

# OFF SHELL $\pi N$ AMPLITUDE AND THE $pp \rightarrow pp\pi^0$ REACTION NEAR THRESHOLD

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## Abstract

We have used a conventional model for the  $pp \rightarrow pp\pi^0$  reaction consisting of the Born term plus the  $s$ -wave rescattering term. As a novelty we have introduced the off shell dependence of the  $\pi N$   $s$ -wave isoscalar amplitude. This amplitude is appreciably enhanced when one moves to the off shell situations met in the problem and, as a consequence, the  $pp \rightarrow pp\pi^0$  cross section becomes considerably larger than with the use of the  $\pi N$  on shell amplitudes. Two different models for the off shell extrapolation found in the literature have been used and the cross sections obtained are large enough to account for the experimental data, although uncertainties remain due to the incomplete knowledge of the off shell extrapolation.

The  $\pi N$   $s$ -wave amplitude is commonly derived from an effective Hamiltonian [1]

$$H = 4\pi \frac{\lambda_1}{m_\pi} \bar{\Psi} \vec{\phi} \vec{\phi} \Psi + 4\pi \frac{\lambda_2}{m_\pi^2} \bar{\Psi} \vec{\tau} \cdot \vec{\phi} \times \vec{\Pi} \Psi \quad (1)$$

and  $\lambda_1, \lambda_2$  are related to the scattering lengths. A remarkable feature of this interaction is the large cancellation between amplitudes which leads to very small values of  $\lambda_1$  compared to  $\lambda_2$ . By using H6hler's results [2] one obtains  $\lambda_1 = 0.0075, \lambda_2 = 0.053$  for on shell  $\pi N$  scattering at threshold.

One of the consequences of the smallness of  $\lambda_1$  is the serious disagreement with the data [3] of the computed cross section for  $pp \rightarrow pp\pi^0$ , based on the Born term and the rescattering terms of fig. 1 using the threshold on shell  $\pi N$  scattering amplitude [4, 5].

A plausible solution was given in [6], based on a relativistic description of the  $NN$  interaction which generates pair terms where the  $N\bar{N}$  components are connected by the isoscalar part of the interaction.

Although this procedure is univocally defined when dealing with  $N$  positive energy components [7, 8], this is not the case when extrapolating the results to negative energy components where large ambiguities are present. As an example the authors of ref. [9], who try to get the relativistic potentials from information of amplitudes involving both nucleons and antinucleons, get a strength for the vector and scalar potentials of about half the results in [8]. Furthermore, modern versions of the  $NN$  potential [10] where the intermediate range attraction ( $\sigma$  exchange in most models) is obtained explicitly from the exchange of two correlated pions, would lead to different results in the  $\bar{N}N$  sector which would reduce the effects found in [6] if pseudovector couplings are used for the  $NN\pi$  vertex.

The purpose of the present work is to provide an alternative explanation for the unexpectedly large  $pp \rightarrow pp\pi^0$  cross section. The idea is based on the findings of ref. [11], where  $s$ -wave pion absorption for pionic atoms was studied. It was found there that, because in the rescattering term of fig. 1b (for an incoming pion) the  $\pi N$   $s$ -wave amplitude appears half off shell, the term proportional to  $\lambda_1^2$  in the absorption rate was appreciably enhanced due

to the off shell amplitude met in the process, which is considerably larger than the on shell one. One should also note that because the absorption rate for charged pions is dominated by a term proportional to  $\lambda_2^2$ , the effects due to the off shell extrapolation of  $\lambda_1$  were small [11] and absorption experiments (of charged pions) never felt the need for it [12].

The increase of the isoscalar amplitude as one moves off shell is a feature of all known models for the off shell extrapolation which can be derived from basic arguments using constraints of current algebra and PCAC [13]. This is also the case in a recent model for the  $\pi N$  scattering which treats again the  $\sigma$  exchange as a correlated two pion exchange [14].

The amplitudes corresponding to figs. 1a, 1b close to pion threshold are given, in Mandl Shaw normalization [15], by

$$-it^{(a)}(p_1 p_2, p'_1 p'_2 p_\pi) = \frac{f}{m_\pi} \frac{p_\pi^0}{2M} \langle s'_2 | \vec{\sigma}_2 (\vec{p}_2 + \vec{p}'_2) | s_2 \rangle + (1 \leftrightarrow 2) + exchange \quad (2)$$

$$-it^{(b)}(p_1 p_2, p'_1 p'_2 p_\pi) = -\frac{f}{m_\pi} \langle s'_1 | \vec{\sigma}_1 \vec{q} | s_1 \rangle F(q) \frac{i}{q^2 - m_\pi^2} \\ \langle s'_2 | (-i) 4\pi \frac{2\lambda_1(q, p_\pi)}{m_\pi} | s_2 \rangle + (1 \longleftrightarrow 2) + exchange \quad (3)$$

where the exchange is done in the two incoming protons (a symmetry factor  $\frac{1}{2}$  will appear in the standard formula for the integrated cross section because of the identity of the two final protons).  $F(q)$  is the monopole form factor with  $\Lambda = 1250 \text{ MeV}$  from [16].

The isoscalar  $\pi N$  amplitude  $\lambda_1(q, p_\pi)$  appears half off mass shell,  $q^0 = 70 \text{ MeV}$ ,  $q = 370 \text{ MeV}/c$ . At this off shell momenta this amplitude is, in most models, about five times bigger than the on shell amplitude.

We have taken two different off shell extrapolations to get an idea of the uncertainties that one can have in the final cross section.

The first model is due to Hamilton and the  $\pi N$  isoscalar amplitude is due to  $\sigma$ -exchange plus a short range piece [17]. In this model we have

$$\lambda_1(q, p_\pi) = -\frac{1}{2}(1 + \epsilon)m_\pi \left[ a_{sr} + a_\sigma \frac{m_\sigma^2}{m_\sigma^2 - (q - p_\pi)^2} \right] \quad (4)$$

with  $\epsilon = m_\pi/M, a_\sigma = 0.220 m_\pi^{-1}, a_{sr} = -0.233 m_\pi^{-1}, m_\sigma = 550 MeV$ .

The second model is the one of ref. [18], where it is shown that for not too large values of the kinematical variables, the isoscalar amplitude satisfying current algebra constraints can be written as

$$\begin{aligned} \overline{F}^+(\nu, t; q^2, p_\pi^2) &= \left(\frac{t}{m_\pi^2} - 1\right) \frac{\sigma(t)}{f_\pi^2} \\ &+ (p_\pi^2 + q^2 - t) \frac{\overline{F}^+(0, m_\pi^2; m_\pi^2, m_\pi^2)}{m_\pi^2} + f_3^+ \nu^2 \end{aligned} \quad (5)$$

with

$$t = (q - p_\pi)^2, \nu = (q + p_\pi)(p + p')/4M$$

where  $p = (M + m_\pi/2, -\vec{q}), p' = (M, \vec{0})$ , are the initial and final nucleon momenta. The first term on the right hand side of the equation is the Adler term and  $\sigma(t)$  is the  $\pi N$  sigma commutator given by

$$\sigma(t) = \frac{\sigma(0)}{\left(1 - \frac{t}{m_1^2}\right)^2 \left(1 - \frac{t}{m_2^2}\right)} \quad (6)$$

with  $m_1 = 8.24m_\pi$ ,  $m_2 = 7.5m_\pi$ ,  $\sigma(0) = 25MeV, f_\pi = 93MeV, f_3^+ = 0.82m_\pi^{-3}$  and  $\overline{F}^+(0, m_\pi^2; m_\pi^2, m_\pi^2) = -0.30m_\pi^{-1}$  [18, 19]. The normalization of the amplitude in eq. (5) is such that for on shell pions at threshold

$$\overline{F}^+(\nu, t; q^2, p_\pi^2) \equiv -\frac{4\pi^2\lambda_1}{m_\pi}(q, p_\pi) \quad (7)$$

As we said, the values of  $\overline{F}^+$  needed correspond to half off shell situations with  $q \simeq 370 MeV/c$ . Eq. (5) for higher momenta would become progressively inaccurate [18, 19], but these momenta do not play a role in the cross section. For this reason, and in order to have well behaved Fourier transforms in our computational scheme, we regularize  $\overline{F}^+$  multiplying it by the function  $\exp(-\vec{q}^2/11m_\pi^2)$  which does not affect the region of momenta of relevance to the problem. The results are rather insensitive to this regularizing factor. Indeed, if we change  $\overline{F}^+$  at momenta  $q$  above  $q = 500 MeV/c$ , making it fall down much faster for instance, the cross sections change only at the level of 2%. The off shell extrapolation of  $\overline{F}^+$  obtained with the quoted regularizing

factor resembles very much the results of model (2) of ref. [14] which are depicted in [20]. Hence we should expect cross sections from that latter model similar to those obtained here with the current algebra extrapolation.

In fig. 2 we show the two off shell extrapolations as a function of  $[-(p_\pi - q)^2]^{1/2}$  for  $\vec{p}_\pi = 0, p_\pi^0 = m_\pi$  and  $q^0 = m_\pi/2$ . This simplified kinematics, appropriate at threshold where we study the reaction, is used to evaluate the three dimensional integrals which appear when we include the initial and final state interaction, as we shall discuss below.

Chiral perturbation theory [21] offers a natural framework to account for, and extend, the results of current algebra, but still has problems in the baryon sector [22, 23] and is restricted to low momenta. The  $\pi N$  amplitude within this framework has been the subject of debate [24, 25] but extensions of the method could prove most useful in providing reliable off shell extrapolations in the region of interest to this problem.

From ref. [4, 5, 6] we know that taking into account initial and final state interaction of the protons in this process near threshold is very important. In order to implement this we follow the standard procedure [1] and recall that the pion goes out in  $s$ -wave and the final state of the two protons is  $L = 0, S = 0, T = 1, J = 0$ . Consequently the initial state is  $J = 0, T = 1, L = 1, S = 1$ , because of angular momentum and parity conservation. We work in coordinate space. Technically this requires to Fourier transform the amplitude of eqs. (2), (3), which are given in momentum space, and then perform integrals in coordinate space using the proper initial and final  $pp$  wave functions. We use radial wave functions solutions of the Schrödinger equation using the  $NN$  interaction of the Paris [26] and Bonn [16] potentials.

The Coulomb interaction is known to decrease the cross section calculated omitting it by an amount which ranges from 30% at  $\eta = 0.2$  to 10% at  $\eta = 0.5$  ( $\eta = p_{\pi max}/m_\pi$ ) [4] [27]. We implement these corrections to the results obtained using the strong potential alone.

In figs. 3 and 4 we show the result for the  $pp \rightarrow pp\pi^0$  cross sections close to threshold using the Paris and Bonn  $NN$  potentials respectively. As found in earlier work [4, 5, 6] we see that using the on shell  $\pi N$  isoscalar amplitude leads to cross sections which are very small compared with experiment. How-

ever, when the off shell extrapolation is used, the cross section is considerably increased. With the model of Hamilton we obtain results close to experiment. With the current algebra model for the  $\pi N$  interaction we obtain results which exceed the experimental cross section by about a factor of two.

The differences between the results obtained with the Paris and Bonn  $NN$  potentials in figs. 3 and 4 are mostly due to the differences appearing in the Born term, which is about 50% bigger with the Bonn  $NN$  potential.

All this is telling us that the present knowledge of the off shell  $\pi N$  extrapolation does not allow us to be very precise on the predictions for the cross section of the  $pp \rightarrow pp\pi^0$  reaction close to threshold, but with present uncertainties it is clear that this off shell extrapolation can by itself explain the experimental data.

It would be interesting to pursue research in the direction of ref. [9] to make the  $NN$ ,  $N\bar{N}$ ,  $\bar{N}\bar{N}$  models more consistent with experimental data on all these channels, or exploit theoretical models like the one of ref. [10], with correlated two pion exchange, extrapolating the model to negative energy states. This would bring new light into the relativistic potentials which are the base of the previous theoretical interpretation of the present problem [6]. If the relativistic scalar and vector potentials are smaller than those used in [6], as claimed in [9], or hinted with the use of correlated two pion exchange in the negative energy sector, the off shell extrapolation of the  $\pi N$  isoscalar amplitude would stand as a likely candidate for the explanation of the  $pp \rightarrow pp\pi^0$  data. This should stimulate theoretical and experimental work to put further constraints on the off shell extrapolation of the  $\pi N$  amplitude in order to reduce the uncertainties which we still have in this cross section, as we have shown in this paper.

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figure captions

fig. 1. Feynman diagrams considered in the  $pp \rightarrow pp\pi^0$  reaction near threshold. a) Born term; b) rescattering term.

fig. 2. Off shell extrapolation of the  $\pi N$  isoscalar amplitudes in the Hamilton [16] (solid line) and current algebra model (regularized at high momenta) [18] as a function of  $\sqrt{-t}$  (dashed-dotted).

fig. 3. Cross section for  $pp \rightarrow pp\pi^0$  near threshold as a function of  $\eta(p_{\pi max}/\mu)$  using the Paris  $NN$  potential for the initial and final state interaction of the two protons. Dashed line: Born term and rescattering term with on shell  $\pi N$  isoscalar amplitude. Solid: with off shell  $\pi N$  amplitude from Hamilton's model [17]. Dashed dotted line: same with the current algebra  $\pi N$  extrapolation [18]. Coulomb effects are included as discussed in the text.

fig. 4. Same as fig. 3 using the Bonn  $NN$  potential for the initial and final state interaction of the two protons

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