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A Flipped SO(10) GUT Model and the Fermion Mass Hierarchy

Stefano Ranfone* **and José W. F. Valle** †

Instituto de Fisica Corpuscular - C.S.I.C.

Department de Fisica Teòrica, Universitat de València

46100 Burjassot, València, SPAIN

Abstract

* E-mail RANFONE@evalvx.ific.uv.es
† E-mail VALLE@flamenco.ific.uv.es

Introduction

Motivated by the nice features of the "flipped" $SU(5) \otimes U(1)$ model [1,2], we have embedded such a model in SUSY $SO(10)$. There are many reasons for having considered $SO(10)$ [3]. First of all, in this model all matter supermultiplets of each generation fit into a single irreducible representation; in addition, it includes left-right symmetry and yields a more interesting physics for neutrino masses. The main motivation for merging the idea of "flipping" with the $SO(10)$ GUT's lies in the fact that it gives the possibility of solving naturally in these models the problem related to the so-called "doublet-triplet" mass splitting. We recall that this problem, common to most of the GUT theories, is essentially the need of splitting the mass of the standard Higgs doublets with respect to the colour triplets, which usually sit in the same supermultiplet. The reason for this requirement is that light colour-triplet scalars would mediate a too-fast proton decay. On the other hand, from the data on $\sin^2\theta_W$ [10] , we also know that only one Higgs-doublet pair must remain light, developing the *standard* vacuum expectation values (VEV's), v_u and v_d. One of the best solutions of this problem is based on the so-called "Missing Partner Mechanism" of Dimopoulos and Wilczek [4]. However, the implementation of this idea in realistic GUT models is not always so easy, and often requires considerable and unnatural modifications [5]. On the contrary, the flipped $SU(5)$ model even in its minimal versions has naturally the nice feature of pushing up to the GUT scale M_G the mass of the colour-triplets, leaving light the standard Higgs doublets [1].

It was early realized that the $SO(10)$ gauge group, among the many possible chains of symmetry breaking, could also break down to the "flipped" version of the $SU(5)$ model [6]. In particular, this occurs if the **45**-adjoint Higgs supermultiplet, Σ, develops [1] a large VEV, $M_{10} \geq M_5 \sim 10^{16}$ GeV, along its B component, transforming as $(1,0)$ under "flipped"- $SU(5) \otimes U(1)$. In the present paper we shall study the "effective" flipped-$SU(5)$ model which is left over after the breaking of $SO(10)$, between M_5 and M_{10}, with particular attention to the resulting fermion and Higgs mass matrices. In spite of the many advantages of starting from $SO(10)$, some difficulties may arise because some of the

[1] M_{10} and M_5 represent, respectively, the breaking scales of $SO(10)$ down to $SU(5) \otimes U(1)$, and of $SU(5) \otimes U(1)$ down to the Standard Model (SM).

Yukawa-type couplings of the $SU(5)$ superpotential have to be equal to each other [7], due to the embedding in $SO(10)$. In particular, at a multi-generation level, it is not possible to generate any Cabibbo-Kobayashi-Maskawa type of quark mixing, unless one enlarges correspondingly also the Higgs doublet sector. In its turn, this means that also the solution of the above mentioned "doublet-triplet" splitting problem must be modified.

In this paper, we shall present a self-consistent model which solves simultaneously all these difficulties in a rather minimal way. In the last part of the paper, we will show how this model may be useful for constructing a consistent scenario, based on the Georgi-Jarlskog type of texture [8]. We show that, if the first two generations are assumed to get their masses via non-renormalizable effective operators, the model can reproduce the observed quark mass hierarchy and mixing, in addition to providing a viable seesaw scheme for neutrino masses.

The Model

The superfield content of the model is specified in Table 1. The motivations for our specific choices will be clarified during the discussion. For a more detailed description of the "minimal" flipped $SU(5)$ models we refer to the existing literature [1,2,7,9].

All standard model fermions plus the right-handed (RH) neutrino, ν^c, and their supersymmetric partners, are accommodated into three copies of an $SO(10)$-spinorial **16** superfield:

$$\Psi_i \sim \mathbf{16}_i \rightarrow F_i(10,1) \oplus \bar{f}_i(\bar{5},-3) \oplus {l^c}_i(1,5)\,, \tag{1}$$

where $i = 1,2,3$ is the generation index and where we have specified its decomposition into $SU(5) \otimes U(1)$. The F_i, $\bar{f}_i$, and ${l^c}_i$ are the usual flipped $SU(5)$ matter superfields, whose particle content may be obtained from the corresponding assignment of the standard $SU(5)$ GUT model by means of the *"flipping"* $u^{(c)} \leftrightarrow d^{(c)}$, $\nu^{(c)} \leftrightarrow e^{(c)}$:

$$\begin{cases} F_i(10,1) = (d,u;d^c;\nu^c)_i\,, \\ \bar{f}_i(\bar{5},-3) = (u^c;e,\nu)_i\,, \\ {l^c}_i(1,5) = {e^c}_i\,. \end{cases} \tag{2}$$

The main feature of this "flipped" assignment is the presence of ν^c into the 10-dimensional supermultiplet, while e^c is the singlet superfield. The electric charge is specified as $Q =$

$T_{3L} - \frac{1}{5}Z + \frac{1}{5}X$, where Z is the generator of $SU(5)$ which commutes with $SU(3)_c \otimes SU(2)_L$, and X is the generator of the extra $U(1)$ symmetry [6].

The first stage of symmetry breaking, $SO(10) \to SU(5) \otimes U(1)$, occurs by assuming that the adjoint-**45** Higgs supermultiplet Σ, which decomposes under $SU(5) \otimes U(1)$ as:

$$\Sigma \sim \mathbf{45} \ \to S(24,0) \oplus T(10,-4) \oplus \overline{T}(\overline{10},4) \oplus B(1,0)\,,$$

gets a VEV along the direction of $B(1,0)$, $< B(1,0) >\equiv M_{10} \geq M_5$.

In the present paper the $SU(5)\otimes U(1) \to SM$ breaking is triggered by two pairs of 16-dimensional Higgs supermultiplets, of which *only one is complete*. The other is assumed to be "incomplete", as in the usual minimal-type of flipped models [1,2,7], in order to implement the missing-partner mechanism [4]. More explicitly, we assume the presence of the following superfields. The "complete" $\mathbf{16} \oplus \overline{\mathbf{16}}$ pair $\Theta_1 \oplus \overline{\Theta}_1$ decomposes as follows:

$$\Theta_1 \sim \mathbf{16}_1 \to \ H_1(10,1) \oplus \bar{\eta}_1(\bar{5},-3) \oplus {\xi_1}^c(1,5)\ , \tag{3a}$$

where:

$$H_1(10,1) = (d_H, u_H; d^c_H; \nu^c_H)_1\,, \ \ \bar{\eta}_1(\bar{5},-3) = (u^c_H; e_H, \nu_H)_1\,, \ \ \xi^c_1(1,5) = e^c_{H_1}\,; \tag{3b}$$

$\overline{\Theta}_1$ of course may be obtained from these formulas by just introducing the "bar" over the symbols of the different superfields. The "incomplete" $\mathbf{16} \oplus \overline{\mathbf{16}}$ pair $\Theta_2 \oplus \overline{\Theta}_2$ is, on the other hand, identical to the one present in the minimal models:

$$\begin{aligned} \Theta_2 &\sim \mathbf{16}_2 \to \ H_2(10,1) \oplus \text{"decoupled fields"}\,, \\ \overline{\Theta}_2 &\sim \overline{\mathbf{16}}_2 \to \ \bar{H}_2(\overline{10},-1) \oplus \text{"decoupled fields"}\,. \end{aligned} \tag{4}$$

Their VEV's (assumed to be of order $M_5 \sim 10^{16}$ GeV), are responsible for the symmetry breaking down to the SM,

$$< \Theta_a >=< H_a >\equiv< \nu^c_{H_a} >\equiv V_a\,, \ \ (a=1,2)\,,$$
$$< \overline{\Theta}_a >=< \bar{H}_a >\equiv< \bar{\nu}^c_{H_a} >\equiv \bar{V}_a\,, \ \ (a=1,2)\,,$$

and are constrained by the D-term flat condition as follows: ${V_1}^2 + {V_2}^2 = \bar{V}_1^2 + \bar{V}_2^2$. The presence of the additional GUT-Higgs supermultiplets, taken here to be a pair of complete $\mathbf{16} \oplus \overline{\mathbf{16}}$ of $SO(10)$, is one of the novelties of the present model.

The final step of symmetry breaking, down to $U(1)_{em}$, is due to the VEV's developed by two different 10-dimensional $SO(10)$ superfields:

$$\Delta_\alpha \sim \mathbf{10}_\alpha \rightarrow h_\alpha(5,-2) \oplus \bar{h}_\alpha(\bar{5},2)\,, \tag{5}$$

$(\alpha = 1,2)$, in strict analogy with the standard type of $SO(10)$ GUT models. The multiplets h_α and $\bar{h}_\alpha$ contain the SM Higgs doublets plus colour-triplets and anti-triplets, respectively:

$$h_\alpha(5,-2) = (D; h^-, h^o)_\alpha\,, \quad \bar{h}_\alpha(\bar{5},2) = (\bar{D}; \bar{h}^o, \bar{h}^+)_\alpha\,. \tag{6}$$

Notice the different $U(1)$ quantum numbers of these 5-dimensional representations with respect to $\bar{f}_i$ of the matter supermultiplets.

The last ingredient of our model is an $SO(10)$ singlet superfield $\Phi \sim \mathbf{1} \rightarrow \phi(1,0)$, whose VEV, σ, will be used for generating the observed hierarchical pattern of fermion masses, as we discuss in detail in the last part of the paper.

In order to further specify our model we start writing the general renormalizable cubic superpotential as follows:

$$\begin{aligned}\mathcal{W}_{10}^R = & A_\alpha^{ij}\Psi_i\Delta_\alpha\Psi_j + B_\alpha^{ab}\Theta_a\Delta_\alpha\Theta_b + C_\alpha^{ab}\overline{\Theta}_a\Delta_\alpha\overline{\Theta}_b + D^{i,a}\Psi_i\overline{\Theta}_a\Phi \\ & + E_{\alpha\beta}\Delta_\alpha\Delta_\beta\Phi + F_\alpha^{i,a}\Psi_i\Theta_a\Delta_\alpha + G\Phi^3 + I_{ab}\Theta_a\overline{\Theta}_b\Phi + L_{\alpha\beta}\Sigma\Delta_\alpha\Delta_\beta\ ,\end{aligned} \tag{7}$$

where A, B, C, E, L are symmetric matrices, $i, j = 1, 2, 3$ label the different matter generations, $a, b = 1, 2$ ("1" and "2" representing respectively the "complete" and the "incomplete" spinorial superfields), and α, $\beta = 1, 2$ label the two different 10-dimensional Higgs supermultiplets. As we will show later, phenomenology and the requirement of a *correct* doublet-triplet mass-splitting will constrain the structure of the above superpotential. Under the $SO(10) \rightarrow SU(5) \otimes U(1)$ breaking, induced by $< \Sigma >$, we have:

$$
\begin{aligned}
\mathcal{W}_{10}^R \to \mathcal{W}_5^R = & A_\alpha^{ij}(F_iF_jh_\alpha + F_i\bar{f}_j\bar{h}_\alpha + \bar{f}_il_j^ch_\alpha) + B_\alpha^{ab}(H_aH_bh_\alpha + H_a\bar{\eta}_b\bar{h}_\alpha\delta_{b1} \\
& + \bar{\eta}_a\xi_b^ch_\alpha\delta_{a1}\delta_{b1}) + C_\alpha^{ab}(\bar{H}_a\bar{H}_b\bar{h}_\alpha + \bar{H}_a\eta_bh_\alpha\delta_{b1} + \eta_a\bar{\xi}_b^c\bar{h}_\alpha\delta_{a1}\delta_{b1}) + D^{i,a}(F_i\bar{H}_a \\
& + \bar{f}_i\eta_a\delta_{a1} + l_i^c\bar{\xi}_a^c)\phi + E_{\alpha\beta}h_\alpha\bar{h}_\beta\phi + F_\alpha^{i,a}(F_iH_ah_\alpha + \bar{f}_iH_a\bar{h}_\alpha + F_i\bar{\eta}_a\bar{h}_\alpha\delta_{a1} \\
& + \bar{f}_i\xi_a^ch_\alpha\delta_{a1} + l_i^c\bar{\eta}_ah_\alpha\delta_{a1}) + G\phi^3 + I_{ab}(H_a\bar{H}_b + \bar{\eta}_a\eta_b\delta_{a1}\delta_{b1} + \xi_a^c\bar{\xi}_b^c\delta_{a1}\delta_{b1})\phi \\
& + L_{\alpha\beta}(T\bar{h}_\alpha\bar{h}_\beta + \bar{T}h_\alpha h_\beta + Bh_\alpha\bar{h}_\beta + Sh_\alpha\bar{h}_\beta)\,,
\end{aligned}
\tag{8}
$$

where δ_{ab} is the Kronecker symbol.

Since the particle content of the present model is quite different with respect to the minimal versions already studied in the literature, it may be worthwhile to list the Goldstone bosons eaten by the gauge bosons and the left-over (physical) fields which will enter in the mass matrices. In the symmetry breaking $SU(5) \otimes U(1) \to SM$ the Goldstone bosons "absorbed" by the X, Y gauge bosons[2] are a linear combination of $(d_{H_1}\,,\, u_{H_1}) \in H_1(10,1) \subset \Theta_1$ and $(d_{H_2}\,,\, u_{H_2}) \in H_2(10,1) \subset \Theta_2$. Their orthogonal combinations remain "uneaten" and will mix, in general, with the standard *up*- and *down*-type squarks. We shall denote these fields simply by $(d_H,\, u_H)$ and $(\bar{d}_H,\, \bar{u}_H)$. On the other hand, the heavy neutral gauge boson gets its large mass by absorbing a linear combination of $\nu_{H_1}^c$, $\nu_{H_2}^c$, $\bar{\nu}_{H_1}^c$ and $\bar{\nu}_{H_2}^c$.

At this point we may study the "doublet-triplet" splitting problem, starting from the $SU(2)_L$-*doublet* mass matrix $\mathcal{M}_2$. The model contains six "down-type" doublets [3]: $\bar{f}_i \equiv (e,\nu)_i \subset \Psi_i\,,\ (i = 1,2,3)$; $h_\alpha \equiv (h_\alpha^-, h_\alpha^o) \subset \Delta_\alpha\,,\ (\alpha = 1,2)$; and $\bar{\eta} \equiv (e_H, \nu_H) \subset \Theta_1$. On the other hand, there are only three "up-type" doublets: $\bar{h}_\alpha \equiv (\bar{h}_\alpha^o, \bar{h}_\alpha^+) \subset \Delta_\alpha\,,\ (\alpha = 1,2)$, and $\eta \equiv (\bar{\nu}_H, \bar{e}_H) \subset \bar{\Theta}_1$. The mass terms for these fields generated by the superpotential $\mathcal{W}_5^R$ yield the following 3×6 doublet mass matrix:

[2] Similarly, their conjugate fields $\bar{X}$ and $\bar{Y}$ will absorb a corresponding combination of $(\bar{d}_{H_1}, \bar{u}_{H_1}) \in \bar{H}_1(\overline{10},-1) \subset \bar{\Theta}_1$ and $(\bar{d}_{H_2},, \bar{u}_{H_2}) \in \bar{H}_2(\overline{10},-1) \subset \bar{\Theta}_2$.

[3] For notational simplicity, sometimes we will represent the "doublet" part by the same symbol used for the corresponding **5**-plet (*e.g.*, $\bar{f}$, $\bar{h}$, etc.).

$$
\mathcal{M}_2 \; = \; \begin{array}{c} \\ \bar{h}_\beta \\ \eta \end{array} \begin{array}{c} \begin{array}{ccc} \bar{f}_i & h_\alpha & \bar{\eta} \end{array} \\ \begin{pmatrix} F^{i,a}_\beta V_a & E_{\alpha\beta}\sigma + L_{\alpha\beta} M_{10} & B^{a1}_\beta V_a \\ D^{i,1}\sigma & C^{a1}_\alpha \bar{V}_a & I_{11}\sigma \end{pmatrix} \end{array} . \tag{9}
$$

The phenomenological constraints on the entries of this matrix are the following. First of all, the data on $\sin^2\theta_W$ [10] require that only one Higgs doublet pair must remain light (at the weak scale), in order to develop the two electroweak VEV's, v_u and v_d (such that $v_u^2 + v_d^2 = v_{SM}^2 \simeq (246\ \text{GeV})^2$). All other pairs are, on the other hand, required to be very heavy, so as to prevent them from acquiring a non-zero VEV. This means that $\mathcal{M}_2$ must be a *rank* = 2 matrix. Finally, it is desirable to avoid any mixing ($\propto M_G$) between "matter" and "Higgs" superfields. All these requirements may be satisfied by imposing:

$$
F^{i,a}_\alpha = D^{i,a} = E_{\alpha\beta} = L_{\alpha\beta} = 0 \,, \tag{10}
$$

in the superpotential (7,8). In this case, leaving aside the massless matter doublet superfields ($\bar{f}_i$) which decouple from the other doublets, we get a *reduced* matrix $\tilde{\mathcal{M}}_2$:

$$
\tilde{\mathcal{M}}_2 \; = \; \begin{array}{c} \\ \bar{h}_1 \\ \bar{h}_2 \\ \eta \end{array} \begin{array}{c} \begin{array}{ccc} h_1 & h_2 & \bar{\eta} \end{array} \\ \begin{pmatrix} 0 & 0 & B_1 \\ 0 & 0 & B_2 \\ C_1 & C_2 & I_1 \end{pmatrix} \end{array} , \tag{11}
$$

where we have set:

$$
B_a = \sum_{b=1}^{2} B^{b1}_a V_b \,, \;\; C_a = \sum_{b=1}^{2} C^{b1}_a \bar{V}_b \,, \;\; (a = 1, 2) \,, \;\; \text{and} \;\; I_1 \equiv I_{11}\sigma \,. \tag{12}
$$

Clearly, $\tilde{\mathcal{M}}_2$ is a singular rank-2 matrix. The corresponding (hermitian) squared mass matrix $\tilde{\mathcal{M}}_2^\dagger \tilde{\mathcal{M}}_2$ will yield only two non-zero eigenmasses, for the two heavy doublet pairs:

$$
m^2_{2,3} = \frac{1}{2}\left(\Sigma^2 + I_1^2\right) \pm \frac{1}{2}\sqrt{\Pi^4 + 2\Sigma^2 I_1^2 + I_1^4} \,,
$$

where: $\Sigma^2 = {B_1}^2 + {B_2}^2 + {C_1}^2 + {C_2}^2$, and $\Pi^2 = {B_1}^2 + {B_2}^2 - {C_1}^2 - {C_2}^2$. Of course, the other eigenvalue of $\tilde{\mathcal{M}}_2^\dagger \tilde{\mathcal{M}}_2$ is zero, corresponding to the MSSM single pair of light Higgs doublets.

As we shall discuss in the last part of the paper, we ascribe the observed hierarchy among the fermion masses to the existence of non-renormalizable terms in the superpotential, proportional to powers of the "suppression factor" $< \Phi > /M_P$ (M_P being the Planck mass). The singlet field VEV $\sigma \equiv < \Phi >$ can then be fixed by using the measured value of the Cabibbo angle [4]:

$$\lambda \equiv \frac{\sigma}{M_P} \sim \sin\theta_C \simeq 0.22\,, \tag{13}$$

which gives $\sigma \sim 10^{18}$ GeV. This value is sufficiently larger than the GUT scale, M_5, so that we can assume $I_1 \gg B_i\,,\ C_i\,.$ As a result we can expand the formulas for the eigenmasses of the heavy Higgs doublet pairs, and get approximate expressions for the corresponding eigenvectors. A simple calculation gives:

$$m_1 = 0\,, \quad m_2 \simeq \frac{1}{2I_1}\sqrt{\Sigma^4 - \Pi^4} = \frac{1}{I_1}\sqrt{({B_1}^2 + {B_2}^2)\,({C_1}^2 + {C_2}^2)}\,, \quad m_3 \simeq I_1\,, \tag{14}$$

which shows that the two massive doublet pairs have masses of order $m_2 \sim {M_5}^2/\sigma \sim \mathcal{O}(10^{14}\,\mathrm{GeV})$ and $m_3 \sim \sigma \sim \mathcal{O}(10^{18}\,\mathrm{GeV})$, respectively. Then, we may also get the expressions for all the (left and right) eigenvectors of $\tilde{\mathcal{M}}_2$. In particular, we find that the heaviest fields correspond to the η and $\bar{\eta}$ coming from the "complete" GUT-Higgs $\mathbf{16} \oplus \overline{\mathbf{16}}$ and have a mass $m_3 \sim \sigma$. Moreover, they do not mix, up to terms of order λ, with the other doublets. On the other hand, the MSSM Higgs bosons (with $m_1 = 0$) and the heavy doublets with a mass $m_2 \sim {M_5}^2/\sigma$, correspond to two orthogonal combinations of the fields h_α and $\bar{h}_\alpha$. Denoting these mass eigenstates as $H_{u(d)}$ and $\Phi_{u(d)}$, respectively, we find:

$$\begin{aligned} H_u &= \bar{h}_1 \cos\chi - \bar{h}_2 \sin\chi\,, \\ \Phi_u &= \bar{h}_1 \sin\chi + \bar{h}_2 \cos\chi\,, \end{aligned} \tag{15a}$$

[4] We recall that, within specific "texture" models, $\sin\theta_C \sim \sqrt{m_1/m_2}$, where m_1 and m_2 are, respectively, the masses of the first and the second generation (either *up*- or *down*-) quarks.

and, analogously:

$$
\begin{aligned}
H_d &= h_1 \cos\xi - h_2 \sin\xi \,, \\
\Phi_d &= h_1 \sin\xi + h_2 \cos\xi \,,
\end{aligned}
\tag{15b}
$$

where the two mixing angles are given by:

$$
\tan\chi = \frac{B_1}{B_2}\,, \quad \tan\xi = \frac{C_1}{C_2}\,. \tag{16}
$$

The "doublet" mass Lagrangian will then be written as:

$$
\mathcal{L}_2 = m_1 H_u H_d + m_2 \Phi_u \Phi_d + m_3 \eta\bar{\eta}\,.
$$

This concludes our study of the doublet sector.

Let's turn now our attention to the (colour) "triplets". The fields left over after the GUT symmetry breaking transforming as $\mathbf{3}$ under $SU(3)_c$ are the following: $(u_i, d_i) \in F_i(10,1) \subset \Psi_i$ $(i = 1,2,3)$, coming from the ordinary matter supermultiplets, (u_H, d_H) (which is the linear combination, coming from $H_1(10,1) \oplus H_2(10,1) \subset \Theta_1 \oplus \Theta_2$, orthogonal to the Goldstone bosons eaten by the (X,Y) gauge bosons, as we have mentioned above), $\bar{u}^c_H \in \eta_1(5,3) \subset \bar{\Theta}_1$, $\bar{d}^c_{H_a} \in \bar{H}_a(\overline{1}0,-1) \subset \bar{\Theta}_a\,,$ $(a = 1,2)$, and $D_\alpha \in h_\alpha(5,-2) \subset \Delta_\alpha\,,$ $(\alpha = 1,2)$ (from the electroweak Higgs superfields). Of course, we also have the corresponding antiparticles transforming as $\bar{\mathbf{3}}$ under $SU(3)_c$. Taking into account the conditions given in eq.(10) and using the fact that η and $\bar{\eta}$, being very heavy fields, cannot develop a non-zero VEV, we may easily get from the superpotential the mass matrices for the *up*- and the *down*-type colour-triplets. In particular, we notice that the matter superfields $(d_i\,,\ d^c_i\,,\ u_i\,,\ u^c_i\,)$ do not receive any contribution proportional to the GUT scale and do not mix with the other states, so that they remain exactly massless before the electroweak symmetry breaking. This fact allows us to express these mass matrices simply in the following form:

$$
\mathcal{M}_{3,d} = \begin{array}{c} \\ d_H \\ D_\alpha \\ \bar{d}^c_{H_a} \end{array} \begin{array}{c} \begin{array}{ccc} \bar{d}_H & \bar{D}_\beta & d^c_{H_b} \end{array} \\ \begin{pmatrix} I\sigma & 0 & 0 \\ 0 & 0 & B^{ab}_\alpha V_a \\ 0 & C^{ab}_\alpha \bar{V}_a & I_{ab}\sigma \end{pmatrix} \end{array}, \quad (a, b, \alpha, \beta = 1, 2), \qquad (17a)
$$

and

$$
\mathcal{M}_{3,u} = \begin{array}{c} \\ u_H \\ \bar{u}^c_H \end{array} \begin{array}{c} \begin{array}{cc} \bar{u}_H & u^c_H \end{array} \\ \begin{pmatrix} I\sigma & 0 \\ 0 & I_{11}\sigma \end{pmatrix} \end{array}, \qquad (17b)
$$

where I stands for some combination of the coefficients I_{ab}. Let us analyse these matrices. First of all, we see that the "uneaten" combinations of the Higgs superfields, d_H, u_H, and their antiparticles, get a very large mass proportional to $\sigma \sim 10^{18}$ GeV. In addition, we see that in general both the 5×5 matrix $\mathcal{M}_{3,d}$ and the 2×2 matrix $\mathcal{M}_{3,u}$ will be non-singular, ensuring that all the corresponding triplet fields will get a mass at the GUT scale. More precisely, in the "down" sector, three states will get a mass of order σ, while the other two will have a mass proportional to $V\bar{V}/\sigma \sim M_G^2/\sigma$. On the other hand, $\mathcal{M}_{3,u}$ suggests that both the "up"-type fields will have a mass of order $\sim \sigma$. The heaviness of all these colour triplets prevents the presence of unsuppressed dimension five operators [20] which might mediate a too fast proton decay. The fact that we have been able to get a single light Higgs doublet pair, while ensuring the absence of dangerous light colour triplets, is essentially the solution of the doublet-triplet splitting problem, for our "extended" flipped model.

Let us now turn our attention to the Yukawa couplings responsible for generating the observed masses for the standard model fermions. All fermion masses arise from the A-term in the superpotential given in eqs.(7-8). Rewriting the "interaction" eigenstates h_α and $\bar{h}_\alpha$ in terms of the "physical" states $H_{u(d)}$ and $\Phi_{u(d)}$ by means of eqs.(15), we may write the Yukawa Lagrangian as:

$$\mathcal{L}_Y = (\bar{e}_{Li}e_{Rj} + \bar{d}_{Li}d_{Rj})\left[A_1^{ij}(H_d\cos\xi + \Phi_d\sin\xi) + A_2^{ij}(-H_d\sin\xi + \Phi_d\cos\xi)\right]$$
$$+ \diamond[6\ (\bar{u}_{Li}u_{Rj} + \bar{\nu}_{Li}\nu_{Rj})\left[A_1^{ij}(H_u\cos\chi + \Phi_u\sin\chi) + A_2^{ij}(-H_u\sin\chi + \Phi_u\cos\chi)\right] . \tag{18}$$

Since in the electroweak symmetry breaking only the light fields H_u and H_d may develop non-zero VEVS, v_u and v_d, respectively, from eq.(18) we get the following mass matrices:

$$\begin{aligned} M_d^{ij} = M_e^{ij} &= (A_1^{ij}\cos\xi - A_2^{ij}\sin\xi)v_d\,, \\ M_u^{ij} = M_{\nu D}^{ij} &= (A_1^{ij}\cos\chi - A_2^{ij}\sin\chi)v_u\,, \end{aligned} \tag{19}$$

where $A_{1(2)}$ are the 3×3 Yukawa coupling matrices, the mixing angles ξ and χ were given in terms of the parameters of the superpotential through eqs.(16) and (12), and as usual, $v_u/v_d \equiv \tan\beta$.

Eq.(19) shows that, since in general $\xi \neq \chi$, the quark mass matrices M_u and M_d will *not* be proportional to each other, resulting in a non-trivial KM mixing. We recall that this fact, in the context of our flipped $SO(10)$ model, is the result of enlarging the electroweak Higgs sector to two **10**-plets. The main result of the work carried out in this first part of the paper has been the extension of the "missing partner" solution of the doublet-triplet splitting problem to this non-minimal model. This extension has been obtained by introducing a "complete" (second) pair of $\mathbf{16}\oplus\overline{\mathbf{16}}$ GUT Higgs superfields. We have also taken into account the constraint, derived from the data on $\sin^2\theta_W$, of having only one light electroweak Higgs doublet pair.

Non-Renormalizable terms, Textures, and the quark mass hierarchy

One of the more successful *ansatze* for the up- and the down-quark mass matrices is the one proposed several years ago by Georgi and Jarlskog [8], according to which:

$$M_u = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}\ ,\quad M_d = \begin{pmatrix} 0 & de^{i\phi} & 0 \\ de^{-i\phi} & f & 0 \\ 0 & 0 & g \end{pmatrix}\ M_e = \begin{pmatrix} 0 & de^{i\phi} & 0 \\ de^{-i\phi} & -3f & 0 \\ 0 & 0 & g \end{pmatrix}\ , \tag{20}$$

where all parameters a, b, c, d, f, g, ϕ are taken to be real. We recall that in the Georgi-Jarlskog scheme it is possible to fit the mass for all generations of charged leptons, in view of the -3 factor which multiplies the parameter f in the (22)-element of the matrix M_e. In the framework of the standard $SU(5)$ GUT models, such -3 factor may be obtained via the contribution of a 45-dimensional Higgs supermultiplet (coming from the **126** of $SO(10)$). However, such a large representation is not required for breaking the "flipped" $SU(5)$ gauge group [1,2], and is not even allowed if we derive the model from the superstring (at a Kac-Moody algebra level equal to one).

Here we shall not try to explain the masses of the first two generations of charged leptons. We simply keep the simple mass relation of minimal GUT models, $m_{e_i} = m_{d_i}$ (at the GUT scale), which is very successful for the third generation, leading to the prediction $m_b \simeq r m_\tau$ at low-energy, $r(\simeq 2.7)$ being the appropriate renormalization parameter. In other words, in this paper we choose not to use the full Georgi-Jarlskog ansatz including the charged leptons. For further simplicity we shall also assume all mass matrices to be real, thus neglecting CP violation.

A way for understanding the observed fermion mass hierarchy in the Georgi-Jarlskog scheme is to assume a corresponding hierarchical pattern for the parameters of the mass matrices: $a \ll b \ll c$ and $d \ll f \ll g$. In this case one can easily check that the quark masses at the GUT scale (*i.e.*, the eigenvalues of M_u and M_d) are given by the following formulas:

$$m_t \simeq c\,, \quad m_c \simeq \frac{b^2}{c}\,, \quad m_u \simeq \left(\frac{a}{b}\right)^2 c\,, \quad m_b \simeq g\,, \quad m_s \simeq f\,, \quad m_d \simeq \frac{d^2}{f}\,. \tag{21}$$

The corresponding diagonalizing matrices for the *up* and the *down* quark sectors can be expressed in terms of three mixing angles, θ_i , as follows:

$$\begin{aligned} V_u &= \begin{pmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}\,, \\ V_d &= \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\,, \end{aligned} \tag{22}$$

where we have set $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ $(i = 1, \ldots, 3)$. These are given approximately by:

$$s_1 \simeq \sqrt{\frac{m_d}{m_s}} \simeq \frac{d}{f}\,, \quad s_2 \simeq \sqrt{\frac{m_u}{m_c}} \simeq \frac{ac}{b^2}\,, \quad s_3 \simeq \sqrt{\frac{m_c}{m_t}} \simeq \frac{b}{c} \quad . \tag{23}$$

The elements of the ordinary KM mixing matrix V_{KM}, which is just the product $V_u^\dagger\, V_d$, may then be written as: $|V_{us}| \simeq |V_{cd}| \simeq \sin\theta_C \simeq s_1$, $|V_{cb}| \simeq s_3$, and $|V_{ub}/V_{cb}| \simeq s_2$. Renormalizing the quark masses from M_G down to low-energy it is found that these formulas give a sufficiently good agreement with the present experimental data on quark mixing [11]: $|V_{us}| \simeq |V_{cd}| \simeq 0.221 \pm 0.003$, $|V_{cb}| \simeq 0.040 \pm 0.008$, and $|V_{ub}| \simeq 0.003 \pm 0.002$. This means that in the framework of the Georgi-Jarlskog ansatz, the understanding of the quark mixing pattern is reduced to that of the hierarchical structure of the quark mass spectrum.

A standard strategy for facing the problem related to the explanation of the fermion mass hierarchy in the context of GUT's is to assume that the mass is generated via the standard Higgs mechanism only for the third family fermions, while the first two generations get their mass only through higher-dimensional non-renormalizable operators in the superpotential. In the present model we shall employ such non-renormalizable operators also for producing an "effective" *seesaw* mechanism, needed for suppressing the neutrino masses.

The simplest possibility is to assume that all the hierarchy is due to powers of a single "suppression" factor, λ, equal to the ratio of the VEV of the singlet field, $< \Phi > \equiv \sigma$, and the Planck mass, M_P, which may be considered as the ultimate cut-off of the theory.

In what follows it is important to notice that all quark mass ratios, normalised at the same scale M_G, can be expressed in terms of the universal parameter $\lambda \sim \sin\theta_C \simeq 0.22$ (*i.e.*, the "λ-parameter" of the Wolfenstein parametrisation for the quark mixing matrix), according to the following pattern:

$$\begin{aligned} \frac{m_u}{m_t} &\simeq \left(\frac{a}{b}\right)^2 \sim \lambda^8\,, \quad \frac{m_d}{m_b} \simeq \left(\frac{d^2}{fg}\right) \sim \lambda^4\,, \\ \frac{m_c}{m_t} &\simeq \left(\frac{b}{c}\right)^2 \sim \lambda^4\,, \quad \frac{m_s}{m_b} \simeq \left(\frac{f}{g}\right) \sim \lambda^2\,, \end{aligned} \tag{24}$$

These eqs. yield the following relations:

$$b \simeq \lambda^2 c, \;\; a \simeq \lambda^6 c, \;\; f \simeq \lambda^2 g, \;\; d \simeq \lambda^3 g \,, \tag{25}$$

from which we can write the two quark mass matrices as follows:

$$M_u \simeq \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} m_t \,, \quad M_d \simeq \begin{pmatrix} 0 & \lambda^3 & 0 \\ \lambda^3 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_b \,. \tag{26}$$

As we see, only the third generation quarks get their mass via the standard Higgs mechanism, implemented in the renormalizable superpotential given in eqs.(7-8).

From eqs.(23-25) we easily get:

$$\frac{< \Phi >}{M_P} \equiv \lambda \sim \sqrt{\frac{m_d}{m_s}} \sim \sin\theta_C \simeq 0.22 \,; \tag{27}$$

The fact that the suppression factor λ is approximately equal to the Cabibbo angle fixes the VEV of our singlet superfield, $< \Phi > \equiv \sigma \simeq 0.22\, M_P \sim \mathcal{O}(10^{18})$ GeV, very near the Planck scale, and much larger than the GUT scale, M_G, thus justifying the expansion with respect to the ratio M_G/σ we used in eq.(14).

At this point we wish to show how it is possible to get the mass matrices given in eqs.(26) in the framework of the present model. From the comparison of eqs.(19) and (26) we get for the two (3×3) Yukawa-coupling matrices A_1 and A_2 the following expressions:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \mathcal{D}^{-1} \begin{pmatrix} -\sin\chi & \sin\xi \\ -\cos\chi & \cos\xi \end{pmatrix} \begin{pmatrix} \mu_d \\ \mu_u \end{pmatrix} \,, \tag{28}$$

where $\mathcal{D} \equiv \sin(\xi - \chi)$ and $\mu_{u(d)} \equiv M_{u(d)}/v_{u(d)}$. Writing explicitly the matrix elements we find:

$$\begin{aligned} A_1^{33} &= \mathcal{D}^{-1} \left(-y_b \sin\chi + y_t \sin\xi\right) \,, \\ A_1^{23} &= A_1^{32} = \mathcal{D}^{-1}\, y_t \lambda^2 \sin\xi \,, \\ A_1^{22} &= -\mathcal{D}^{-1}\, y_b \lambda^2 \sin\chi \,, \\ A_1^{13} &= A_1^{31} = 0 \\ A_1^{12} &= A_1^{21} = \mathcal{D}^{-1} \left(-y_b \lambda^3 \sin\chi + y_t \lambda^6 \sin\xi\right) \,, \\ A_1^{11} &= 0 \,, \end{aligned} \tag{29a}$$

and

$$A_2^{ij} = A_1^{ij}(\sin\xi \to \cos\xi\,,\ \sin\chi \to \cos\chi)\,, \tag{29b}$$

where $y_b \equiv m_b/v_d$ and $y_t \equiv m_t/v_u$ are the bottom and top quark Yukawa couplings. These formulas completely fix the structure we need to assume for the non-renormalizable (NR) terms in the superpotential, required in order to reproduce the correct quark mass spectrum and mixing.

The part of the superpotential responsible for generating fermion masses (apart from the term needed for implementing the neutrino seesaw mechanism, which will be discussed later) may then be expressed as a series expansion in powers of the suppression factor λ as follows:

$$\mathcal{W}_Y^{10} = \underbrace{A^{ij}_{(R),\alpha}\,\Psi_i\Delta_\alpha\Psi_j}_{\text{renormal.}} + \underbrace{\sum_n A^{ij}_{(n),\alpha}\left(\frac{\Phi}{M_P}\right)^n \Psi_i\Delta_\alpha\Psi_j}_{\text{non}-\text{renormal.}}\ . \tag{30}$$

In the above equation

$$A_{(R),\alpha} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_\alpha^{33} \end{pmatrix}\ ,\quad (\alpha = 1, 2)\,, \tag{31}$$

(A_1^{33} and A_2^{33} have been given in eqs.(29)), showing that at the renormalizable level only the third generation fermions get mass. For the non-renormalizable part, eqs.(29) give the following non-vanishing Yukawa-type couplings $A^{ij}_{(n),\alpha}$:

$$\begin{aligned} A_{(2),1} &= \mathcal{D}^{-1}\begin{pmatrix} 0 & 0 & 0 \\ 0 & -y_b\sin\chi & y_t\sin\xi \\ 0 & y_t\sin\xi & 0 \end{pmatrix}\ , \\ A_{(3),1} &= \mathcal{D}^{-1}\begin{pmatrix} 0 & -y_b\sin\chi & 0 \\ -y_b\sin\chi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ , \\ A_{(6),1} &= \mathcal{D}^{-1}\begin{pmatrix} 0 & y_t\sin\xi & 0 \\ y_t\sin\xi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ , \end{aligned} \tag{32a}$$

and:

$$A_{(n),2} = A_{(n),1}(\sin\chi \to \cos\chi\,, \quad \sin\xi \to \cos\xi)\,. \tag{32b}$$

These couplings fix completely the superpotential in eq.(30), responsible for the generation of the charged fermion mass hierarchy.

We now turn to the neutrino masses. As we have seen above in eq.(19), the "Dirac" neutrino mass matrix $M_{\nu D}$ is expected to be equal to the up-quark mass matrix M_u at the GUT scale. This means, of course, that we need to implement a seesaw-type of mechanism, in order to suppress the neutrino masses consistently with the present phenomenological (astrophysical and cosmological) bounds [12].

As is well known, a key ingredient of the seesaw mechanism [13], at least in the framework of the ordinary GUT's, is the presence of a large Majorana mass for the right-handed (RH) neutrinos (ν^c). In the standard $SO(10)$ models, for example, this mass can be generated via the contribution of the $SU(2)_L$-singlet Higgs field sitting in the **126**-representation. In the non-supersymmetric models, it may also be induced radiatively at the two-loop level via the so-called Witten mechanism [14], even in the absence of the **126** Higgs multiplet. In such a case, it is sufficient to employ just the VEV at the GUT scale of a **16** Higgs multiplet, also responsible for breaking the $SO(10)$ gauge group down to $SU(5)$. On the other hand, in flipped string-inspired models based on $SU(5)\otimes U(1)$, due to the lack of large Higgs representations, it is not possible to generate a Majorana mass for the RH neutrino. However, an effective seesaw mechanism may be produced by means of the mixing at the GUT scale between the ν^c and the fermionic component of extra singlet superfields, ϕ_i [15] In these minimal type models, this mixing was produced by a term of the type $F_i < \bar{H} > \phi_j \to \nu^c_i\, \phi_j\, M_G$. Unfortunately, such a term disappears in our extended ($SO(10)$-embedded) model, since, as can be seen from eq.(8), it is proportional to the Yukawa-type couplings $D^{i,a}$, which had to be set to zero according to the conditions (10). Therefore, we need to construct a seesaw type of model in a different way. Consistently with the philosophy adopted in our study of the charged fermion mass hierarchy we shall assume that also the seesaw mechanism is due to NR terms in the superpotential [16]. A minimal choice, as was discussed in ref. [7], is the following:

$$\mathcal{W}_{\nu^c} \; = \; \frac{1}{M_P}\,\Gamma^{ij}_{ab}\,\Psi_i\bar{\Theta}_a\bar{\Theta}_b\Psi_j\,, \quad (a,b=1,2;\;\; i,j=1,2,3). \tag{33}$$

Since only the **10**-components ($\bar{H}_a$) of $\bar{\Theta}_a$ may develop a non-vanishing [5] VEV ($< \bar{H}_a > \equiv < \bar{\nu}^c_{H_a} > \equiv \bar{V}_a\,, \;\; (a\,=\,1,2)$), it is easy to see that the term $\mathcal{W}_{\nu^c}$ may affect only the RH neutrino component of the matter superfields Ψ_i, resulting in an effective Majorana mass of the type:

$$M_R^{ij} \; = \; \frac{1}{M_P}\,\Gamma^{ij}_{ab}\,\bar{V}_a\bar{V}_b\,. \tag{34}$$

Recalling that, having set to zero the F and the D couplings in the superpotential (7,8), the standard neutrinos and anti-neutrinos cannot mix with the three (heavy) uneaten linear combinations of $\nu^c_{H_1}\,,\;\; \nu^c_{H_2}\,,\;\; \bar{\nu}^c_{H_1}$ and $\bar{\nu}^c_{H_2}$ we can write the neutrino mass matrix simply in the standard (ν_i, ν^c_j) basis[6]:

$$\mathcal{M}_\nu \; = \begin{pmatrix} 0 & M_u \\ M_u^T & M_R \end{pmatrix}. \tag{35}$$

This is just the ordinary seesaw mass matrix which, in the general case where M_R is non-singular, results in three light neutrinos with a mass of order:

$$m_{\nu_i} \;\sim\; \frac{m^2_{u_i}}{M_G^2}\,M_P\,. \tag{36}$$

This formula will lead to a phenomenologically interesting neutrino mass spectrum: m_{ν_e} : $m_{\nu_\mu} : m_{\nu_\tau} \simeq\; 10^{-9} : 10^{-3} - 10^{-4} : 1 - 10$ eV. For suitable choices of mixing angles, one may have MSW oscillations $\nu_e \rightarrow \nu_\mu$ in the sun [18] and we might ascribe the hot component of the Dark Matter of the universe to a τ-neutrino with a mass of a few eV [19].

Discussion

We have presented a viable version of the SO(10) model in which the breaking occurs via the flipped rather than the usual Georgi-Glashow SU(5). The model is consistent with

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[5] In fact, as we have seen from eq.(11), the **5**-component of $\bar{\Theta}_1$, η , having a very large mass $m_3 \simeq I_1 \sim \mathcal{O}(\sigma)$, cannot develop a non-zero VEV.

[6] The situation is quite different in the previous minimal-type of $SU(5)$-flipped models, where the F and the D terms were present.

the phenomenological requirements of having a non-trivial quark mixing matrix, natural doublet-triplet splitting, and a single pair of light electroweak Higgs doublet scalar bosons. We recall that these requirements are in conflict with the minimal version of the flipped models embedded in SO(10). This conflict can not be solved by simply duplicating the doublet-triplet splitting mechanism characteristic of flipped models, through the use of a second pair of incomplete $\mathbf{16} \oplus \overline{\mathbf{16}}$ Higgs multiplets. We have remedied this situation by adding instead a pair of complete spinorial multiplets.

We have also shown how, in the presence of suitable non-renormalizable superpotential terms, the model can reproduce the hierarchy observed in quark masses and mixings, as well as an acceptable neutrino masses generated via the seesaw mechanism.

As a final comment we note that, so far in this paper we have assumed *ad hoc* the vanishing of the "dangerous" D, E, F, and L Yukawa-type coupling constants in the superpotential of eqs.(7,8). However, it is easy to see that it is possible to get rid of all these couplings by just introducing a Z_2 discrete symmetry in the superpotential, under which the $SO(10)$ superfields of the model transform as follows [7]:

$$\Psi_i \to \Psi_i\,, \quad \Delta_\alpha \to \Delta_\alpha\,, \quad \Theta_a \to -\Theta_a\,, \quad \bar{\Theta}_a \to \bar{\Theta}_a\,, \quad \Phi \to -\Phi\,, \quad \Sigma \to -\Sigma\,. \tag{37}$$

This symmetry also allows the presence of the non-renormalizable operators needed for implementing the neutrino seesaw mechanism, as discussed above. At a single generation level, this Z_2-symmetry is sufficient for the construction of a self-consistent model based on $SO(10) \to SU(5)_{fl} \otimes U(1)$. Unfortunately, in a more realistic multi-generational scenario, our assumed discrete symmetry is not sufficient to derive the particular structure of the non-renormalizable operators needed for reproducing the correct charged fermion mass hierarchy in the framework of the Georgi-Jarlskog *texture*. In this case such a texture should follow from some underlying symmetry of the model. Moreover, the explanation of the masses of the first two generation charged leptons will require some extension of our scheme. Another difficulty of our present model, is related to the μ problem [17]. Here

[7] Actually, this is not the only possible choice; another solution, for example, is: $\Psi_i \to -\Psi_i\,, \quad \Delta_\alpha \to \Delta_\alpha\,, \quad \Theta_a \to \Theta_a\,, \quad \bar{\Theta}_a \to -\bar{\Theta}_a\,, \quad \Phi \to -\Phi\,, \quad \Sigma \to -\Sigma\,.$

we have ensured the absence of a dangerously large μ-term, but in a more complete model one should be able to derive the correct value $\mu \sim M_W$.

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References

[1] I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, Phys. Lett. **194B** (1987) 231; **205B** (1988) 459; **208B** (1988) 209; **231B** (1989) 65.

[2] I. Antoniadis, G.K. Leontaris and J. Rizos, Phys. Lett. **245B** (1990) 161.
G.K. Leontaris, J. Rizos and K. Tamvakis, Phys. Lett. **243B** (1990) 220; **251B** (1990) 83;
I. Antoniadis, J. Rizos and K. Tamvakis, Phys. Lett. **278B** (1992) 257; **279B** (1992) 281;
J.L. Lopez and D.V. Nanopoulos, Nucl. Phys. **B338** (1990) 73; Phys. Lett. **251B** (1990) 73.
D. Bailin and A. Love, Phys. Lett. **280B** (1992) 26.

[3] For a review see, *e.g.*, G. G. Ross, Grand-Unified Theories, Benjamin, 1985; R. N. Mohapatra, Unification and Supersymmetry, Springer, 1986

[4] S. Dimopoulos, S. Wilczek, report N. NSF-ITP-82-07, Aug. 1981 (unpublished); R. Cahn, I. Hinchliffe, L. Hall, Phys. Lett. **109B** (1982) 426.

[5] K. S. Babu, R. N. Mohapatra, Phys. Rev. Lett. **74** (1995) 2418.

[6] S. M. Barr, Phys. Lett. **112B** (1982) 219.

[7] E. Papageorgiu and S. Ranfone, Phys. Lett. **282B** (1992) 89.

[8] H. Georgi, C. Jarlskog, Phys. Lett. **86B** (1979) 297.

[9] S. Ranfone and E. Papageorgiu, Phys. Lett. **295B** (1992) 79.

S. Ranfone, Phys. Lett. **324B** (1994) 370.

[10] M. Martinez, talk at *Elementary particle Physics: Present and Future*, Ed. A. Ferrer and J. W. F. Valle, World Scientific, in press.

[11] Particle Data Group, Phys. Rev. **D50** (1994) 1173.

[12] J. W. F. Valle, talk at TAUP 95, Toledo, ed. A. Morales *et al.*, Nucl. Phys. Proc. Suppl. (in press); A. Yu. Smirnov, talk at *Elementary particle Physics: Present and Future*, Ed. A. Ferrer and J. W. F. Valle, World Scientific, in press.

[13] M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, ed. D. Freedman et al. (1979); T. Yanagida, in *KEK lectures*, ed. O. Sawada et al. (1979).

[14] E. Witten, Phys. Lett. **B91** (1980) 81.

[15] E. Witten, Nucl. Phys. **B258** (1985) 75; R. Mohapatra, J. W. F. Valle, Phys. Rev. **D34** (1986) 1642; J. W. F. Valle, Nucl. Phys. Proc. Suppl. **B11** (1989) 118; see also refs. [1,2,7,9] above.

[16] J-P. Derendinger, L. Ibanez and H. P. Nilles, Nucl. Phys. **B267** (1986) 365; F. del Aguila et al. Nucl. Phys. **B272** (1986) 413; S. Nandi, U. Sarkar, Phys. Rev. Lett. **55** (1986) 566; J. W. F. Valle, Phys. Lett. **186** (1987) 78

[17] C. Munoz, Proceedings of SUSY 94, Ann Arbor, USA, C. Kolda, J. Wells, editors.

[18] S. Mikheyev and A. Smirnov, Sov. J. Nucl. Phys., **42** (1986) 1441; L. Wolfenstein, Phys. Rev., **D17** (1978) 2369; **D20** (1979) 2634.

[19] G. F. Smoot et al., Astr. J. **396** (1992) L1; E.L. Wright et al., Astr. J. **396** (1992) L13; R. Rowan-Robinson, proceedings of the *International School on Cosmological Dark Matter*, Ed. A. Perez and J. W. F. Valle, World Scientific, 1994, p. 7-13

[[20]] N. Sakai, T. Yanagida, Nucl. Phys. **B197** (1982) 533; S. Weinberg, Phys. Rev. **D26** (1982) 287; J. Ellis, D. Nanopoulos, S. Rudaz, Nucl. Phys. **B202** (1982) 46; S. Dimopoulos, S. Raby, F. Wilczek, Phys. Lett. **B112** (1982) 133