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CHIRAL UNITARY APPROACH TO THE K^- DEUTERON SCATTERING LENGTH

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Abstract

Starting from a recent model where the $\bar{K}N$ amplitudes are evaluated from the chiral Lagrangians using a coupled channel unitary method, we evaluate here the scattering length for K^- deuteron scattering. We find that the double scattering contribution is very large compared to the impulse approximation and that the charge exchange contribution of this rescattering is as large as the sequential K^- scattering on the two nucleons. Higher order rescattering corrections are evaluated using coupled channels Faddeev equations with K^- and $\bar{K}^0$. The higher order corrections involving intermediate pions and hyperons are found negligible.

I. INTRODUCTION

The low energy scattering of K^- with deuterium has been the subject of much study in the past [1,2] and it is one of the processes where the impulse approximation is manifestly insufficient, the rescattering terms being quite large. The input in all these studies is elementary amplitudes for $\bar{K}N$ scattering which are either taken from experiment or evaluated within theoretical models. The theoretical models for $\bar{K}N$ are rather involved since there are many coupled channels which have to be dealt with consistently (concretely, 10 physical channels in the K^-p channel, K^-p, $\bar{K}^0n$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\pi^0\Sigma^0$, $\pi^0\Lambda$, $\eta\Sigma^0$, $\eta\Lambda$, $K^+\Xi^-$, $K^0\Xi^0$). Theoretical studies with coupled channels were used in Refs. [3–5] fitting the input to the data. In Ref. [5] the strength of the different transition potentials was determined from fits to the data allowing only modifications of up to 50 percent from the SU(3) relations.

The introduction of chiral Lagrangians in the meson baryon sector [6] has allowed one to deal with this interaction from the modern chiral perspective. Yet, a unitary treatment

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with coupled channels is necessary in this case since perturbation theory cannot be applied, among other reasons due to the presence of the $\Lambda(1405)$ resonance below the $\bar{K}N$ threshold. Coupled channel Lippmann Schwinger equations are used in Refs. [7,8] including the channels which are physically open, and some terms from higher order chiral Lagrangians are obtained from fits to experiment. In Ref. [9] the $\eta\Sigma$ and $\eta\Lambda$ channels are also included and a good description of the low energy data is obtained, amongst them the properties of the $\Lambda(1405)$ resonance, which is generated dynamically from the lowest order chiral Lagrangian with the coupled channel equations. In this later case only the lowest order Lagrangian was used together with a cut off which was the only free parameter of the theory. A justification of the success of the method omitting the higher order Lagrangians can be seen by comparing the similar success in the meson meson sector of the coupled channel equations using the lowest order Lagrangians [10] and the more refined Inverse Amplitude Method in coupled channels [11], case which also includes the $O(p^4)$ Lagrangian (see also discussions to this respect in Refs. [12] and [13]).

The case of K^- deuteron scattering requires also the explicit treatment of the coupled channels. The Faddeev equations rely already on partial summations over the different channels which lead to the $\bar{K}N$ t matrix on each individual nucleon, but even then the explicit channels appear in the multiple collisions with two different nucleons and the Faddeev equations can be generalized to these channels [14–16]. In the present work we follow these lines but we observe that the relevant channels in the Faddeev equations are the K^-N and $\bar{K}^0N$ channels. The channels involving $\pi\Sigma$, and other inelastic ones by analogy, require at least three successive collisions on the nucleons of the deuteron and provide a negligible contribution to the deuteron scattering length. The Faddeev equations with the $\bar{K}N$ channels lead to an analytical formula involving several of the scattering lengths of $\bar{K}N$, which improves on the one used in Ref. [17], where only the elastic collisions of the K^- with the nucleons are considered and the charge exchange is neglected.

The measurement of the K^- deuteron scattering length can thus provide information on some of these amplitudes provided the others are already known. Assuming isospin symmetry, the knowledge of the K^-p scattering amplitude allows one to obtain the K^-n scattering length using the Faddeev formula. However, one of the findings of this work and the one of Ref. [9] is that isospin symmetry is not very accurate for energies close to threshold so one has to admit certain uncertainties when extracting the elementary amplitudes from the deuteron scattering data. In any case the deuteron results will provide extra checks of accuracy of the modern chiral theories used for the $\bar{K}N$ interaction.

Our treatment involves only the evaluation of the strong interaction scattering length. Coulomb corrections to the Deser formula [18] to extract the scattering length from the measurement of the width and shift of the 1s level of the K^- deuteron atom planned at Frascati [19] have been worked out in Ref. [17].

II. K^-N SCATTERING LENGTHS

As mentioned in the introduction, the dynamics of $\bar{K}N$ scattering at low energies is dominated by the presence of the $\Lambda(1405)$ resonance and needs to be described by non-perturbative methods. In this section we review the approach followed in Ref. [9] and present the results for the scattering lengths of the elementary $\bar{K}N$ reactions needed in the calculation of the K^-d scattering amplitude, namely $K^-p \to K^-p$, $K^-n \to K^-n$, $\bar{K}^0 n \to \bar{K}^0 n$ and $K^-p \to \bar{K}^0 n$.

The starting point is the lowest-order chiral Lagrangian coupling mesons and baryons, which in the case of meson-baryon transition amplitudes reduces to

$$L_1^{(B)} = \langle \bar{B} i \gamma^\mu \frac{1}{4f^2} [(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi) B - B (\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi)] \rangle \ , \tag{1}$$

where Φ and B denote the octets of pseudoscalar mesons and $1/2^+$ baryons, respectively, and the symbol $\langle\rangle$ stands for the trace of SU(3) matrices.

From the Lagrangian of Eq. (1) one derives all possible transition amplitudes between the different meson-baryon states of a given charge and strangeness that can be built from the meson and baryon octets. There are ten such channels for K^-p scattering, namely K^-p, $\bar{K}^0 n$, $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $\eta\Lambda$, $\eta\Sigma^0$, $K^+\Xi^-$ and $K^0\Xi^0$, and six in the case of K^-n scattering, namely K^-n, $\pi^0\Sigma^-$, $\pi^-\Sigma^0$, $\pi^-\Lambda$, $\eta\Sigma^-$ and $K^0\Xi^-$. At low energies all the possible amplitudes reduce to the form

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k_j^0 + k_i^0) \ , \tag{2}$$

where k_j^0, k_i^0 are the initial, final energies of the mesons and the explicit values of the coefficients C_{ij} can be found in Ref. [9].

Using average masses for each particle multiplet, it is also possible to work in isospin formalism. Making the appropriate basis transformation, the transition coefficients of good isospin, D_{ij} for $I=0$ channels ($\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, $K\Xi$) and F_{ij} for $I=1$ ones ($\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$, $K\Xi$), can be easily derived from the C_{ij} coefficients involving K^-p and related channels and are also given in Ref. [9].

The lowest-order amplitudes of Eq. (2) are then inserted in a coupled-channel Bethe-Salpeter equation

$$t_{ij} = V_{ij} + V_{il}\, G_l\, t_{lj} \ , \tag{3}$$

from where one extracts the elastic and transition scattering amplitudes. The indices i, l, j run over all possible meson-baryon channels and G_l is the loop function containing the propagators of the meson and baryon in the intermediate states. Although in the former equation the last term on the right hand side involves in principle the off-shell dependence of the amplitudes, the simple form of V_{ij} in Eq. (2) allows to reabsorb the off-shell pieces of the amplitude into renormalization of the coupling constant f, as shown in Ref. [9]. Therefore, the V and t amplitudes simply factorize on-shell out of the loop integral and the problem reduces to one of solving a coupled set of algebraic equations, with G_l given by

$$\begin{aligned} G_l(\sqrt{s}) &= i \int \frac{d^4q}{(2\pi)^4} \frac{M_l}{E_l(-\vec{q})} \frac{1}{\sqrt{s} - q^0 - E_l(-\vec{q}) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon} \\ &= \int_{|\vec{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_l(\vec{q})} \frac{M_l}{E_l(-\vec{q})} \frac{1}{\sqrt{s} - \omega_l(\vec{q}) - E_l(-\vec{q}) + i\epsilon} \ , \end{aligned} \tag{4}$$

where M_l, E_l and m_l stand, respectively, for the baryon mass, the baryon energy and the meson mass in the intermediate state, and $\sqrt{s}$ is the total energy in the center-of-mass (CM) frame.

The approach of Ref. [9] and summarized here depends on one parameter, the loop regularization cut-off, $q_{\rm max}$. Using the particle basis, a value of 630 MeV was adjusted to reproduce the K^-p scattering branching ratios at threshold. At the same time the weak decay constant was slightly modified to $f = 1.15 f_\pi$, a value lying in between the empirical pion and kaon weak decay constants, in order to optimize the position of the $\Lambda(1405)$ resonance. The scattering cross sections, which were not used in the fit, were shown to be in good agreement with the low energy data.

The scattering lengths are obtained from the amplitudes t_{ij} through

$$a_{ij} = -\frac{1}{4\pi}\frac{m_N}{\sqrt{s}} t_{ij} \ , \tag{5}$$

and the relevant ones for the study of K^-d scattering are shown in Table 1 for two different energies. The value 1432.6 MeV corresponds to $m_K + m_N$, where m_K is the K^- mass and m_N an average of the neutron and proton masses. The value 1431.49 includes binding effects by assigning half of the deuteron binding energy of 2.22 MeV to each nucleon. We also give results for the scattering lengths from the isospin formalism, obtained by first solving the Bethe-Salpeter equation for the various isospin channels and then transforming the isospin amplitudes back to the particle basis. Note that the scattering lengths more affected by a slight change in the energy value or by the particular basis used are a_p, a_x and a_n^0, which contain the isospin $I = 0$ component where the $\Lambda(1405)$ resonance shows up.

III. MULTIPLE SCATTERING SERIES

It is well known that the impulse approximation fails to describe the K^- deuteron scattering length. Furthermore even the contribution of a few terms of the multiple scattering series does not give accurate values for the scattering length. This indicates that the multiple scattering series does not converge rapidly for K^-d elastic scattering at low energies. Therefore, in this case more sophisticated approaches based on the solution of the Faddeev equations are required.

On other hand, there are many difficulties in the solution of the Faddeev equations for the K^-d elastic scattering. The first one is related to the coupling to many inelastic channels with Σ, Λ, and Ξ baryons which make the solution of the problem technically difficult. The second problem is related to the isospin symmetry which is often used in the solution of the Faddeev equations. Indeed, as seen from the considerations in the former section, isospin for K^-N scattering is a good quantum number only with accuracy of 20%. In such a situation the use of the physical channels for the coupled equations becomes more realistic than using an isospin formalism. This of course increases the number of the coupled channels making the numerical procedure more complicated.

Thus getting an unambiguous information about K^-N scattering lengths from elastic K^-d scattering becomes a difficult task. However, below we present one theoretical scheme which should considerably facilitate the solution of this problem.

A. Single scattering (impulse) approximation

We will start our considerations with the well known results of the *impulse approximation* where only contributions from one (single) kaon scattering are taken into account. In this case, we can get the following expression for the s-wave K^-d scattering t-matrix (T_{Kd}) in terms of the elementary s-wave t-matrices which describe K^-N scattering on the proton (t_p) and neutron (t_n):

$$T_{Kd}(k',k) \;=\; \left[t_p(k',k) + t_n(k',k)\right] F_d(Q)\,, \tag{6}$$

where $\mathbf{Q} = (\mathbf{k}' - \mathbf{k})/2$ is the momentum transfer with initial and final kaon momentum $\mathbf{k}$ and $\mathbf{k}'$, respectively. $F_d(Q)$ is the elastic deuteron form factor

$$F_d(Q) = \int e^{-i\mathbf{Q}\cdot\mathbf{r}} \;\mid \phi_d(\mathbf{r}) \mid^2 \; d\mathbf{r} \tag{7}$$

normalized to unity at $Q = 0$. Therefore $\mid \phi_d(\mathbf{r}) \mid^2 = \mid u(r) \mid^2 + \mid w(r) \mid^2$, where $u(r)$ and $w(r)$ are the S- and D-components of the deuteron wave functions taken from Ref. [20].

For the low energy limit, when $\mathbf{k}$, $\mathbf{k}' \to 0$, taking into account the relations between t-matrices and scattering lengths (or amplitudes)

$$T_{Kd} = -\frac{4\pi(m_K + M_d)}{M_d} A_{Kd}, \qquad t_{p,n} = -\frac{4\pi(m_K + m_N)}{m_N} a_{p,n} \tag{8}$$

we obtain the following simple expression for the K^-d scattering length in the impulse approximation

$$A^{IA}_{Kd} = \frac{M_d}{m_K + M_d}\left(1 + \frac{m_K}{m_N}\right)(a_p + a_n) = (-0.49 + i\,2.08)\,\mathrm{fm}\,, \tag{9}$$

where M_d and m_N are the deuteron and nucleon masses, respectively, and m_K is the kaon mass. The numerical value was obtained using the elementary amplitudes a_p and a_n in the physical basis at W = 1431.49 MeV, which includes the effects of the deuteron binding energy, 2.22 MeV, and given in Table 1.

Note that in general within the impulse approximation the effects from the motion of the nucleons has to be taken into account in the evaluation of the elementary t-matrix. However, numerous investigations [21–24] show that the substitution for the nucleon momentum $\mathbf{p}_N \to \mathbf{p}_{eff.} = -(\mathbf{k} - \mathbf{k}')/2$ is a very good approximation. In the case of S-shell nuclei such approximation is even exact for the linear terms in $\mathbf{p}_N$. Therefore, we expect that in the limit $\mathbf{k} \to 0$ the static approximation, $\mathbf{p}_N = 0$, is reliable.

B. Double scattering contribution

The first correction to the impulse approximation is related to the contributions coming from the diagrams depicted in Fig. 1. We evaluate them using Feynman diagram rules. Then for the S-matrix we get

$$S_{Kd}^{(2)} = \int\int d^4x\, d^4x' \,\frac{1}{\sqrt{2V\omega_K}}\,\frac{1}{\sqrt{2V\omega_{K'}}}\,\varphi_p^*(x)\,\varphi_n^*(x')\,e^{-iKx}\,e^{iK'x'}\,\varphi_p(x)\,\varphi_n(x')$$
$$\times \int \frac{d^4q}{(2\pi)^4}\,\frac{i\,e^{iq(x-x')}}{q^2-m_K^2+i\epsilon}\,(-it_p)\,(-it_n)\,, \tag{10}$$

where $K=(\omega_K,\mathbf{k})$ and $K'=(\omega_K',\mathbf{k}')$ are kaon 4-momenta in the initial and final states and $\varphi_{p(n)}$ is the proton (neutron) wave function normalized to unity. The plane waves are normalized to unity in the volume V. The space part of the deuteron wave function in Eq. (10) can be written in terms of CM and relative coordinates, i.e. $\varphi_p(\mathbf{x})\,\varphi_n(\mathbf{x}') = \frac{1}{\sqrt{V}}e^{i\mathbf{K}_d\cdot\mathbf{R}}\varphi_d(\mathbf{r})$. Then the S-matrix can be related to the kaon-deuteron scattering T-matrix in the following way

$$S_{Kd}^{(2)} \,=\, 1 \,-\, i\frac{(2\pi)^4\,M_d}{V^2\sqrt{2\omega_K\,2\omega_K'\,E_d\,E_d'}}\,\delta(K+K_d-K'-K_d)\,T_{Kd}^{(2)}\,, \tag{11}$$

where E_d and E_d' are the total energies of the deuteron with momentum $\mathbf{K}_d$ and $\mathbf{K}_d'$ in the initial and final states, respectively.

In the CM frame and low energy limit $\mathbf{k}$, $\mathbf{k}'\to 0$ we obtain the following expressions and numerical values for the contribution from diagrams (a) and (b) of Fig. 1

$$A_{Kd}^{(2,a)} = \frac{M_d}{m_K+M_d}\left(1+\frac{m_K}{m_N}\right)^2 2\,a_p\,a_n\left\langle\frac{1}{r}\right\rangle = (-2.03-i0.17)\,\mathrm{fm}\,, \tag{12a}$$

$$A_{Kd}^{(2,b)} = -\frac{M_d}{m_K+M_d}\left(1+\frac{m_K}{m_N}\right)^2\,a_x^2\left\langle\frac{1}{r}\right\rangle = (-1.07+i1.46)\,\mathrm{fm}\,, \tag{12b}$$

where

$$\left\langle\frac{1}{r}\right\rangle = \frac{2}{\pi}\int_0^\infty F_d(q)\,dq = \int d\mathbf{r}\,\mid\varphi_d(r)\mid^2\,\frac{1}{r} = 0.449\,\mathrm{fm}^{-1} \tag{13}$$

and a_x is the scattering length (or amplitude) for the kaon charge exchange reaction $K^-p\to\bar{K}^0n$. Thus we can see that the contribution to the real part of A_{Kd} from the double scattering is larger than that from the impulse approximation. This is mainly due to the cancellation of the proton and neutron contributions from the single scattering.

C. Triple scattering and coupling with the $\Sigma\pi$ channel

Now let us estimate the contribution from the diagrams depicted in Fig. 2a. The corresponding S-matrix is

$$S_{Kd}^{(3)} = \int\int\int d^4x\, d^4x'\, d^4x''\,\frac{1}{\sqrt{2V\omega_K}}\,\frac{1}{\sqrt{2V\omega_{K'}}}\,\varphi_p^*(x'')\,\varphi_n^*(x')\,e^{-iKx}\,e^{iK'x''}\,\varphi_p(x)\,\varphi_n(x')$$
$$\times\int\frac{d^4q}{(2\pi)^4}\,\frac{i\,e^{iq(x-x')}}{q^2-m_K^2+i\epsilon}\int\frac{d^4q'}{(2\pi)^4}\,\frac{i\,e^{iq'(x'-x'')}}{q'^2-m_K^2+i\epsilon}\,(-it_p)\,(-it_n)\,(-it_p)$$
$$\times\int\frac{d^4p}{(2\pi)^4}\,\frac{i\,e^{ip(x-x'')}}{p^0-E(\mathbf{p})+i\epsilon}\,. \tag{14}$$

First let us evaluate the contribution from triple kaon scattering including also charge exchange processes. In contrast to the case of the double scattering considered above, now we have two mesons and one baryon propagators. As a first step we evaluate exactly the energy variable integration. After this we make the assumption of heavy baryons, i.e. $E(\mathbf{p})$ in the baryon propagator is replaced by the baryon mass. Then the integration over the three momentum of the baryon $\mathbf{p}$ gives rise to a $\delta^3(\mathbf{x}-\mathbf{x}'')$ function which brings together the $\mathbf{x}$ and $\mathbf{x}''$ coordinates. Formally this is equivalent to the so called *fixed scatterer approximation* often used in the literature. Using this approximation and taking into account that at low energies the on-shell kaon energy $\omega_K \to m_K$

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{e^{-i\mathbf{q}\cdot\mathbf{r}}}{\omega_K^2 - m_K^2 - \mathbf{q}^2 + i\epsilon} \to -\frac{1}{4\pi r} \;, \tag{15}$$

we obtain the following expression and numerical value for the triple kaon scattering contribution

$$\begin{aligned} A_{Kd}^{(3)} &= \frac{M_d}{m_K + M_d} \left(1 + \frac{m_K}{m_N}\right)^3 \left[a_p a_n (a_p + a_n) - a_x^2 (2a_n - a_n^0)\right] \left\langle \frac{1}{r^2} \right\rangle \\ &= (-3.21 - i0.43)\,\mathrm{fm}\,, \end{aligned} \tag{16}$$

where $< 1/r^2 >= \int d\mathbf{r} \mid \varphi_d(r) \mid^2 /r^2 = 0.289\,\mathrm{fm}^{-2}$ and the amplitude a_n^0 describes elastic scattering of the $\bar{K}^0$ meson on the neutron. Note, by comparing Eq. (16) with Eqs. (9), (12a), (12b) that the convergence of the multiple scattering series is rather poor.

To estimate the contribution which would come from the coupling with the $\Sigma\pi$ channel we, as an example, consider the $\Sigma^+\pi^-$ channel. The corresponding diagram is depicted in Fig. 2b. For its evaluation we will use again the fixed scatterer approximation. The only difference is in the treatment of the pion propagator. Now the on-shell pion energy in the intermediate state is $\omega_\pi = 220$ MeV and it is well known that in this region the p-wave contribution related with the excitation of the $\Delta(1232)$ resonance dominates, especially in the $\pi^- n$ elastic channel. Therefore, for the estimation we shall consider the contribution only from the Δ resonance taking $t_{\pi^- n} \approx t^{\Delta}_{\pi^- n}\, \mathbf{q}\cdot\mathbf{q}'$. In the description of the pion propagation we will use the so called K-matrix (or on-shell) approximation, i.e

$$\frac{1}{\omega^2 - m_\pi^2 - \mathbf{q}^2 + i\epsilon} \to -i\pi\,\delta(\omega_\pi^2 - m_\pi^2 - \mathbf{q}^2)\,. \tag{17}$$

For these energetic pions this approximation allows one to take into account the largest part of the pion rescattering contribution. The final expression obtained in this way for the contribution to the scattering length is the following

$$A_{Kd}^{(\Sigma)} = -\frac{M_d}{m_K + M_d} \left(1 + \frac{m_K}{m_N}\right)^2 \left(1 + \frac{m_\pi}{m_N}\right) a_\Sigma^2\, a^{(p)}_{\pi^- n} \left\langle q^2\, j_1^2 \right\rangle \;, \tag{18}$$

where $q = 174$ MeV is the on-shell pion momenta, $j_1(z)$ is the spherical Bessel function and

$$\left\langle q^2\, j_1^2 \right\rangle = q^2 \int d\mathbf{r} \mid \varphi_d(r) \mid^2 \, j_1^2(qr) = 0.083\,\mathrm{fm}^{-2}\,. \tag{19}$$

The value of the p-wave part of the $\pi^- n$ elastic scattering amplitude is $a^{(p)}_{\pi^- n} = 0.50 + i0.09$ fm which corresponds to the $\Delta(1232)$ contribution at $\omega_\pi = 220$ MeV. Now if we take for the amplitude of the $K^- p \to \Sigma^+ \pi^-$ reaction the value $a_\Sigma = -0.39 + i0.04$ fm from Ref. [9], we obtain that $A^{(\Sigma)}_{Kd} = -0.015 + i0.000$ fm which is only 0.5% of the contribution from the triple scattering given by Eq. (16). This estimation allows with a good accuracy to neglect the contributions from the coupling with inelastic channels, but we must keep the coupling with the kaon charge exchange channel. For this purpose in the next section we derive a formalism based on the Faddeev equations.

IV. SOLUTION OF THE FADDEEV EQUATION

As we have demonstrated in the previous section the convergence of the multiple scattering series is very poor. Therefore, we can not use the iteration procedure to calculate the $K^- d$ scattering length. For this purpose we follow a more general scheme based on the solution of the Faddeev equations. Note that since isospin is not a good quantum number, we will write these equations in the physical basis and present the elastic scattering T-matrix as a sum of the two Faddeev partitions

$$T_{Kd} \,=\, T_p \,+\, T_n \,, \tag{20}$$

where T_p and T_n describe the interaction of the K^--mesons with the deuteron starting with a first collision on a proton and a neutron, respectively.

Graphically these interactions are illustrated in Fig. 3 and they satisfy the following system of integral equations

$$\begin{aligned} T_p &= t_p + t_p\, G_0\, T_n + t^x_p\, G_0\, T^x_n \,, \\ T_n &= t_n + t_n\, G_0\, T_p \,, \\ T^x_n &= t^x_n + t^0_n\, G_0\, T^x_n + t^x_n\, G_0\, T_n \,, \end{aligned} \tag{21}$$

where G_0 is the free kaon propagator and t_p and t_n are the t-matrices for $K^- p$ and $K^- n$ elastic scattering, respectively. Note that for the proton partition, T_p, we have also a contribution from the charge exchange channel with elementary t-matrix, t^x_p, and the third Faddeev partition, T^x_n, which describes the $\bar{K}^0 nn \to K^- pn$ transition including multiple rescattering in the intermediate inelastic states. Through this term the coupling with the break-up channel is realized and it is expressed via the additional elementary charge exchange, t^x_n, and elastic $\bar{K}^0 n$ scattering, t^0_n, matrices.

The equations (21) are a set of operator equations. On the other hand, the final expression for the scattering length appears as an expectation value of the scattering operator with the deuteron ground state, i.e.

$$A_{Kd} \,=\, \frac{M_d}{m_K + M_d} \int d\mathbf{r} \;\mid \varphi_d(\mathbf{r}) \mid^2\; \hat{A}_{Kd}(r)\,, \qquad \hat{A}_{Kd}(r) = \hat{A}_p(r) + \hat{A}_n(r) \tag{22}$$

Indeed, the analytical expressions for the amplitudes $\hat{A}_p(r)$ and $\hat{A}_n(r)$ in Eq. (22) can be determined by the solution of the Faddeev equations (21). For that, let us apply recipes which we have found in the calculations of the multiple scattering series. First, following Eq. (15) the integral over the kaon propagator G_0 is replaced by $-1/4\pi r$. Second, using the relations (8), all the elementary matrices t_p, t_n, $t^x_p = t^x_n$ and t^0_n are replaced by their threshold values of the corresponding scattering lengths a_p, a_n, a_x and a_n^0, respectively, up to a factor. Then we get the following system of equations for the amplitudes $\hat{A}_p(r)$ and $\hat{A}_n(r)$:

$$\begin{aligned}
\hat{A}_p(r) &= \tilde{a}_p + \tilde{a}_p \frac{1}{r}\,\hat{A}_n(r) - \tilde{a}_x \frac{1}{r}\,\hat{A}_n^x(r)\,, \\
\hat{A}_n(r) &= \tilde{a}_n + \tilde{a}_n \frac{1}{r}\,\hat{A}_p(r)\,, \\
\hat{A}_n^x(r) &= \tilde{a}_x - \tilde{a}_n^0 \frac{1}{r}\,\hat{A}_n^x(r) + \tilde{a}_x \frac{1}{r}\,\hat{A}_n(r)\,,
\end{aligned} \tag{23}$$

where $\tilde{a} = a\,(1 + m_K/m_N)$. Note that there is a minus sign in the terms which lead to np configuration in the final state due to the fact that this configuration appears with minus sign in the isospin zero wave function of the deuteron, $(pn - np)/\sqrt{2}$.

After the solution of the system of equations (23) the amplitude $\hat{A}_{Kd}$ can be written in an analytic form

$$\hat{A}_{Kd}(r) = \frac{\tilde{a}_p + \tilde{a}_n + (2\tilde{a}_p\tilde{a}_n - b_x^2)/r - 2b_x^2\tilde{a}_n/r^2}{1 - \tilde{a}_p\tilde{a}_n/r^2 + b_x^2\tilde{a}_n/r^3}\,, \tag{24}$$

where $b_x = \tilde{a}_x/\sqrt{1+\tilde{a}_n^0/r}$ is the charge exchange amplitude renormalized due to the $\bar{K}^0 n$ rescattering. If we keep only terms of order $1/r$ in this solution we get

$$\hat{A}_{Kd}^{(1)}(r) = \tilde{a}_p + \tilde{a}_n + (2\tilde{a}_p\tilde{a}_n - \tilde{a}_x^2)\frac{1}{r} \tag{25}$$

which brings us to the *(IA + double scattering)* results in the multiple scattering approach [see Eqs. (9), (12a), (12b)]. In a similar way, by expanding up to order $(1/r)^2$ we can easily obtain our previous results for triple scattering.

The comparison of the full solution (24) with the results of the first iteration (25) shows that the main difference is at the small distance: at $r \to 0$ we have $A_{Kd} \to 0$ and $A_{Kd}^{(1)} \to \infty$. This difference is illustrated in Fig. 4. Thus we can conclude that at low energies the multiple rescattering essentially reduces the contribution from the short distances and hence the scattering length becomes less sensitive to short range correlation effects than one might expect from the truncation of the series at the level of $A_{Kd}^{(1)}$ and $A_{Kd}^{(2)}$, which involve $< 1/r >$ and $< 1/r^2 >$, respectively.

In Table 2 we collect our final results obtained using the elementary amplitudes in the physical and isospin basis. Here we again demonstrate the poor convergence of the multiple scattering series. The calculations are done at two energies, one where the deuteron binding effects are ignored, $W = 1432.6$ MeV, and another one where we take the physical mass of the deuteron, $W = 1431.49$ MeV. The energy dependence of the scattering amplitude makes the results sensitive to the 2.22 MeV binding of the deuteron, with differences of up to 20%.

We can see that the results obtained here using elementary scattering amplitudes that relied upon isospin symmetry differ somewhat from those obtained using the elementary amplitudes calculated with the physical basis. Particularly, the imaginary parts of the scattering length differ by about 40%. In addition, note that even if we take the a_n and a_p amplitudes from the physical basis and for the others we use the isotopic relations $a_x =$

$a_p - a_n$ and $a_n^0 = a_p$, we get $A_{Kd} = -2.33 + i\,2.48$. Comparison with the full results, $-1.99 + i\,2.28$, obtained within the physical basis further demonstrates the consequences of the isospin violation effects for K^--deuteron scattering.

With respect to the approach of Ref. [17], which uses a similar method to ours, we have included the charge exchange channels. We can see from Eqs. (12a),(12b) that the charge exchange double scattering is rather important and this is also the case when the full multiple scattering series is summed, as one can see in Table 2. There we show the results obtained from the multiple scattering series neglecting the charge exchange contribution ($b_x = 0$) in Eq. (24), which we call "only el.resc.". The "charge exch." results in the table denote the changes induced by the term b_x, i.e. A_{Kd}(charge exch.)= A_{Kd}(total)$-A_{Kd}$(only el.resc.). Our input for the elementary amplitudes is also slightly different than that used in Ref. [17]. Our result for the scattering length, $A_{Kd} = -1.99 + i\,2.28$ fm, has larger strength for both the real and imaginary parts that those found in Ref. [17], around $-0.7 + i\,1.2$ fm or those of Ref. [14], $-1.47 +\ 1.08$ fm.

V. CONCLUSION

We have studied K^- scattering on the deuteron at low energies and have evaluated the K^-d scattering length. The input consisted on elementary $\bar{K}N$ amplitudes previously calculated using chiral Lagrangians and a coupled channels unitary scheme. We found that the multiple scattering series on the deuteron was poorly convergent which forced us to sum it by means of Faddeev equations. We found that we needed to include the charge exchange channels in the Faddeev approach, but we could omit intermediate inelastic channels (involving for instance $\Sigma\pi$ states) which, however, were relevant in the evaluation of the elementary scattering matrices in Ref. [9].

We have found a K^-d scattering length of the order $-2.0 + i\,2.3$ fm which has somewhat larger strength, both in the real and imaginary parts, than in other approaches. We also found here that isospin is only an approximate symmetry for K^-d scattering and violation of the isospin symmetry can be as large as 40%, hence, one should not rely upon isospin considerations when evaluating the K^-d scattering length.

Comparison of the present results with the experimental results expected from the DEAR experiment at Frascati should bring light on some of the issues involved in the problem, like chiral symmetry and partial isospin breakup.

The findings of this paper should also be of much of use when trying to extract information on elementary amplitudes from the deuteron data. The formulas which we obtain would allow one to deduce a_n from A_{Kd} using isospin relationships, but, as discussed above, this would induce uncertainties of up to 40%. We have seen that the general formulas, without assuming isospin symmetry, rely upon four scattering lengths a_p, a_n, a_x, a_n^0. Knowledge of three of them from other experiments and the use of the deuteron data would allow one to obtain information on the fourth. Conversely, we can say that the deuteron data will introduce a further check of consistency between elementary amplitudes determined either experimentally or theoretically.

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TABLES

TABLE I. K^-N scattering lengths (in fm) in the physical and isospin bases

reactions	Physical basis $W = 1432.60$ MeV	Physical basis $W = 1431.49$ MeV	Isospin basis $W = 1431.49$ MeV
a_p $(K^-p \to K^-p)$	$-0.789 + i\,0.929$	$-0.983 + i\,1.148$	$-0.818 + i\,1.301$
a_n $(K^-n \to K^-n)$	$0.574 + i\,0.619$	$0.579 + i\,0.572$	$0.537 + i\,0.495$
a_x $(K^-p \to \bar{K}^0n)$	$-1.099 + i\,0.522$	$-1.318 + i\,0.669$	$-1.355 + i\,0.806$
a_n^0 $(\bar{K}^0n \to \bar{K}^0n)$	$-0.387 + i\,1.159$	$-0.593 + i\,1.192$	$-0.818 + i\,1.301$

TABLE II. K^--deuteron scattering length (in fm) calculated using different approximations

approximations	Physical basis $W = 1432.60$ MeV	Physical basis $W = 1431.49$ MeV	Isospin basis $W = 1431.49$ MeV
IA	$-0.260 + i\,1.872$	$-0.489 + i\,2.079$	$-0.339 + i\,2.172$
IA + double resc.	$-2.735 + i\,2.895$	$-3.585 + i\,3.709$	$-3.115 + i\,4.468$
IA + double+triple resc.	$-4.929 + i\,2.084$	$-6.794 + i\,3.274$	$-6.679 + i\,5.223$
A_{Kd} (only el.resc.)	$-1.161 + i\,1.336$	$-1.441 + i\,1.443$	$-1.343 + i\,1.652$
A_{Kd} (charge exch.)	$-0.454 + i\,0.573$	$-0.552 + i\,0.837$	$-0.497 + i\,1.122$
A_{Kd} (total)	$-1.615 + i\,1.909$	$-1.993 + i\,2.280$	$-1.840 + i\,2.774$

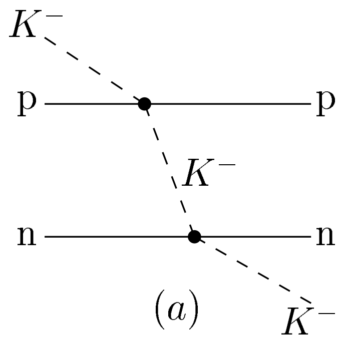

FIG. 1. Graphical illustration of the double scattering contributions

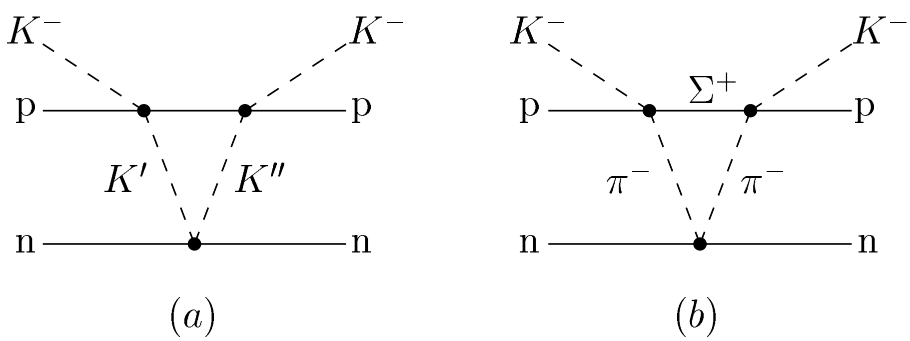

FIG. 2. Graphical illustration of the triple scattering contributions

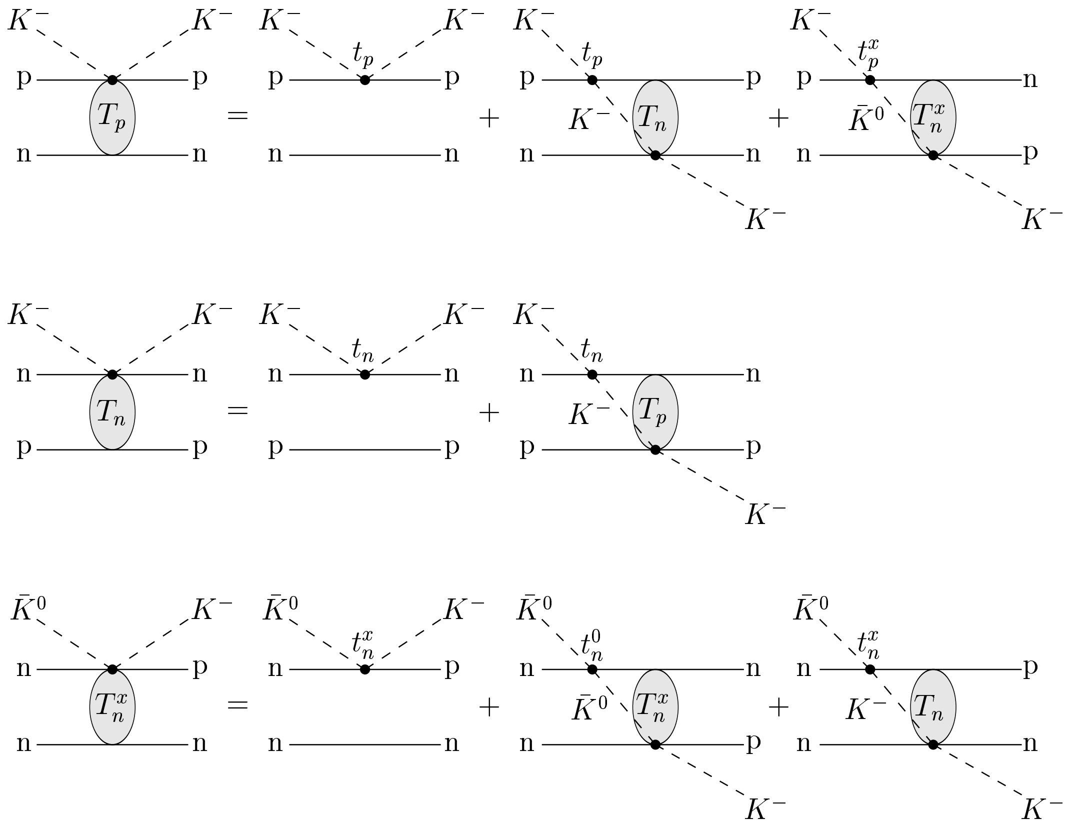

FIG. 3. Graphical illustration of the Faddeev partitions in kaon-deuteron scattering

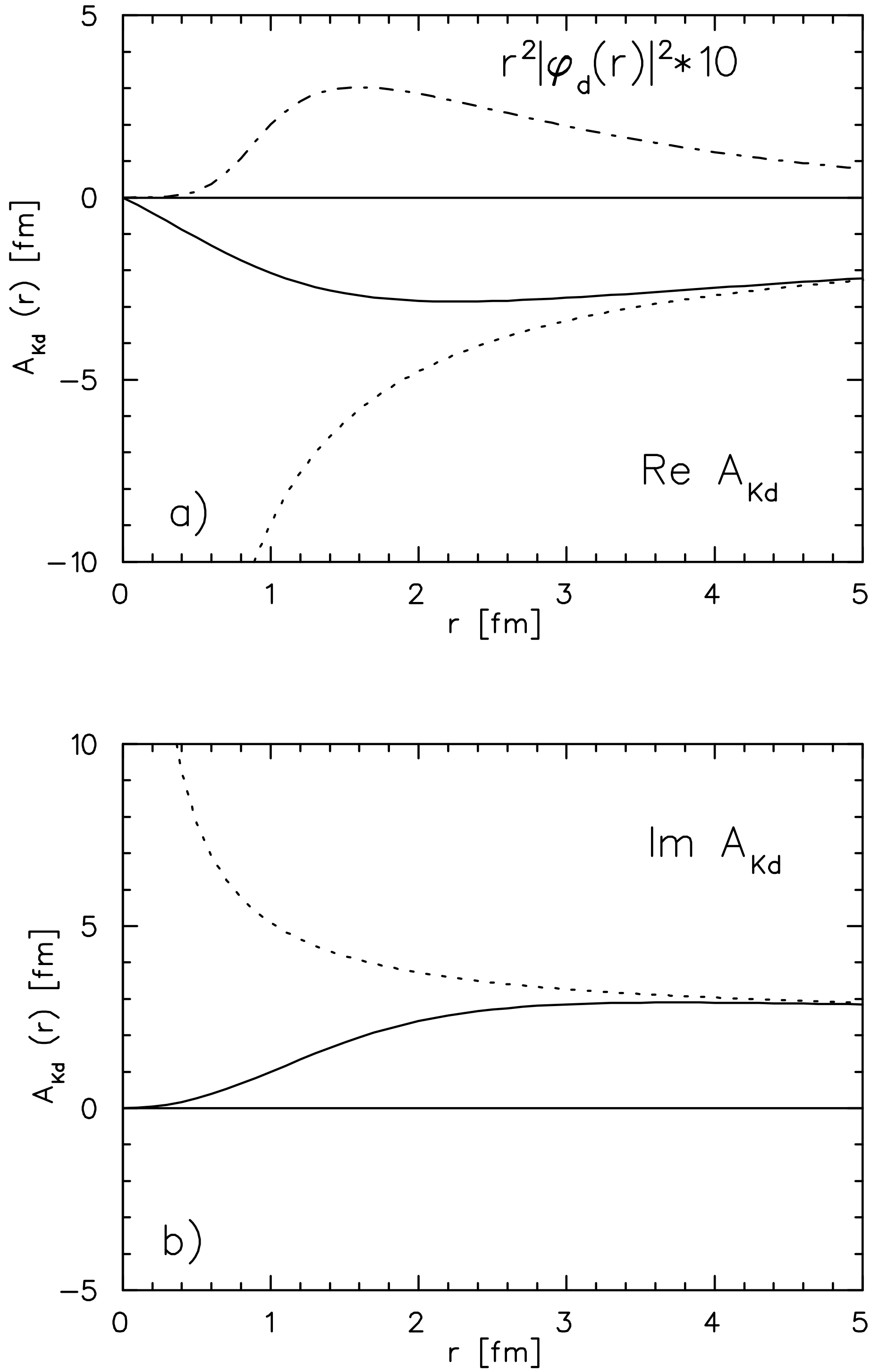

FIG. 4. $\hat{A}_{Kd}(r)$ (solid curves) and $\hat{A}^{(1)}_{Kd}(r)$ (dotted curves). In the upper panel we also show the deuteron wave function