

THE AXIAL CHARGE RENORMALIZATION IN A RELATIVISTIC DESCRIPTION OF FINITE NUCLEI

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Abstract

Starting from a realistic One-Boson-Exchange model of the nucleon nucleon interaction the relativistic mean field for nucleons is determined within the Dirac Brueckner Hartree Fock approach for finite nuclei. The matrix elements of the axial charge operator evaluated for the solutions of the Dirac equation with this selfenergy are investigated. These matrix elements are enhanced with respect to the equivalent non relativistic ones obtained from the solutions of the Schrödinger equation with the non relativistic equivalent potential. The present results confirm at a qualitative level the results for the axial charge renormalization obtained with perturbative approaches. However, the results obtained differ in size from those of the perturbative approach and are nucleus and state dependent.

1 Introduction.

As suggested in ref. [1] the presence of a large scalar potential in a relativistic version of the nucleon selfenergy in the nucleus [2] leads to a sizeable renormalization of the axial charge in nuclei. This renormalization, which is also sometimes referred to as the heavy meson exchange current contribution, must be considered in addition to the conventional meson exchange currents studied earlier [3, 4, 5, 6]. More quantitative evaluations of this renormalization, following the idea of [1], have been recently provided in [7, 8, 9]. In ref. [7] a perturbative approach is used starting from a relativistic description of the NN potential and taking direct and exchange terms. The strong short-range and tensor components of a realistic NN interaction give rise to significant two-nucleon correlations. The effects of NN correlations are taken into account in the investigations of ref. [8] by using the Brueckner G-matrix. The estimates reported in [7] and [8] were made for the system of infinite nuclear matter.

The investigations of ref. [9] are performed directly for finite nuclei. Also in this case the effect of the nucleon selfenergy is treated in a perturbative way. The operators are reduced to a bispinor representation and the calculations are carried out in a nonrelativistic frame. The single-particle wavefunctions are represented by oscillator wavefunctions and the effect of correlations are included in terms of a simple local correlation function.

In the present work we want to consider the relativistic features, the effects of correlations and the single-particle wavefunctions consistently. For that purpose we employ the results of the relativistic Dirac Brueckner Hartree Fock (DBHF) calculations of ref. [10]. These calculations are based on the version A of the relativistic One-Boson-Exchange potential of [11]. The results of the calculation of the ground-state properties of double closed-shell nuclei are in good agreement with the experimental data and the resulting self-energy yields a real part for the optical potential of low-energy nucleon nucleus scattering, which is close to the empirical analysis [12].

After this short introduction we will review the perturbative treatment of the heavy meson exchange current contribution in nuclear matter. The self-consistent DBHF calculations are discussed in section 3, while section 4 contains a discussion of the non-relativistic reduction. The results are presented and discussed in section 5 and the final section summarizes the main conclusions.

2 Perturbative renormalization of the axial charge in a relativistic approach.

A realistic NN interaction contains a large attractive scalar isoscalar component (due to σ -exchange in OBE model) and a repulsive vector isovector component (ω -exchange). Evaluating the selfenergy of a nucleon in a medium

of nuclear matter with such an interaction using the mean field approximation, one finds that it contains a large attractive scalar component and a repulsive component, which under Lorentz transformation transforms like the timelike component of a vector [2]

$$\Sigma = \Sigma^s \frac{\rho}{\rho_0} + \Sigma^v \gamma^0 \frac{\rho}{\rho_0} \quad (1)$$

with ρ the nuclear density and ρ_0 the saturation density of nuclear matter ($\rho_0 = 0.17 \text{ fm}^{-3}$). Taking into account the Fock-exchange terms in the Hartree-Fock approximation or accounting for correlation effects in the DBHF approximation one obtains a small spacelike vector component and finds that all the terms depend slightly on the momentum of the nucleon [10]. We now want to calculate matrix elements for the axial operator $g_A \gamma^\mu \gamma_5$, concentrating on the axial charge $g_A \gamma^0 \gamma_5$ for nucleons moving in the nuclear medium. We can immediately write the perturbative corrections to the axial charge due to the nucleon selfenergy, which are depicted diagrammatically in fig. 1, where we have separated the contribution from positive and negative intermediate states in the nucleon propagator. Analytically this decomposition is given by

$$\frac{\not{p} + M}{p^2 - M^2} = \frac{M}{E(\vec{p})} \left\{ \frac{\sum_r u_r(\vec{p}) \bar{u}_r(\vec{p})}{p^0 - E(\vec{p}) + i\epsilon} + \frac{\sum_r v_r(-\vec{p}) \bar{v}_r(-\vec{p})}{p^0 + E(\vec{p}) - i\epsilon} \right\} \quad (2)$$

where $M, E(\vec{p})$ are the mass and on shell energy of the free nucleon and u_r, v_r the ordinary free spinors in Mandl-Shaw representation [13]. The axial charge matrix element is reduced to a bispinor representation assuming $E(\vec{p}) \simeq M$ by means of

$$\bar{u}(\vec{p}') \gamma^0 \gamma_5 u(\vec{p}) = \chi' \frac{\vec{\sigma}(\vec{p} + \vec{p}')}{2M} \chi \quad (3)$$

Now the a) and b) diagrams from fig. 1 with positive intermediate nucleon components are automatically absorbed into the calculation with dressed non relativistic wave functions but genuine corrections from the negative intermediate states c) and d) remain. One can easily see that the renormalization with the $\Sigma^v \gamma^0$ term of (1) vanishes identically and only the renormalization with the Σ^s term remains. One immediately gets a renormalized axial charge matrix element corresponding to bare matrix element plus figs. 1c and 1d given by

$$g_A \left(1 - \frac{\Sigma^s \rho}{M \rho_0}\right) \chi' \frac{\vec{\sigma}(\vec{p} + \vec{p}')}{2M} \chi \quad (4)$$

or equivalently a renormalization of the axial charge by the amount $(1 - \frac{\Sigma^s \rho}{M \rho_0})$. This is the result obtained in [7]. Note that since the relativistic potential of (1) implicitly accounts for direct and exchange terms no further corrections have to be done in contrast to [7] where, because one starts from a NN interaction, direct and exchange terms are explicitly evaluated. With standard values of

Σ^s of the order of -400MeV and taking $\rho \simeq \rho_0$ one obtains a renormalization factor of the order of 1.4 in qualitative agreement with [7, 9].

Another way to arrive at eq.(4) is to realize that the solution of the Dirac equation for with a selfenergy of the kind displayed in eq.(1) yields Dirac spinors for the nucleons in the nuclear medium, which are identical to Dirac spinors of free nucleons, except that the mass of the nucleon M has to be replaced by an effective mass $M^* = M + \Sigma^s \rho / \rho_0$. Calculating the matrix element for the axial charge operator with these dressed Dirac spinors and reducing it to a bispinor representation one finds as in eq.(3)

$$\begin{aligned} \bar{u}(p') \gamma^0 \gamma_5 \tilde{u}(p) &= \chi' \frac{\vec{\sigma}(\vec{p} + \vec{p}')}{2 \left(M + \Sigma^s \frac{\rho}{\rho_0} \right)} \chi \\ &= \left[1 - \frac{\Sigma^s}{M} \frac{\rho}{\rho_0} + \left(\frac{\Sigma^s}{M} \frac{\rho}{\rho_0} \right)^2 + \dots \right] \chi' \frac{\vec{\sigma}(\vec{p} + \vec{p}')}{2M} \chi \end{aligned} \quad (5)$$

It should be noted that this non-perturbative treatment of the the heavy meson exchange current contribution to the renormalization of the axial charge yields an effect which is considerably larger than the perturbative treatment of eq.(4). Using again $\Sigma^s = -400\text{MeV}$ and taking $\rho \simeq \rho_0$ one obtains a factor of 1.7 rather than 1.4 (see above).

3 Finite nuclei renormalization.

Solving the Dirac equation directly for the finite nucleus the diagrams in fig. 1 plus all terms of higher order in the nucleon selfenergy insertions can automatically be taken into account by evaluating the matrix elements of the $\gamma^0 \gamma_5$ operator between the solutions of the Dirac equation. The relativistic selfenergy for the finite nucleus is calculated following the scheme defined as Hartree approximation in ref. [10]. In this scheme we assume an effective Lagrangian for nucleons, a scalar σ meson and a vector ω meson. This effective Lagrangian is defined to be used in Dirac Hartree calculations for finite nuclei. The effects of correlations and the Fock-exchange terms are taken into account by assuming coupling constants for the meson nucleon interaction terms, which are density dependent and which are determined such that a Dirac Hartree calculation of nuclear matter reproduces the results of microscopic DBHF calculations for the OBE potential A at all densities.

In this scheme the correlation effects are deduced from nuclear matter and treated in a local density approximation. The investigations of ref.[10] demonstrate that these Dirac Hartree calculations yield results which are close to DBHF calculations in which the correlation effects are treated directly for the finite nucleus under consideration. Furthermore the predictions of this Dirac Hartree approximation for binding energy and radius of nuclei like ^{16}O

and ^{40}Ca are close to the empirical data. Furthermore it is worth noting that this approach leads to scalar and vector components of the self-energy

$$\Sigma(r) = \Sigma^s(r) + \gamma^0 \Sigma^v(r) \quad (6)$$

which are local and depend on the radial distance r only. Using this selfenergy in the Dirac equation

$$[i\vec{\gamma}\vec{\nabla} + M + \Sigma^s(r) + \gamma^0 \Sigma^v(r)]\Psi = \gamma^0 E \Psi \quad (7)$$

we obtain solutions for the energy E and Dirac spinors with the various quantum numbers for the orbital angular momentum l and total angular momentum j . By following the nomenclature of Itzykson and Zuber [14] we write the relativistic wave functions as

$$\Psi_{jm}^l(\vec{r}) \equiv \left\{ \begin{array}{cc} i \frac{G_{lj}(r)}{r} & \psi_{jm}^l \\ \frac{F_{lj}(r)}{r} & \vec{\sigma} \cdot \hat{r} \psi_{jm}^l \end{array} \right\} \quad (8)$$

where ψ_{jm}^l are the spin wave functions given by

$$\begin{aligned} \psi_{jm}^{(+)} &= \left(\frac{1}{2j}\right)^{1/2} \left\{ \begin{array}{cc} (j+m)^{1/2} & Y_{j-1/2}^{m-1/2} \\ (j-m)^{1/2} & Y_{j-1/2}^{m+1/2} \end{array} \right\} \text{ for } j = l + 1/2 \\ \psi_{jm}^{(-)} &= \left(\frac{1}{2j+2}\right)^{1/2} \left\{ \begin{array}{cc} (j+1-m)^{1/2} & Y_{j+1/2}^{m-1/2} \\ -(j+1+m)^{1/2} & Y_{j+1/2}^{m+1/2} \end{array} \right\} \text{ for } j = l - 1/2 \quad (l > 0) \end{aligned} \quad (9)$$

With a little bit of algebra the matrix elements of $\gamma^0 \gamma_5$ between Dirac wave functions are readily evaluated and the results are shown in the appendix.

Since the axial charge renormalization is checked in the $0^+ \leftrightarrow 0^-$ first forbidden β -decay transitions we have performed the calculations for the relevant matrix elements in these transitions in two nuclei ^{16}O and ^{40}Ca , in order to see the difference of the renormalization for nuclei with different mass number.

For such double closed shell nuclei a 0^- state is formed with a ph excitation for l_p and l_h with different parity and $j_p = j_h$. We have

$$\begin{aligned} |0^- \rangle &= \sum_m (-1)^{j_h - m} C(j_p j_h 0; m, -m) a_{j_p m}^\dagger a_{j_h m} |0^+ \rangle \\ &= \sum_m \sqrt{\frac{1}{2j_h + 1}} a_{j_p m}^\dagger a_{j_h m} |0^+ \rangle \end{aligned} \quad (10)$$

The matrix element for the $0^+ \leftrightarrow 0^-$ transition is readily obtained from the formulas in the appendix by setting $j = j', m = m'$ summing over m and multiplying by $(2j+1)^{-1/2}$. The $0^+ \leftrightarrow 0^-$ transition can only be done with cases b) and c) and in both cases we get a simplified solution which is

$$\langle 0^+ | \gamma^0 \gamma_5 e^{i\vec{q}\vec{r}} | 0^- \rangle =$$

$$\sqrt{2j+1}(-i) \int r^2 dr \left[\frac{G_{\nu j'}(r)}{r} \frac{F_{lj}(r)}{r} - \frac{F_{\nu j'}(r)}{r} \frac{G_{lj}(r)}{r} \right] j_0(qr) \quad (11)$$

We construct the 0^- state in ^{16}O as a ph excitation with the orbitals $1p_{1/2}$ and $2s_{1/2}$. We also consider the component with $1p_{3/2}$ and $1d_{3/2}$ in order to see whether the renormalization is state dependent or not. In the case of ^{40}Ca we take the orbitals $1d_{3/2}$ and $2p_{3/2}$. In all cases we consider a proton hole state and a neutron particle state, as it corresponds to β transitions. The neutron states in the $2s_{1/2}$ and $1d_{3/2}$ orbitals in ^{16}O and the $2p_{3/2}$ orbital of ^{40}Ca are all bound states in the potential used.

4 Non relativistic calculation.

In order to see the effects of the renormalization due to the relativistic structure of the potential (6) we solve the Schrödinger equation with the equivalent nonrelativistic potential [15, 12]

$$U_{SEP}(r) = \Sigma^s(r) + \frac{E}{M} \Sigma^v(r) + \frac{(\Sigma^s(r))^2 - (\Sigma^v(r))^2 + U_{\text{Darwin}}(r)}{2M} \quad (12)$$

with

$$\begin{aligned} U_{\text{Darwin}}(r) &= \frac{3}{4} \left[\frac{1}{D(r)} \frac{dD(r)}{dr} \right]^2 - \frac{1}{rD(r)} \frac{dD(r)}{dr} - \frac{1}{2D(r)} \frac{d^2D(r)}{dr^2}, \\ D(r) &= E + \Sigma^s(r) - \Sigma^v(r). \end{aligned} \quad (13)$$

The single-particle wavefunctions obtained from the solution of the Schroedinger equation with U_{SEP} are used to evaluate the matrix elements of the $\vec{\sigma}(\vec{p} + \vec{p}')/2M$ operator and we find

$$\begin{aligned} \langle 0^+ | \vec{\sigma} \frac{(\vec{p} + \vec{p}')}{2M} | 0^- \rangle = \\ i(-1)^{j'+l'+1/2} \frac{1}{2M} \sqrt{2} F(n'l'j', nlj; \lambda = 0) \end{aligned} \quad (14)$$

where the function F is defined in the appendix.

5 Results and discussion.

In fig. 2 we show the matrix elements of the axial charge for the relativistic and non relativistic cases of eqs. (11) and (14) respectively as a function of

q . We show the results for the $1p_{1/2} \rightarrow 2s_{1/2}$ and $1p_{3/2} \rightarrow 1d_{3/2}$ transitions on ^{16}O . One observes that the strength of the latter transition is about a factor 2 larger than for the first one. In both cases the relativistic calculation of the matrix element yields larger values than the non-relativistic one derived from the equivalent non relativistic potential. We also observe that the matrix elements are weakly dependent on the momentum transfer q up to values of $q \simeq 100 \text{ MeV}/c$. In fig. 3 we show the ratio of the relativistic versus non relativistic matrix elements as a function of q for the two transitions in ^{16}O . The values of the ratios at $q = 0$, relevant to β decay, are about 1.33 and 1.20 for the $1p_{1/2} \rightarrow 2s_{1/2}$ and $1p_{3/2} \rightarrow 1d_{3/2}$ transitions, respectively.

The strength of $\Sigma^s(r)$ from eq. (6) at $r = 0$ is in our case $\Sigma^s \simeq -384 \text{ MeV}$, close to the value typical for nuclear matter at saturation ($\Sigma^s \simeq -400 \text{ MeV}$) which we have considered in our estimates of section 2. In the perturbative approach of section 2 we would have obtained a ratio of 1.41 whereas the non perturbative nuclear matter estimate would even yield a ratio of 1.69 for this value of Σ^s . We can see that the calculations performed directly for the finite nuclei yields results which are significantly smaller than those estimates from nuclear matter. The reason for this difference is the fact that the calculation of matrix elements for finite nuclei requires a radial integration which is dominated by the integrand at the surface. This is due to the fact that the integrand contains a product of wavefunctions for a particle- and a hole-state. The nuclear density at the surface, however, is smaller than for $r = 0$ or the saturation density of nuclear matter. Consequently also the relativistic effects due scalar potential Σ^s , leading to an enhancement of the small component of the Dirac spinor, will be smaller at these relevant densities than at the center of the nucleus or at the saturation density of nuclear matter. Similar, although a bit smaller, reductions with respect to the nuclear matter approach were also found in the finite nuclei perturbative approach of [9], though the results were found to be sensitive to the short range correlations assumed. Here short range correlations are incorporated in the problem in a selfconsistent way.

From these considerations we can also understand that the relativistic renormalization of the axial charge operator in the case of the $1p_{3/2} \rightarrow 1d_{3/2}$ transition is smaller than the one in the $1p_{1/2} \rightarrow 2s_{1/2}$ case. The smaller renormalization in the case of the $1p_{3/2} \rightarrow 1d_{3/2}$ transition can be interpreted in terms of the centrifugal barrier which pushes the d state more to the surface of the nucleus where the potential Σ^s is weaker. Furthermore we observe a slight increase of the renormalization as a function of q . At a larger momentum transfer one tends to probe more the higher densities in the interior of the nucleus.

These results are confirmed by our calculations for the nucleus ^{40}Ca . In fig. 4 we show the relativistic and non relativistic matrix elements for the $1d_{3/2} \rightarrow 2p_{3/2}$ transition in ^{40}Ca and in fig. 5 the ratio of the relativistic to non relativistic matrix elements. The ratio is of the order of 1.23, rather independent on the momentum transfer. This result for the renormalization

of the axial charge is very similar but slightly larger than the ratio obtained for the $1p_{3/2} \rightarrow 1d_{3/2}$ in ^{16}O . Once again the centrifugal barrier is responsible for a reduced renormalization compared to the expectations of nuclear matter approach.

6 Conclusions

We have analyzed in detail the renormalization of the axial charge in nuclei by evaluating the matrix elements of the axial charge operator with relativistic wave functions, solutions of the Dirac equation with the relativistic potential, and with non relativistic wave function, solutions of the Schrödinger equation with an equivalent non relativistic potential. We have found renormalization effects due to the use of the relativistic wave functions, enhancing the axial charge in the direction found in earlier perturbative approaches for nuclear matter. However, the quantitative results differ from the estimates derived for nuclear matter significantly. Using the G-matrix derived from a realistic meson exchange model of the NN interaction [11] a perturbative estimate of the heavy meson exchange current contribution to the axial charge at nuclear matter saturation density [8] would yield a renormalization factor of 1.4 and a non perturbative treatment would lead to enhancement as large as 1.7. For finite nuclei the enhancement factors considerably smaller, of the order of 1.2 - 1.3. We argue that this reduction of the renormalization effect is due to the smaller densities at the surface of finite nuclei, which are relevant for the evaluation of actual matrix elements. From these considerations we can also understand the dependence of the renormalization factor on the momentum transfer and on the transition actually considered.

The amount of axial charge renormalization depends on the model for the NN interaction. We have employed a relativistic meson exchange model (Potential version A of the Bonn potential [11]), which has been derived to reproduce NN scattering data. It is fair to quote at this point that using this potential in the present case there is the assumption that the relativistic potential constructed to reproduce NN scattering of on shell nucleons can be extrapolated to deal with negative energy states and on shell and off shell conditions. This is certainly a strong assumption from which all the microscopically constructed relativistic potentials suffer, and indeed different parametrizations of the NN amplitude on shell lead to different relativistic potentials [16]. Some efforts have been done to constrain the relativistic potential to be consistent with the $\bar{N}N$ elementary amplitudes [17] and this leads to potentials like the one obtained here but about one half their strength. Even then this potential is constructed at the level of the impulse approximation or low density limit, $t\rho$, and many body effects should modify it. It is clear that many efforts are still necessary to be able to claim that an unambiguous microscopical relativistic potential has been determined. On the other hand one can take a more phenomenological approach and say that a certain relativistic potential

has a wide degree of phenomenological success, providing fair nuclear binding energies, spin-orbit splitting, nucleon nucleus cross sections and polarization observables, etc. [10, 12]. The potential we have used is one of such and provides empirical support for the axial charge renormalization found, but this does not exclude the possibility of other potentials with the same degree of phenomenological success and still providing different axial charge renormalization. The ultimate answer to this question is tied to the progress in our understanding of the meaning and accurate strength of the relativistic potential. Meanwhile, by using a fair and plausible model we have done detailed calculations and showed that the results are sufficiently different from the perturbative results to encourage the use of the present approach in future works dealing with the problem.

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Appendix: Matrix elements of the axial charge operator.

A) Relativistic case: We write here the matrix element for the $\gamma^0\gamma_5$ operator between relativistic wave functions

$$\langle n'l'j'm'|\gamma^0\gamma_5e^{iq\vec{r}}|nljm\rangle \quad (a.1)$$

We distinguish 4 cases

$$\begin{aligned} a)j' &= l' + 1/2, & j &= l + 1/2 \\ b)j' &= l' + 1/2, & j &= l - 1/2 \\ c)j' &= l - 1/2, & j &= l + 1/2 \\ d)j' &= l - 1/2, & j &= l - 1/2 \end{aligned}$$

and the resulting matrix element is

$$\begin{aligned} \sqrt{4\pi} \sum_{\lambda} i^{\lambda}(-i) \int r^2 dr \left[\frac{G_{\nu j'}(r)}{r} \frac{F_{l j}(r)}{r} - \frac{F_{\nu j'}(r)}{r} \frac{G_{l j}(r)}{r} \right] j_{\lambda}(qr) \\ (2\lambda + 1)^{1/2} Y_{\lambda, m' - m}^*(\hat{q}) A_i \end{aligned} \quad (a.2)$$

where A_i is given for each of the cases a) b) c) d) listed above by

$$A_a = \frac{1}{2j'} C(j + 1/2, \lambda, j' - 1/2; 000)$$

$$\{(j' + m')^{1/2}(j + 1 - m)^{1/2} C(j + 1/2, \lambda, j' - 1/2; m - 1/2, m' - m)$$

$$- (j' - m')^{1/2}(j + 1 + m)^{1/2} C(j + 1/2, \lambda, j' - 1/2; m + 1/2, m' - m)\} \quad (a.3)$$

$$A_b = \frac{1}{2j'} C(j - 1/2, \lambda, j' - 1/2; 000)$$

$$\{(j' + m')^{1/2}(j + m)^{1/2} C(j - 1/2, \lambda, j' - 1/2; m - 1/2, m' - m)$$

$$+ (j' - m')^{1/2}(j - m)^{1/2} C(j - 1/2, \lambda, j' - 1/2; m + 1/2, m' - m)\} \quad (a.4)$$

$$A_c = \frac{1}{2j' + 2} C(j + 1/2, \lambda, j' + 1/2; 000)$$

$$\{(j' + 1 - m')^{1/2}(j + 1 - m)^{1/2} C(j + 1/2, \lambda, j' + 1/2; m - 1/2, m' - m)$$

$$+(j' + 1 + m')^{1/2}(j + 1 + m)^{1/2}C(j + 1/2, \lambda, j' + 1/2; m + 1/2, m' - m)\} \quad (a.5)$$

$$A_d = \frac{1}{2j' + 2}C(j - 1/2, \lambda, j' + 1/2; 000)$$

$$\{(j' + 1 - m')^{1/2}(j + m)^{1/2}C(j - 1/2, \lambda, j' + 1/2; m - 1/2, m' - m)$$

$$-(j' + 1 + m')^{1/2}(j - m)^{1/2}C(j - 1/2, \lambda, j' + 1/2; m + 1/2, m' - m)\} \quad (a.6)$$

B) Non relativistic case: we evaluate matrix elements of the $\vec{\sigma}(\vec{p} + \vec{p}')/2M$ operator between non relativistic states

$$\langle n'l'j'm' | \vec{\sigma}(\vec{p} + \vec{p}')/2M | nljm \rangle \quad (a.7)$$

The derivation of this matrix elements requires a bit more algebra than the non relativistic case. With the help of some useful formulas from the appendix of ref. [18] we obtain the following result

$$\begin{aligned} & \langle n'l'j'm' | \vec{\sigma} \frac{(\vec{p} + \vec{p}')}{2M} e^{i\vec{q}\vec{r}} | nljm \rangle = \\ & i(-1)^{j'+l'+1/2} \sqrt{4\pi} \frac{1}{2M} \sum_{\lambda} C(j\lambda j'; 1/2, 0, 1/2) \\ & C(j\lambda j'; m, m' - m) i^{\lambda} Y_{\lambda, m' - m}^*(\hat{q}) \left[\frac{2(2\lambda + 1)}{2j' + 1} \right]^{1/2} \\ & F(n'l'j', nlj; \lambda) \end{aligned} \quad (a.8)$$

with $\lambda + l + l'$ an odd number, where the last function is given by

$$\begin{aligned} & F(n'l'j', nlj; \lambda) = \\ & \delta_{j, l+1/2} (l + 1)^{1/2} \int_0^{\infty} r^2 dr \phi_{l'}(r) \left[\frac{d\phi_l(r)}{dr} - \frac{l}{r} \phi_l(r) \right] j_{\lambda}(qr) \\ & - (-1)^{j-j'+l-l'} \delta_{j', l'+1/2} (l' + 1)^{1/2} \left(\frac{2j + 1}{2j' + 1} \right)^{1/2} \\ & \int_0^{\infty} r^2 dr \left[\frac{d\phi_{l'}(r)}{dr} - \frac{l'}{r} \phi_{l'}(r) \right] \phi_l(r) j_{\lambda}(qr) \end{aligned}$$

$$\begin{aligned}
& -\delta_{j,l-1/2} l^{1/2} \int_0^\infty r^2 dr \phi_{l'}(r) \left[\frac{d\phi_l(r)}{dr} + \frac{l+1}{r} \phi_l(r) \right] j_\lambda(qr) \\
& + (-1)^{j-j'+l-l'} \delta_{j',l'-1/2} l'^{1/2} \left(\frac{2j+1}{2j'+1} \right)^{1/2} \\
& \int_0^\infty r^2 dr \left[\frac{d\phi_{l'}(r)}{dr} + \frac{l'+1}{r} \phi_{l'}(r) \right] \phi_l(r) j_\lambda(qr)
\end{aligned} \tag{a.9}$$

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Figure captions:

Fig. 1 Diagrams appearing in the perturbative approach to the renormalization of the axial charge.

a) b) involving positive energy intermediate nucleon states: c) d) involving negative energy intermediate states.

Fig. 2 Axial charge form factor for the $0^+ \rightarrow 0^-$ transition in ^{16}O from the orbitals $1p_{1/2} \rightarrow 2s_{1/2}$ and $1p_{3/2} \rightarrow 1d_{3/2}$ with relativistic and equivalent non relativistic wave functions.

Fig. 3 Ratio of the relativistic to non relativistic matrix elements of fig. 2 as a function of the momentum transfer.

Fig. 4 Same as fig. 3 for ^{40}Ca and the transition $0^+ \rightarrow 0^-$ from the orbitals $1d_{3/2} \rightarrow 2p_{3/2}$.

Fig. 5 Ratio of relativistic to non relativistic matrix elements for the transition in fig. 4.

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