 is to increase the cross section in around 20% at 320 MeV, both in $\gamma\gamma \rightarrow \pi^+\pi^-$ and in the invariant mass distribution of the two photons in the $(\pi^-, \gamma\gamma)$ reaction (lines 2). In $\gamma\gamma \rightarrow \pi^+\pi^-$ the *a* term (line 1) is the only significative one at this energy since *b* and *c* (line 3) tend to 0 at threshold. Thus the total effect of the chiral terms in the cross section is a 20% close to $2m_\pi$ (change between line 5 and 6). But in $(\pi^-, \gamma\gamma)$ the *b* and *c* terms (as well as *d* and *e*, not present in $\gamma\gamma \rightarrow \pi^+\pi^-$) no longer vanish (line 3) because the virtual pion momenta are not zero. Actually it is 7 times larger than the *a* term (line 1) at 320 MeV. When we add the chiral terms, the interference (line 4) is opposite to the

one of a and similar in size, compensating therefore the effect (line 6) when we add the chiral terms to the a , b and c terms together (line 5). This is the reason why the effect of the chiral terms, accounting for the dynamical generation of the σ , is so small in the $(\pi^-, \gamma\gamma)$ reaction. In summary: the large size of the pion-Bremsstrahlung terms and its opposite interference with the chiral ones mask the effect of the σ meson.

In spite of this discouraging result, we can try to magnify the effect of the σ meson by making some kind of filters or kinematical cuts in order to remove the hindering terms b , c , d and e of Fig. 1. If we turn off numerically these terms the effects that we would expect should be those shown in the right plot of Fig. 10:

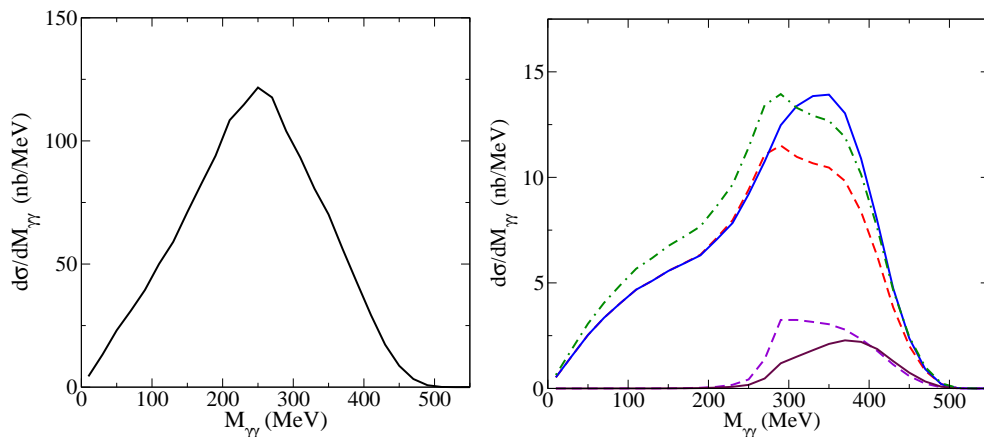


Figure 10: Left panel: invariant mass distribution for $(\pi^-, \gamma\gamma)$ reaction in ^{208}Pb with the full model. Right panel: without the inclusion of the mechanisms with infrared divergence. See the text for a whole explanation.

In the right panel of Fig. 10 we have plotted the invariant mass distribution of the two photons for the reaction in ^{208}Pb , setting explicitly to zero the b , c , d and e terms of Fig. 1 (those that manifest the infrared divergence). The plot has been evaluated at $T_\pi = 380$ MeV and $\omega_{min} = 50$ MeV. We can observe how much strength do these contributions represent comparing to the full model at the same energy and photon energy cut shown in the left panel. The solid line of the right plot represents the model with only the a term plus the chiral terms but these latter ones calculated at zero density, that is if we turn off the density in the rescattering of the pions in the scalar-isoscalar channel. If we turn on the density dependence of these rescattering terms we obtain the dashed line. The dashed-dotted line is the same plot but renormalized to the peak of the continuous line in order to compare both curves. A shift of strength to low invariant masses when the medium effects are considered can be observed. The two lower curves represent the case when only the chiral terms are considered (diagrams of Fig. 7 removing the first two diagrams of column one) both at zero and at true density (both magnified 7 times in order to have the curves within the scale of the figure). But Fig. 10 is only a theoretical exercise that shows the best effect we can expect, provided one could eliminate experimentally the b , c , d and e mechanisms, the last four terms in Eq. (1) respectively. Looking at Eq. (1) we can see that, should we go to very low pion momentum, these terms would vanish, but we would be in a kinematical region very far from $2m_\pi$ where the shift in the $\pi\pi$ amplitude in the medium occurs according to Fig. 5. There are, however, some possible experimental set ups to remove these hindering terms:

The first one is to keep only those events where the two photons are produced in the direction of the incident π^- . This would force the polarization vectors \vec{e}_1 and \vec{e}_2 to be orthogonal to \vec{q} and then these terms would vanish. The photons so chosen can go in the same direction or back-to-back. We have also to remove those which go in the same direction because they have zero invariant mass.

When this filter is actually implemented, one has to choose a finite acceptance angle for the deviation of the photons with respect to the direction of the incident pion. The results when we implement this filter with an acceptance angle of 5 degrees are shown in Fig. 11.a.

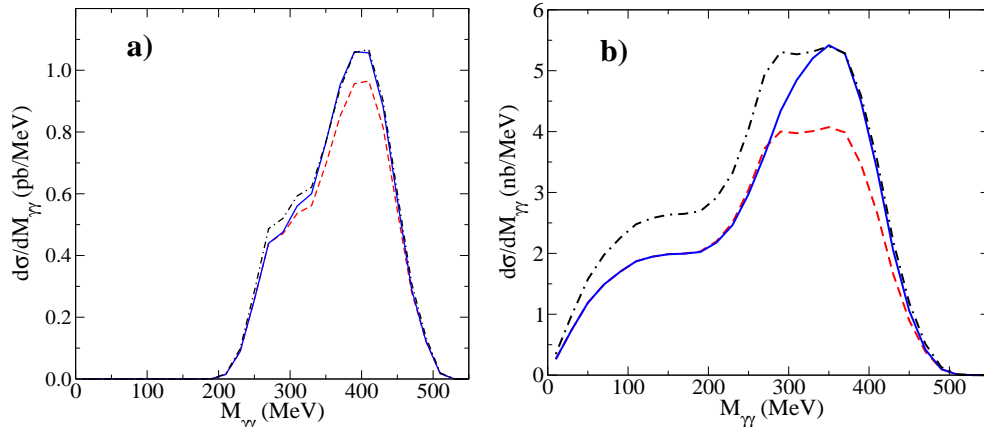


Figure 11: Results after implementing the filters. Left panel: only accepted events when the photons go back-to-back and parallel to direction of the incident pion, which an acceptance angle of 5 degrees. Right panel: only accepted events when the polarization of both photons are orthogonal to the direction of the incident pion.

The continuous line is calculated with zero density for the chiral terms, the dashed one is the full model and the dashed-dotted line is the full model normalized to the continuous line. We can see that the effect of the density in the chiral terms is visible in the size but barely noticeable in the shape of the strength because this cut favours the accumulation of events at high invariant masses, thus minimizing the $2m_\pi$ region. The other problem with this filter is the strong decrease of the statistics since the cross section is reduced in nearly five orders of magnitude, mostly due to the reduced phase space acceptance.

Another possibility to eliminate the hindering terms, according to Eq. (1), is to implement a filter on the polarization of the photons in such a way that only those events with photon polarization orthogonal to the momentum of the initial π^- are accepted. In Fig. 11.b we can see that the implementation of this latter filter causes the invariant mass distribution to look much more like that of Fig. 10. The shift of the shape between the continuous and dashed-dotted lines manifests the modification due to the nuclear medium of the $\pi\pi$ interaction in the σ channel. But this procedure has the strong inconvenience of having to filter the polarization of photons in the final state.

In addition to this problem, one should point out that by filtering the photon polarization the cross section has been reduced in a factor 20, hence reducing appreciably the statistics of a cross section already small to start with. This also has the inconvenience that, given the fact that an eventual polarization of the photons would necessarily have a certain uncertainty in this polarization, this uncertainty, allowing the contribution from

the pion-Bremsstrahlung terms, could drastically enhance the cross section with respect to the one calculated in Fig. 11.b, thus masking the σ in medium effects. In the same direction, the Bremsstrahlung terms on the nucleon lines, which are small compared to the dominant terms of the full model as we pointed out, would no longer be small compared to those of the filtered amplitude, which could also make the cross section bigger than evaluated in Fig. 11.b, once again blurring the effects of the σ in the medium.

5 Conclusions

We have made a theoretical analysis of the $(\pi^-, \gamma\gamma)$ reaction on the proton and nuclei beyond the scope of its application to pionic atoms. After explaining the mechanisms considered for the free reaction, we have summarized the procedure used to study the reaction on nuclei, based on very well tested many body techniques. As an immediate application we have studied the viability of the reaction to test the shift of the σ meson pole in nuclear matter, using a model which provides the dynamical generation of the σ meson in the $\pi\pi$ rescattering in $(I = 0, L = 0)$ within the framework of a unitary extension of χPT and its modification in nuclear matter [21]. We have implemented in the $(\pi^-, \gamma\gamma)$ reaction the model of [29] for the $\pi^-\pi^+ \rightarrow \gamma\gamma$ reaction where the dynamical generation of the σ plays an important role. We have investigated whether a shift of strength in the invariant mass distribution of the two photons appears when we pass from the free reaction to the in-medium case. The first observation is that the very strong background due to the pion-Bremsstrahlung mechanisms hides the desired effect. We have tested several experimental filters and kinematical cuts in order to reduce the influence of these pion-Bremsstrahlung terms, but even then we have obtained a small signal of the desired effects and, furthermore, these experimental set ups seem not easy to implement. All this said, we must conclude that the $(\pi^-, \gamma\gamma)$ reaction on nuclei is not a very suitable reaction to test the in-medium modification of the properties of the σ meson, in spite of testing bigger nuclear densities than the $(\gamma, \pi\pi)$ or $(\pi, \pi\pi)$ reactions.

Leaving this negative result aside, the present work has performed a realistic model for the $(\pi^-, \gamma\gamma)$ reaction in nuclei, extrapolating the results for the same reaction in pionic atoms done in [1] to the region of pions in the continuum. Comparison with the experiment can serve as a further test of the many body techniques used here which have been successfully used for other reactions, providing extra confidence in these easy to implement and, so far, quite accurate methods, which can be applied to most nuclear reactions at intermediate energies.

Acknowledgments

We are specially grateful to M. J. Vicente Vacas for fruitful discussions and his careful reading of the manuscript. One of us, L.R., acknowledges support from the Consejo Superior de Investigaciones Científicas. This work is partly supported by DGICYT contract numbers BFM2000-1326, and the E.U. EURIDICE network contract no. HPRN-CT-2002-00311.

References

- [1] A. Gil and E. Oset, Phys. Lett. B **346** (1995) 1 [arXiv:nucl-th/9410032].
- [2] T. E. Ericson and C. Wilkin, Phys. Lett. B **57** (1975) 345.
- [3] S. Barshay, Phys. Lett. B **78** (1978) 384 [Erratum-ibid. B **79** (1978) 514].
- [4] L.I. Lapidus and M.M. Musakhanov, Sov. J. Nucl. Phys. 15(1972)558
- [5] E. M. Nyman and M. Rho, Nucl. Phys. A **287** (1977) 390.
- [6] L. L. Salcedo, E. Oset, M. J. Vicente-Vacas and C. Garcia-Recio, Nucl. Phys. A **484** (1988) 557.
- [7] E. Oset and M. J. Vicente-Vacas, Nucl. Phys. A **454** (1986) 637.
- [8] Proc. of the Workshop on the "Possible existence of the σ meson and its implication in hadron physics", Kyoto June, 2000, Ed. S. Ishida et al., web page <http://amaterasu.kek.jp/YITPws/>.
- [9] E. v. Beveren, G. Rupp, N. Petropoulos and F. Kleefeld, arXiv:hep-ph/0211411.
- [10] J. Gasser and U. G. Meissner, Nucl. Phys. B **357**, 90 (1991).
- [11] U. G. Meissner, Comments Nucl. Part. Phys. **20**, 119 (1991).
- [12] N. A. Tornqvist and M. Roos, Phys. Rev. Lett. **76** (1996) 1575.
- [13] A. Dobado, M. J. Herrero and T. N. Truong, Phys. Lett. B **235** (1990) 134.
- [14] A. Dobado and J. R. Pelaez, Phys. Rev. D **47** (1993) 4883 [arXiv:hep-ph/9301276].
- [15] J.A. Oller, E. Oset and J. R. Peláez, Phys. Rev. Lett. **80** (1998) 3452; Phys. Rev. D **59** (1999) 074001 [Erratum-ibid. D **60** (1999) 099906].
- [16] J. A. Oller and E. Oset, Phys. Rev. D **60** (1999) 074023.
- [17] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 517 (1985).
- [18] P. Schuck, W. Noerenberg and G. Chanfray, Z. Phys. A **330** (1988) 119.
- [19] R. Rapp, J. W. Durso and J. Wambach, Nucl. Phys. A **596** (1996) 436 [arXiv:nucl-th/9508026].
- [20] Z. Aouissat, R. Rapp, G. Chanfray, P. Schuck and J. Wambach, Nucl. Phys. A **581** (1995) 471.
- [21] H. C. Chiang, E. Oset and M. J. Vicente-Vacas, Nucl. Phys. A **644** (1998) 77 [arXiv:nucl-th/9712047].
- [22] J. G. Messchendorp *et al.*, Phys. Rev. Lett. **89** (2002) 222302 [arXiv:nucl-ex/0205009].
- [23] F. Bonutti *et al.* [CHAOS Collaboration], Nucl. Phys. A **638** (1998) 729.
- [24] A. Starostin *et al.* [Crystal Ball Collaboration], Phys. Rev. Lett. **85** (2000) 5539.
- [25] A. Starostin *et al.* [Crystal Ball Collaboration], arXiv:nucl-ex/0204002.
- [26] M. J. Vicente Vacas and E. Oset, Phys. Rev. C **60** (1999) 064621 [arXiv:nucl-th/9907008].
- [27] L. Roca, E. Oset and M. J. Vicente Vacas, Phys. Lett. B **541** (2002) 77 [arXiv:nucl-th/0201054].
- [28] S. Chiku and T. Hatsuda, Phys. Rev. D **58** (1998) 076001 [arXiv:hep-ph/9803226].
- [29] J. A. Oller and E. Oset, Nucl. Phys. A **629** (1998) 739 [arXiv:hep-ph/9706487].

- [30] J. B. Cammarata, Phys. Rev. C **20** (1979) 705.
- [31] D. Beder, Phys. Rev. C **26** (1982) 555.
- [32] R. C. Carrasco and E. Oset, Nucl. Phys. A **536** (1992) 445.
- [33] H. C. Chiang, E. Oset and P. Fernandez de Cordoba, Nucl. Phys. A **510** (1990) 591.
- [34] S. K. Singh and E. Oset, Nucl. Phys. A **542** (1992) 587.
- [35] A. Gil, J. Nieves and E. Oset, Nucl. Phys. A **627** (1997) 543 [arXiv:nucl-th/9711009].
- [36] J. Nieves, E. Oset and C. Garcia-Recio, Nucl. Phys. A **554** (1993) 554.
- [37] E. Oset and D. Strottman, Phys. Rev. C **42** (1990) 2454.
- [38] E. Oset and W. Weise, Phys. Lett. B **60** (1976) 141.
- [39] J. A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438 [Erratum-ibid. A **652** (1999) 407] [arXiv:hep-ph/9702314].