

# Isoscalar Roper Excitation in $p(\alpha, \alpha')$ Reactions in the 10 – 15 $GeV$ Region

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## Abstract

Recent experiments at Saturne at 4  $GeV$  showed that the  $(\alpha, \alpha')$  reaction on the proton shows two distinctive peaks, which were associated to  $\Delta$  projectile excitation and Roper target excitation. A subsequent theoretical analysis has shown that this picture is qualitatively correct but there are important interference effects between the two mechanisms. Furthermore, at this energy the ratio of strengths for the Roper and  $\Delta$  peak is about 1/4. In the present paper we show that by going to the 10 – 15  $GeV$  region the interference effects become negligible, the signal for the Roper excitation is increased by more than an order of magnitude and the ratio of cross sections at the peaks for Roper and  $\Delta$  excitation becomes of the order of unity, thus making this range of energies ideal for studies of isoscalar Roper excitation.

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The  $(\alpha, \alpha')$  reaction on proton targets at kinetic energy  $T_\alpha = 4.2 \text{ GeV}$  was studied at SATURNE [1] and two distinctive peaks were identified (see Fig. 1), which were associated to  $\Delta$  excitation in the  $\alpha$  projectile and Roper excitation in the proton target (see Fig. 2). In a recent theoretical analysis we showed that the two mechanisms of Fig. 2 were dominant in the reaction and that other possible mechanisms, like Roper excitation on the projectile or two  $\Delta$  excitation, were negligible [2]. However, it was found that the interference between the two mechanisms in Fig. 2 was appreciable and it was important to consider for a proper analysis of the data and the excitation of the isoscalar  $NN \rightarrow NN^*$  transition amplitude.

In Fig. 1 one can see the results for the projectile  $\Delta$  excitation, Roper target excitation and interference. One observes there that the interference term is large and that the strength of the Roper is about 1/4 of the strength of the  $\Delta$  excitation at their peaks.

It would be interesting to have other experiments which magnified the strength of Roper excitation, both in absolute terms and relative to the  $\Delta$  and if possible diminished the interference term, which makes a theoretical model necessary in order to separate the Roper contribution. All these things are accomplished by performing the  $(\alpha, \alpha')$  reaction at higher energies, as we explain here.

We take the same model which was used in [2] to analyse the  $(\alpha, \alpha')$  reaction at 4  $\text{GeV}$ . The cross section for the processes

$$\alpha + p \rightarrow \alpha + p + \pi^0$$

$$\alpha + p \rightarrow \alpha + n + \pi^+ \quad (1)$$

is given by

$$\begin{aligned} \frac{d^2\sigma}{dE_{\alpha'}d\Omega_{\alpha'}} &= \frac{p_{\alpha'}}{(2\pi)^5} \frac{M_{\alpha'}^2 M^2}{\lambda^{1/2}(s, M^2, M_{\alpha'}^2)} \int d^3p_{\pi} \frac{1}{E_{N'}\omega_{\pi}} \\ &\times \bar{\Sigma}\Sigma|T|^2\delta(E_{\alpha} + E_N - E_{\alpha'} - E_{N'} - \omega_{\pi}), \end{aligned} \quad (2)$$

where  $\lambda(\dots)$  is the Källén function and  $s$  the Mandelstam variable for the initial  $p - \alpha$  system.

By means of eq. (2) we can take into account the mechanisms of  $\Delta$  excitation in the projectile, Fig. 2a, and the Roper excitation in the target, Fig. 2b, with the Roper decaying into a nucleon and a pion (which accounts for about 65% of the  $N^*$  free width). The contribution of the Roper decay into  $\pi\pi N$  is also accounted for in [2] and is included in the final results here but it does not interfere with the amplitude of  $\Delta$  excitation in the projectile, since the final states are different.

The  $T$  matrix for the diagram 2a is evaluated taking into account  $\pi + \rho$  exchange together with the Landau Migdal induced correction. For the diagram of Fig. 2b, which enforces the exchange of an isoscalar object, we take an effective " $\sigma$ " exchange, which incorporates the possible exchange of an  $\omega$  meson and the effect of nuclear correlations. The strength of this isoscalar exchange piece is determined by making a fit to the experiment of ref. [1]. The expressions for the  $\Delta$  and Roper terms and the interference can be seen in eq. (4) of ref. [3] and eqs. (4), (20) of ref. [2]. Their reproduction here is not necessary to understand the results.

We have evaluated the cross section for the  $(\alpha, \alpha')$  reaction on the proton, using the same model, for kinetic energies 10 *GeV* and 15 *GeV* of the  $\alpha$  particle. We show in Fig. 3 the results obtained for 15 *GeV*. Those at 10 *GeV* are qualitatively similar but the Roper and  $\Delta$  peaks have a strength of about 4 [*mb/(sr·MeV)*]. In Fig. 3 we show the results of the Roper excitation (with decay of the Roper into  $\pi N$ ), those of the  $\Delta$  projectile excitation, their interference and the sum, which includes also the contribution of the  $N^* \rightarrow \pi\pi N$  decay (with the distortion of the two pions by the  ${}^4\text{He}$  nucleus which must remain unbroken). Comparison of Fig. 3 with Fig. 1 shows the welcome feature of the 15 *GeV* reaction:

- i) The cross section for the Roper excitation is increased by more than an order of magnitude with regard to the one at 4 *GeV*.
- ii) The strength of the Roper and  $\Delta$  peaks is similar, while at 4 *GeV* the former had a strength of about 1/4 of the latter.
- iii) The interference term is practically negligible compared to the Roper contribution. This is in contrast with the 4 *GeV* case where the strength of these two terms was similar.

A situation like the one in Fig. 3 makes experimentally much easier the extraction of information on the properties of Roper excitation by an isoscalar source. Such experiments can be easily implemented in the Synchrophasotron of Dubna which accelerates nuclei up to  $T_{kin} \simeq 4 \text{ GeV}/A$  and in the new superconductive synchrotron, the Nuclotron, which accelerates nuclei up to  $T_{kin} \simeq 6 \text{ GeV}/A$  [4]. In fact in a related experiment carried out at Dubna

on the  $C(d, d')X$  reaction [5] at  $8.9 \text{ GeV}/c$ , a reanalysis of the data in terms of  $M_X$  calculated for the  $p(d, d')$  kinematics shows a clear peak around the Roper mass [6].

It is relatively easy to understand the features observed in the results at  $15 \text{ GeV}$ . In the first place the small interference. It is easy to see that as the energy of the beam increases it becomes progressively more difficult to have the same kinematic configuration of  $\alpha, N, \pi$  in the final state for the two mechanisms of Fig. 2. Indeed, in the lab. system the pion coming from the decay of the Roper in the mechanism of Fig. 2b will be distributed in a wide range of angles ( it would be isotropic for the  $N^*$  decay at rest, but the effective  $\sigma$  brings some momentum along). However, the pion coming from the  $\Delta$  decay in the mechanism of Fig. 2a will be directed in a very narrow cone along the direction of motion of the  $\alpha$  particle in the frame where the initial proton is at rest. The cone becomes narrower as the  $\alpha$  particle energy increases and, hence, the overlap of the final state configurations in the two mechanisms of Fig. 2 (and the interference term) becomes smaller as  $T_\alpha$  increases.

In order to understand the change of strength of the Roper and  $\Delta$  excitations and their relative weight we must look at another factor. The reason in this case lies in the nucleus form factor which one has in this reaction.

Indeed, in both the mechanisms of Fig. 2 the amplitude contains the nuclear form factor [3]

$$F_{He}(\vec{k}) = \int d^3r \rho_{He}(\vec{r}) \exp \left[ -\frac{1}{2} \int_{-\infty}^{\infty} \sigma_{NN} \rho_{He}(\vec{b}, z') dz' \right] e^{i\vec{k}\cdot\vec{r}}$$

$$\times \exp \left[ -\frac{i}{2} \int_0^\infty \frac{1}{p_\pi} \Pi(p_\pi, \rho_{He}(\vec{r}')) d\ell \right], \quad (3)$$

where

$$\begin{aligned} \vec{r}' &= \vec{r} + \frac{\vec{p}_\pi}{|\vec{p}_\pi|} \ell, \\ \vec{k} &= \vec{p}_\alpha - \vec{p}_{\alpha'}. \end{aligned} \quad (4)$$

The momenta  $\vec{p}_\alpha, \vec{p}_{\alpha'}, \vec{p}_\pi$  appearing in eqs. (3), (4) are evaluated in the frame where the initial  $\alpha$  particle is at rest. In eq. (3)  $\rho_{He}(\vec{r})$  is the Harmonic - Oscillator density distribution of the  $\alpha$  particle,  $\sigma_{NN}$  the nucleon-nucleon total cross section and  $\Pi(p_\pi, \rho)/2\omega_\pi$  is the pion nuclear optical potential, taken from ref. [7] up to  $T_\pi \simeq 250 MeV$  and extrapolated at high energies when needed using the lowest order optical potential [2].

The form factor of eq. (3) is the  ${}^4He$  nuclear form factor incorporating the distortion of the proton and pion waves, both in the eikonal approximation. Now when  $T_\alpha$  increases, for a same energy transfer the momentum transfer is smaller. Indeed in the forward direction of the  $\alpha'$  we have

$$\begin{aligned} p_\alpha - p_{\alpha'} &= \sqrt{E_\alpha^2 - M_\alpha^2} - \sqrt{E_{\alpha'}^2 - M_\alpha^2} \\ &\simeq E_\alpha - E_{\alpha'} - \frac{M_\alpha^2}{2E_\alpha} + \frac{M_\alpha^2}{2E_{\alpha'}} = (E_\alpha - E_{\alpha'}) \left( 1 + \frac{M_\alpha^2}{2E_\alpha E_{\alpha'}} \right). \end{aligned} \quad (5)$$

Hence, the invariant four momentum transfer squared will be

$$-q^2 \simeq (E_\alpha - E_{\alpha'})^2 \left( \left[ 1 + \frac{M_\alpha^2}{2E_\alpha E_{\alpha'}} \right]^2 - 1 \right) \quad (6)$$

which decreases as  $E_\alpha$  increases. The magnitude  $-q^2$  is equivalent to  $\vec{k}^2$  in the Breit frame of the nucleus and is essentially also  $\vec{k}^2$  of eqs. (3), (4) for the  ${}^4\text{He}$  at rest. Hence, we should expect an increase of the value of the nucleus form factor,  $F_{\text{He}}(\vec{k})$  as  $E_\alpha$  increases. Furthermore the relative increase in the form factor (think in terms of the undistorted  $\exp[-\vec{k}^2/4\alpha^2]$  form factor of  ${}^4\text{He}$  with  $\alpha^2 = 0.76\text{fm}^{-2}$ ) will be bigger if the excitation energy  $E_\alpha - E_{\alpha'}$  is bigger. This is actually what we see in Fig. 4, where we plot the ratio of  $|F_{\text{He}}(\vec{k})|^2$  at two different  $T_\alpha$  energies, 10 GeV and 4.2 GeV, and 15 GeV and 4.2 GeV.

We observe in Fig. 4 that at  $\omega = E_\alpha - E_{\alpha'}$  around 200 MeV, where the  $\Delta$  peak appears, the increase of the form factor is moderate. However, at  $\omega = 550$  MeV, where the Roper peak appears, the ratio of form factors squared has a value of the order of four to five. This factor is the one responsible for the relative increase of strength of the Roper excitation versus the  $\Delta$  excitation at  $T_\alpha = 15$  GeV with respect to the experiment of [1] at 4.2 GeV.

The absolute increase in strength both for the Roper and  $\Delta$  excitation can be traced back both to the form factor effect and the phase space factor  $p_{\alpha'}$  in the numerator of the cross section formula of eq. (2).

One might think that performing more exclusive experiments, *i.e.*, detecting a pion and a nucleon in coincidence and making a plot in terms of the  $\pi N$  invariant mass, one would magnify the Roper peak with respect to a background. We have checked theoretically, within our model, that this is

not the case and the invariant mass distribution which one obtains resemble very much the  $\omega$  distributions of Fig. 1 and 3.

In our theoretical model we have neglected any dependence of the interaction on the energy, since we have no elements to think that this might be the case. Obviously a certain energy dependence cannot be ruled out, an information which would be provided by the same experiment and which would be much useful to help construct microscopic models for the interaction.

The results obtained here should encourage the implementation of the experiments. After decades of studies around the  $\Delta$  region the time has come to study in detail the properties of the next nucleon excitation. Quark models have difficulties to explain the properties of the Roper [8]; the authors of ref. [1] suggested that the Roper could be interpreted as a monopole excitation of the nucleon (breathing mode); the decay of the Roper into two pions in S-wave plays an important role in the  $\pi N \rightarrow \pi\pi N$  reaction close to threshold [9, 10], which must be brought under control in order to make predictions about the  $\pi\pi$  scattering length, *etc.*

The proposed experiments exciting the Roper with an isoscalar source will bring new information about this resonance, its decay and its coupling to different hadronic components and will pose new challenge to models of this resonance.

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## Figure Caption

**Fig. 1** Calculated cross sections of the target Roper process [2] and the projectile  $\Delta$  process [3] at  $E_\alpha = 4.2 \text{ GeV}$  and  $\theta_{Lab} = 0.8^\circ$ . The variable  $\omega$  is the energy transfer defined as  $\omega = E_\alpha - E_{\alpha'}$ . The thick line indicates the sum of all contributions. Experimental data are taken from ref. [11]. Here we used  $g_{\sigma NN^*}^2/4\pi = 1.33$ ,  $M^* = 1430 \text{ MeV}$ ,  $\Gamma^*(s = M^{*2}) = 300 \text{ MeV}$ .

**Fig. 2** Diagrams for the  $(\alpha, \alpha')$  reaction which we consider in this paper. They are (a)  $\Delta$  excitation in the projectile [3], (b) Roper excitation in the target [2]. The  $\sigma$  exchange must be interpreted as an effective interaction in the  $T = 0$  exchange channel [2].

**Fig. 3** Same as Fig. 1. Here  $E_\alpha = 15 \text{ GeV}$  and  $\theta_{Lab} = 0^\circ$ .

**Fig. 4** The squared ratio of the  $\alpha$  form factor is plotted as a function of the energy transfer  $\omega = E_\alpha - E_{\alpha'}$ . The line (1) indicates the squared ratio of the form factor of  $E_\alpha = 10 \text{ GeV}, \theta_{Lab} = 0^\circ$  case to  $E_\alpha = 4.2 \text{ GeV}, \theta_{Lab} = 0.8^\circ$  case, and the line (2) the squared ratio of  $E_\alpha = 15 \text{ GeV}, \theta_{Lab} = 0^\circ$  to  $E_\alpha = 4.2 \text{ GeV}, \theta_{Lab} = 0.8^\circ$ . The form factor  $F$  is defined by eq. (3) in text.

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