A THEORETICAL APPROACH TO THE $(\pi^-, \gamma\gamma)$ REACTION IN NUCLEI

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Abstract

We have studied the π^- capture in nuclei leading to two photons, using improved many body methods which have been tested with success in one photon π^- radiative capture, μ^- capture and ν scattering. The qualitative features of the experimental data are reproduced but there are still some disagreement at small relative photon angles. The reaction is a potential source of information on details of the $\gamma\gamma\pi\pi$ vertex where chiral corrections can be relevant. New and more precise experiments are planned which make calculations like the present one relevant and opportune.

The $(\pi^-, \gamma\gamma)$ reaction in nuclei was the subject of much attention in the last decade [1, 2, 3, 4]. One of the things which stimulated this research was the possibility of finding precritical phenomena [5, 6], tied to pion condensation [7, 8], through the nuclear renormalization of the virtual pions with small energy and a finite momenta which appear in the driving mechanism for the process [2, 4]. The experimental work on this reaction has been sparse, with only two devoted experiments which provide the angular correlations of this pionic capture mode from pionic atoms [9] [10]. The agreement of the qualitative results of [1, 2] with experiment was only rough, with discrepancies of the order of a factor two to four, and a poor reproduction of the angular dependence. A more quantitative approach was followed in ref. [11] using pionic wave functions appropriate for finite nuclei. Yet the approach relies upon the closure sum, although some corrections to improve it have also been done, and uses approximate pionic wave functions which rely upon the concept of Z_{eff} and distortion factors tested in μ^- capture or radiative pion capture. The average nuclear excitation energy is also taken as suited for μ^- capture, but in this case the nuclear excitation energy is smaller since the two photons carry most of the energy of the pion. The results of [11] agree with experiment at small relative angles between the photons but disagree in about a factor four at large angles and the shape of the angular distribution is poorly reproduced.

With time passing there are general reasons to look back to this reaction: the renormalization of virtual pions in the medium is an important issue to complement our knowledge acquired in the study of real pions in the meson factories. This knowledge is important to evaluate the renormalization of weak currents in nuclei among others. These renormalizations are important to get proper rates in reaction like μ^- capture or neutrino scattering on nuclei. With the advent of new neutrino detectors, precise evaluations of neutrino nucleus cross sections are necessary to calibrate such detectors and the cross sections are rather sensitive to these nuclear renormalizations [12]. From the theoretical side, chiral perturbation theory provides corrections from hadronic loops which can be tested experimentally. Concretely in the $\gamma\gamma \rightarrow \pi\pi$ process, which enters the driving term of our reaction, as we shall see, these corrections have been done [13], although in a different channel than the one occurring here, and compared to recent DAPHNE measurements [14].

Improved techniques allow now to make more complete and precise measurements of the $(\pi^-, \gamma\gamma)$ reaction and new proposals are under way [15]. On the other hand in the last years theoretical progress has been done which allows a more accurate evaluation of the capture rates than it was possible in the past.

We follow here a procedure which has proved very accurate and simple to evaluate radiative pion capture [16], μ^- capture [17] and ν scattering on nuclei [18]. The method consists in evaluating (let us take μ^- capture as an example) the decay width of a μ^- in infinite nuclear matter but taking into account Pauli blocking, Fermi motion and the explicit sum over occupied states, with proper account of the energy of all nucleon states. Hence the closure sum, and consequently the dependence of the results on the average excitation energy, are avoided here. One has also the advantage of working with the relativistic operators throughout without the need to make the usual nonrelativistic reduction. At the end the width is evaluated as a function of the nuclear density and a mapping into finite nuclei is made via the local density approximation.

For our particular case let us assume a π^- in dilute nuclear matter to begin with (proper corrections will be implemented later). The pion decay width into the 2γ emission channel is given by

$$\Gamma = \sigma v_{rel} \rho_p \tag{1}$$

with ρ_p the proton density and σ the $\pi^- p \to \gamma\gamma n$ cross section. We take the model of fig. 1 for the amplitude of this latter reaction. Other terms with radiation from the nucleons are much smaller than these [4, 11]. In addition we shall work in the Coulomb gauge, $\epsilon^0 = 0, \quad \vec{\epsilon}(k)\vec{k} = 0$, and, since the momenta of the pionic atoms is small, the terms of fig. 1b, 1c become negligible. They are exactly zero when $\vec{q} = 0$, and for $\vec{q} \neq 0$ they are of the order of $|\vec{q}/\mu|^2$, with μ the pion mass, which is very small for pionic atoms. (Do not confuse these terms with ∇^2 terms in [11] which come from the way the closure sum is done). The expression for σv_{rel}, with $M/E \simeq 1$ for nucleons, is

$$\begin{aligned} \sigma v_{rel} \quad &= \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \frac{1}{2k_1} \frac{1}{2k_2} \frac{1}{2\omega_\pi} \frac{1}{2} \overline{\sum} \sum |T|^2 . \\ & 2\pi\delta(q^0 + E_p - E_n - k_1 - k_2) \end{aligned} \tag{2}$$

where

$$-\, iT = 2ie^2 \epsilon^\mu(k_1)\epsilon_\mu(k_2) \frac{i}{q'^2 - \mu^2} \frac{f}{\mu} \sqrt{2} \vec{\sigma}\vec{q}' \tag{3}$$

with $q' = q - k_1 - k_2$ and the factor $\frac{1}{2}$ in front of the sum and average over polarizations stands because of the symmetry of the two photons. Note that because we work now in a particular gauge

$$\sum_{i=1,2} \epsilon_i(k_1)\epsilon_j(k_1) = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{4}$$

Next step consists in replacing $-\pi\rho_p\delta(q^0 + E_p - E_n - k_1 - k_2)$ by $Im\bar{U}(q^0 - k_1 - k_2, -(\vec{k}_1 + \vec{k}_2))$, the Lindhard function for ph excitation of the np type given by

$$\bar{U}(q') = 2 \int \frac{d^3p}{(2\pi)^3} \frac{n_1(\vec{p})[1 - n_2(\vec{p} + \vec{q}')]}{q^0 + E_p(\vec{p}) - E_n(\vec{p} + \vec{q}') + i\epsilon} \tag{5}$$

with $n_1(\vec{p})$ the occupation number for protons and $n_2(\vec{p} + \vec{q}')$ the occupation number for neutrons. This substitution takes into account the finite density corrections, accounting explicitly for Fermi motion and Pauli blocking. After the proper substitutions we obtain

$$\begin{aligned}\frac{d\Gamma}{d\Omega_{12}} &= \frac{1}{\mu}\frac{1}{(2\pi)^5}\left(\frac{f}{\mu}\right)^2 e^4 \int k_1 dk_1 \int k_2 dk_2 \vec{q}'^2 (1+cos^2\theta_{12}). \\ &\quad \left(\frac{1}{q'^2-\mu^2}\right)^2 (-2) Im\bar{U}(q',\rho_p,\rho_n) \end{aligned} \tag{6}$$

with θ_{12}, Ω_{12}, the relative angle and solid angle of the two photons. In the actual evaluation of eq.(6) we consider a lower threshold for k_1 and k_2 of 25 MeV in order to compare with the data of the experiment [9]. For π^- capture from a particular pionic orbit we have

$$\Gamma_{nl} = \int d^3r |\phi_{nl}(\vec{r}\,)|^2 \Gamma(\rho_p(\vec{r}\,)\rho_n(\vec{r}\,)) \tag{7}$$

which makes explicit use of the local density approximation. $\phi_{nl}(\vec{r}\,)$ are the pionic wave functions which we obtain by solving the Klein Gordon equation with the potential of ref. [19].

Finally, since the experiment does not distinguish the decay from individual pionic orbits, a weighed average like in radiative π^- capture must be done and we get

$$R^{\gamma\gamma} = \sum_{nl} \frac{\Gamma_{nl}^{\gamma\gamma}}{\Gamma_{nl}^{abs}} \omega_{nl} \tag{8}$$

We take ω_{nl} from [20] (see also [16]) and Γ_{nl}^{abs} from several experiments tabulated in [17].

We can also take into account the renormalization of the process due to medium effects in the pion propagator, in analogy to the propagation of *ph* components in the longitudinal channel. The details can be seen in ref. [17] and it amounts to the change

$$Im\bar{U} \to \frac{Im\bar{U}}{|1-UV_l|^2} \tag{9}$$

with U the Lindhard function for ph plus $\triangle h$ excitation (and different normalization than $\bar{U}$, an extra factor 2 in the ph excitation part to account for isospin) and V_l the longitudinal part of the ph interaction. One must be cautious here. Since the two photons will carry most of the pion energy, the energy left for nuclear ph excitation is small. Then if the photons go back to back, the momentum transfer to the ph excitation is also small. This situation, with q^0,$\vec{q}$ small leads to unrealistic values of ReU_N, from ph excitation, if standard formulas [21] are used. Indeed, for q^0=0 and $\vec{q} \rightarrow 0$, ReU_N has a finite limit which is fallacious since the response in finite nuclei is strictly zero in closed shell nuclei. The discrepancies appear because one has a ratio of a numerator which is zero and a denominator which contains the ph excitation energy. The latter one is finite in finite nuclei, but zero in the continuum spectrum of infinite matter. The problem is solved if a realistic excitation gap energy is considered in the evaluation of ReU_N and this is done in [22]. We use for ReU_N the expressions of this latter reference.

Our results, compared with the experimental ones of ref. [9] are shown in fig. 2 for ^9Be and fig. 3 for ^{12}C. Our results approximately agree with experiment for angles above 90^0. In the case of ^{9}Be the results are on the upper side of the data, while for ^{12}C they are on the lower side. At small angles, however, our results are consistently below the data. The shape of the angular distribution is qualitatively correct, something that did not appear in the previous theoretical calculations [1, 2, 11]. The renormalization of the longitudinal response, eq. (9), in this case reduces the results, particularly at large angles, leading to a better agreement with the data.

We hope, however, that the improved techniques in the new proposals [15] lead to a new wave of very precise data from where one could take more seriously the discrepancies with theoretical models which would allow us to make progress on details of the elementary $\gamma\gamma\pi\pi$ vertices or possible missing many body effects.

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References

[1] T.E.O. Ericson and C. Wilkin, Phys. Lett. 57B(1975)345

[2] S. Barshay, Phys. Lett. 78B(1978)384; 79B(1978)514

[3] L.I. Lapidus and M.M. Musakhanov, Sov. J. Nucl. Phys. 15(1972)558

[4] E. M. Nyman and M. Rho, Nucl. Phys. A287(1977)390

[5] J. D. Delorme and M. Ericson, Phys. Lett. 76B(1978)182

[6] H. Toki and W. Weise, Phys. Lett. 92B(1980)265

[7] A. B. Migdal, Rev. Mod. Phys. 50(1978)107

[8] G.E. Brown and W. Weise, Phys. Reports 27(1976)1

[9] M. Lebrun, J. Deutsch, D. Favart, P. Lipnik, P. Macq and R. Prieels, Phys. Rev. C30(1984)383

[10] E. Mazzucato et al., Phys. Lett. B96(1980)43

[11] P. Christillin and T.E.O. Ericson, Phys. Lett. B87(1979)163

[12] J. Engel, S. Pittel and P. Vogel, Phys. Rev. Lett. 67(1991)426; ibid, 1994 preprint submitted to Phys. Rev.

[13] S. Bellucci, in the DAPHNE Physics Handbook, Vol. II, Ed. L. Maiani, G. Pancheri and N. Paver, pag. 419

[14] D. Morgan and M.R. Pennington, Phys. Lett. B192(1987)207

[15] J. Deutch, private communication

[16] H.C. Chiang, E. Oset, R. Carrasco, J. Nieves and J. Navarro Nucl. Phys. A510(1990)573; erratum Nucl. Phys. A514(1990)749

[17] H.C. Chiang, E. Oset and P. Fernández de Córdoba, Nucl. Phys. A510(1990)591

[18] S. H. Singh and E. Oset, Nucl. Phys. A542(1992)587; ibid, Phys. Rev. C48(1993)1246

[19] J. Nieves, E. Oset and C. García-Recio, Nucl. Phys. A554(1993)509

[20] H. W. Baer, K.M. Crowe and P. Trüol, Advances in nuclear physics, ed. M. Baranger and E. Vogt, vol. 9, 1977, and references therein.

[21] A.L. Fetter and J.D. Walecka, *Quantum theory of many-particle systems*, McGraw-Hill, 1971

[22] E. Oset, D. Strottman, H.Toki and J. Navarro, Phys. Rev. C48(1993) 2395

Figure Captions

Fig.1 Feynman diagrams for the amplitude.

Fig.2 Comparison between experimental data and our theoretical results for ^{9}Be. Energy resolution in the experimental data is 25 MeV photon threshold. Energy resolution is included in the theoretical results for 25 MeV (solid line) and 17 MeV (long dashed dotted line) photon threshold.

Fig.3 Comparison between experimental data and our theoretical results for ^{12}C. Energy resolution in the experimental data is 25 MeV (boxes) and 17 MeV (crosses) photon threshold. Energy resolution is included in the theoretical results for 25 MeV (solid line) and 17 MeV (long dashed dotted line) photon threshold.