SHORT RANGE CORRELATIONS IN THE PION S-WAVE SELF-ENERGY OF PIONIC ATOMS

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Abstract

We evaluate the contribution of second order terms to the pion-nucleus s-wave optical potential of pionic atoms generated by short range nuclear correlations. The corrections are sizeable because they involve the isoscalar s-wave πN amplitude for half off-shell situations where the amplitude is considerably larger than the on-shell one.

In addition, the s-wave optical potential is reanalyzed by looking at all the different conventional contributions together: lowest order, Pauli corrected rescattering term, second order absorptive effects, terms from the interaction of pions with the virtual pion cloud (chiral corrections) and correlation effects. Different off-shell extrapolations for the πN amplitude are used and it is found that, although some individual terms are sensitive to the extrapolation, the sum of them is rather insensitive. The results are compared with empirical values from best fits to the data and are found to be compatible, within theoretical and empirical uncertainties. The results do not rule out further contributions but they put stringent constraints on their strength.

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The original work of Ericson-Ericson on pionic atoms [1] set already the two largest contributions to the s-wave pion nucleus optical potential. Its real part is given by

$$\mathrm{Re}\Pi^{(s)} = -4\pi[(1+\epsilon)(b_0 + \Delta b_0)\rho + (1+\epsilon)b_1(\rho_n - \rho_p) + (1+\frac{1}{2}\epsilon)\mathrm{Re}B_0\rho^2] \quad (1)$$

with $\epsilon = \frac{m_\pi}{M_N}$, $\rho = \rho_n + \rho_p$ and

$$b_0 = \frac{1}{3}(a_1 + 2a_3); \quad b_1 = -\frac{1}{3}(a_1 - a_3) \quad (2)$$

where a_1, a_3 are the isospin $\frac{1}{2}, \frac{3}{2}$ scattering lengths given by [2]

$$a_1 = (0.171 \pm 0.004)\, m_\pi^{-1}; \quad a_3 = (-0.105 \pm 0.003)\, m_\pi^{-1} \quad (3)$$

The magnitude Δb_0, introduced in [1] and subsequently rederived in different formalisms with exactly the same results [3, 4, 5], accounts for the Pauli corrected second order rescattering terms. Its derivation within the many-body framework used in the present paper is shown in appendix A of ref. [5] and its value is given by

$$\Delta b_0 = -\frac{6k_F}{\pi m_\pi^2}\frac{1}{1+\epsilon}(\lambda_1^2 + \lambda_2^2); \quad \lambda_{1,2} = -\frac{1}{2}m_\pi(1+\epsilon)b_{0,1} \quad (4)$$

In addition, there are other terms of higher order in the nuclear density, accounted for in terms of $\mathrm{Re}B_0$, which in ref. [1] were fit to the data, but which have been subsequently evaluated. We discuss them below.

One of them is the real part corresponding to the absorption diagrams, also known as dispersive correction, which is calculated in [5, 6]. The results were obtained using two different off-shell extrapolations, one from Hamilton [7] and the other one from LMM [8]. The uncertainties from different sources were estimated to be of the order of 30%, the largest one coming from the off-shell extrapolation. Indeed, the results obtained there in terms of the second order parameter $\mathrm{Re}B_0$ were

$$\delta\mathrm{Re}B_{0,d} = 0.032\, m_\pi^{-4}\,[5]; \quad \delta\mathrm{Re}B_{0,d} = 0.017\, m_\pi^{-4}\,[6] \quad (5)$$

using the off-shell extrapolations of ref. [7] (Hamilton) and ref. [8] (LMM) respectively.

The findings in ref. [9] on the effective density $\rho_{\mathrm{eff}} = 0.5\,\rho_0$ ($\rho_0 = -0.17\,\mathrm{fm}^{-3}$), for the s-wave part of the optical potential, allow one to talk alternatively in terms of an equivalent $\delta b_{0,d}$ parameter, multiplying $\delta B_{0,d}$ by $0.23\, m_\pi^3$.

On the other hand, recent evaluations of the contribution to the s-wave self-energy from the interaction of pions with the virtual pion cloud [10, 11], within the context of Weinberg chiral Lagrangians [12], gave rise to a moderate repulsion. These terms are additional to those considered in [5, 6], which are based on the πN interaction alone. Their derivation and interpretation within

the many-body scheme of this paper is given in [11]. In terms of an equivalent $\delta b_{0,\text{ch}}$ parameter, using again the effective density of [9], one obtains

$$\delta b_{0,\text{ch}} = -0.0022 \pm 0.0002 \ m_\pi^{-1} \tag{6}$$

where the 10% error is an estimate due to second order effects in the modern chiral perturbation expansion [13], which improves upon the Weinberg results.

The present paper adds to these, the effects of short range nuclear correlations in conjunction with the off-shell extrapolation of the πN scattering amplitude. Short range correlations are taken into account in the evaluation of the p-wave pion self-energy. Their effects are very important and lead to the famous Lorentz-Lorenz effect [1]. The interpretation of this effect in terms of nuclear correlations modifying the $\pi + \rho$ exchange was given in [14]. In spite of its relevance in the modification of the p-wave self-energy, a similar work on the modification of the s-wave self-energy by the nuclear short range correlations has never been done and this is the purpose of this paper. We follow the ideas of [14] to perform this work.

In fig. 1 we show diagrammatically how a pion exchange in a rescattering term is modified due to the simultaneous exchange of other mesons (in the meson exchange picture) which generate the nuclear correlations. It is well-known [15] that the effect of this multiple exchange of mesons, simultaneously with the pion, can be approximated by means of a correlation function such that, if the diagram of fig. 1a can be represented by a potential $V(\vec{r})$, the sum of 1a plus all those of the type 1b can be represented by $\tilde{V}(\vec{r})$, given by

$$\tilde{V}(\vec{r}) = V(\vec{r})g(\vec{r}) \tag{7}$$

where $g(\vec{r})$ is an appropriate correlation function incorporating the repulsion at short distances. Hence

$$g(\vec{r}) = 1 - f(\vec{r}) \qquad \text{with} \qquad f(0) = 1, \quad f(\infty) = 0 \tag{8}$$

Since in nuclear matter it is easier to apply Feynman rules in momentum space, we have instead

$$V(\vec{q}) \rightarrow \tilde{V}(\vec{q}) = \int \frac{d^3k}{(2\pi)^3} V(\vec{q} - \vec{k}) \, \Omega(\vec{k}) \tag{9}$$

where

$$
\begin{aligned}
V(\vec{k}) &= \int d^3r \, e^{i\vec{k}\vec{r}} V(\vec{r}) \\
\Omega(\vec{k}) &= \int d^3r \, e^{i\vec{k}\vec{r}} g(\vec{r}) = (2\pi)^3 \delta(\vec{k}) - B(\vec{k}) \, ; \quad B(\vec{k}) = \int d^3r \, e^{i\vec{k}\vec{r}} f(\vec{r})
\end{aligned}
\tag{10}
$$

Hence

$$\tilde{V}(\vec{q}) = V(\vec{q}) + V_c(\vec{q}) \, ; \quad V_c(\vec{q}) = -\int \frac{d^3k}{(2\pi)^3} V(\vec{q} - \vec{k}) B(\vec{k}) \tag{11}$$

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In this way we separate in $\tilde{V}(\vec{q})$ one piece which corresponds to the ordinary one pion exchange and a second one induced by the correlations.

If we particularize the diagrams of fig. 1 to the s-wave πN interaction, we have the series for the pion propagator depicted in fig. 2. The terms connected by a one pion exchange line are summed up automatically by the Dyson equation, by writing the pion propagator which incorporates the proper pion self-energy. This self-energy corresponds to the irreducible diagrams (i.e. not connected by a pion line) in figs. 2b and 2d. There we see the lowest order s-wave pion self-energy (2b) and a new piece generated by the correlations (2d) where the wavy line corresponds to $V_c(\vec{q})$ of eq. (11).

The interesting thing to observe is that, in the correlated potential, V enters as a function of $\vec{q} - \vec{k}$, with $\vec{k}$ an integration variable, and it is well-known that the isoscalar part of the s-wave πN amplitude (the one that enters in fig. 2d for isospin saturated nuclei) is small on-shell due to PCAC, but grows very fast as one moves to off-shell situations [7, 8, 16].

In order to make a detailed evaluation of this piece let us recall that the s-wave πN t-matrix (diagonal in spin) is written as [17]

$$-it_{\beta\alpha}(q, q') = -i4\pi \left\{ \frac{2\lambda_1(q, q')}{m_\pi} \delta_{\beta\alpha} - i\frac{\lambda_2(q, q')}{m_\pi^2}(q^0 + q'^{\,0}) \, \epsilon_{\beta\alpha\lambda}\tau^\lambda \right\} \qquad (12)$$

with α, β, λ pionic isospin indices. The lowest order pion self-energy corresponding to fig. 2b is written as

$$\Pi(q) = 4\pi \frac{2\lambda_1(q, q)}{m_\pi} \rho \qquad (13)$$

and the second order term, induced by the correlations, corresponding to fig. 2d is given by

$$\Pi_c(q) = -\int \frac{d^3k}{(2\pi)^3} \left[4\pi \frac{2\lambda_1(q, k - q)}{m_\pi} \rho \right]^2 \frac{1}{(q - k)^2 - m_\pi^2} B(\vec{k}) \qquad (14)$$

We observe that the isoscalar amplitude enters there as a function of the half off-shell variables. For pionic atoms we have $q \equiv (m_\pi, \vec{0})$ and $q - k \equiv (m_\pi, -\vec{k})$, since we are assuming the exchange of mesons generating the correlations to be static (a fair approximation if they are of sufficiently short range).

In order to get a quantitative result for $\Pi_c(q)$ with some estimation of the errors we have taken three off-shell extrapolations of the isoscalar amplitude and three different correlation functions. The three models used for the extrapolation are those of refs. [7, 8] and the recent one of the Jülich group [18]. The first one assumes for the isoscalar amplitude, of interest here, a short range piece together with σ exchange and has been used in several many-body calculations [5, 19, 20]. It gives, at $q^0 = q'^{\,0} = m_\pi$,

$$\lambda_1(\vec{q}, \vec{q}') = -\frac{1}{2}(1 + \epsilon)m_\pi \left[a_{\text{sr}} + a_\sigma \frac{m_\sigma^2}{m_\sigma^2 + (\vec{q} - \vec{q}')^2} \right] \qquad (15)$$

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with $a_\sigma = 0.220\ m_\pi^{-1}$, $a_{sr} = -0.233\ m_\pi^{-1}$ and $m_\sigma = 550$ MeV.

The second one is based on a separable model for the πN interaction and the use of dispersion relations. It can be parametrized in the region of interest to us as [20]

$$\lambda_1(\vec{q}, \vec{q}\,') = \lambda_1(0)(1 + c_1 p^2 + c_2 p^4 + c_3 p^6) \exp(-cp^2) \tag{16}$$

with $p = |\vec{q} - \vec{q}\,'|$, $\lambda_1(0) = 0.0075$, $c_1 = 4.98\ \text{fm}^2$, $c_2 = 0.726\ \text{fm}^4$, $c_3 = 0.203\ \text{fm}^6$, and $c = 0.462\ \text{fm}^2$.

The third model [18] uses (direct and crossed) nucleon and delta-isobar pole diagrams together with correlated 2π-exchange in the σ ($J{=}0$) and ρ ($J{=}1$) channels. It describes quantitatively all relevant πN scattering phase shifts over the whole elastic region. We take model (2) of [18] which, for reasons indicated in that paper, is preferred. The normalization of the T-matrix of [18] needed for our computation is the one of eq. (1) of [18] with the equivalence $T^{(s)} \equiv 8\pi\lambda_1/m_\pi$. In the region of interest to us this amplitude can be parametrized as

$$\lambda_1(\vec{q}, \vec{q}\,') = (l_1 + l_2 p + l_3 p^2) \exp(-(p - p_0)^2/\omega^2) \tag{17}$$

with $p = |\vec{q} - \vec{q}\,'|$, $l_1 = 3.254\ 10^{-3}$, $l_2 = 0.1156\ \text{GeV}^{-1}$, $l_3 = 0.1588\ \text{GeV}^{-2}$, $p_0 = 0.204$ GeV, and $\omega = 0.530$ GeV. The quantity λ_1 is plotted in fig. 3 for the three models.

On the other hand we have chosen three different correlation functions suited to calculations of nuclear matter:

$$
\begin{aligned}
\text{I)} \quad & g(r) = 1 - j_0(q_c r) \\
\text{II)} \quad & g(r) = 1 - \exp(-r^2/R^2) \\
\text{III)} \quad & g(r) = (1 - \exp(-r^2/a^2))^2 + br^2 \exp(-r^2/c^2)
\end{aligned} \tag{18}
$$

with $q_c = 780$ MeV, $R = 0.75$ fm, $a = 0.5$ fm, $b = 0.25\ \text{fm}^{-2}$ and $c = 1.28$ fm.

The results for Π_c are shown in Table I where we give the correction in terms of a coefficient $\delta\text{Re}B_{0,c}$. We can see that the contribution from this correlated piece is systematically repulsive, and the strength depends on the extrapolation and the correlations used.

For a given off-shell extrapolation there is a certain dispersion of the results using different correlation functions. We use this to obtain an estimate of the uncertainties. Hence we take the average value and an error which allows us to reach the extremes.

The results obtained with the Hamilton and Jülich extrapolations are very similar, but they differ appreciably from those of the LMM model. Although we would favour the results of the Jülich model, the large sensitivity to the off-shell extrapolation is an unpleasant feature. But we must recall here that the dispersion corrections of eqs. (5) are also dependent on the off-shell extrapolation. An interesting finding of the present work is the realization that there are cancellations between the two pieces which depend on the off-shell

extrapolation, the dispersion and the correlation corrections, and that when the terms are added, the sum is small compared to $b_0 + \Delta b_0$ and the dependence on the off-shell extrapolation is weak. This can be seen in Table II. The dispersive correction column has been completed with the results of the Jülich model using the theoretical framework of [5] [6] and the Jülich extrapolation. For this purpose the extrapolation of λ_2 is also needed and we find for the region of interest to our problem

$$\lambda_2(\vec{q}, \vec{q}') = \lambda_2(0)(1 + ap^2)\exp(-bp^2) \tag{19}$$

with $p = |\vec{q} - \vec{q}'|$, $\lambda_2(0) = 0.053$, $a = 9$ GeV^{-2}, $b = 3.9$ GeV^{-2}.

In the dispersive results of Table II we have included errors of 20% due to uncertainties other than the off-shell extrapolation [5, 6]. The sum of the dispersive and correlation corrections appears in the third column of Table II and we observe that the results are: i) small, (of the order of 15% of the $b_0 + \Delta b_0$ term), ii) rather similar for the different off-shell extrapolations and iii) compatible within the theoretical errors. We take an average of this sum and its errors as representative of this combined contribution and, after transforming it into a $\delta b_{0,d+c}$ term, we show it below, together with the other contributions.

If we add up all the contributions discussed in this paper, with their corresponding uncertainties, to yield the theoretical equivalent value, $\bar{b}_0^{\text{th}}$ we find:

$$
\begin{aligned}
b_0 &= -0.0130 \pm 0.0033 \; m_\pi^{-1}, \quad \text{lowest order} \\[2mm]
\Delta b_0 &= -0.0140 \pm 0.0008 \; m_\pi^{-1}, \quad \text{second order Pauli} \\
& \qquad\qquad\qquad\qquad\qquad\quad \text{corrected rescattering} \\[2mm]
\delta b_{0,d+c} &= +0.0039 \pm 0.0020 \; m_\pi^{-1}, \quad \text{dispersion + correlation} \\[2mm]
\delta b_{0,\text{ch}} &= -0.0022 \pm 0.0002 \; m_\pi^{-1}, \quad \text{chiral} \\[2mm]
\bar{b}_0^{\text{th}} &= -0.0253 \pm 0.0063 \; m_\pi^{-1}, \quad \text{total}
\end{aligned}
\tag{20}
$$

The parameter Δb_0 is evaluated from eq. (4) using again the effective density. The errors in b_0 and Δb_0 are those coming from the experimental errors in a_1 and a_3.

It is interesting to compare these results to empirical values obtained from best fits to pionic atom data. Using the potential of Seki and Masutani [9], the one of Meirav et al. [21] and the one of Nieves et al. [22], and using again the concept of effective density to write all of them in terms of an equivalent b_0 parameter, we find

$$
\begin{aligned}
b_0 &= -0.0285 \pm 0.0007 \, m_\pi^{-1} \quad [9] \\[2mm]
b_0 &= -0.0350 \pm 0.0062 \, m_\pi^{-1} \quad [21] \\[2mm]
b_0 &= -0.0323 \pm 0.0007 \, m_\pi^{-1} \quad [22]
\end{aligned}
\tag{21}
$$

where the errors are the statistical errors coming from the best fit (see section 5 of ref. [23] for a reanalysis with these potentials and the determination of the statistical errors). The results in eqs. (21) mean that there are also uncertainties in the empirical value of b_0 tied to different assumptions about the shape of the potential. It is also interesting to observe that although the theoretical result of eqs. (20) lies a little below the empirical results, it is compatible with all of them if uncertainties are considered. This becomes more obvious if a weighted average [24] of the empirical results of eqs. (21) is done, which leads to $b_0^{\mathrm{emp}} = -0.0304 \pm 0.0005 m_\pi^{-1}$, where again the error is only statistical. The dispersion of results in eqs. (21) indicates that there is an error of the order of $0.002 m_\pi^{-1}$ tied to the assumptions in the shape of the potentials, such that the empirical value with its error becomes $b_0^{\mathrm{emp}} = -0.0304 \pm 0.0025 m_\pi^{-1}$. The central value of the theoretical results is 17% below the empirical one, but if uncertainties are considered the results are compatible.

One last comment should be made about the role of the πNN form factor in the present calculations. We have used throughout a monopole form factor with a cut off mass $\Lambda = 1300$ MeV from the full Bonn NN potential [25]. The form factor affects only the dispersion and chiral terms. There are indications that the cut off mass, Λ, could be smaller [26]. In this case the strength of both terms is reduced by a similar percentage. Since the dispersive term is about three times larger than the chiral term, the net effect is to increase $|\bar{b}_0^{\mathrm{th}}|$ and bring it even closer to the empirical value.

The results of this paper have shown that correlation effects in the s-wave potential are sizeable and that the off-shell dependence of the πN amplitude must be considered in its evaluation. The correlation correction is then sensitive to the off-shell extrapolation used. However, when this correction is added to the dispersive correction, similarly sensitive to the off-shell behaviour, the sum of the two effects is small and rather insensitive to the extrapolation used.

The present results should lead us to reflexion. In the past many theoretical efforts have been devoted to the search of the "missing repulsion" (see section 4 of ref. [11] for a thorough and critical discussion of the different theoretical works). The results obtained here indicate that the "missing repulsion", if any, cannot be large. They certainly do not exclude other sources of repulsion, like those tied to the renormalization of pions and nucleons in the inner structure of the πN models [27, 28], but puts severe constraints on their strength.

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Table I : Correlation correction to the optical potential parameter $\mathrm{Re}B_0[m_\pi^{-4}]$ (eq. (1)) derived from the pion-nucleon amplitudes of Hamilton [7] , LMM [8] and the Jülich group [18] using three different correlation functions (eqs. (18)).

$\delta\mathrm{Re}B_{0,c}[m_\pi^{-4}]$	I	II	III
Hamilton	-0.0127	-0.0133	-0.0141
Jülich	-0.0051	-0.0146	-0.0094
LMM	-0.0001	-0.0080	-0.0012

Table II: Dispersive and correlation corrections plus their sum in terms of the $\mathrm{Re}B_0$ parameter of eq. (1). The second column is the average of the correlation effects of Table I for the three correlation functions.

$[m_\pi^{-4}]$	$\delta\mathrm{Re}B_{0,d}$	$\delta\mathrm{Re}B_{0,c}$	sum
Hamilton [7]	0.032 ± 0.006 [5]	-0.013 ± 0.001	0.019 ± 0.007
Jülich [8]	0.028 ± 0.006	-0.010 ± 0.005	0.018 ± 0.011
LMM [18]	0.017 ± 0.004 [6]	-0.003 ± 0.005	0.014 ± 0.009

FIGURE CAPTIONS:

Fig. 1. (a) Second order term in πA scattering mediated by pion exchange alone. (b) Schematic representation of simultaneous exchange of other mesons together with pion exchange, which are taken into account in terms of a correlation function.

Fig. 2. Diagrammatic representation of the pion propagator led by the pion s-wave self-energy. The piece (d) is mediated by the correlated potential of eq. (11).

Fig. 3. The value of $\lambda_1(0, \vec{k})$ defined in eqs. (15), (16) and (17) as a function of the momentum transfer in units of pion mass, for pionic atom kinematics. (a) Hamilton model from ref. [7]. (b) Jülich group model (2) from ref. [18]. (c) LMM model from ref. [8].
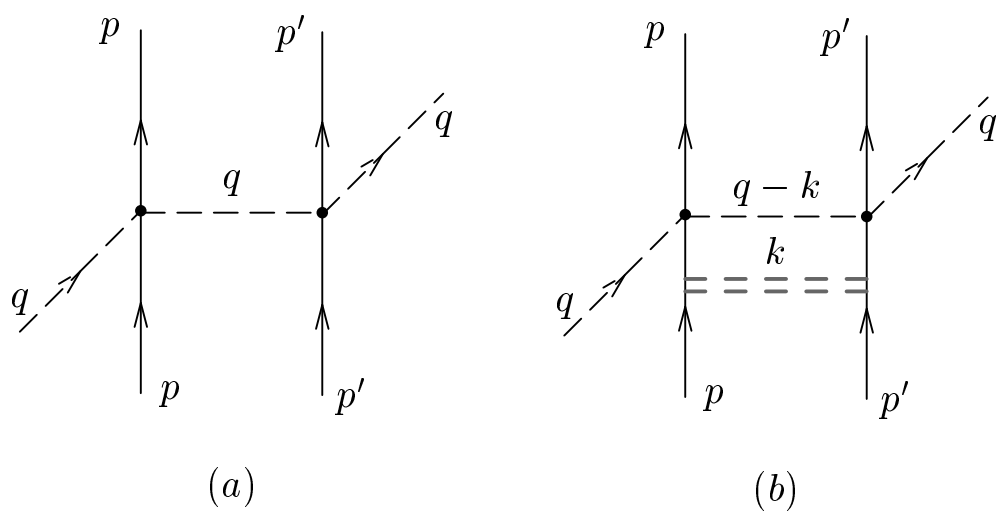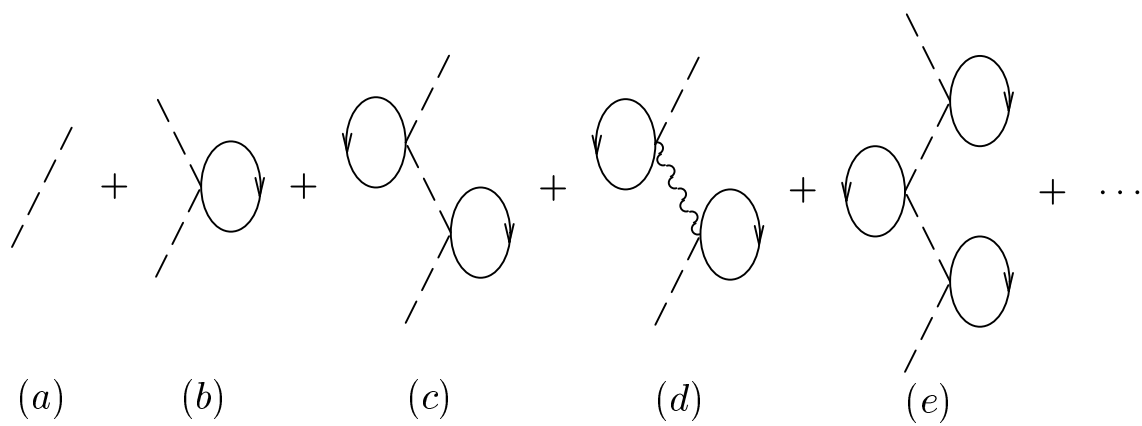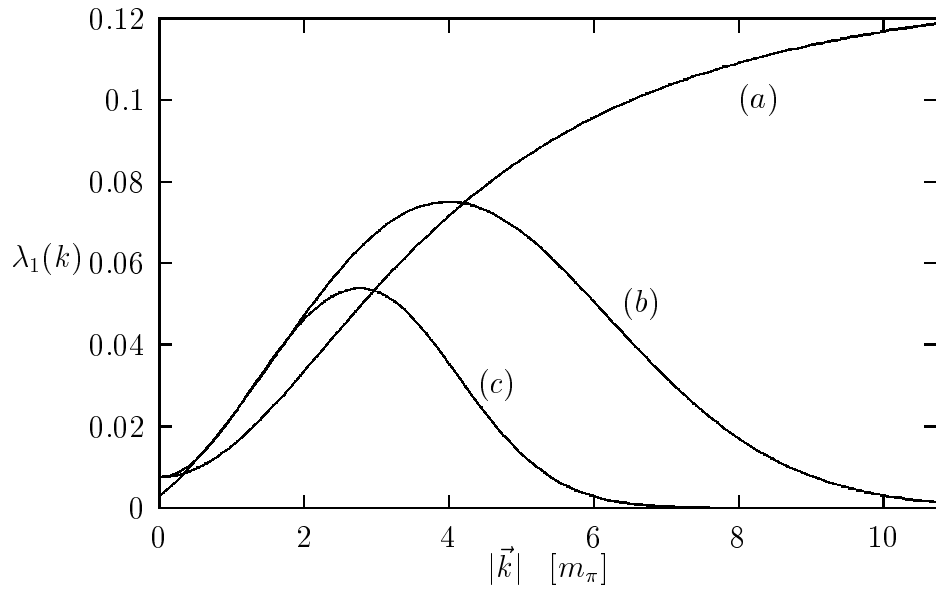
Fig. 1

Fig. 2

Fig. 3