Chiral loops and VMD in the $V \to PP\gamma$ decays

J. E. Palomar[1], S. Hirenzaki[1,2] and E. Oset[1]

[1] *Departamento de Física Teórica and IFIC,*
Centro Mixto Universidad de Valencia-CSIC,
Ap. Correos 22085, E-46071 Valencia, Spain
[2] *Department of Physics, Nara Women's University, Nara 630-8506, Japan*

Abstract

We evaluate radiative decays of ρ and ω going to two neutral mesons, $\pi^0\pi^0$ and $\pi^0\eta$. We use the sequential vector decay mechanisms in addition to chiral loops and ρ-ω mixing. The chiral loops are obtained using elements of $U\chi PT$ successfully applied in the study of meson-meson interactions up to 1.2 GeV. The chiral loops are found very important in the case of the $\rho \to \pi^0\pi^0\gamma$ decay and small in the other cases. A good agreement with present measurements of $\rho \to \pi^0\pi^0\gamma$ and $\omega \to \pi^0\pi^0\gamma$ is obtained and predictions are made for the other decays where the rates obtained are rather small.

1   Introduction

The radiative vector meson decay into two pseudoscalar mesons has attracted continuous attention. It has been a case for tests of vector meson dominance (VMD), through the sequential mechanism $V \to PV \to PP\gamma$ [1, 2], but more recently it has been advocated as a source of information on the meson scalar sector in order to learn about the controversial nature of these states. One of the clearests examples is the $\phi \to \pi^0\pi^0\gamma$ decay where the experiment [3, 4, 5] shows very clearly a peak for the $f_0(980)$ excitation. Similarly the $\phi \to \pi^0\eta\gamma$ reaction shows a clear peak for the $a_0(980)$ excitation [6]. These decays of the ϕ are particularly interesting since the contribution of the sequential processes in the VMD model [2] is negligible and the decay width can be attributed to the excitation of the scalar mesons. Attempts to obtain the rates for these reactions and for $\phi \to K^0\bar{K}^0\gamma$ were done including loops of charged kaons to which the photons could couple [7, 8]. A link to chiral perturbation theory (χPT) was established in [9], by using the lowest order

chiral meson-meson scattering amplitude. An important step in this direction was given in [10] where elements of unitarized chiral perturbation theory ($U\chi PT$) were used, which directly lead to the excitation of the $f_0(980)$ and $a_0(980)$ resonances in these reactions from the consideration of the chiral loops in coupled channels. The excitation of these resonances from the chiral loops was made possible because previously it had been found [11] that the loop iteration provided by the Bethe-Salpeter equation using a kernel (potential) from the lowest order chiral Lagrangian, with an appropriate loop regularization, provides a very good description of the meson-meson interaction in the scalar sector, including the dynamical generation of the $\sigma(500)$, $f_0(980)$ and $a_0(980)$ resonances.

The Bethe-Salpeter approach along the lines of [11] has been further used to study meson-meson and meson-baryon interaction in [12, 13]. More elaborate steps taking into account explicitly the contribution of higher order chiral Lagrangians [14], or generating them from the explicit exchange of meson resonances, were done in [15, 16] respectively. At the same time these works provided an explanation of why the Bethe-Salpeter equation with the explicit use of only the lowest order chiral Lagrangian in [11] is so successful in the scalar sector. The interesting thing in all these works is that the scalar resonances are generated dynamically from the multiple scattering implicit in the unitary approach without the need to introduce them as genuine resonances (those which would survive in the large N_c limit). The nature of the $\sigma(500)$ as a meson-meson scattering resonance was already advocated in [17, 18]. Other approaches would start from quark components for the mesons but the unitarization dresses them with a large cloud of meson-meson components [19]. The unitarization of chiral perturbation theory has thus brought a new insight into the nature of the scalar resonances, which continues to be a subject of strong debate [20, 21].

The purpose of the present paper is to complement the study initiated in [10] and extend it to other vector meson decays. Here we study the decays $\rho \to \pi^0\pi^0\gamma$, $\omega \to \pi^0\pi^0\gamma$, $\rho \to \pi^0\eta\gamma$ and $\omega \to \pi^0\eta\gamma$. All these decays were calculated also in ref. [22] using chiral Lagrangians and the extended Nambu-Jona-Lasinio model. The study of the first two reactions has been done recently in [23] combining the sequential vector meson decay mechanisms with loop contributions. The second of the reactions has also been revisited recently including effects of ρ-ω mixing in [24]. The latest two reactions were studied in [2] using again the sequential vector meson decay mechanisms. Here we also include the loop corrections and ρ-ω mixing in the case of the $\rho \to \pi^0\eta\gamma$ reaction. The first of the reactions has also been studied in [25, 26] assuming $\rho \to \sigma\gamma$ decay, although apparently a too large transition amplitude was used in the approach, see [23] for details.

The present work is also stimulated by the recent measurement of the $\rho \to \pi^0\pi^0\gamma$ decay by the SND Collaboration [27] where a value for the branching

ratio

$$B(\rho \to \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.2) \times 10^{-5} \tag{1}$$

is obtained. The work of [23] finds that there are approximately equal contributions to the $\rho \to \pi^0\pi^0\gamma$ process from the sequential mechanism and from the pion loops. This latter contribution is very interesting since, given the fact that the pion-pion scalar isoscalar amplitude factorizes on shell in this mechanism, the process is sensitive to the $\pi\pi$ interaction in the region where the sigma meson is produced. Furthermore, the phase space for the process and the dynamical factors in the total amplitude make the information appearing there a complement to the one obtained from other processes from where the $\pi\pi$ phase shifts are measured. Also, the interference between the loop contribution and the sequential process adds new information about the pion-pion interaction and the properties of the σ meson. In ref. [23] a simple analytical model for the $\pi\pi$ interaction was used in which $\sigma(500)$ and $f_0(980)$ exchange are explicitly included in the $\pi\pi$ amplitude. Then values of the $\sigma(500)$ mass and width from the recent experimental determination [28] are used. These values are $m_\sigma = 478^{+24}_{-23} \pm 17$ MeV and $\Gamma_\sigma = 324^{+42}_{-40} \pm 21$. In addition the paper shows that the results are quite sensitive to the mass and width of the σ, concluding that precise measurements of the process can provide valuable information on these two magnitudes.

Our aim here is different. We would like to use the reaction as a further test of the $U\chi PT$ approach to the meson-meson interaction. Indeed, analogously to the way the scalar resonances are generated in that approach, in the present case the reaction mechanisms immediately lead to the excitation of these resonances without the need to include them explicitly in the formalism. The theoretical framework allows one to make predictions for production processes, once the basic parameters of the theory, just one regularizing cut off in [11], have been fixed by fits to the scattering data.

For the sequential mechanism we follow the approach of [2], but for the loop contributions we follow the approach of [10] where the chiral tensor formalism for the vector mesons [29] is used, as done in [30] in the study of the $\rho \to \pi^+\pi^-\gamma$ decay. In [10] it was found that the results depend both on the G_V and F_V coupling constants which are a bit different than the values provided by VMD where $F_V = 2G_V$. Actually, as shown in [31], the use of the empirical values of G_V and F_V extracted from $\rho \to \pi\pi$ and $\rho \to e^+e^-$ decays or from $\phi \to \pi\pi$ and $\phi \to e^+e^-$ decays leads to appreciable differences in the results in the case of the $\phi \to \pi^0\pi^0\gamma$ decay.

2 Chiral loop contributions

The chiral loop contribution to the $\rho \to \pi^0\pi^0\gamma$ decay was already formulated in ref. [10], where the chiral unitary approach was used to deal with the final state interaction of the meson-meson system. We follow the same procedure here and show the explicit form of the transition amplitudes for all the decays considered in this paper, $\rho \to \pi^0\pi^0\gamma$, $\rho \to \pi^0\eta\gamma$, $\omega \to \pi^0\pi^0\gamma$ and $\omega \to \pi^0\eta\gamma$.

We shall make use of the chiral Lagrangian for vector mesons of ref. [29] and follow the lines of ref. [30] in the treatment of the radiative meson decay. The Lagrangian coupling vector mesons to the pseudoscalar mesons and photons is given by ref. [29],

$$\mathcal{L} = \frac{F_V}{2\sqrt{2}} < V_{\mu\nu} f_+^{\mu\nu} > + \frac{iG_V}{\sqrt{2}} < V_{\mu\nu} u^\mu u^\nu >, \tag{2}$$

where $V_{\mu\nu}$ is a 3×3 matrix of antisymmetric tensor fields representing the octet of vector mesons, $f_+^{\mu\nu}$ is related to the photon field, u^μ are SU(3) matrix involving derivatives of the pseudoscalar meson fields and $< >$ denotes the trace in flavour space. The couplings G_V and F_V are deduced from the $\rho \to \pi^+\pi^-$ and $\rho \to e^+e^-$ decays, and taken to be $G_V = 69$ MeV and $F_V = 154$ MeV [30]. A singlet field, ω_1, is introduced through the substitution $V_{\mu\nu} \to V_{\mu\nu} + I_3 \times \frac{\omega_{1,\mu\nu}}{\sqrt{3}}$ with I_3 the 3×3 unit matrix, and the physical ϕ and ω meson fields are defined by assuming the ideal mixing. Here the F_V term include the $VPP\gamma$ (V vector and P pseudoscalar) couplings and the G_V term include VPP and $VPP\gamma$ couplings.

From the Lagrangian, the basic couplings to evaluate the loop contributions to the present decays are,

$$\begin{aligned}
t_{\omega K^+ K^-} &= -\frac{1}{2}\frac{G_V M_\omega}{f_\pi^2}(p - p')_\mu \epsilon^\mu(\omega) \\
t_{\omega K^+ K^-\gamma} &= e\frac{G_V M_\omega}{f_\pi^2}\epsilon_\nu(\omega)\epsilon^\nu(\gamma) \\
&+ \frac{e}{M_\omega f_\pi^2}\left(\frac{F_V}{2} - G_V\right) P_\mu \epsilon_\nu(\omega)[k^\mu \epsilon^\nu(\gamma) - k^\nu \epsilon^\mu(\gamma)] \tag{3}
\end{aligned}$$

with p_μ, p'_μ are K^+, K^- momenta, P_μ the ω meson momentum and k_μ the photon momentum. There are no couplings $\omega\pi^+\pi^-$ nor $\omega\pi^+\pi^-\gamma$. The couplings for ρ meson are given explicitly in ref. [10]. The pion decay constant f_π is taken to be 92.4 MeV.

The ρ^0 and ω decays into neutral mesons and photon can take place through the loop contributions shown in figure 1. The technology to introduce the final state interaction is developed in ref. [32, 33] and applied to the
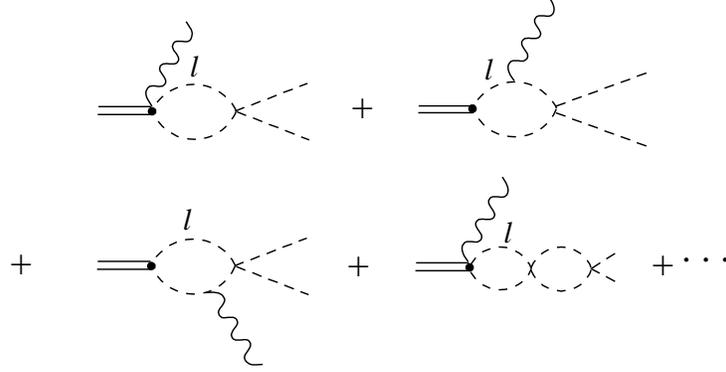
Figure 1: Loop diagrams included in the chiral loop contributions. The intermediate states in the loops can be K^+K^- or $\pi^+\pi^-$.

radiative decay of ρ^0 and ϕ mesons in ref. [10]. Using gauge invariance arguments one finds that the loop function is finite and that the meson-meson scattering amplitude factorizes in the loop integral with its on shell value. Following the procedure in ref. [10], we can write the explicit expression for the transition amplitudes for $\rho \to \pi^0\pi^0\gamma$ and $\omega \to \pi^0\pi^0\gamma$ as,

$$
\begin{aligned}
t^{loop}_{\rho \to \pi^0\pi^0\gamma} &= 2e\frac{G_V M_\rho}{f_\pi^2}\left(\tilde{G}_{\pi\pi}t_{\pi^+\pi^-,\pi^0\pi^0} + \frac{1}{2}\tilde{G}_{KK}t_{K^+K^-,\pi^0\pi^0}\right)\epsilon_\mu(\rho)\epsilon^\mu(\gamma) \\
&+ \frac{2e}{f_\pi^2}\left(\frac{F_V}{2} - G_V\right)q\left(G_{\pi\pi}t_{\pi^+\pi^-,\pi^0\pi^0} + \frac{1}{2}G_{KK}t_{K^+K^-,\pi^0\pi^0}\right)\epsilon_\mu(\rho)\epsilon^\mu(\gamma) \\
t^{loop}_{\omega \to \pi^0\pi^0\gamma} &= 2e\frac{G_V M_\omega}{f_\pi^2}\frac{1}{2}\tilde{G}_{KK}t_{K^+K^-,\pi^0\pi^0}\epsilon_\mu(\omega)\epsilon^\mu(\gamma) \\
&+ \frac{2e}{f_\pi^2}\left(\frac{F_V}{2} - G_V\right)q\frac{1}{2}G_{KK}t_{K^+K^-,\pi^0\pi^0}\epsilon_\mu(\omega)\epsilon^\mu(\gamma)
\end{aligned}
\tag{4}
$$

where q is the photon momentum in the vector meson rest frame, $\tilde{G}$ the loop function with the photon attached, G the ordinary two meson propagator function, and $t_{M_1 M_2, M_1' M_2'}$ is the strong transition matrix element. The $\tilde{G}$ is defined in ref. [10] and has the analytic expression given in [32]. The two meson propagator function, G is the one appearing in the Bethe-Salpeter equation for the meson-meson scattering and is regularized in ref. [11] using a cut-off parameter. The strong transition matrix element, $t_{M_1 M_2, M_1' M_2'}$, is also evaluated in ref. [11].

The transition amplitudes to the $\pi^0\eta\gamma$ final state are readily obtained by omitting the pion loop contribution for ρ meson and replacing the $t_{K^+K^-,\pi^0\pi^0}$ into $t_{K^+K^-,\pi^0\eta}$ for both ρ and ω mesons.
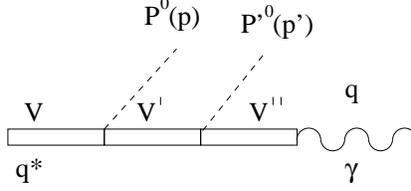
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Figure 2: Feynman diagram corresponding to the vector meson exchange contribution. V and V' are either ρ or ω mesons, and P, P' can be either π^0 or η mesons.

3   Sequential mechanism contributions

Apart from the loop contributions studied in section 2, there is another decay mechanism based on vector meson exchange whose contribution is as important as the one coming from the loops in the case of $\rho \to \pi^0\pi^0\gamma$, and dominant in the rest of the decays studied here. This mechanism has been studied in [2, 9, 23] and in this section we will follow closely these references.

The Feynman diagrams corresponding to this mechanism, $V(q^*, \epsilon^*) \to P(p)P'(p')\gamma(q, \epsilon)$, are of the form shown in figure 2. The vertices involved in these diagrams come from the Lagrangians:

$$\begin{aligned}
\mathcal{L}_{VVP} &= \frac{G}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle \\
\mathcal{L}_{V\gamma} &= -4f^2 eg A_\mu \langle Q V^\mu \rangle
\end{aligned} \tag{5}$$

where $G = \frac{3g^2}{4\pi^2 f}$ is the $\rho\omega\pi$ coupling and g can be related to the G_V coupling of the chiral resonance Lagrangians of eq. (2), $g = -\frac{G_V M_\rho}{\sqrt{2}f^2}$. The total amplitude has the form:

$$\mathcal{A}(V \to P^0 P'^0 \gamma) = C_{VPP'\gamma}\left(\frac{G^2 e}{\sqrt{2}g}\right)\left\{ \frac{P^2\{a\} + \{b(P)\}}{M_{V_1}^2 - P^2 - iM_{V_1}\Gamma_{V_1}} + \frac{P'^2\{a\} + \{b(P')\}}{M_{V_2}^2 - P'^2 - iM_{V_2}\Gamma_{V_2}} \right\} \tag{6}$$

where the $\{a\}$ and $\{b\}$ functions and the $C_{VPP'\gamma}$ coefficients are defined in ref. [2] and P and P' are the momenta of the intermediate resonance in the $t-$ and $u-$ (V_1 and V_2) channels respectively. The amplitude $\mathcal{A}$ corresponds to our t matrices changing the sign.

From the amplitude of eq. (6) one can calculate the branching ratios of the different decay modes. The results are given in table 4. In the calculation of the ω decay channels we have used a momentum-dependent width for the intermediate ρ meson in the propagators, which leads to an enhancement of the decay width of around 12% when compared to the calculation performed with a constant width, as pointed out in refs. [23, 24].

3.1 ρ-ω mixing effects

In addition to the VMD contribution, the incorporation of isospin violation effects (due to quark mass differences and electromagnetic corrections) allowing the mixing of the ρ and ω resonances is readily possible. This mixing is well known and it has been seen to be relevant in processes like the $\omega \to \pi^+\pi^-$ decay or in the pion form factor in the ω region [34, 35, 36]. A treatment of this mixing within χPT was done in [37]. Here we will follow the study of its effects in the radiative decays of vector resonances done in [23, 24], where the mixing allows the transition $V \to V'$ in the process: $V \to V' \to PP'\gamma$. Thus, the amplitudes can be written as $\mathcal{A}_0(V \to PP'\gamma) + \epsilon\mathcal{A}(V' \to PP'\gamma)$, where $\mathcal{A}_0(V \to PP'\gamma)$ includes the contributions coming from VMD and the loops and ϵ is the mixing parameter:

$$\epsilon \equiv \frac{\Theta_{VV'}^2}{M_V^2 - M_{V'}^2 - i(M_V\Gamma_V - M_{V'}\Gamma_{V'})} \tag{7}$$

with [37]

$$\Theta_{VV'}^2 = \frac{M_V^2}{M_{V'}^2}\left[-(m_{K^0}^2 - m_{K^+}^2) + (m_{\pi^0}^2 - m_{\pi^+}^2) + \frac{e^2 F_V^2}{3}\right] \tag{8}$$

We have considered this effect in the calculation of the $\omega \to \pi^0\pi^0\gamma$, $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ decays. There is still another effect of this mixing, which is that it modifies the V' propagator in $\mathcal{A}_0$ by:

$$\frac{1}{D_{V'}(s)} \to \frac{1}{D_{V'}(s)}\left(1 + \frac{g_{V\pi\gamma}}{g_{V'\pi\gamma}}\frac{\Theta_{VV'}^2}{D_V(s)}\right) \tag{9}$$

where $D_V = s - M_V^2 + iM_V\Gamma_V$. This effect is relevant in the case of the ρ propagator since $g_{\omega\pi\gamma}/g_{\rho\pi\gamma} = 3$, according to $SU(3)$ symmetry, and in fact makes the branching ratio a 8% larger in the $\omega \to \pi^0\pi^0\gamma$ case and a 11% larger in the $\omega \to \pi^0\eta\gamma$ case.

4 Numerical results

Using the transition amplitudes described in the previous sections, we can calculate the differential decay widths of the ρ and ω mesons as,

$$\frac{d\Gamma}{dM_I} = \frac{1}{64\pi^3} \int_{m_\pi}^{M_V - q - m'} d\omega \frac{M_I}{M_V^2} \bar{\sum} \sum |t|^2 \theta(1 - A^2), \tag{10}$$

where M_I is the invariant mass of the final two mesons, M_V the initial vector meson mass (M_ρ or M_ω), m' is the pion mass for the $\pi\pi\gamma$ decay and η mass for the $\pi\eta\gamma$ decay and q is the photon momentum in the initial vector meson rest frame. A accounts for the cosine of the angle between the π^0 and the photon and it is defined as,

$$A = \frac{1}{2pq} \left[(M_V - \omega(p) - q)^2 - m'^2 - p^2 - q^2 \right], \tag{11}$$

where p and $\omega(p)$ are the π^0 momentum and energy in the initial vector meson rest frame. A symmetry factor $1/2$ must be implemented in eq. (10) in the case of $\pi^0\pi^0$ in the final state.

The spin sum and average of the transition amplitudes, $\bar{\sum} \sum |t|^2$, can be expressed using the contravariant tensor F^{ij} as:

$$\bar{\sum} \sum |t|^2 = \frac{1}{3} \left[F^{ij} F^{ij*} - \frac{1}{|q|^2} (F^{ij} q_j)(F^{ij'*} q_{j'}) \right], \tag{12}$$

where the tensor expression F^{ij} of the transition amplitude t is defined as

$$t \equiv F^{ij} \epsilon_i(V) \epsilon_j(\gamma). \tag{13}$$

We show the total decay widths obtained in table 4. There we can see, in agreement with [23], that the dominant contribution is the one corresponding to the sequential mechanism in all cases except for the $\rho \to \pi^0\pi^0\gamma$ decay, where the loop contribution is comparable. As we can see in figure 3 the interference between these two contribution is constructive. It is worth noting that the final shape of the mass distribution with the sum of the sequential and loop contributions is rather different from the one obtained with either of the two mechanisms. Experimental information on this observable would thus be most welcome. The loop contribution for the $\omega \to \pi^0\pi^0\gamma$ decay was estimated small in [23] using qualitative arguments. Here we corroborate this claim performing the actual calculation and find it also small in the case of the $\rho \to \pi^0\eta\gamma$ and $\omega \to \pi^0\eta\gamma$ decays, due to the relatively high mass of the kaons. The ρ-ω mixing effects are negligible in the $\rho \to \pi^0\pi^0\gamma$ decay, but relevant in the rest of the decays. Although the mixing contribution is by itself small, it has important interferences with the sequential contribution, and in addition modifies the resonance propagators involved, as was already mentioned in section 3.1.
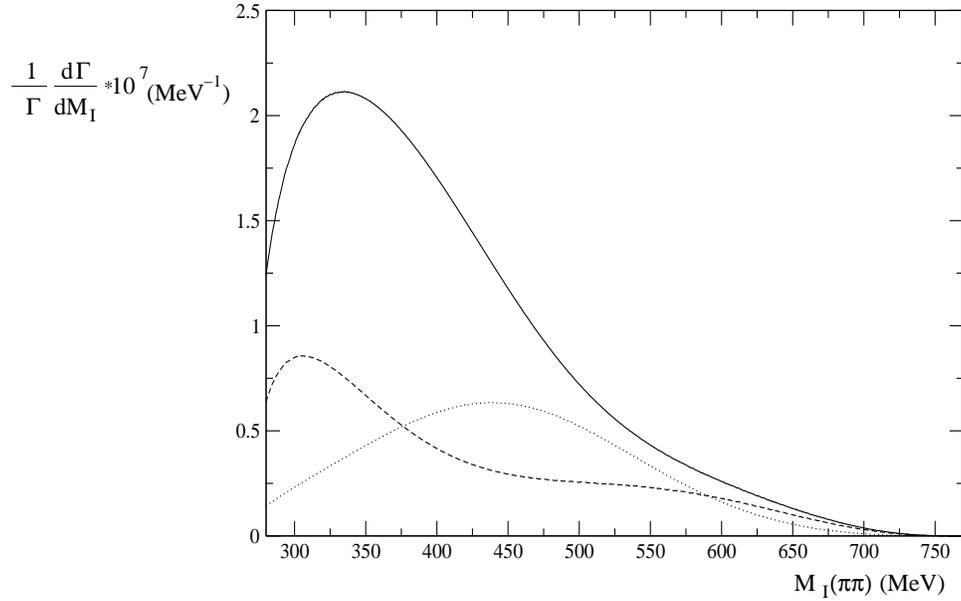
Figure 3: $dB(\rho \to \pi^0\pi^0\gamma)/dM_I$ as a function of the invariant mass of the two pions. Dashed line: sequential contribution; dotted line: loop contribution; solid line: sum of both. The sequential and loop contributions interfere constructively.

BR	$\rho \to \pi^0\pi^0\gamma$	$\rho \to \pi^0\eta\gamma$	$\omega \to \pi^0\pi^0\gamma$	$\omega \to \pi^0\eta\gamma$
sequential	$1.5 \cdot 10^{-5}$	$6.6 \cdot 10^{-10}$	$4.3 \cdot 10^{-5}$	$3.0 \cdot 10^{-7}$
loops	$1.5 \cdot 10^{-5}$	$5.4 \cdot 10^{-11}$	$4.3 \cdot 10^{-7}$	$2.2 \cdot 10^{-9}$
sequential + ρ-ω mixing	not evaluated	$7.5 \cdot 10^{-10}$	$4.8 \cdot 10^{-5}$	$3.4 \cdot 10^{-7}$
Total	$4.2 \cdot 10^{-5}$	$7.5 \cdot 10^{-10}$	$4.7 \cdot 10^{-5}$	$3.3 \cdot 10^{-7}$

Table 1: Branching ratios due to the different contributions to the $V \to P^0 P'^0 \gamma$ decays considered.

The result obtained here for the $\rho \to \pi^0\pi^0\gamma$ branching ratio is in good agreement with the recent SND collaboration measurement [5]: $B(\rho \to \pi^0\pi^0\gamma) = (4.8^{+3.4}_{-1.8} \pm 0.2) \times 10^{-5}$. The same collaboration obtained for the $\omega \to \pi^0\pi^0\gamma$ decay a branching ratio $B(\omega \to \pi^0\pi^0\gamma) = (7.8 \pm 2.7 \pm 2.0) \times 10^{-5}$, thus confirming the previous and more accurate measurement of the GAMS collaboration [38], $B(\omega \to \pi^0\pi^0\gamma) = (7.2 \pm 2.5) \times 10^{-5}$. Our predicted branching ratio for this decay is within the error bars of these experimental values, as we can see in table 4. We should also mention that although we do not give errors in our numbers, an estimate of 20% theoretical error is realistic in view of the accuracy of the chiral Lagrangians used in eq. (5) to provide the radiative decays of the vector mesons [39]. For the case of the $\rho \to \pi^0\eta\gamma$ and $\omega \to \pi^0\eta\gamma$ decays there are not experimental data for the branching ratios.

Finally, in table 2 we compare our results with other analyses and also with the experiment. The theoretical approaches followed in the literature are quite varied. In ref. [40] current algebra, hard pions and Ward identities were used. In [41] an approach with low energy effective Lagrangians with gauged Wess-Zumino terms was followed. A different procedure was followed in [22] using chiral Lagrangians and the extended Nambu-Jona-Lasinio model to fix the couplings of the resonance contribution (the results given in table 2 are the largest ones in the intervals given in [22], which are still low compared with experiment). In [2] only the sequential VMD mechanisms were used and the results were improved in [9] for the $\rho \to \pi^0\pi^0\gamma$ decay including the one loop χPT contribution. This latter point is further improved in [23], where a more realistic $\pi\pi$ isoscalar amplitude is used. In [10] only the loop contributions were evaluated, and here the sequential VMD mechanisms are considered in addition. Finally, in [24], where only the $\omega \to \pi^0\pi^0\gamma$ decay is evaluated, the sequential VMD mechanisms are considered including the $\omega - \rho$ mixing, but only the sequential mechanism is used for the ρ decay in this latter term. The order of magnitude in the different approaches is similar, with the exception of the results in [22] which seem abnormally low. Yet, the rates obtained in [23] and in the present work match better with the experimental $\rho \to \pi^0\pi^0\gamma$ decay width. Table 2 shows also the evolution of the approaches and the results with the time, and how the original VMD mechanisms suggested in [1] have survived, while the advent of χPT and its unitary extensions have brought the mechanisms needed to obtain a satisfactory result for the $\rho \to \pi^0\pi^0\gamma$ decay, as seen in [23] and the present work. With respect to this last reference our approach adds the novel thing of using directly the $\pi\pi$ amplitudes from $U\chi PT$, while in [23] a phenomenological model for the T matrix accounting explicitly for the σ and $f_0(980)$ mesons was used. In our approach both mesons are dynamically generated from the multiple scattering of pions and kaons driven by the

Work	$\rho \to \pi^0\pi^0\gamma$	$\rho \to \pi^0\eta\gamma$	$\omega \to \pi^0\pi^0\gamma$	$\omega \to \pi^0\eta\gamma$
[41]	2.9×10^{-5}	4.0×10^{-6}	8.2×10^{-5}	6.3×10^{-6}
[40]	1.1×10^{-5}	$---$	2.7×10^{-5}	$---$
[22]	4.7×10^{-6}	2.0×10^{-10}	1.4×10^{-5}	8.3×10^{-8}
[2]	1.1×10^{-5}	4×10^{-10}	2.8×10^{-5}	1.6×10^{-7}
[9]	2.6×10^{-5}	4×10^{-10}	2.8×10^{-5}	1.6×10^{-7}
[10]	1.4×10^{-5}	$---$	$---$	$---$
[24]	$---$	$---$	$(4.6 \pm 1.1) \times 10^{-5}$	$---$
[23]	3.8×10^{-5}	$---$	$(4.5 \pm 1.1) \times 10^{-5}$	$---$
This work	4.2×10^{-5}	7.5×10^{-10}	4.7×10^{-5}	3.3×10^{-7}
Experiment	$(4.8^{3.4}_{-1.8} \pm 0.2)$ $\times 10^{-5}$ [5]		$(7.8 \pm 2.7 \pm 2.0) \times 10^{-5}$ [5] $(7.2 \pm 2.5) \times 10^{-5}$ [38]	

Table 2: Branching ratios of the different $V \to P^0 P'^0 \gamma$ decays in the literature.

dynamics of the lowest order chiral Lagrangian.

The unitary approach followed here for the meson meson interaction leads to the $f_0(980)$ and $a_0(980)$ resonances for the $\pi^0\pi^0$ and $\pi^0\eta$ final states, respectively, without having to introduce them explicitly. One may wonder what would be the contribution of the $f_1(1285)$, $a_1(1260)$, $f_2(1270)$ and $a_2(1320)$ resonances as intermediate states. Avoiding the discussion whether they could or could not be generated dynamically as their f_0, a_0 partners [42], we can deal with those mechanisms by considering tree level Feynman diagrams which rely upon empirical couplings of these resonances to vector mesons, pseudoscalars and photons. One of the possible mechanisms would then be the one of fig. 2 where V' is substituted by any of those resonances (with zero charge). However, the VRP vertex, with V any neutral vector meson, $P = \pi^0, \eta$ and $R = f_1, f_2, a_1, a_2$ (with zero charge), is not allowed by charge conjugation [43]. Analogously one can see in the explicit Lagrangians involving the a_1, f_1 resonances that such terms vanish [22]. Thus, we are left with the diagrams where a photon is produced in the first place, see fig 4. The topology of this diagram is actually the same one as that in the f_0, a_0 production considered so far. Next we see that if R is the f_1 or a_1 resonance the RPP' vertex with $P, P' = \pi^0, \eta$ is forbidden by parity reasons, because the decay must proceed in p-wave and f_1, a_1 have positive parity.

Finally we are left with the mechanism of fig. 4 with the f_2 or a_2 resonances. In order to estimate the contribution of these mechanisms to the decay processes studied here, or to the radiative ϕ decay of [10], we rely upon the results obtained in [44] in the study of the $\gamma\gamma \to \pi^0\pi^0$, $\pi^0\eta$, with obvious similarity to the $V \to \gamma P P'$ process studied here. In this work the contribution of the f_2 and a_2 resonances was considered and found to be very
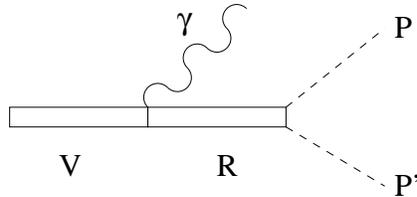
Figure 4: Feynman diagram corresponding to the a_2, f_2 meson exchange contribution. V is a vector meson, R is a f_2 or a_2 resonance and P, P' can be either π^0 or η mesons.

important in the regions around the corresponding resonance poles. However, as the calculation provides, or one can see explicitly in figs. 7 and 10 of [44], the extrapolation of the resonance contribution down to energies below the ρ and ω mass, which we have in the decays studied here, is negligible. At these energies the effect of the f_0, a_0 resonances which are closer in energy are relatively more important, and even then they do not play a significant role in the ρ and ω radiative decay.

Although we have not studied here the radiative ϕ decay, it is worth taking advantage of the former discussion to estimate the effects of the f_2, a_2 intermediate states in this decay. Once again the figures of ref. [44] mentioned above are illustrative. Fig. 10 shows that for $\pi^0\eta$ production the a_2 contribution would be a small fraction of the a_0 contribution at energies below 1020 MeV found in this decay. The case of the f_2 resonance is more subtle because as one can see in fig. 7 of [44] the f_0 resonance shows up only weakly in the $\gamma\gamma \to \pi^0\pi^0$ reaction and the background of the a_2 resonance at energies around 1 GeV does not seem negligible. However, the small signal of the f_0 resonance in this reaction is due, as noted in [44], to a strong cancellation between the f_0 contribution and the one with an ω in the intermediate state in the $\gamma\gamma \to \pi^0\pi^0$ process. However, while the $\gamma\omega\pi^0$ vertex is sizeable, the $\phi\omega\pi^0$ vertex is not allowed by isospin conservation, and hence the cancellation does not appear in the radiative decay of the ϕ, making the f_0 production sizeable as clearly seen in the experimental data [3, 4, 5] which are dominated by the $\phi \to f_0\gamma$ decay. In the absence of that cancellation the f_2 contribution to the radiative decay of the ϕ into $\pi^0\pi^0\gamma$ is also very small compared to the dominant $\phi \to f_0\gamma$ contribution.

5   Conclusions

We have studied the radiative decays of the ρ and ω mesons into two neutral mesons including the mechanisms of sequential vector meson decay, ρ-ω mixing and chiral loops. In the case of the ω decays we find, confirming previous findings, that the sequential mechanism is dominant. We also find that the

loop contribution is very small for the $\rho \to \pi^0 \eta \gamma$ case. The ρ-ω mixing was found non negligible for the $\rho \to \pi^0 \eta \gamma$, $\omega \to \pi^0 \pi^0 \gamma$ and $\omega \to \pi^0 \eta \gamma$ decays. The loop contribution is very important in the case of the $\rho \to \pi^0 \pi^0 \gamma$ and the branching ratio obtained in this case, with the sum of the sequential and loop mechanisms, is about three times larger than with either mechanism alone, leading to values compatible with present experimental measurements. The shape of the $\pi\pi$ mass distribution is also found rather different to that with either of the mechanisms alone. We have also estimated the effects of the f_2, a_2 intermediate resonances concluding that they are negligible for the decays studied here. In the case of the $\omega \to \pi^0 \pi^0 \gamma$ decay our predicted branching ratio is also in agreement with the experimental values. The results obtained for the $\rho \to \pi^0 \pi^0 \gamma$ decay provide a further consistency check of the $U\chi PT$ approach to meson-meson interaction and its underlying interpretation of the nature of the scalar mesons. Further measurements on invariant mass distributions would provide extra tests of these ideas, allowing us to gain further insight on the controversial nature of the scalar mesons.

Acknowledgments

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