MAJORONS: A SIMULTANEOUS SOLUTION TO THE LARGE AND SMALL SCALE DARK MATTER PROBLEMS

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A B S T R A C T

It is shown that the existence of Majorons, which enable a heavy neutrino, 500 eV $\lesssim m_{\nu_H} \lesssim 25$ keV to decay into a light neutrino $m_{\nu_L} \lesssim 8$ eV and a Majoron, with lifetime 10^4 yr $\lesssim \tau_{\nu_H} \lesssim 10^8$ yr can solve both the large and small scale dark matter problems. For a primordial "Zeldovich" spectrum of fluctuations the limits are $m_{\nu_H} \lesssim 550$ eV and $\tau_{\nu_H} \gtrsim 10^7$ to 10^8 yr (the ranges $m_{\nu_H} \lesssim 500$ eV and $\tau_{\nu_H} \gtrsim 10^8$ yr are allowed by the model but galaxy formation becomes problematic). The large scale dark matter problem is how to achieve the critical density as implied by inflation, the small scale problems deal with the haloes of galaxies and galaxy formation and perturbation growth. The heavy neutrino could provide the solution to the small scale problem by initiating perturbation growth before decoupling. The decay products will be fast and thus not bound to the initial clumps, thus solving the large scale problem. The low mass relic neutrinos that were not decay products would remain bound in the gravitational potentials which grew from the initial perturbations. The resulting universe would be radiation dominated, which is consistent with present observations if $H_0 \lesssim 40$ km/sec/Mpc. An alternative solution can occur when $m_{\nu_L} \approx 10$ eV: the universe can again become matter dominated in the present epoch. This solution still allows $H_0 \sim 50$ km/sec/Mpc. The Majoron model parameters which best fit the dark matter considerations are presented.

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It has been pointed out from a variety of view points[1,2,3,4], that there are two kinds of "dark matter" problems. One is the large scale problem associated with achieving a critical density parameter, $\Omega = 1$ (or more precisely a flat universe) as required by inflationary cosmologies.[5] The other is the small scale problem which is associated with halos of galaxies and with galaxy formation timescales. Because Big Bang Nucleosynthesis[6] requires $\Omega_{baryon} \lesssim 0.15$, it is clear that the large scale problem would require non-baryonic matter (if it is a real problem). The need for non-baryonic matter on the small scale problem is more subtle. Dynamics of galaxies imply galactic masses which are significantly larger than visible masses; however, up to the scales of binaries and small groups, the total implied Ω is still $\lesssim 0.1$, thus there is no direct need for non-baryonic matter. On the larger scales of super clusters, dynamical arguments[7,8] lead to an implied Ω of ~ 0.1 to ~ 0.5 but the uncertainties are large. Thus from dynamic arguments alone it is not obvious that this dark matter is required to be non-baryonic (although some[9] have argued in that direction). The situation changes if one looks at how galaxies probably form.

Primordial fluctuations in density which eventually grow into galaxies and clusters should be of the adiabatic variety if they are to be consistent with grand unified theories[10]. For adiabatic fluctuations variations in temperature, δT will accompany variations in baryon density, $\delta \rho_b$. Current limits on the anisotropy of the 3°K background[11] argue that $\frac{\delta T}{T}$ at recombination is $\lesssim 6 \times 10^{-5}$. thus $\frac{\delta \rho_b}{\rho_b}$ at recombination must be $< 2 \times 10^{-4}$ if the fluctuations are adiabatic. But from recombination to the present epoch is only a factor of $\sim 10^3$ in redshift. Thus if all the matter is baryonic $\frac{\delta \rho}{\rho}$ cannot grow by more than 10^3 which would not enable $\frac{\delta \rho}{\rho} \sim 1$ to exist at the present, contrary to present observations. The solution to this problem (see refs. 2 and 3 and references therein) is to have non-

baryonic matter be dominant so that its fluctuations can start growing before recombination and after decoupling the baryons fall into the pre-existing potential wells.

If the non baryonic matter is only of the "hot" variety[3] and only condenses on large scales (clusters of galaxies and larger) then galaxies will form by fragmentation after clusters[12]. This scenario seems to have difficulties[13] because it results in galaxies forming too late. Whereas, if the non-baryonic matter is all of the "cold" variety[3] and can condense on small (dwarf galaxies or smaller) as well as large scales, then clusters build up hierarchically from the small scales and galaxy formation works. However, the density of such cold matter will then be totally measured by the dynamics of large clusters[1] which yield Ω's of a few tenths and certainly < 1 which is contrary to inflation's need. On the other hand, hot matter easily gives $\Omega = 1$ and naturally explains the mass scale of superclusters[12].

In the past, attempts were made to find a single type of particle to solve all the dark matter problems. Some examples were low mass neutrinos, axions, photinos, gravitinos, etc. However no single particle seemed to simultaneously solve the large scale and small scale dark matter problems. This has led to composite proposals[1] of 2 different particles, one hot (neutrinos?) the other cold (axions?) or to have other unattractive solutions such as a non-zero cosmological constant[14] or to have non-random phase initial fluctuations[15,16], or to have light not be a good tracer of mass concentrations.

In this paper we resurrect another alternative solution involving a heavy neutrino decaying to a lighter one plus a Majoron[17]. The idea of galaxy formation involving decaying massive neutrinos is not new.[18,19,20] However, some earlier work was perhaps overly restrictive in not allowing the present universe to be radiation dominated. We will return to this point later (see also the discussion in

Ref. 14 who also are resurrecting decaying ν's). A further motivation for the present paper is that we now have a workable physical theory[21] that can enable heavy neutrinos to decay to light ones without producing observationally unallowed photons. Let us now examine what is necessary in order for neutrinos to decay and still satisfy the cosmological constraints.

Neutrinos with only standard electroweak interactions would decay producing $e^+ e^-$ pairs (if $m_\nu > 1$ MeV) or γ's whose abundances would violate cosmological and astrophysical bounds if heavy neutrinos are unstable on cosmological time scales[18,20,22]. They should be considered as stable ($\tau > t_{universe}$) and, therefore, their masses constrained to be $m \lesssim 100$ eV or $m \gtrsim 5$ GeV by considerations of their present mass density.[22,23] Neutrinos with some additional spontaneously broken global interaction could, however, easily circumvent these mass limits. Massive neutrinos may decay into lighter ones by the emission of a Goldstone boson, $\nu' \rightarrow \nu G$. Goldstone bosons are the zero mass, spin zero particles generated by the spontaneous violation of a global symmetry. There is one of them per every broken generator and their couplings to fermions are derivative. Thus, their coupling constants are inversely proportional to the breaking scale of the global symmetry.

There are two models of this kind in which neutrinos could be cosmologically relevant[24,25,21] (i.e. where the annihilation $\nu\nu \rightarrow GG$ does not eliminate cosmic neutrinos as soon as they become nonrelativistic). In one of them[24,25] the global symmetry is interfamiliar or, as it is called, horizontal: it relates leptons and quarks of the same electric charge into multiplets[F1]. The same operator which changes μ into e within a multiplet transforms c into u, s into d, and ν_μ into ν_e within the others. Therefore the decays $\mu \rightarrow ef$, $K \rightarrow \pi f$, and $\nu_\mu \rightarrow \nu_e f$ are related. We use f for "familons" (following Wilczek[25]) the neutral Goldstone bosons characteristic of the spontaneous breakdown of global interfamiliar sym-

metries. The experimental bounds on the first two decays imply that the break-down scale, F, must be $F \gtrsim 10^{10}$ GeV. If the interfamiliar symmetry contains the U(1) of Peccei and Quinn[26] (it is not necessarily so) an "invisible" axion[27] will appear together with the familons. Cosmological implications of axions force another bound $F \lesssim 10^{12}$ GeV[28]. A comparison between the first and the last mentioned decays leads to a lifetime

$$\tau\left(\nu' \rightarrow \nu f\right) \cong 10^2 \left[\frac{F}{10^{10} \text{ GeV}}\right]^2 \left[\frac{100 \text{keV}}{m_{\nu'}}\right]^3 \text{ yr} \tag{1}$$

for the decay of a massive neutrino ν' into another one ν whose mass is negligible. Notice, that, since there are many familons, every one mediating a particular decay, there are no relevant mixing factors contributing to eq (1)[F1].

There is another model in which invisible decays of this kind are produced[21]. The global symmetry is now the $U_L(1)$ of lepton number (thus, it does not involve quarks). Moreover, the Goldstone boson, called here the Majoron[17] (designated by J in the following) does not even couple at the tree level to the charged leptons. It couples only to the singlet (right handed) components of neutrinos. The model consists of adding to the standard Glashow-Weinberg-Salam model, one singlet right-handed neutrino per generation and to have more than one singlet Higgs field (two is enough) coupled to them. The lepton number of the singlet Higgs fields must not be the same, therefore the lepton number of the three families must differ. Neutrinos of a given generation have both lepton number conserving Dirac masses m_D, and lepton number violating Majorana masses, M. The vacuum expectation values of the non standard Higgs fields spontaneously violate lepton number and produce large Majorana masses of order M for the singlet neutrinos (hence the origin of the name Majoron). The masses m_D are of the order of the charged fermion masses. Two types of mass eigenstate Majorana neutrinos are found: superheavy ones, mainly composed of singlet

current eigenstate neutrinos, whose masses are of order M, and light ones, the observed ones, mainly composed of the current eigenstate neutrino members of doublets, with masses $m_D \sim \dfrac{m_0^2}{M}$. The superheavy neutrinos are at scales beyond the range of present experiments. The Majoron is a certain combination of the singlet Higgs fields components, and, thus couples strongly to the superheavy neutrinos, very weakly to the usual ones (only through the small component of the singlet current neutrinos that they contain) and essentially negligibly to matter (not at tree level). Hence the existence of this kind of Majoron cannot be ruled out by experiments; it has only cosmological implications.

This model for the generation of neutrino masses was proposed long ago by Chikashige, Mohapatra and Peccei[17]. However, because it contained only one singlet Higgs field (the same would happen if there were many but all of them with equal quantum numbers) it failed to produce fast neutrino decays on a cosmological time scale[29]. In any event, these authors analyzed[30] some of the cosmological consequences of such a decay. They have worried mainly, as we will do below, about the constraint that the energy density of Majorons, ρ_M, should not exceed the critical density now $(\rho_M)_0 < \rho_c$. There are important mixing factors and Yukawa coupling constants, designed generically as s and h, which contribute to the decay rate of the neutrinos and depend on the particular details of the model used. In terms of the parameter K,

$$K = \frac{M}{sh} \tag{2}$$

(notice that K is an upper bound to M, because sh < 1) the lifetime for a heavier ν' decaying into a much lighter one and the Majoron[24,30] is

$$\tau(\nu' \rightarrow \nu J) \cong 32\,\pi\,\frac{K^2}{m_{\nu_H}^3} \cong 10^2 \left[\frac{K}{10^{10}\text{GeV}}\right]^2 \left[\frac{100\text{keV}}{m_{\nu'}}\right]^3 \text{yr} \tag{3}$$

The difference between (1) and (3) is that, while K could be $< 10^{10}$ GeV (naturally one would expect $\dfrac{m_\nu}{m_D} = \dfrac{m_D}{M}$) and mixing angles could be easily chosen to allow for some decays and forbid others without involving quarks, the scale of familons must be $F \gtrsim 10^{10}$ GeV. See figure (1) for the values of K implied by Eq. (3).

Axions could be present or absent in both models (though they are completely disconnected to Majorons and would fit naturally with familons). We will not consider them. We speak about Majorons in the following but everything applies, in principle, for familons as well.

The general picture we have in mind is that heavy massive neutrinos, ν_H, could dominate the density of the universe for a certain period. This period must begin after they become non relativistic. Unlike baryons, which are frozen in the radiation field up to recombination, these neutrinos are free to cluster. Therefore density enhancement in fluctuations can begin before the recombination era, but still satisfying the limits on $\delta T/T$ at recombination. Around the lumps formed by the heavier neutrinos, the lighter neutrinos can cluster, when they become non relativistic. The baryons will eventually fall into these lumps after the recombination era. After that, the heavier neutrinos decay leaving a hot population of relativistic light neutrinos ν_L, and Majorons. The decay cannot occur until $\delta\rho_{\nu_H}/\rho_{\nu_H}$ has grown by a factor $\gtrsim 5 \times 10^3$. Thus the temperature at matter domination $T_M \sim m_{\nu_H}/10$ must exceed the temperature at the time of decay, T_r, by a factor of $\gtrsim 5 \times 10^3$. (Neutrinos of mass m_ν don't dominate the radiation density until $\sim m_\nu/10$.) In principle, at this point there are two possibilities: either this population is still relativistic (and so the density of Majorons now will be around half of the total, $\rho_b + \rho_{\nu_L} + \rho_M = \rho_b + \rho_{\nu_L}{}^{NR} + 2\rho_M \approx \rho_c$, where $\rho_{\nu_L}{}^{NR}$ is the mass density of non-relativistic ν_L's); or the light neutrinos ν_L pro-

duced in the decay have become non relativistic at a temperature $T_{\nu_L}^{NR}$ and dominate the energy density of the universe now (then $2\rho_{\nu_L}^{NR} + \rho_b \approx \rho_c$ and the density of Majorons should be much smaller than ρ_c). In the first case the universe would now be radiation dominated (by Majorons and ν_L), in the second it would be matter dominated (by ν_L). In either case the large scale dark matter problem could be satisfied by the decay-product ν_L's (and Majorons in the radiation dominated cases) and galaxies would have formed from the ν_H's and retained those primordial ν_L's that initially fell into the ν_H lumps along with the baryons.

Let us estimate $(\rho_M)_0$. We follow the analysis of Dicus, Kolb and Teplitz[18] for a radiation dominated universe (Dicus and Teplitz[20] have done a similar analysis for familons.) The dominant interaction among neutrinos is through the standard weak interaction neutral current process $\nu_H + \bar\nu_H \rightarrow \nu_L + \bar\nu_L$, as they assume.[18] Therefore we use their calculations of temperature T_D, and number density $n_{\nu_H}(T_D)$ at the moment when the heavy neutrino decouple: for $m_{\nu_H} < 1 MeV$, $T_D = 3.4 \times 10^{10}\,^{\circ}K$, $n_{\nu_H} = 3 \times 10^{32}$ and thus

$$\rho_M = 2\, n_{\nu_H}(T_D) \left[\frac{1.9\,^{\circ}K}{T_D}\right]^3 \int_{t_D}^{t_u} dt \left[\frac{m_{\nu_H}}{2}\right] \left[\frac{t}{t_u}\right]^{\frac{1}{2}} \frac{e^{-(t-t_D)/\tau}}{\tau} \tag{4}$$

where t_D is the ν_H decoupling time, t_u is the age of the universe now, $t_u \approx 1.3$ to 1.9×10^{10} yr. The $(1.9\,^{\circ}K/T_D)^3$ factor accounts for the volume expansion from t_D up to now, $\dfrac{m_{\nu_H}}{2}$ is the initial energy of Majorons and $\left[\dfrac{t}{t_u}\right]^{\frac{1}{2}}$ accounts for its red shift; $n_H(T_D)\ e^{-(t-t_D)/\tau}$ is the ν_H density at the time t (at which the decay occurs), while $\dfrac{1}{\tau}$ is the probability that ν_H has decayed . Since $t_D \sim 1sec$ the time of decay will be $t_D + \tau \approx \tau$ and for $\tau \ll t_u$ the previous expression can be approximated by

$$\rho = n_H(T_D) \left[\frac{1.9\,^\circ K}{T_D} \right]^3 \frac{\sqrt{\pi}}{2} \, m_{\nu_H} \left(\frac{\tau}{t_u} \right)^{\frac{1}{2}} . \qquad (5)$$

This corresponds to the approximation that all the particles decay at a time τ. Now we require that

$$\rho_b + (\rho_{\nu_L} + \rho_M) \sim \rho_b + \rho_{\nu_L}{}^{NR} + 2\rho_M \lesssim \rho_c = 12 \, h_0^2 \, \text{keV cm}^{-3} \qquad (6A)$$

where $h_0 \equiv H_0/100 \text{km/sec/Mpc}$ and $\rho_{\nu_L}{}^{NR} \sim 100 \, m_{\nu_L}/\text{cm}^3$ which we will see is $\lesssim 1\text{keV/cm}^3$, from nucleosynthesis[6] we know that $0.15 < \rho_b < 0.4 \, \text{keV/cm}^3$. If we neglect $\rho_b + \rho_{\nu_L}{}^{NR}$ relative to ρ_c then

$$2\,\rho_M \lesssim 10 \, h_0^2 \, \text{KeV/cm}^3 \qquad (6B)$$

which assumes $h_0{}^2 > 0.1$, then relation 6B holds for

$$\left[\frac{K}{10^{10} \, \text{GeV}} \right] \lesssim 2. \left[\frac{m_{\nu_H}}{1 \, \text{KeV}} \right]^{\frac{1}{2}} h_0^2 \qquad (7)$$

therefore

$$\tau(\nu_H \to \nu_L J) \lesssim 4 \left[\frac{1 \, \text{KeV}}{m_{\nu_H}} \right]^2 10^8 \, h_0^4 \, \text{yr} . \qquad (8)$$

For familons similar constraints hold.[20] However, for familons the value $m_{\nu_H} = 10 \, \text{KeV}$ is only marginally acceptable and any lower value is in contradiction with bounds on $\mu \to ef$ and $K \to \pi f$[25,20]. Thus long decay times are difficult, but not impossible for familons (see in figure (1) the range of values of τ and m_{ν_H} for which a value $K \gtrsim 10^{10} \, \text{GeV}$ might still be compatible with the bound (8).)

Relatively long lifetimes are required because, as mentioned earlier, fluctuations must grow during the ν_H matter dominated epoch by a factor $\gtrsim 5 \times 10^3$. That is $(m_{\nu_H}/T_\tau) \gtrsim 5 \times 10^4$, or equivalently $(t_u/\tau)^{\frac{1}{2}} \lesssim m_{\nu_H}/(5 \times 10^4 \, T_0)$. Thus for $T_0 = 3\,^\circ K$ we obtain

$$\tau \gtrsim 2 \times 10^{-4} \, t_u \left[\frac{1 \, KeV}{m_{\nu_H}}\right]^2 \gtrsim \frac{2.6 \times 10^6}{(m_{\nu_H}/keV)^2} \, yr. \tag{9}$$

where we have used the constraint that $t_u > 1.3 \times 10^{10}$ yr from Globular cluster ages[31]. Relation (9) is consistent with (8) for all $h_0 \gtrsim 0.3$. It is also required that ν_H's survive beyond recombination at t_{rec} so that baryons can fall into the $\delta\rho/\rho \gtrsim 1$ clumps. In a radiation dominated universe $t_{rec} = t_u \left[\frac{T_0}{T_{rec}}\right]^2 \approx 10^{-6} \, t_u$. Thus we have the additional bound that

$$\tau > 10^{-6} \, t_u \, \Phi \gtrsim 1.3 \times 10^4 \, \Phi \, yr. \tag{10}$$

where the factor Φ is limited by $1 < \Phi < 10^4$ to allow enough time for baryons to non-linearly cluster around the ν_H clumps before ν_H decay. For $m_{\nu_H} > (14/\Phi^{\frac{1}{2}})$ KeV the bound from (10) supersedes the bound from (9). When the bound of (10) is applied to equation (8) one obtains the bound on m_{ν_H}

$$m_{\nu_H} < 1.7 \times 10^2 \, h_0^2 \, / \, \Phi^{\frac{1}{2}} \, KeV \tag{11}$$

For a radiation dominated universe $t_u \sim \frac{1}{2H_0} \sim \frac{10^{10} \, yr}{2h_0}$ (actually it will be $\sim 10\%$ greater than this due to baryons and non relativistic ν_L's being present) thus the bound of 1.3×10^{10} yr on t_ν requires $h_0 < 0.4$, and we obtain the limit $m_{\nu_H} \lesssim 25$ KeV. However, we will see below, that this limit can be strengthened if we assume something about the primordial fluctuation spectrum.

The requirement that the ν_H decay take place before the present so as to enable the decay products to be distributed outside the clumps, requires $\tau \lesssim 10^9$ yr. Thus from equation (8) $m_{\nu_H} \gtrsim 0.6 \, h_0^2$ KeV. If we assume[34] $h_0 \gtrsim 0.3$ then $m_{\nu_H} \gtrsim 60$ eV, however below we will supersede this requirement.

The characteristic objects which could be formed when the ν_H's dominate

the mass of the universe have Jean s masses

$$M_{Jeans} \sim \frac{3 \times 10^{12} \, M_\odot}{\left(\frac{m_{\nu_H}}{keV}\right)^2} \tag{12}$$

and thus are greater than $5 \times 10^9 \, M_\odot$ which is in good agreement with galaxy sizes. If we want the ν_H's to truly solve the small scale problems then we need $M_{Jeans} \lesssim 10^{13} \, M_\odot$ which tightens our lower bound to be $m_{\nu_H} \gtrsim 500$ eV.

If we assume a Zeldovich spectrum of primordial fluctuations then the power, $\delta\rho/\rho$ on scales larger than the horizon falls off as $M^{-2/3}$. Linear growth of the fluctuations begins when the ν_H's dominate at $T_M \sim m_{\nu_H}/10$. The mass of baryons, M_B, within the horizon at this time is

$$M_B \sim \rho_{b_0} \frac{T_M^3}{T_0^3} \frac{4}{3} \pi \, (ct_M)^3 \tag{13A}$$

where t_M is the age at T_M. Since $t_M \approx 10^6/T_M^2(KeV)$ sec and $T_0 \sim 3K \sim 3 \times 10^{-7}$ KeV and $\rho_b \lesssim 0.4$ KeV/cm^3 then

$$M_B \lesssim \frac{2 \times 10^{72}}{m_{\nu_H}^3(KeV)} \, KeV \sim \frac{1.5 \times 10^9}{m_{\nu_H}^3(KeV)} \, M_\odot \tag{13B}$$

As emphasized by ref (14) scales larger than this are suppressed in their initial fluctuation by $M^{-2/3}$ relative to the initial $\delta\rho/\rho \lesssim 2 \times 10^{-4}$. Logarithmic growth by a factor of ~ 2.5 occurs[14] during the remaining radiation dominated universe. Thus M_B is increased to

$$M_{B'} = M_B(2.5)^{3/2} \approx \frac{6 \times 10^9}{m_{\nu_H}^3(KeV)} \, M_\odot \tag{13C}$$

To produce present day bound ($\delta\rho/\rho \gtrsim 1$) clumps of baryons larger than $M_{B'}$ requires growth factors of $> 5 \times 10^3$. Since galaxies contain $\gtrsim 10^{11} \, M_\odot$ of equilibrated baryons it is important that the baryon scale on which $\delta\rho/\rho$ today is

$\gtrsim 1$ be $\gtrsim 10^{11} \, M_\odot$. Thus there is a need to increase $M_{B'}$ by a factor of $\gtrsim 17$ over the coefficient of eq. (13C). This can be accomplished by either (1) having $m_{\nu_H} \lesssim 400 \, eV$[F2] or by (2) having τ increase enough to have an additional growth of $\gtrsim (17)^{2/3} \approx 6.5$ which increases τ by a factor of ~ 40 or by (3) having some intermediate combination of the (1) and (2). Since the resultant upper bound on T_r goes linearly with $m_{\nu_H}^{-1}$ the combination of these constraints yields a consistant solution for all $m_{\nu_H} \lesssim 550 \, eV$. Combining this with our lower bound on m_{ν_H} from Jeans mass arguments yields a solution for the Zeldovich spectrum only for m_{ν_H} close to 500 eV.[F3] (Of course, if the assumption of a Zeldovich spectrum is dropped then our previous upper limit of $\sim 25 \, KeV$ still applies.) Note from equation (13B) that $m_{\nu_H} \approx 500 \, eV$ yields the smallest primordial clump of baryons, M_B, to be $\sim 10^{10} M_\odot$ which would imply that dwarf galaxies form via fragmentation during collapse rather than as primordial entities. The τ_{ν_H} lifetime for $m_{\nu_H} \sim 500 \, ev$ consistent with $0.3 \lesssim h_0 \lesssim 0.4$ is from $\sim 10^7$ to $\sim 4 \times 10^7 \, yr$.

We have assumed that the ν_L produced in the decay is still relativistic. The temperature at which the hot population of ν_L's which has received an energy $\dfrac{m_{\nu_H}}{2}$ at the time τ (when the temperature was $T_r = T_0 \left[\dfrac{t_u}{\tau}\right]^{\frac{1}{2}}$) becomes nonrelativistic depends on its mass (we define $T_{\nu_L}^{NR}$ as the temperature at which the energy $E_{\nu_L} \approx \dfrac{m_{\nu_L}}{3}$):

$$T_{\nu_L}^{NR} = \frac{2m_{\nu_L}}{3m_{\nu_H}} \, T_0 \left[\frac{t_u}{\tau}\right]^{\frac{1}{2}} . \tag{14}$$

The requirement $T_{\nu_L}^{NR} < T_0$ implies a bound on m_{ν_L}, $m_{\nu_L} \ll \dfrac{3m_{\nu_H}}{2} \left[\dfrac{\tau}{t_u}\right]^{\frac{1}{2}}$.

From eq. (8) this yields $m_{\nu_L} < 0.2 h_0^2 \, KeV$. For $h_0 < 0.4$ this yields

$m_{\nu_L} \lesssim 30$ eV, however the arguments of Ref (1) on neutrino mass densities with $h_0 < 0.4$ supersede and yield $m_{\nu_L} \lesssim 8$ eV.

The ν_L's left over from the initial decoupling will become non-relativistic when $T < m_{\nu_L}$ and will also fall into the density perturbations of the ν_H's before the ν_H's decayed, to the degree allowed by the phase space constraints[32]. This latter constraint will enable the non-relativistic ν_L's to form dark matter halos in clusters of galaxies with a small low velocity fraction of their distribution being marginally able to cluster as dark halos around single galaxies but from phase space arguments they cannot provide dark matter around dwarf spheroidals.[33] Of course, some baryonic matter must also be dark[6] since $\rho_b > \rho_{visible}$ and this dark baryonic matter can form halos of dark spheroidals as well as normal galaxies. It is important to remember that for $h_0 = 0.4$, $\Omega_b \equiv \rho_b/\rho_c$ can be as large as ~ 0.25 and thus dark baryons could even provide the dark cluster material, if necessary. However, the present density of ν_L's is $\left(\rho_{\nu_L}^{NR}\right)_0 \sim 10^2\, m_{\nu_L}$ cm^{-3}, thus it would be $\sim 0.4\, \rho_c$ for the extreme mass of $m_{\nu_L} \sim 8$ eV, and $h_0 \sim 0.4$, also in agreement with the needs of the dark matter problem in clusters of galaxies, with the lesser amounts clustered on smaller scales being consistent with a trend of rising mass/light ratios from single galaxies to clusters[7]. The ν_H decay products could constitute the large scale dark matter yielding a hot unclustered background.

We can identify the ν_H's as ν_τ's and the ν_L's as the intermediate mass eigenstate (say ν_μ), and the lightest neutrino (say ν_e) plays a nonimportant role. There would be two populations of ν_μ: the cold one formed by the ν_μ which were in equilibrium with the radiation and became nonrelativistic at a temperature $T \sim m_{\nu_\mu}$, and a hot one, produced together with the Goldstone bosons. The cold population of ν_μ's (and, maybe ν_e's) cluster around the perturbations and form

the dark halos.

A slightly different picture, but with the same features, could be a three neutrino scheme, in which the heavier ν_H's decay only into the lighter ones, $\nu_\tau \to \nu_e J$. The halos would still be composed of the intermediate ν_μ's, but the bulk of the missing energy density would be in the relativistic very light neutrinos, ν_e. It is straightforward to get such patterns within the Majoron model described at the beginning (see Appendix).

In each of the above options the present universe is radiation dominated even though galaxies form in an epoch when the universe went through a matter dominated phase. Critical density radiation dominated universes have $t_u \approx \dfrac{1}{2H_0}$. As mentioned before this requires $H_0 \lesssim 40$ km/sec/Mpc. Although published values for H_0 range from 50 - 100 km/sec/Mpc, the advocates[34] of 50 have assured us that values as low as 30 cannot be categorically dismissed. Clearly low values for H_0 are an important requisite for these models. Remember that, as H_0 decreases, the limit from nucleosynthesis on ρ_b can be translated into a larger fraction of ρ_c and even more of the dark matter can be baryonic. Such a high fraction of baryons would increase t_u to slightly above the $\dfrac{1}{2H_0}$ estimate[14].

There is another totally different possibility: the ν_L product of the ν_H decay could have already become non relativistic and dominate the energy density of the universe. This situation is compatible with a lifetime constraints of (9) and (10) only if the universe again became matter dominated very recently so as to provide sufficient cosmological expansion to cool the Majorons and ν_L's. Thus we can still use the evaluation of ρ_M for a radiation dominated universe, then, if we ask for ρ_M, to be a fraction A of ρ_c, the lifetime becomes $\tau \lesssim A^2 \, h_0^4 \left[\dfrac{1 \text{ keV}}{m_{\nu_H}} \right]^2 10^9 \, \text{yr}$, which is consistent with (10) for

$$A > \frac{10^{-3}}{1KeV} \frac{m_{\nu_H}}{h_0^2} \Phi^{\frac{1}{2}} \left[\frac{t_u}{10^{10}\,yr}\right]^{\frac{1}{2}}$$. Since even in this case the universe spends

much time being radiation dominated, h_0 must still be $\lesssim 0.5$. Thus

$A \gtrsim \dfrac{10^{-2}\,m_{\nu_H}\,\Phi^{1/2}}{1KeV}$ and to be matter dominated we have $A \ll \Omega_b + 2\Omega_{\nu_L}$

where $\Omega_{\nu_L} \sim \dfrac{100 m_{\nu_L}(eV)}{h_0^2}$.

Thus, for the range in m_{ν_H} given previously, a matter dominated solution is possible if

$$m_{\nu_L} > \frac{3m_{\nu_H}}{2} \left[\frac{\tau}{t_u}\right]^{\frac{1}{2}} . \tag{15A}$$

Using the bounds from eq. (10) in (15A) yields

$$m_{\nu_L} \gtrsim 1.5 \times 10^{-3}\,m_{\nu_H}\,\Phi^{\frac{1}{2}} \tag{15B}$$

For a $m_{\nu_H} \sim 500\,eV$ this yields a matter dominated solution when $m_{\nu_L} > 0.75\Phi^{1/2}\,eV$. However, in order for the total density to be equal to the critical density for $h_0 \approx 0.5$ it is required that $m_{\nu_L} \approx 10\,eV$. This requirement is consistent with the previous limit for $\Phi \lesssim 300$. This latter matter dominated solution has the appealing feature of being compatible with $h_0 \sim 0.5$ rather than requiring $h_0 < 0.4$. In such models a $500\,eV$ ν_H has a lifetime $\sim 10^8\,yr$ from eq (8). As before, this solution still requires fragmentation to make dwarf galaxies and must use dark baryons as their halos and as some fraction of the normal galactic halos.

It is interesting to note from equations 2, 3, and 7, that the optimal solution with $m_{\nu_H} \approx 500\,eV$ and $\Omega = 1$ requires the Majoron parameter $K \approx 2 \times 10^9\,GeV$, for the radiation dominated solution ($m_{\nu_L} < 8\,eV$;

$0.3 < h_0 < 0.4$) and $K \approx 3 \times 10^9$ GeV for the matter dominated solution ($m_{\nu_L} \approx 10$ eV; $h_0 \approx 0.5$).

In summary we have shown that Majoron decay of heavy neutrinos to light ones can simultaneously solve both the large and small scale dark matter problems. Solutions are only possible for small values for the Hubble constant. The required value for the Majoron coupling constants is allowed by all present experiments. Of course, proving the existence of such a Majoron is another problem.

Acknowledgements

Appendix

We give here the details of a Majoron model in which the heaviest (among the light) neutrino (call it ν_H) decays into a lighter one (called ν_L) while the other neutrino is stable (we choose this one to coincide with the current eigenstate ν_μ; it could be chosen to be the ν_e, for example, by exchanging the first and second families below).

Let us take the value of the lepton number for the 1st, 2nd and 3rd families to be 1, q, and (2q - 1) respectively and consider the case in which the two singlet Higgs fields added to the standard model, x and x', carry the lepton numbers -2 and -2q respectively. The doublet Higgs field ϕ present in the standard model must have lepton number zero, since it couples to the three families; therefore its couplings, which give origin to the Dirac mass terms, are diagonal. We call these Dirac masses α, β and γ for the 1st, 2nd, and 3rd family respectively (they are all proportional to the vacuum expectation value $<\phi> \sim 250$ GeV) The V.E.V. $<x>$ and $<x'>$, which are much larger than $<\phi>$, produce Majorana masses for the right handed neutrinos: the coupling of x produces only a diagonal mass term for the 1st family, a, while x' produces a diagonal mass for the 2nd one, b, as well as a mixing between the 1st and 3rd families, d. This full 6 x 6 matrix for the neutrinos is

$$
\begin{pmatrix}
000 & \alpha 00 \\
000 & 0\beta 0 \\
000 & 00\gamma \\
\hline
\alpha 00 & a0d \\
0\beta 0 & 0b0 \\
00\gamma & d00
\end{pmatrix}
\qquad \text{A.1}
$$

The second family does not mix with the other two, thus we find that the ν_μ current eigenstate is also a mass eigenstate with mass $m_{\nu_\mu} = \beta^2/b$. Let us diagonalize the 4 x 4 matrix which remains when the 2nd family is eliminated from A-1,

$$\left(\begin{array}{c|c} 0 & D \\ \hline D & M \end{array}\right) \qquad \text{A.2}$$

where

$$D = \begin{pmatrix} \alpha\,0 \\ 0\,\gamma \end{pmatrix} \ \text{ and } \ M = \begin{pmatrix} a\,d \\ d\,0 \end{pmatrix} \qquad \text{A.3}$$

we consider all the elements real, for simplicity. The matrix A.2 is diagonalized by following matrix

$$\left(\begin{array}{c|c} V_1 & DM^{-1}V_2 \\ \hline -M^{-1}DV_1 & V_2 \end{array}\right) \qquad \text{A.4}$$

Where V_1 and V_2 are 2 x 2 othogonal matrices (Since some of the eigenvalues could be negative, in general A.4 should be multiplied by a diagonal matrix of phases in order to make all eigenvalues positive). We are interested in V_1, which diagonalizes the sector of light neutrinos

$$V_1^T \left(D\, M^{-1}\, D \right) V_1 = (m)_{\text{diagonal}} \qquad \text{A.5}$$

It is defined by

$$V_1 = \begin{pmatrix} \cos\theta\,-\sin\theta \\ \sin\theta\,\cos\theta \end{pmatrix} \ \text{ with } \ \frac{\sin\theta\,\cos\theta}{(\cos^2\theta - \sin^2\theta)} = \frac{d\alpha}{a\gamma} \qquad \text{A.6}$$

Recalling that the Majoron J is

$$J = \frac{\text{Im}(<x> x + q<x'>x')}{\sqrt{<x>^2 + q^2<x'>^2}} \qquad \text{A.7}$$

(where Im stands for imaginary part) it is now straight forward to obtain the coupling constant g of the vertex $\nu_H \nu_L J$:

$$g = \frac{(m_{\nu_H} + m_{\nu_L})}{\sqrt{<x>^2 + q^2<x'>^2}}\, 2(q-1)\sin\theta\,\cos\theta \qquad \text{A.8}$$

Comparing the decay rate

$$\Gamma\left(\nu_H \to \nu_L J\right) = \frac{1}{32\pi}\, g^2 m_{\nu_H} \qquad \text{A.9}$$

when m_{ν_L} is neglected, with equation (3), we can identify the parameter K of eq (2)

$$K \equiv \frac{\sqrt{<x>^2 + q^2 <x'>^2}}{2(q-1)\,\sin\theta\,\cos\theta} \qquad A.10$$

From A.8 we can easily check that if the lepton number of the generations coincide, $l_3 - l_1 = 2(q-1) = 0$, the coupling vanishes at this order (A.8 gives the coupling at leading order in the expansion parameters $<\phi>/<x>$ and $<\phi>/<x'>$).

FOOTNOTES

[F1] There is one exception. This is the case of an Abelian horizontal group, that is a U(1) group under which each family transforms with a different charge. In this case there would be a unique familon whose couplings with quarks, charged leptons and neutrinos would include important mixing factors, call them generically $\sin\Theta_{qq'}$, $\sin\Theta_{ll'}$, $\sin\Theta_{\nu\nu'}$. The scale F in Eq. (1) should be replaced by $F/\sin\Theta_{\nu\nu'}$. This quantity is not strongly bounded by the experimental limits which indeed would apply to $F/\sin\Theta_{\mu e}$ of $F/\sin\Theta_{\pi K}$. This case would be identical to the Majoron model in relation to neutrino decays.

[F2] For values $m_{\nu_H} < 500$ eV the natural Jeans mass is larger than typical galaxy masses, and thus to produce galaxies some fragmentation process should occur.

[F3] In this case, the lower bound on τ almost coincides with the upper bound of Eq. (7) for the value $h_0 = 0.4$ (the maximum possible if the universe is still now radiation-dominated). Thus the dotted line in Fig. 1 should be raised so that it is almost in contact with the dashed one corresponding to $h_0 = 0.4$, allowing only for the small region shown by the arrow as "preferred solutions".

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Figure Caption

Fig.(1)The figure shows the curves of fixed lifetime for the Majoron model, corresponding to eq. 3. The dashed lines are the upper bound of eq (7) for different values of h_0, while the dotted line corresponds to a lower bound equivalent to the one in eq. 9. The upper bounds correspond to the limit from the critical density. The lower bound comes from requiring a long enough period of matter domination. Both bounds would coincide (leaving no allowed values) for $h_0 \sim 0.3$. We see that ranges $10^8 \, y \gtrsim \tau \gtrsim 10^4 \, y$ correspond to masses of few $100eV \lesssim n_{\nu_\mu} \lesssim 25KeV$. The preferred solution with the Zeldovich fluctuation spectrum yields K of a few 10^9 GeV.