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## ABSTRACT

Constraints on Additional  $Z'$  Gauge Bosons  
from a Precise Measurement of the  $Z$  Mass<sup>\*†</sup>

We analyse the constraints on the mass and mixing of a superstring inspired  $E_6$   $Z'$  neutral gauge boson that follow from the recent precise  $Z$  mass measurements and show that they depend very sensitively on the assumed value of the  $W$  mass and also, to a lesser extent, on the top quark mass.

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Despite the impressive success of the standard  $SU(3) \otimes SU(2) \otimes U(1)$  model in describing the observed interactions of quarks and leptons, there are good reasons for believing that it is not the complete theory and there are many motivations to seek for extensions. Many of these extensions such as left-right symmetric,  $SO(10)$  or  $E_6$  theories predict the existence of additional neutral gauge bosons at low energy. Here we note that the recent more precise measurements of the  $Z$  boson mass by the MARKII and CDF collaborations [1, 2] offer a valuable test on the gauge structure of the electroweak interaction. In fact, it is well known that the standard electroweak theory predicts a definite correlation between the gauge boson masses and the electroweak mixing angle. Mixing with an extra  $Z'$  gauge boson affects this correlation and on this basis we can constrain these possibilities.

Here we focus on the simplest theoretically well-motivated case where the gauge sector contains an additional  $U(1)$  gauge symmetry at low energies. Of special interest in this regard are the models where the new  $U(1)$  hypercharge quantum numbers are derived from an underlying  $E_6$  symmetry at a sufficiently high energy scale. This situation typically occurs in superstring models based on Calabi-Yau compactification [3]. Another motivation for choosing this class of models is that they predict to lowest order approximation that the  $\rho$  parameter measuring the ratio between the strength of charged to neutral currents is one as in the standard model. Here we analyse the impact of the new  $Z$  mass determination on the possible existence of such an additional neutral gauge boson. We study the constraints both on the  $Z'$  mass and on its mixing angle. Although somewhat model-dependent, we find that these constraints always depend very sensitively on the assumed value of the  $W$  mass. Relatively large  $W$  masses (certainly consistent with UA1 data) may be easily accommodated provided the  $Z'$  is relatively light.

Alternatively for the case of low  $W$  masses (as already indicated by preliminary CDF results) we find that the new experimental results for the  $Z$  mass imply stringent limits on the  $Z'$  mass. In addition in the simplest versions of  $E_6$  superstring inspired models we have stringent limits on the  $Z'$  mixing angle. We also analyse the effect a heavy top quark would have upon these constraints, through radiative corrections. Our results highlight the importance of precision measurements of the  $Z$  mass at LEP/SLC and of a better  $W$  mass determination from hadron colliders.

Being a rank six group  $E_6$  contains in general two neutral gauge bosons beyond those of the standard model. These couple to two new hypercharges which may be taken to be those corresponding to the  $U(1)$  symmetries in  $E_6/SO(10)$  or  $SO(10)/SU(5)$ , denoted  $\psi$  and  $\chi$ , respectively. These hypercharges are given in table 1.

For proper normalization the standard hypercharge  $Y$  values should be scaled by a factor  $\sqrt{3/5}$  while the new hypercharges  $Y_\chi$  and  $Y_\psi$  should be scaled by factors  $1/\sqrt{40}$  and  $1/\sqrt{24}$ , respectively.

Here we will focus on the gauge sector of these models.  $W_c$  will assume that only one combination of the  $\chi$  and  $\psi$  symmetries survives at low energies. This still leaves a continuum of possible models with an extra  $U(1)$ . For example one might take any linear combination of the above hypercharges [6] *i.e.*

$$Y(\beta) = \cos \beta Y_\chi + \sin \beta Y_\psi. \quad (1)$$

Which particular combination is realized at low energies depends on the assumed pattern of symmetry breaking starting from the original  $E_6$ .

If  $E_6$  is broken all the way in one step via a non-abelian flux factor then

the following value is selected for the angle  $\beta$ :

$$\begin{aligned}\cos \beta &= \sqrt{3/8} \\ \sin \beta &= -\sqrt{5/8}\end{aligned}\quad (2)$$

leading to the  $\eta$  model considered in ref [8].

If, on the other hand, the assumed manifold discrete symmetry is abelian, then there are several rank six choices for the resulting intermediate gauge symmetry  $G$ . To ensure the possibility of further breaking at a very large energy scale via the Higgs mechanism we require that light  $SU(3) \otimes SU(2) \otimes U(1)$  singlets in the  $\mathbf{27}$  and  $\overline{\mathbf{27}}$  should survive the process of compactification [9, 10]. Several intermediate gauge groups can arise this way [11]. Here we focus on the simplest of these possibilities where  $G = SU(3) \otimes SU(2)_L \otimes U(1)^3$ . One of the  $U(1)$ 's in  $G$  (the one in  $E_6/SO(10)$ ) can then break due to a large vacuum expectation value  $\langle n \rangle$  along a suitably D-flat direction [12]. This way we are led to the  $\chi$  model described in ref [13]. It is defined by the  $U(1)_X$  hypercharge of table 1, *i.e.*

$$\begin{aligned}\cos \beta &= 1 \\ \sin \beta &= 0\end{aligned}\quad (3)$$

In this model it is in principle possible to suppress proton decay and flavour changing neutral currents by the large intermediate symmetry scale  $\langle n \rangle$ .

Arbitrary values of  $\beta$  (such as the one that defines the “ $\psi$  model” having  $Y_\psi$  as hypercharge) are possible outcomes of a primordial  $E_6$  symmetry but are not realized in the context of the restricted class of  $E_6$  models that arise in string theories [7].

We will now study the constraints on the mass and mixing angle of the  $Z_\chi$  and  $Z_\eta$  that arise from the new experimental  $Z$  mass measurements.

	$SU(3) \otimes SU(2) \otimes U(1)_Y \otimes U(1)_X \otimes U(1)_\psi$
$Q$	$(3, 2, 1/6, -1, 1)$
$u^c$	$(\bar{3}, 1, -2/3, -1, 1)$
$e^c$	$(1, 1, 1, -1, 1)$
$d^c$	$(\bar{3}, 1, 1/3, 3, 1)$
$\ell$	$(1, 2, -1/2, 3, 1)$
$H_d$	$(1, 2, -1/2, -2, -2)$
$g^c$	$(\bar{3}, 1, 1/3, -2, -2)$
$H_u$	$(1, 2, 1/2, 2, -2)$
$g$	$(3, 1, -1/3, 2, -2)$
$\nu^c$	$(1, 1, 0, -5, 1)$
$n$	$(1, 1, 0, 0, 4)$

Table 1: Quantum Numbers of the particles in the  $\mathbf{27}$  of  $E_6$  with respect to the gauge group  $SU(3) \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \otimes U(1)_\psi$ .

For this we need to specify the symmetry breaking. In the present models this should occur around the TeV energy region. The electrically neutral scalars responsible for symmetry breaking are also restricted, since (a) there are only doublets and singlets under  $SU(2)_L$  and (b) they lie in the  $\mathbf{27}$  of  $E_6$ . The singlets have the quantum numbers of  $n$  and  $\nu^c$ , given in table 1 and may acquire relatively large VEVs (in the TeV region), *i.e.*  $\langle \nu \rangle^c \neq 0$  [14] and/or  $\langle n \rangle \neq 0$ , in order to break the new  $U(1)$ . The doublets have the quantum numbers of  $H_u$ ,  $H_d$  and  $\ell$  and their VEVs are responsible for electroweak breaking. It is straightforward then to work out from table 1 the neutral vector boson mass matrix in these models. In order to properly identify the massless photon field and the correct electric charge we must require

$$\tan \theta_W = g'/g \quad (4)$$

where  $g' = \sqrt{3/5} g_1$  gives the relation between the standard hypercharge gauge coupling constant at low energy and the constant  $g_1$  corresponding to the properly normalized  $E_6$  generator. Here  $g$  is the  $SU(2)_L$  gauge coupling. The resulting  $2 \times 2$  mass matrix has the form

$$\begin{pmatrix} m_{Z^0}^2 & \mu^2 \\ \mu^2 & M^2 \end{pmatrix} \quad (5)$$

where  $m_{Z^0}^2$  would be the  $Z$  mass in the absence of mixing with the extra  $Z'$ ,

$$m_{Z^0}^2 = \frac{m_W^2}{\cos^2 \theta_W} \quad (6)$$

and

$$\cos^2 \theta_W = 1 - A^2/m_W^2 \quad (7)$$

where the parameter

$$A^2 = \frac{\pi\alpha}{\sqrt{2}G_F} \simeq (37.281 \text{ GeV})^2 \quad (8)$$

is well determined from Thomson scattering ( $\alpha$ ) and  $\mu$  decay ( $G_F$ ). The mixing parameter  $\mu^2$  is given by

$$\mu^2 = m_{Z^0}^2 \sin \theta_W \left[ \frac{\sqrt{10}}{3} (1 - 2\xi) \sin \beta - \sqrt{2/3} \cos \beta \right] \quad (9)$$

and depends on the chosen model through the angle  $\beta$  and also through the dynamical parameter  $\xi$  [17]

$$\xi = \frac{\langle H_d \rangle^2}{\langle H_u \rangle^2 + \langle H_d \rangle^2} \quad (10)$$

Similarly  $M^2$  is another model-dependent parameter related to the symmetry breaking scale of the extra  $U(1)$ .

So far we have neglected radiative corrections. These are of two types. The dominant source of corrections is the running of  $\alpha$  from  $\alpha(q^2 = 0)$  up its short distance value relevant to us. Another potentially large contribution may arise *e.g.* from a heavy top quark. The net effect of these corrections is to rescale the parameter  $A^2$  in eq. (8), so that eq. (7) should be replaced by

$$\sin^2 \theta_W = \frac{A^2}{m_W^2 (1 - \Delta r)} \quad (11)$$

In the figures we display the constraints on the  $Z'$  mass and mixing obtained from eq. (5) corresponding to a representative top mass of 90 GeV and a Higgs mass of 100 GeV, using the value  $\Delta r = 0.0606$  taken from ref [18]. For the mass of the  $Z$  we use the central value and error given by the MARKII collaboration [1], *i.e.*  $m_Z = 91.17 \pm 0.18 \text{ GeV}$ .

In fig 1 we plot the dependence of the mass of  $Z'$  on the mass of the  $W$  in both the  $\chi$  and  $\eta$  models given the new  $Z$  mass measurements.

For *large*  $W$  masses there is a narrow band of relatively low  $Z'$  masses which is allowed by the gauge boson mass data and in this case a non-zero mixing should exist, as seen from fig 2. If however, as is already

indicated by preliminary CDF results [2], the  $W$  mass turns out in the *low* side, then one expects to be very close to the standard model. This is exactly what the figures show: we obtain a stringent lower limit on the  $Z'$  mass and a stringent upper limit for the mixing angle as can be seen from figure 2. To obtain the constraints on the  $Z'$  parameters in the  $\eta$  model we need to assume a value for the dynamical  $\xi$  parameter, and we have chosen a reasonable range, recommended in ref [7] with  $\xi$  varying between  $\xi = 0.04$  and  $\xi = 0.27$ . Uncertainties in the detailed dynamics in these models could allow for a larger value of  $\xi \simeq 0.5$  that would somewhat weaken our constraints for the  $\eta$  model.

In both models we derive relatively stringent bounds on the  $Z'$  mass, depending on the assumed value of the  $W$  mass. The recent measurement repeated by CDF,  $m_W = 80.0 \pm 0.6 \text{ GeV}$  would imply a 95 percent confidence lower limit on the  $Z'$  mass of about  $270 \text{ GeV}$  in either the  $\chi$  or  $\eta$  case. For heavier top quark and a fixed value of the  $W$  mass the constraints on the extra  $Z'$  become *more stringent*. However they depend only to a very mild degree, on the unknown Higgs mass.

The limits obtained above should be complemented with others similar to those of ref [4, 5] obtained by combining  $W, Z$  mass data with low energy neutral current data. We expect such combined constraints to leave very little room for new superstring  $E_6$  gauge bosons. This is largely due to the restricted set of Higgs scalars present in string models. Since the sign of the  $Z - Z'$  mixing angle is determined by that of  $\mu^2$  it crucially depends on dynamics, *e.g.* in the case of the  $\eta$  model, on the allowed values of the parameter  $\xi$ . The mixing angle could only become positive (in our sign conventions) in situations where  $R$  parity is substantially broken [15] through a non-zero expectation value for the left handed sneutrino, *i.e.*  $\langle \ell \rangle^0 \neq 0$

but it is not clear to what extent this would be phenomenologically permissible. As a result in both the  $\chi$  and  $\eta$  models (with no sneutrino VEV and no renormalization of the  $U(1)$  gauge couplings) the allowed mixing angle values are *precisely those for which the neutral current constraints are the strongest*. We therefore expect that superstring  $E_6$  gauge bosons are excluded unless the  $Z - Z'$  mixing is extremely small and the  $Z'$  mass extremely large. In fact, if one simply compares our results with the pre-SLC combined limits, given in ref [4, 5] taking the correct sign into account one obtains an improvement by a factor 2-3 [19] relative to what the bounds would be in a nonsuperstring  $E_6$  model. This agrees with the results of ref [5] but disagrees with the interpretation of the bounds of ref [4] presented in ref [20]. However, to do a careful quantitative determination of the combined constraints on the  $Z'$  mass and mixing it would be desirable to have a detailed study of the neutral currents along the lines of ref [4, 5] but incorporating the improved  $Z$  mass in a consistent way throughout the analysis.

To conclude, the increased precision expected from low energy neutral current measurements of  $\sin^2\theta_W$  and from the  $W$  mass determination at the Tevatron will substantially improve our understanding of the gauge structure of the electroweak interaction. Further improvement may come from more refined  $e^+e^-$  experiments, including the possible study of polarization asymmetries, such as suggested at the SLC. The work described here highlights the importance and complementarity of these experiments in further constraining the new physics possibilities suggested by superstring models and presumably in discriminating between different options.

Finally the limits obtained here should serve as useful guides for planning direct searches of new  $Z'$ 's at hadron colliders such as the SSC.

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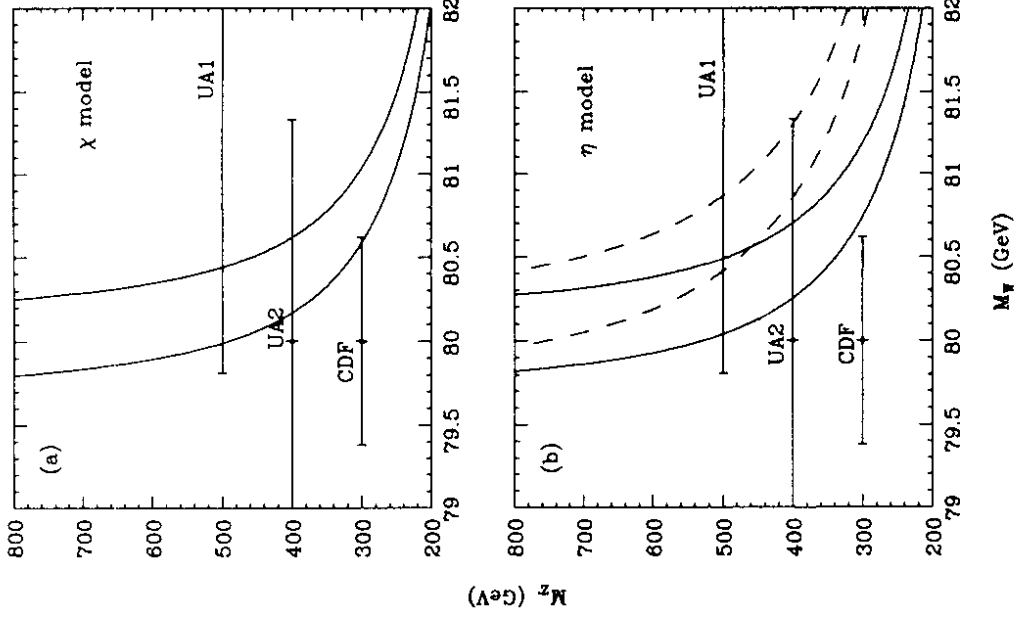
### Figure Captions

**Fig 1:**

The allowed region of the  $Z'$  mass plotted as a function of the  $W$  mass for a top quark mass of  $90\text{GeV}$ . The upper curve corresponds to  $m_z = 91.35\text{GeV}$  while the lower one is for  $m_z = 90.99\text{GeV}$ . Fig. (a) corresponds to the  $\chi$  model while (b) is for the  $\eta$  model. The dashed curves correspond to  $\xi = 0.04$  while the solid curves are for  $\xi = 0.27$ . We also show the various existing  $W$  mass measurements with their errors.

**Fig 2:**

The allowed region of the  $Z'$  mixing angle plotted as a function of the  $W$  mass for a top quark mass of  $90\text{GeV}$ . The upper curve corresponds to  $m_z = 91.35\text{GeV}$  while the lower one is for  $m_z = 90.99\text{GeV}$ . Fig. (a) corresponds to the  $\chi$  model while (b) is for the  $\eta$  model. The dashed curves correspond to  $\xi = 0.04$  while the solid curves are for  $\xi = 0.27$ . We also show the various existing  $W$  mass measurements with their errors.



**Fig. 1**

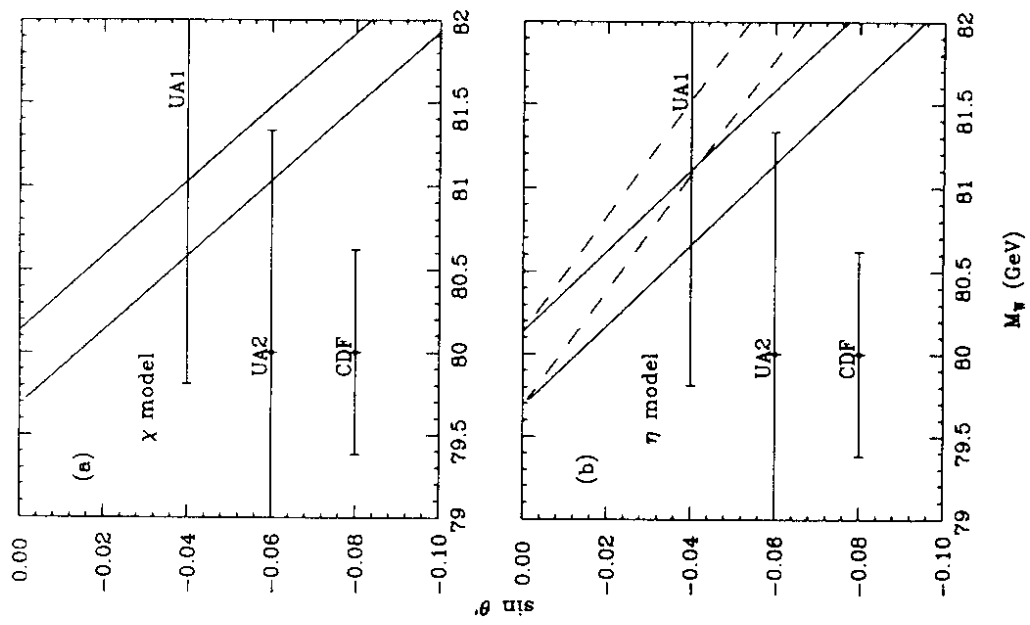


Fig. 2