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Fitting Simpson's Neutrino into the Standard Model

J W F Valle*

Rutherford Appleton Laboratory
Chilton, Didcot, Oxon OX11 OQX, UK

Abstract

I show how to accommodate the 17 Kev state recently reported by Simpson as one of the neutrinos of the standard model. Experimental constraints can only be satisfied if the μ and τ neutrino combine to a very good approximation to form a Dirac neutrino of 17 Kev leaving a light ν_e. Neutrino oscillations will provide the most stringent test of the model. The cosmological bounds are also satisfied in a natural way in models with Goldstone bosons. Explicit examples are given in the framework of Majoron type models. Constraints on the lepton symmetry breaking scale which follow from astrophysics, cosmology and laboratory experiments are discussed.

*On leave from University of Brasilia, Brazil

I. The neutrino mass problem is again in the news. Simpson has recently reported evidence for a distortion of the Curie plot corresponding to the β decay of tritium which may be interpreted as the emission of a 17 Kev state with a 3% mixing probability with the electron neutrino [1]. The effects of such a state would show up in all β spectra where their emission is kinematically allowed so it should soon be possible to obtain confirmation. If this exciting result is there to stay, then the obvious question is, can such a state be accounted for by one of the three light SU(2) doublet neutrinos or not? The answer to this is non trivial due to the many phenomenological constraints on the neutrino masses and mixing parameters which come from several experiments and also cosmology. The main constraints are listed below.

1. Upper limit on the ν_e mass from tritium β decay, which gives $m_{\nu_e} < 50$ev [F1].
2. Limits from neutrino oscillation searches [3].
3. Absence of neutrinoless double β decay ($\beta\beta_{o\nu}$) which implies "$m_{\nu e}$" < 6eV where the parameter "m_{ν_e}" is a model dependent combination of neutrino masses and mixing parameters [4].
4. The necessity to rid the universe from excess relic neutrinos, which would otherwise overclose the present universe implies m < 100 ev [5].

These constraints severely restrict models that try to relate Simpson's neutrino to the known neutrinos. Because Simpson's neutrino is strongly mixed with the ν_e one can see that in order to obey the requirements above the leptonic mixing pattern must not leave any state unmixed, but must be more complex. The combined solution for these requirements singles out a model in which ν_μ and ν_τ (two-component neutrinos) combine very accurately to form a quasi-Dirac neutrino [6] and the ν_e is a very light Majorana particle. The effective neutrino mass matrix has an approximate non-standard lepton symmetry $L_e - L_\mu + L_\tau$ [7]. To the extent to which this symmetry is a good symmetry we have consistency with all existing data. Refined oscillation experiments will provide the most stringent test.

Moreover the only natural way to obey the cosmological bound (requirement 4) is to invoke the presence of Goldstone bosons ensuing from the spontaneous breaking of global symmetries [8,10].

We consider various Majoron-type models which can realise the symmetry suggested by the experimental data. Models with familons [11] offer another possibility which, however, is phenomenologically rather constrained and appears in its simplest form to be ruled out by cosmological considerations.

II. The mass matrix

We now construct the mass matrix under the above requirements. It is easy to see that in order to enforce natural cancellation of the $\beta\beta_{o\nu}$ decay amplitude and, at the same time, have a light ν_e (requirements 1 and 3) no state should be left unmixed in the leptonic charged current. Moreover, the stringent limits from oscillation searches further specify the effective mass matrix for the left handed neutrinos to be predominantly of the basic form [7]:

$$m_L = \begin{pmatrix} 0 & \varepsilon & 0 \\ \varepsilon & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m \qquad (1)$$

where we have pulled out the factor $m \simeq 17$ kev which sets the overall scale and, for simplicity, have assumed CP conservation. To fit the experimental results of Simpson we set $\varepsilon \simeq .17$. Diagonalization (to first order in ε) yields a massless state

$$\nu_1 \approx \nu_e - \varepsilon\nu_\tau \qquad (2)$$

and a pair of degenerate 17 KeV states of opposite CP

$$-i\sqrt{2}\nu_2 \approx - \varepsilon\nu_e + \nu_\mu - \nu_\tau \qquad (3a)$$

$$\sqrt{2}\nu_3 \approx \varepsilon\nu_e + \nu_\mu + \nu_\tau \qquad (3b)$$

which make up a Dirac neutrino. It is easy to see that the $\beta\beta_{o\nu}$ amplitude is proportional to

$$\text{amp}(\beta\beta_{o\nu}) \propto \sum_{\alpha=1}^{3} K^2_{e\alpha}\, m_\alpha \equiv m_{Lee} \qquad (4)$$

so it vanishes naturally. In eq (4) $K_{a\alpha}$ (a=e,μ,τ are flavor eigenstates; α=1,2,3 are mass eigenstates) is the matrix describing the charged current weak interaction

$$\frac{ig}{\sqrt{2}} W_\mu \sum_{a,\alpha} \tilde{e}_a K_{a\alpha} \gamma_\mu \left(\frac{1+\gamma_5}{2}\right) \nu_\alpha + \text{h.c.} \qquad (5)$$

The $K_{a\alpha}$ can be read off from eq (2)and (3). With this pattern of mixing, the only oscillations are between ν_e and ν_τ, with a probability factor

$$P(\nu_e \to \nu_\tau ; L) \approx 2\varepsilon^2 \left[1 - \cos \frac{m^2 L}{2E}\right] \qquad (6)$$

(where L is the distance and E >> m is the beam energy). Since present experiments are not sensitive to oscillations on this $(17\ \text{Kev})^2$ scale it is legitimate to average them out so as to obtain a direct bound on the mixing coefficient ε. The present experimental limit on $\nu_e\nu_\tau$ oscillations is not very stringent so that the large mixing $\varepsilon \sim .17$ required by the Simpson data is still marginally acceptable [3]. The other oscillations between ν_e and ν_μ as well as ν_μ and ν_τ are absent in the limit where the symmetry of eq (1) is exact. A slight departure from the symmetry limit will give rise to large oscillations between ν_μ and ν_τ. (Notice in this scheme it is not possible to account for the smallness of the flux of solar neutrinos seen in Davis' experiment).

So far we have only discussed the first three requirements. In order to obey in a natural way requirement 4, that the contribution of Simpson's neutrino to the present density of the universe does not exceed the critical value, we now appeal to Goldstone bosons.

III. Majoron Models

III(a) Triplet Models

I first try to fit Simpson's neutrino in the framework of the Higgs triplet Majoron model of neutrino masses, where global lepton number is spontaneously broken at a scale low compared with the weak scale [8]. The Majoron (denoted J) is dominantly coupled to the left handed neutrinos. By virtue of this coupling there are fast annihilation processes of neutrinos into Majorons that deplete the universe from relic neutrinos when its temperature falls below their mass. Thus the Lee-Weinberg bound is automatically avoided. The Majoron also couples through its Higgs doublet admixture, to all charged fermions, with strength

$$g_{ffJ} \simeq \mp \frac{2\langle h\rangle}{\langle\phi\rangle^2} m_f \qquad (7)$$

where m_f is the diagonal fermion mass matrix, $\langle\phi\rangle$ and $\langle h\rangle$ are the doublet and triplet vacuum expectation values (vev's), respectively.
An important constraint comes from astrophysics and energy loss processes in stars involving the emission Majorons [12]. Recently Dugan et al [7] reanalysed the calculation of ref [12] and gave the following bound on the triplet vev,

$$\langle h\rangle < 1 \text{ Mev} \qquad (8)$$

In this simplest model (containing only one triplet) the mass matrix of the (left handed) neutrinos is the most general one. Thus there are no predictions for mixings, no natural reason for destructive interference in the neutrinoless double β decay amplitude, etc. We then move on to the two triplet case, first considered in ref. [9]. With the assignments given in the table, the resulting mass matrix for the left handed neutrinos is precisely of the form given in eq(1). In order to ensure the absence of any additional global symmetry beyond the $U(1)_{global}$ displayed in the table one may add another triplet h", neutral under $U(1)_{global}$, possessing the additional couplings $\ell_e^T C\, \tau_2 h'' \ell_\tau$, $\ell_\mu^T C\, \tau_2 h'' \ell_\mu$ and in the Higgs potential, terms like $\phi^+ h'' \tau_2 \phi^*$ and Tr $(h^+ h'' h'^+ h'')$ allowed by our assignments. In order to ensure that this additional triplet does not spoil the symmetric form of the mass matrix, eq (1) we choose a positive mass squared term for it in the Higgs potential. In this case a nonzero vev will still be generated for h" (due to the cubic term) but it will be small. In addition the Goldstone boson associated to $U(1)_{global}$ (call it J, as before) will have only a negligible component along h". Clearly, by construction, this model obeys requirements 1, 2 and 3 discussed above. In addition, the cosmological bound $\rho_\nu < \rho_c$ is automatically satisfied [8]. However, in this model J also couples to charged fermions through its Higgs doublet component

$$- \frac{2}{\langle\phi\rangle} \frac{-\langle h\rangle^2 + \langle h'\rangle^2}{[\langle h\rangle^2 + \langle h'\rangle^2]^{\frac{1}{2}}} \text{ Im } \phi \qquad (9)$$

so we expect (barring an unnatural cancellation between $\langle h\rangle^2$ and $\langle h'\rangle^2$) that the same bound given in eq (8) should also hold. This means that in order to provide a 17 Kev mass the relevant Yukawa coupling $g_{e\tau}$ in eq (1) must obey $g_{e\tau} > 1.7 \times 10^{-2}$. For such a value one expects Majoron bremsstrahlung in the decays of π and K mesons to occur at levels marginally consistent with present experimental limits on rare weak decays [13]. As emphasised in ref. [7] improved measurements of the μ spectrum in $K \to \mu\nu$ would either discover this Majoron or exclude these models.

III(b) Singlet Models

As discussed above, the Majoron model based on Higgs triplets is marginally consistent with present experimental information, so one expects a narrow window, if any, of allowed values for the parameters. We now show how consistency with experiment can also be achieved in a model with right handed neutrinos whose large Majorana mass is provided by singlet Higgs fields vevs: lepton number is broken at a high scale. The original model [14] is excluded for a 17 Kev neutrino will be cosmologically stable against decay via Majoron emission [15] [F2]. This is a consequence of the fact that the Majoron couplings to neutrinos are diagonal to order $(m_f/M)^2$ where m_f is a typical Dirac mass and M is the large Majorana mass [15].

We thus consider a model involving two singlets, Φ and Φ' of the type considered in ref [10]. The quantum numbers are given in the table.
The resulting 6x6 neutrino mass matrix,

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 \\ \hline m_1 & 0 & 0 & M_1 & M_2 & M_3 \\ 0 & m_2 & 0 & M_2 & 0 & 0 \\ 0 & 0 & m_3 & M_3 & 0 & M_4 \end{array}\right) \qquad (10)$$

where $m_1 \sim \langle\phi\rangle$ are of the order of a typical Dirac fermion mass and the large Majorana masses are $M_4 >> M_3 \sim \langle\Phi'\rangle >> M_2, M_1 \sim \langle\Phi\rangle$ [The entries M_2 and M_4 might also arise from yet another Higgs singlet, neutral under U(1)]. The relative phase of Φ and Φ' is defined by terms in the potential like $\Phi'^2 \Phi^*$.

matrix for the left handed neutrinos

$$m_L = \begin{pmatrix} 0 & \varepsilon & 0 \\ \varepsilon & \varepsilon'' & 1 \\ 0 & 1 & \varepsilon' \end{pmatrix} m \qquad (11)$$

which, when ε', $\varepsilon'' \to 0$ reproduces eq (1). One can verify that for $\varepsilon', \varepsilon'' < 5 \times 10^{-5}$ one can satisfy the limits from neutrino oscillations [3]. In this case the ν_1 acquires a mass $m_\nu \sim 3 \times 10^{-2}$ev. There is no natural reason however in the present model for why ε', ε'' are so small: the amalgamation of ν_μ and ν_τ into a Dirac particle has to be fine tuned. This is a general feature of this class of models, where the global lepton symmetry is broken only by weak iso-singlets: there is no natural way to obtain the symmetric form eq (1). Unlike the case of triplet models, astrophysics poses no severe constraints since the Majoron coupling to the electron occurs only at one loop level and is rather negligible. Similarly, its coupling to neutrinos is too small to be seen in rare weak decays due to the large lepton symmetry breaking scale. In short, this Majoron is invisible. The cosmological constraints will be discussed in IV.

III (c) Singlet-Triplet Models

We now turn to a hybrid model in which the 17 Kev scale is fed in directly via triplet vev's while the departure from the symmetry limit of the effective left handed mass matrix of eq (1) is associated with large singlet vevs. The quantum numbers for a model containing two Higgs isotriplets and a singlet are given in the table.

The mass matrix is

$$\left[\begin{array}{ccc|ccc} 0 & \varepsilon m & 0 & m_1 & 0 & 0 \\ \varepsilon m & 0 & m & 0 & m_2 & 0 \\ 0 & m & 0 & 0 & 0 & m_3 \\ \hline m_1 & 0 & 0 & M_3 & 0 & M_2 \\ 0 & m_2 & 0 & 0 & M_1 & 0 \\ 0 & & m_3 & M_2 & 0 & 0 \end{array}\right] \qquad (12)$$

where $M_3 \sim \langle\Phi\rangle$ and the other entries M_1 and M_2 need not be proportional to a neutral singlet vev. The relative global phase of Φ, h and h' is fixed by couplings like $Tr(h^+ h')\phi^*$. Diagonalization of eq (12) leads to an

light neutrino mass matrix of the form

$$m_L = \begin{pmatrix} 0 & \varepsilon m & m_f^2/M \\ \varepsilon m & m_{f'}^2/M & m \\ m_f^2/M & m & m_{f''}^2/M \end{pmatrix} \qquad (13)$$

where the dominant eμ and μτ entries are provided by triplet vevs while the others vanish in the limit $M \to \infty$ [F3]. (m_f are typical Dirac masses). By construction, the non-observation of e→μ and μ→τ oscillations is recovered in the limit M→∞ and, at the same time, the electron neutrino becomes massless. How about constraints from astrophysics and cosmology? In a model of this type the Majoron does couple at tree level to charged fermions, because it contains a non zero component along the Higgs doublet direction, very much as in a model involving only triplets. However such admixture is further suppressed to be of order $\langle h\rangle^2/\langle\Phi\rangle\langle\phi\rangle$ where $\langle h\rangle \sim \langle h'\rangle << \langle\phi\rangle << \langle\Phi\rangle$

$$g_{ffJ} \simeq O\left(\frac{1}{\langle\phi\rangle} \max\left[\frac{\langle h\rangle^2, \langle h'\rangle^2}{\langle\Phi\rangle\langle\phi\rangle}\right]\right) m_f \; . \qquad (14)$$

Thus the Majoron coupling to the electron, responsible for stellar energy loss will satisfy the astrophysical limit even for $\langle h \rangle$, $\langle h' \rangle$ as large as 20 Gev, the maximum allowed by the present limits on the W to Z mass ratio (ρ parameter) as long as $\langle\Phi\rangle > 10^6$ Gev.

The coupling of the Majoron to light neutrinos in this model is of the same order of magnitude as in a pure singlet model, (section III(b)) ie $O(m_f^2/M^2)$. Thus, Majoron bremsstrahlung imposes no constraint on models of this type due to the large scale of lepton symmetry breaking.

IV Cosmological Bounds

We now consider the constraints from cosmology on the m ≈ 17 Kev neutrinos. This discussion applies to models (b) and (c) considered above. In order that they do not overclose the present universe these heavy neutrinos must decay with lifetime shorter than the age of the universe. The only allowed fast decay channel involves Goldstone bosons [10]. Indeed, the decays $\nu_{2,3} \to \nu_1 + J$ are present in the models we have considered due to the non-sequential lepton charge assignments [9,10]. The relevant coupling constant can be read off [9] explicitly from the (symmetric part of) the matrix

$$F_{\alpha\beta} \approx \frac{m_\alpha}{M} \sum_{a=1}^{3} K_{a\alpha}\, q_a\, K_{a\beta} \qquad (15a)$$

where q_a are the lepton charges under $U(1)_{global}$, (specified in table (1B)), the coefficients $K_{a\alpha}$ are given by eqs. (2), (3), m_α are the light neutrino masses and M is the parameter which sets the scale of breaking of the global lepton symmetry,

$$M \approx \left[Q^2\langle\phi\rangle^2 + \ldots\right]^{\frac{1}{2}} \qquad (15b)$$

where the sum includes all scalar fields which carry a nonzero (global) charge Q. Notice from eq (15) that for the decay $\nu_{2,3} \to \nu_1 + J$ to be possible one requires (i) that K be nontrivial and (ii) that the global charge assignments distinguish among lepton families. All of these conditions are met for the models considered in III(b) and III(c) above and one expects a lifetime

$$\tau \sim \frac{3\times10^{-2}}{\varepsilon^2}\left(\frac{M}{10^7 GeV}\right)^2 yr \qquad (16)$$

In order that there is enough time to redshift the contributions of the decay products to the present energy density of the universe a short lifetime $\tau < 10^5$ yr is required [17]. This is an absolute upper bound on the lifetime: in the framework of this class of models it implies an upper bound on the lepton symmetry breaking scale, $M < 10^{10}$ Gev. Considerations regarding the growth of structure in the early universe also constrain τ and therefore M. Indeed if the heavy neutrino decays around 10^5 yr it seems unlikely, then that baryonic density fluctuations will ever have a chance to develop so as to be able to condense into galaxies. The point is that baryons remain frozen in the radiation until the recombination epoch at $t \approx 3\times10^5$ yr by which time the 17 Kev neutrinos should have decayed, leaving a radiation dominated universe in which perturbation growth is severely inhibited [18]. Thus we expect a stronger upper limit should be imposed on the lifetime. An extreme requirement would be that the heavy neutrinos

have decayed before they could matter dominate the energy density of the universe, ie $\tau < 2 \times 10^{-2}$ yr. In this case their effect on galaxy formation would be totally negligible. Clearly this cosmologically safe requirement may be too strong.

Following Turner and Steigman [19] we find that a weaker limit, $\tau<1$ yr would be acceptable. Requiring the 17 Kev neutrinos to satisfy this limit implies the upper bound, $M<10^7$ Gev. This limit poses a severe constraint for familon-type models. We know of no such existing model that survives this requirement.

V Familon Models

In models with spontaneously broken global family symmetries there arise, in general, several Goldstone bosons - familons - coupling to divergences of flavor changing neutral currents involving charged fermions [11]. Thus new rare effects would be expected like, for example, decays of the type $\mu \to e + f$ and $K \to \pi + f$ (f denotes a familon) which make these models much more restrictive than the Majoron type models we have been considering so far. The typical lower bound on the scale of breaking of the horizontal symmetry, is $F > 10^{10}$ Gev [20]. Thus we expect that, if the same scale is also responsible for the decay of our 17 Kev neutrinos $\nu_{2,3} \to \nu_1 + f$ their lifetime will not be short enough to allow adequate redshift of their decay products. Thus the simplest versions of the familon models cannot accommodate Simpson's neutrino in a way consistent with cosmology. However it may be possible to produce models that judiciously avoid these particular decay modes and therefore allow for a lower breaking scale.

It is interesting to contrast the situation here with the Majoron models we have considered in III. The point is that in all those models the (small) Majoron coupling to charged fermions is flavor conserving. Therefore the scale of lepton symmetry breaking can be lower than 10^7 Gev.

VI.

In summary, I have pointed out how the 17 Kev neutrino recently reported by Simpson can be accommodated in the framework of the standard model. More refined neutrino oscillation searches will provide a sensitive test of the model. In order to avoid the Lee-Weinberg bound we were led to include Goldstone bosons. Several possibilities were considered. Our specific examples are meant to illustrate general possibilities. They were all formulated in terms of SU(2)xU(1) but similar analysis applies effectively for most theories based on higher gauge groups of interest such as left-right symmetry.

Within the framework of Majoron models with a non sequential U(1) lepton symmetry we have considered 3 cases: (a) low scale breaking, leading to a model whose allowed parameter space, if any, is rather restricted by present experimental limits; (b) high scale breaking, leading to an acceptable model with an invisible Majoron and no immediate phenomenological consequences and, finally, (c) hybrid models with both kinds of breaking. In the latter case the triplet vev can be as high as the 20 Gev upper limit imposed by our present knowledge of the ρ parameter. This is possible thanks to the presence of singlets with large vev's. These vev's should, in any event, be lower than 10^7 Gev so as to satisfy cosmology. Therefore, models with familons are not favored.

Finally we note that in this paper we have not addressed the issue of the naturalness and origin of the various symmetry breaking scales involved.

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Note added

As the work reported here was being completed, I received a copy of the paper by Dugan et al, ref [7]. I am grateful to Graciela Gelmini for sending me a copy of their manuscript prior to publication.

Table (1A)

		III(a)		III(b)		III(c)		
	ϕ	h	h'	Φ	Φ'	h	h'	Φ
SU(2)xU(1)	(2,1)	(3,2)	(3,2)	(1,0)	(1,0)	(3,2)	(3,2)	(1,0)
U(1)global	0	-1	1	2	1	-1	1	2

Table (1B)

	$SU(2)_L xU(1)_Y$	$U(1)_{global}$ (a)	(b)	(c)
$\ell_{eL}=(\nu_e,e)_L^T$	(2,-1)	1	1	1
$\ell_{\mu L}=(\nu_\mu,\mu)_L^T$	(2,-1)	0	-1	0
$\ell_{\tau L}=(\nu_\tau,\tau)_L^T$	(2,-1)	-1	0	-1
e_R	(1,-2)	1	1	1
μ_R	(1,-2)	0	-1	0
τ_R	(1,-2)	-1	0	-1
ν_{eR}	(1,0)	-	-1	-1
$\nu_{\mu R}$	(1,0)	-	1	0
$\nu_{\tau R}$	(1,0)	-	0	1

Table 1: Assignments of the various fields under $SU(2)_L xU(1)_Y xU(1)_{global}$: (A) Higgs fields responsible for mass generation; (B) Lepton families.

Footnotes

[F1] We are discarding the lower bound quoted by the ITEP group which has recently been subjected to criticism by Bergkvist [Ref 2].

[F2] Annihilations of neutrinos into Majorons are negligible in this model.

[F3] It is conceivable that the m_{Lee} entry could also acquire a contributions of order m_f^2/M which do not come directly from the diagonalization, eq. (13).