Dynamical Left–Right Symmetry Breaking

EUGENI AKHMEDOV^{ab}, MANFRED LINDNER^c, ERHARD SCHNAPKA^d
and JOSE W. F. VALLE^e

^{b}International Centre for Theoretical Physics,
Strada Costiera 11, I–34100 Trieste, ITALY

^{c,d} Institut für Theoretische Physik, Technische Universität München,
James–Franck–Strasse, D–85748 Garching, GERMANY

^{e}Instituto de Física Corpuscular - C.S.I.C.
Departament de Física Teòrica, Universitat de València
46100 Burjassot, València, SPAIN

Abstract

We study a left–right symmetric model which contains only elementary gauge boson and fermion fields and no scalars. The phenomenologically required symmetry breaking emerges dynamically leading to a composite Higgs sector with a renormalizable effective Lagrangian. We discuss the pattern of symmetry breaking and phenomenological consequences of this scenario. It is shown that a viable top quark mass can be achieved for the ratio of the VEVs of the bi–doublet $\tan\beta \equiv \kappa/\kappa' \simeq 1.3$–4. For a theoretically plausible choice of the parameters the right–handed scale can be as low as $\sim 20\ TeV$; in this case one expects several intermediate and low–scale scalars in addition to the Standard Model Higgs boson. These may lead to observable lepton flavour violation effects including $\mu \to e\gamma$ decay with the rate close to its present experimental upper bound.

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^{a}On leave from National Research Center "Kurchatov Institute", 123182 Moscow, Russia
^{b}Email: akhmedov@tsmi19.sissa.it
^{c}Email: Manfred.Lindner@Physik.TU–Muenchen.DE
^{d}Email: Erhard.Schnapka@Physik.TU–Muenchen.DE
^{e}Email: valle@flamenco.ific.uv.es

Introduction

Left–right symmetric extensions [1, 2, 3] of the Standard Model are very attractive. In this class of models parity is unbroken at high energies and its non–conservation at low energies occurs through a spontaneous symmetry breakdown mechanism. In addition, it is remarkable how the known fermions fit very economically and symmetrically into representations of the underlying gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. For phenomenological reasons the $SU(2)_R \times U(1)_{B-L}$ symmetry must first be broken above a few TeV to $U(1)_Y$ and in a second step the left–over Standard Model gauge group $SU(2)_L \times U(1)_Y$ is broken as usual to $U(1)_{em}$. In conventional left–right (LR) models this desired symmetry breaking sequence is realized in analogy to the Standard Model with the help of scalar particles and the Higgs mechanism. Therefore suitable scalars are introduced and a renormalizable potential with the required vacuum expectation values (VEV's) is chosen. But like in the Standard Model it is desirable to motivate the existence of such scalars by either adding supersymmetry[1] or to view the Higgs sector as an effective description of some dynamical symmetry breaking mechanism in analogy to superconductivity. We will pursue here the second possibility since the combination of the correct dynamical LR symmetry breaking sequence with the usual nice features of LR models appears especially attractive. Composite Higgs scenarios are moreover more economical and explain nicely why certain Yukawa couplings are of order unity and how Higgs and fermion masses are related. However, a phenomenologically acceptable scenario is not easy to construct and there exist only a few attempts in the literature [5] which try to break the left–right symmetry dynamically in a phenomenologically acceptable way.

The model which is developed in this paper is to our knowledge the first complete and successful attempt of this kind which does not just assume that the left–right breaking dynamics will work correctly. The underlying Lagrangian is essentially a left–right symmetric generalization of the BHL model of top condensation [6] which will be invariant under local $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ transformations. The emerging symmetry breaking pattern can be understood as a sequence of two steps. First a hybrid bi–fermion condensate in the lepton sector (equivalent to a Higgs doublet of $SU(2)_R$) breaks the $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ symmetry and the three Goldstone Bosons are eaten resulting in right–handed gauge boson masses. After this dynamical $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ breaking, a Dirac condensate for the top quark breaks in a second step the gauged $SU(2)_L \times U(1)_Y$ symmetry corresponding to a dynamical $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ breaking. We will establish that this is the preferred symmetry breaking sequence of our model by calculating the effective potential in analogy to the techniques used in the top–condensate approach to the Standard Model [6, 7, 8, 9].

The dynamics of our model will lead to composite Higgs bosons which are responsible for the symmetry breaking in the language of the effective Lagrangian. We show that, unlike in conventional LR models, whether or not parity is spontaneously broken depends just on the fermion content of the initial Lagrangian and not on the choice of parameters of the Higgs potential. Predictions are obtained for masses of the Higgs bosons in terms of fermion masses and in addition certain relations between the masses of scalars are found.

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[1]Some examples are given in Ref. [4].

The model predicts intermediate–scale Higgs scalars (with their masses being one or two orders of magnitude lower than the parity–breaking scale) which are the pseudo–Goldstone Bosons of an accidental global $SU(4)$ symmetry of the effective Higgs potential. It is shown that parity breakdown at a right–handed scale propagates down and eventually causes the electro–weak symmetry breakdown at a lower (electro–weak) scale, i.e. a tumbling scenario of symmetry breaking is operative.

In Section I we start with reviewing some basic features of the top–condensate approach and conventional LR models. Further, we discuss a dynamical LR–symmetry breaking model with the usual fermion content which features composite triplet scalars (i.e. reproduces the LR model with the presently most popular Higgs boson content [3]) and analyze its shortcomings. In Section II we present our model which contains a new singlet fermion and leads to composite Higgs doublets and a Higgs singlet. For this model we discuss the four–fermion terms which are responsible for the dynamical symmetry breakdown. Section III contains results for the effective potential in bubble approximation and in Section IV we present our predictions in this approximation. In the following Section V we obtain and discuss the renormalization group improved predictions, especially for the top quark mass. Section VI contains some phenomenological considerations and in Section VII we give our conclusions. Technical details of our calculations are given in the Appendices.

I Preliminary Considerations

I.1 The BHL Model of Electro–Weak Symmetry Breaking

Let us briefly recall some features of the top–condensate approach [6, 7, 8, 9] following the BHL model [6]. The model Lagrangian consists of kinetic terms for fermions and gauge fields plus attractive four–fermion (4-f) interactions:

$$\mathcal{L} = \mathcal{L}_{kinetic} + G(\overline{Q}_L t_R \overline{t}_R Q_L) \, , \tag{I.1}$$

where Q_L is the third–generation quark doublet. For large enough G symmetry breaking occurs and can e.g. be studied in the NJL approximation[2] [10, 11], i.e. in leading order of a large–N_c expansion with a cutoff Λ. In the auxiliary field formalism eq. (I.1) can be rewritten in terms of a static, non–propagating, scalar doublet $\varphi := -G\overline{t}_R Q_L$ of mass $G^{-\frac{1}{2}}$ such that the Lagrangian eq. (I.1) becomes [12]

$$\mathcal{L}_{aux} = \mathcal{L}_{kinetic} - \overline{Q}_L \varphi t_R - \overline{t}_R \varphi^\dagger Q_L - G^{-1} \varphi^\dagger \varphi \, . \tag{I.2}$$

Due to the dynamics of the original model further terms (including kinetic terms) emerge in the effective Lagrangian at low energy scales μ after one integrates out the degrees of freedom with energies between μ and Λ. For large cutoff Λ only renormalizable terms are allowed such that we obtain

$$\mathcal{L}_{eff} = \mathcal{L}_{aux} + Z_\varphi \left(D_\mu \varphi \right)^\dagger \left(D^\mu \varphi \right) + \delta M^2 \varphi^\dagger \varphi - \frac{\delta\lambda}{2} \left(\varphi^\dagger \varphi \right)^2 - \delta g_t \left(\overline{Q}_L \varphi t_R + \overline{t}_R \varphi^\dagger Q_L \right) + \delta\mathcal{L}_{kinetic} \, . \tag{I.3}$$

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[2]We use the well known term NJL throughout this paper even though the work of Vaks and Larkin was received and published first.

Note that symmetry breaking occurs when $\delta M^2 > G^{-1}$ which is achieved for $G > G_{\text{crit}}$. We can immediately read off those conditions which express the composite nature of the effective scalar Lagrangian: $Z_\varphi \overset{\mu^2 \to \Lambda^2}{\longrightarrow} 0$, $\delta M^2 \overset{\mu^2 \to \Lambda^2}{\longrightarrow} 0$, and $\delta\lambda \overset{\mu^2 \to \Lambda^2}{\longrightarrow} 0$. This expresses simply the fact that all dynamical effects must disappear as μ approaches the scale Λ. In addition we have the normalization conditions $\delta g_t \overset{\mu^2 \to \Lambda^2}{\longrightarrow} 0$ and $\delta\mathcal{L}_{kinetic} \overset{\mu^2 \to \Lambda^2}{\longrightarrow} 0$. We can now use the freedom to rescale the scalar field φ by defining $\varphi := \varphi/\sqrt{Z_\varphi}$ such that the Lagrangian becomes[3]

$$\mathcal{L}_{eff} = \mathcal{L}_{kinetic} + (D_\mu\varphi)^\dagger (D^\mu\varphi) + \frac{\hat{\lambda}v^2}{2}\varphi^\dagger\varphi - \frac{\hat{\lambda}}{2}\left(\varphi^\dagger\varphi\right)^2 - \hat{g}_t\left(\overline{Q}_L\varphi t_R + \overline{t}_R\varphi^\dagger Q_L\right) \ . \quad \text{(I.4)}$$

Here we introduced $\frac{\hat{\lambda}v^2}{2} = \frac{\delta M^2 - G^{-1}}{Z_\varphi}$, $\hat{\lambda} = \frac{\delta\lambda}{Z_\varphi^2}$ and $\hat{g}_t = \frac{1 + \delta g_t}{\sqrt{Z_\varphi}}$ and the effective Lagrangian has now become the Standard Model. From the definition of $\hat{g}_t$, $\hat{\lambda}$ and v we see that the compositeness conditions are

$$\lim_{\mu^2 \to \Lambda^2} \hat{g}_t^{-2}(\mu^2) = 0 \ , \quad \lim_{\mu^2 \to \Lambda^2} \frac{\hat{\lambda}(\mu^2)}{\hat{g}_t^4(\mu^2)} = 0 \ , \quad \lim_{\mu^2 \to \Lambda^2} \frac{\hat{\lambda}(\mu^2)v^2(\mu^2)}{2\hat{g}_t^2(\mu^2)} = -G^{-1} \ , \quad \text{(I.5)}$$

where Λ corresponds to the high energy cutoff of the BHL–model.

The vanishing of the kinetic term of the composite Higgs field is thus equivalent to a Landau singularity of the running Yukawa coupling $\hat{g}_t$. The compositeness conditions eq. (I.5) can therefore be translated into boundary conditions of the renormalization group flow at Λ. Since the effective Lagrangian is identical to that of the Standard Model one can employ the usual one–loop β–functions [13]. The running top Yukawa coupling indeed develops the desired Landau pole above a top mass value of $197\ GeV$, and as m_t is increased the Landau pole moves to lower scales. If the above compositeness condition $\hat{g}_t \to \infty$ is imposed on the full renormalization group equations at e.g. $\Lambda = 10^{15}\ GeV$, then one finds $m_t = 227\ GeV$. For the above Λ already the analysis with one–loop β–functions turns out to be rather reliable since running down from the Landau pole one ends up in the attractive infrared quasi–fixed point [14][4].

Note that the solutions of the renormalization group equations in a limit corresponding to the fermionic bubble approximation also satisfy the compositeness conditions. However in this limit one finds with $\Lambda = 10^{15}\ GeV$ a much lower top quark mass, $m_t = 164\ GeV$. Thus QCD and electro–weak corrections play a non–negligible role in the precise mass predictions. This demonstrates that the β–functions of the full effective Lagrangian are superior to those derived in bubble approximation.

Finally we have to comment on the cutoff Λ regulating loop effects of the BHL model. We imagine that such a cutoff is motivated by new physics which is not specified in detail. In the case of the BHL model theories have been constructed where some new gauge interactions with heavy gauge bosons with masses $M_x \sim \Lambda$ motivate the cutoff [16, 17, 18, 19]. Then, by integrating out heavy bosons, four–fermion terms as in eq. (I.1) with effective couplings $G \sim 1/M_x^2$ emerge as lowest dimensional operators. Throughout this paper we will take a similar attitude, but we will not try to relate our 4-f structures to any renormalizable model.

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[3]Ignoring fermionic wave function contributions which do not play a role for the compositeness conditions.
[4]Note that this quasi-fixed point is related, but not identical to the Pendleton–Ross fixed point [15].

I.2   Left–right symmetric models

We will perform steps similar to those described above in LR symmetric models. In order to remind the reader of the main features of LR symmetry and to introduce the notation, we first describe conventional LR model building before we discuss our scenario.

In LR symmetric models based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [1, 2, 3] quarks and leptons of each generation are symmetrically placed into the doublet representations of $SU(2)_L$ and $SU(2)_R$. We will deal only with the third generation of quarks and leptons for which the assignment is

$$Q_L = \begin{pmatrix} t \\ b \end{pmatrix}_L \sim (3,\ 2,\ 1,\ 1/3)\ , \qquad Q_R = \begin{pmatrix} t \\ b \end{pmatrix}_R \sim (3,\ 1,\ 2,\ 1/3)\ ;$$

$$\Psi_L = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \sim (1,\ 2,\ 1,\ -1)\ , \quad \Psi_R = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_R \sim (1,\ 1,\ 2,\ -1)\ . \qquad \text{(I.6)}$$

The usual Dirac masses of the fermions are generated by the VEV of a bi–doublet Higgs scalar ϕ:

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1,\ 2,\ 2,\ 0)\ ; \quad \langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}\ . \qquad \text{(I.7)}$$

However, to arrive at the phenomenologically required symmetry breaking pattern $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{e.m.}$, one needs additional Higgs multiplets. The simplest possibility is to add two doublets [1, 2]:

$$\chi_L = \begin{pmatrix} \chi_L^0 \\ \chi_L^- \end{pmatrix} \sim (1,\ 2,\ 1,\ -1)\ , \quad \chi_R = \begin{pmatrix} \chi_R^0 \\ \chi_R^- \end{pmatrix} \sim (1,\ 1,\ 2,\ -1)\ . \qquad \text{(I.8)}$$

Then $SU(2)_R$ is broken at the right–handed scale M_R by $\langle \chi_R^0 \rangle = v_R$, and the electro–weak symmetry is broken by the VEVs of ϕ and possibly of χ_L^0 ($\equiv v_L$). In fact, χ_L is not necessary for the above symmetry breakdown pattern; it is usually introduced to ensure the discrete parity symmetry under which [5]

$$Q_L \leftrightarrow Q_R\ , \quad \Psi_L \leftrightarrow \Psi_R\ , \quad \phi \leftrightarrow \phi^\dagger\ , \quad \chi_L \leftrightarrow \chi_R\ , \quad W_L \leftrightarrow W_R\ . \qquad \text{(I.9)}$$

It has been shown [20] that even if the Higgs potential is exactly symmetric with respect to the discrete parity transformation, the vacuum of the model may prefer $v_R \gg v_L$. This would give a very elegant explanation of parity violation at low energies as being the result of spontaneous symmetry breakdown.

The doublets χ_L and χ_R are singlets of either $SU(2)_R$ or $SU(2)_L$ and so cannot interact in a renormalizable way with the usual quarks and leptons. This means that neutrinos are Dirac particles and get their masses in the same way as the other fermions. It is therefore very difficult to understand the smallness of their masses – one needs an extreme fine tuning of the corresponding Yukawa couplings. An attractive solution of this problem was suggested in Ref. [3] where two Higgs triplets $\Delta_L \sim (1,\ 3,\ 1,\ 2)$ and $\Delta_R \sim (1,\ 1,\ 3,\ 2)$ were

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[5]Note that this symmetry also requires the equality of the $SU(2)_L$ and $SU(2)_R$ gauge coupling constants $g_{2L} = g_{2R} \equiv g_2$.

introduced instead of the doublets χ_L and χ_R. Leptons can interact with the Higgs triplets through the Majorana–like Yukawa coupling

$$f(\Psi_L^T C\tau_2 \vec{\tau}\vec{\Delta}_L \Psi_L + \Psi_R^T C\tau_2 \vec{\tau}\vec{\Delta}_R \Psi_R) + h.c. \ , \tag{I.10}$$

where C is the charge conjugation matrix. In this model neutrinos are Majorana particles, and the seesaw mechanism [21] is operative. This mechanism provides a very natural explanation of the smallness of the usual neutrino masses, relating it to the fact that the right–handed scale is much higher than the electro–weak scale. In other words, parity non–conservation at low energies and the smallness of neutrino masses have a common origin in this model.

I.3 Dynamical Symmetry Breaking for the Triplet Model

We now assume, following the approach of Refs. [6, 7, 8, 9] to the Standard Model, that the low–energy ($\mu < \Lambda$) degrees of freedom of our LR model are just fermions and gauge bosons, with no fundamental Higgs fields being present. Therefore the Lagrangian contains only the usual kinetic terms for all gauge fields and fermions. In addition we postulate in a first step the following set of gauge–invariant 4-f interactions to be present at low energies:

$$L_{int} = L_{int1} + L_{int2} \ , \tag{I.11}$$

$$\begin{aligned}
L_{int1} &= G_1(\overline{Q}_{Li}Q_{Rj})(\overline{Q}_{Rj}Q_{Li}) + [G_2(\overline{Q}_{Li}Q_{Rj})(\tau_2\overline{Q}_L Q_R \tau_2)_{ij} + h.c.] \\
&\quad + G_3(\overline{\Psi}_{Li}\Psi_{Rj})(\overline{\Psi}_{Rj}\Psi_{Li}) + [G_4(\overline{\Psi}_{Li}\Psi_{Rj})(\tau_2\overline{\Psi}_L\Psi_R\tau_2)_{ij} + h.c.] \\
&\quad + [G_5(\overline{Q}_{Li}Q_{Rj})(\overline{\Psi}_{Rj}\Psi_{Li}) + h.c.] + [G_6(\overline{Q}_{Li}Q_{Rj})(\tau_2\overline{\Psi}_L\Psi_R\tau_2)_{ij} + h.c.] \ , \tag{I.12} \\
L_{int2} &= \tilde{G}_7[(\Psi_L^T C\tau_2\vec{\tau}\Psi_L)(\overline{\Psi}_L\vec{\tau}\tau_2 C\overline{\Psi}_L^T) + (\Psi_R^T C\tau_2\vec{\tau}\Psi_R)(\overline{\Psi}_R\vec{\tau}\tau_2 C\overline{\Psi}_R^T)] \ . \tag{I.13}
\end{aligned}$$

In analogy to the BHL model the G_a are dimensionful 4-f couplings of the order of Λ^{-2} motivated by some new physics at the high energy scale Λ. The indices i and j refer to isospin and it is implied that the colour indices of quarks are summed over within each bracket. Note that the above interactions are not only gauge invariant, but also (for hermitean G_2, G_4, G_5 and G_6) symmetric with respect to the discrete parity operation (I.9). However, it should be emphasized that the 4-f interactions of eqs. (I.12) and (I.13) do not constitute the most general set of the gauge– and parity–invariant 4-f interactions. We left out undesirable terms which, if critical, would produce electrically charged or coloured condensates. We simply assume that the new physics responsible for the effective 4-f couplings does not produce such terms.

We will use the above Lagrangian and later modifications thereof to break the LR symmetry dynamically. We assume that the heaviest (i.e. the third generation) quarks and leptons play a special role in the symmetry breaking dynamics and so confine ourselves to the discussion of the third generation. In this limit only the third–generation fermions are massive while all the light fermions are considered to be massless. This seems to be a good starting point from where light fermion masses could, e.g., be generated radiatively.

One might expect that, just as in the NJL model or in [6], for strong enough ("critical") G_a the LR symmetry will be dynamically broken and the correct pattern of the LR symmetry breakdown will emerge. However, as we shall see, the situation with the LR model is more complicated.

One way to explore the symmetry breaking in the model and to study the composite Higgs scalars is to consider the four–point Green functions generated by the 4-f couplings of equations (I.12) and (I.13) and study their poles corresponding to the two–particle bound states. This can be done analytically in fermion bubble approximation in which the exact solution can be obtained [6, 10, 11]. The scale Λ plays again the role of a natural cutoff for the divergent diagrams. An alternative approach is to rewrite the 4-f structure in terms of static auxiliary fields (see equations (I.1), (I.2) and Appendix A for details). These static auxiliary fields can acquire gauge invariant kinetic terms and become physical propagating scalar fields.

Loosely speaking, the auxiliary scalars are defined as the square roots of the original 4-f operators. Therefore the 4-f structure of eq. (I.12) can generate the composite bi–doublet Higgs field ϕ which (for small lepton 4-f couplings) has the structure

$$\phi_{ij} \sim \alpha(\overline{Q}_{Rj}Q_{Li}) + \beta(\tau_2\overline{Q}_L Q_R\tau_2)_{ij} \ , \qquad (I.14)$$

while the 4-f terms of equation (I.13) give rise to the composite triplet scalars,

$$\vec{\Delta}_L \sim (\Psi_L^T C\tau_2\vec{\tau}\Psi_L), \quad \vec{\Delta}_R \sim (\Psi_R^T C\tau_2\vec{\tau}\Psi_R) \ . \qquad (I.15)$$

Note that while the VEVs of ϕ are particle–antiparticle condensates, the VEVs of Δ_L and Δ_R are Majorana–like particle–particle condensates[6].

The model based on the 4-f couplings of eqs. (I.12) and (I.13) has thus all the necessary ingredients to produce the composite Higgs fields ϕ, Δ_L and Δ_R. However, as we shall shortly see, it does not lead to the correct pattern of LR symmetry breakdown.

For parity to be spontaneously broken, one needs $\langle\Delta_R\rangle > \langle\Delta_L\rangle$. One can readily make sure that this can only be satisfied provided $\lambda_2 > \lambda_1$ where λ_1 and λ_2 are the coefficients of the $[(\Delta_L^\dagger\Delta_L)^2 + (\Delta_R^\dagger\Delta_R)^2]$ and $2(\Delta_L^\dagger\Delta_L)(\Delta_R^\dagger\Delta_R)$ quartic couplings in the Higgs potential [20]. In the conventional approach, λ_1 and λ_2 are free parameters and one can always choose $\lambda_2 > \lambda_1$. On the contrary, in the composite Higgs approach based on a certain set of effective 4-f couplings, the parameters of the effective Higgs potential are not arbitrary: they are all calculable in terms of the 4-f couplings G_a and the scale of new physics Λ [6]. In particular, in fermion bubble approximation at one loop level the quartic couplings λ_1 and λ_2 are induced through the Yukawa couplings of eq. (I.10) and are given by the diagrams of Fig. 1. It can be seen from Fig. 1b that to induce the λ_2 term one needs the Ψ_L–Ψ_R mixing in the fermion line in the loop, i.e. the lepton Dirac mass term insertions. However, the Dirac mass terms are generated by the VEVs of the bi–doublet ϕ; they are absent at the parity breaking scale which is supposed to be higher than the electro–weak scale. Even if parity and electro–weak symmetry are broken simultaneously (which is hardly a phenomenologically

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[6]A Majorana–like condensate of right handed neutrinos has been considered in [22] in the framework of the Standard Model.
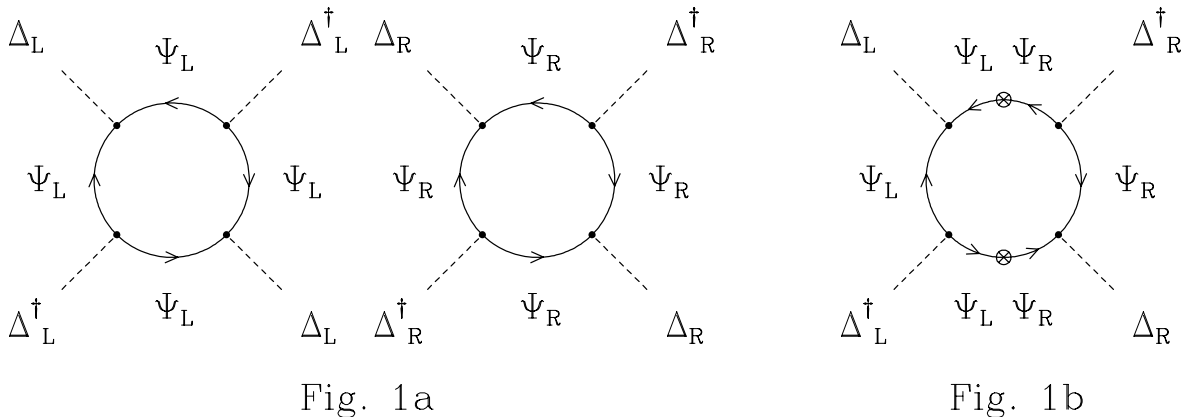
Figure 1: *Fermion loop diagrams contributing to the quartic couplings λ_1 (Fig. 1a) and λ_2 (Fig. 1b) for Higgs triplets.*

viable scenario), this would not save the situation since the diagram of Fig. 1b is finite whereas the one of Fig. 1a is logarithmically divergent and so the inequality $\lambda_2 > \lambda_1$ cannot be satisfied. We also checked that using the full one–loop Coleman–Weinberg [23] effective potential instead of the truncated one which includes up to the quartic terms in the fields does not save the situation. Therefore we will consider a model with a different composite Higgs content. As we have mentioned, the simplest model includes two doublets, χ_L and χ_R, instead of the triplets Δ_L and Δ_R. The question is whether it is possible to get the correct pattern of dynamical symmetry breakdown in this model. As we shall see, this is really the case.

II   The Model

We will now consider the original version of the LR symmetric model with a bi–doublet ϕ and two doublets χ_L and χ_R, as defined in eqs. (I.7) and (I.8). In conventional LR models one can have the doublet Higgs bosons without introducing any other new particles. In our model, however, χ_L and χ_R are composite and cannot be built from the standard fermions. We therefore assume that in addition to the usual quark and lepton doublets of eq. (I.6) there is a gauge–singlet fermion

$$S_L \sim (1,\ 1,\ 1,\ 0) \ . \tag{II.1}$$

To maintain discrete parity symmetry one needs a right–handed counterpart of S_L. This can be either another particle, S_R, or the right–handed antiparticle of S_L, $(S_L)^c \equiv C\overline{S}_L^T = S_R^c$. The latter choice is more economical and, as we shall see, leads to the desired symmetry breaking pattern. We therefore assume that under parity operation

$$S_L \leftrightarrow S_R^c \ . \tag{II.2}$$

With the singlet fermion S_L our interaction Lagrangian becomes

$$L_{int} = L_{int1} + L_{int3} \ , \tag{II.3}$$

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and contains now in addition to (I.12) the following term

$$L_{int3} = G_7[(S_L^T C \Psi_L)(\overline{\Psi}_L C \overline{S}_L^T) + (\overline{S}_L \Psi_R)(\overline{\Psi}_R S_L)] + G_8(S_L^T C S_L)(\overline{S}_L C \overline{S}_L^T) , \qquad (II.4)$$

which is now supposed to substitute for L_{int2} of eq. (I.13) since we no longer want the triplet Higgses to be present in the model. The composite Higgs scalars induced by the 4-f couplings of eq. (II.4) are the doublets χ_L and χ_R and in addition a singlet σ:

$$\chi_L \sim S_L^T C \Psi_L , \quad \chi_R \sim \overline{S}_L \Psi_R = (S_R^c)^T C \Psi_R , \quad \sigma \sim \overline{S}_L C \overline{S}_L^T . \qquad (II.5)$$

Using the Yukawa couplings of the doublets χ_L and χ_R (see eq. (II.6) below), one can now calculate the fermion–loop contributions to the quartic couplings $\lambda_1[(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2]$ and $2\lambda_2(\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R)$ in the effective Higgs potential (Fig. 2). The λ_1 and λ_2 terms are now
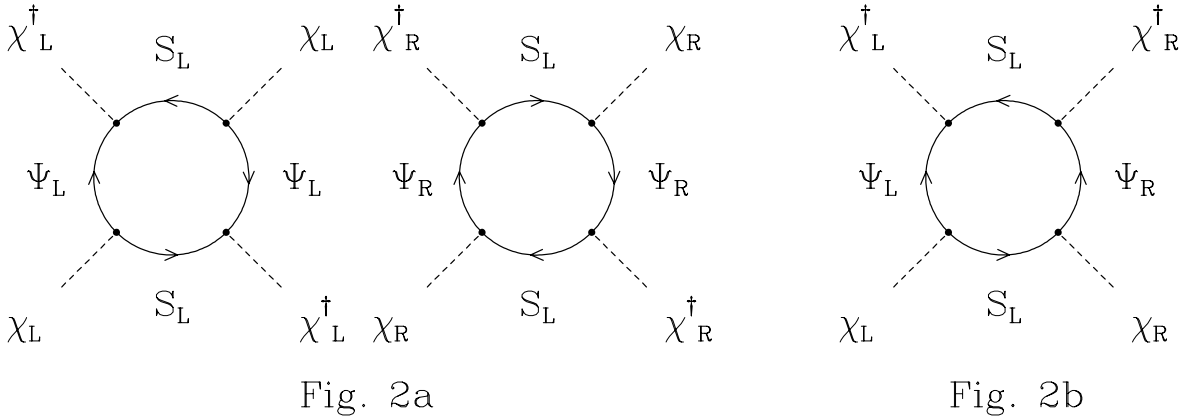

Fig. 2a   Fig. 2b

Figure 2: *Fermion loop diagrams contributing to the quartic couplings λ_1 (Fig. 2a) and λ_2 (Fig. 2b) for the Higgs doublets $\chi_{L/R}$.*

given by similar diagrams. Since the Yukawa couplings of χ_L and χ_R coincide (which is just the consequence of the discrete parity symmetry), Figs. 2a and 2b yield $\lambda_1 = \lambda_2$. Recall that one needs $\lambda_2 > \lambda_1$ for spontaneous parity breakdown. As we shall see, taking into account the gauge–boson loop contributions to λ_1 and λ_2 shown in Fig. 3 will automatically secure this relation.

We did not include into eq. (II.3) a 4-f term of the kind $[(S_L^T C S_L)^2 + (\overline{S}_L C \overline{S}_L^T)^2]$ which is also gauge invariant and parity–symmetric. One possible motivation for not considering such terms is that if one imagines the new physics responsible for the 4-f terms as being related to some new vector boson exchange [16, 17, 18, 19], such terms are never induced. Here we just assume that there is a global symmetry which precludes these terms. For example, this could be a global $U(1)$ symmetry under which S_L has the charge $+1$, the rest of fermions being neutral [χ_L, χ_R and σ will have the charges $+1$, -1 and -2 according to (II.5)]. This will forbid the $(S_L^T C S_L)^2 + (\overline{S}_L C \overline{S}_L^T)^2$ terms and also the bare mass term for S_L which is allowed otherwise by the gauge symmetry, since S_L is a gauge–singlet fermion.

Now we will switch to the auxiliary field formalism, in which the scalars χ_L, χ_R, ϕ and σ have the following bare mass terms and Yukawa couplings:

$$L_{aux} = -M_0^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) - M_1^2 \operatorname{tr}(\phi^\dagger \phi) - \frac{M_2^2}{2} \operatorname{tr}(\phi^\dagger \widetilde{\phi} + h.c.) - M_3^2 \sigma^\dagger \sigma$$

$$- \left[\overline{Q}_L (Y_1 \phi + Y_2 \widetilde{\phi}) Q_R + \overline{\Psi}_L (Y_3 \phi + Y_4 \widetilde{\phi}) \Psi_R + h.c. \right]$$

$$- \left[Y_5 (\overline{\Psi}_L \chi_L S_R^c + \overline{\Psi}_R \chi_R S_L) + Y_6 (S_L^T C S_L) \sigma + h.c. \right] \; . \qquad \text{(II.6)}$$

Here the field $\widetilde{\phi} \equiv \tau_2 \phi^* \tau_2$ has the same quantum numbers as ϕ: $\widetilde{\phi} \sim (1, \, 2, \, 2, \, 0)$. By integrating out the auxiliary scalar fields one can reproduce the 4-f structures of eqs. (II.3) and express the 4-f couplings $G_1, ..., G_8$ in terms of the Yukawa couplings $Y_1, ...Y_6$ and the mass parameters M_0^2, M_1^2, M_2^2 and M_3^2 (the explicit formulas are given in Appendix A). Note that all the mass terms in L_{aux} are positive. In what follows we use the freedom of rescaling the scalar fields to choose $M_1 = M_3 = M_0$. At low energies ($E < \Lambda$) the scalar fields will acquire gauge–invariant kinetic terms, quartic couplings and renormalized mass terms through radiative corrections. This means that the auxiliary fields may become physical propagating fields, and their squared mass terms may become negative at some low energy scale, leading to non–vanishing VEVs of the composite scalars, i.e. to fermion condensates [6].

From eq. (II.6) one can readily find the fermion masses. The masses of the quarks and charged leptons and the Dirac neutrino mass m_D are given by the VEVs of the bi–doublet (we assume all the VEVs to be real):

$$m_t = Y_1 \kappa + Y_2 \kappa' \; , \qquad m_D = Y_3 \kappa + Y_4 \kappa' \; ,$$

$$m_b = Y_1 \kappa' + Y_2 \kappa \; , \qquad m_\tau = Y_3 \kappa' + Y_4 \kappa \; . \qquad \text{(II.7)}$$

We already mentioned that LR models with only doublet Higgs scalars usually suffer from the large neutrino mass problem. It turns out that introducing the singlet fermion S_L not only provides the spontaneous parity breaking in our model, but also cures the neutrino mass problem. In fact, as it was first noticed in [24], with an additional singlet neutral fermion S_L the neutrino mass matrix takes the form (in the basis (ν_L, ν_L^c, S_L))

$$M_\nu = \begin{pmatrix} 0 & m_D & \beta \\ m_D^T & 0 & M^* \\ \beta^T & M^\dagger & \widetilde{\mu} \end{pmatrix} \; , \qquad \text{(II.8)}$$

where the entries β, M and $\widetilde{\mu}$ can be read off from eq. (II.6):

$$\beta = Y_5 v_L \; , \quad M = Y_5 v_R \; , \quad \widetilde{\mu} = 2 Y_6 \sigma_0 \; . \qquad \text{(II.9)}$$

Here $\sigma_0 \equiv \langle \sigma \rangle$. For $v_R \gg \kappa, \kappa', v_L$ and $v_R \gtrsim \sigma_0$ one obtains two heavy Majorana neutrino mass eigenstates with the masses $\sim M$ and a light Majorana neutrino with the mass $m_\nu \simeq \widetilde{\mu}(m_D^2/M^2) - 2\beta m_D/M$ which vanishes in the limit $M \to \infty$. This is the modified seesaw mechanism which provides the smallness of the neutrino mass.

III The Effective Potential in Bubble Approximation

In the auxiliary field formalism we can now study the dynamical symmetry breaking by calculating the effective potential. The effective mass terms and wave–function renormalization constants can be obtained from the 2–point scalar Green functions (Fig. 4), whereas

the quartic terms are given by the 4–point functions (Fig. 5). Higher–order terms are finite in the limit $\Lambda \to \infty$ and therefore relatively unimportant. A convenient way to calculate the potential parameters which automatically takes care of the numerous combinatorial factors in the diagrams of Figs. 4 and Fig. 5 is to calculate the full one–loop Coleman–Weinberg effective potential of the system and then to truncate this potential so as to keep only the terms up to and including quartic ones in the scalar fields. To derive the low energy effective Lagrangian we introduce an infrared cutoff μ in the loop integrals. This procedure and its physical meaning are discussed in Appendix B.

III.1   Spontaneous Parity Breaking

As we have already emphasized, stability of the vacuum with $v_L \neq v_R$ requires $\lambda_2 > \lambda_1$. While at fermion loop level we have $\lambda_2 = \lambda_1$, we will see that the gauge boson loop contributions will automatically secure the relation $\lambda_2 > \lambda_1$. Both λ_1 and λ_2 obtain corrections from $U(1)_{B-L}$ gauge boson loops (see Figs. 3a and 3b), whereas only for λ_1 there are additional diagrams with W_L^i or W_R^i loops (Fig. 3). Since all these contributions have a relative
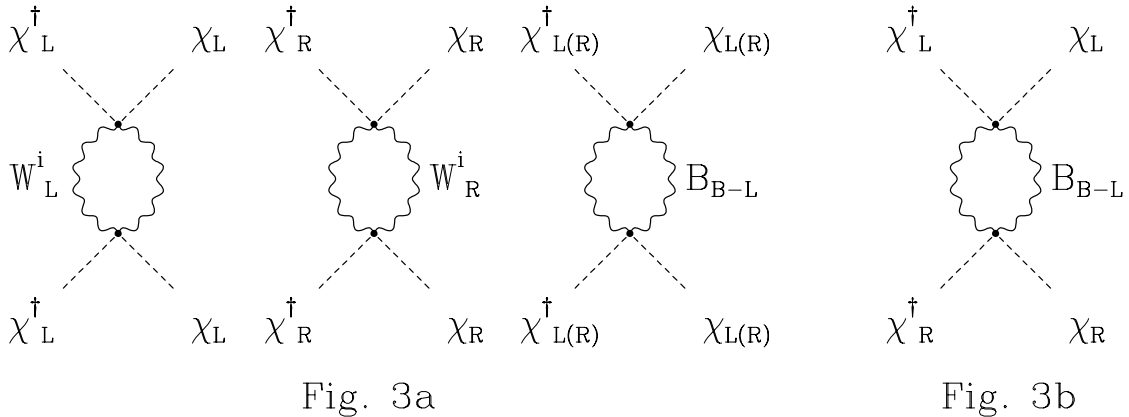

Fig. 3a   Fig. 3b

Figure 3: *Gauge boson loop diagrams contributing to the quartic couplings λ_1 (Fig. 3a) and λ_2 (Fig. 3b) for the Higgs doublets $\chi_{L/R}$ in Landau gauge.*

minus sign compared to those from the fermion loops, one finds $\lambda_2 > \lambda_1$ irrespective of the values of the Yukawa or gauge couplings or any other parameter of the model, provided that the $SU(2)$ gauge coupling $g_2 \neq 0$ [compare the expressions for λ_1 and λ_2 in (B.9)]. Thus the condition for spontaneous parity breaking is automatically satisfied in our model.

We have a very interesting situation here. In a model with composite triplets Δ_L and Δ_R parity is never broken, i.e. the model is not phenomenologically viable [7]. At the same time, in the model with two composite doublets χ_L and χ_R instead of two triplets (which requires introduction of an additional singlet fermion S_L) parity is broken automatically. This means that, unlike in the conventional LR models, in the composite Higgs approach *whether or not parity can be spontaneously broken depends on the particle content of the model rather than on the choice of the parameters of the Higgs potential.*

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[7] At least if one does not introduce triplet or higher–representation fermions.

III.2 Effective mass parameters

Let us now study the symmetry breaking in the model in detail. The gauge and parity symmetries can only be broken if the relevant squared mass terms in the effective potential become negative. These terms are given by the sums of the bare mass terms and the one–loop corrections coming from the diagrams of Fig. 4:

$$\widetilde{M}_0^2 = M_0^2 - \frac{1}{8\pi^2}\left[Y_5^2 - \frac{3}{8}Z_\chi(3g_2^2 + g_1^2)\right](\Lambda^2 - \mu^2) , \tag{III.1}$$

$$\widetilde{M}_1^2 = M_0^2 - \frac{1}{8\pi^2}\left\{\left[N_c(Y_1^2 + Y_2^2) + (Y_3^2 + Y_4^2)\right] - \frac{9}{4}Z_\phi g_2^2\right\}(\Lambda^2 - \mu^2) , \tag{III.2}$$

$$\widetilde{M}_2^2 = M_2^2 - \frac{1}{4\pi^2}(N_c Y_1^2 Y_2^2 + Y_3^2 Y_4^2)(\Lambda^2 - \mu^2) , \tag{III.3}$$

$$\widetilde{M}_3^2 = M_0^2 - \frac{1}{4\pi^2}Y_6^2\,(\Lambda^2 - \mu^2) . \tag{III.4}$$

The factors Z_χ and Z_ϕ multiplying the gauge contributions in the above formulas for $\widetilde{M}^2$ [and also in the expressions for quartic couplings λ, see (B.9)] are the prefactors for the gauge invariant kinetic terms of composite scalars, similar to Z_ϕ in eq. (I.3). They will be given explicitly in eqs. (IV.1) and (IV.2) below and appear in eqs. (III.1), (III.2) and (B.9) because the kinetic terms of the scalars are not yet brought into the canonical form. The running mass terms coincide with the bare masses at $\mu = \Lambda$.

While the bare mass parameters M_i^2 in eq. (II.6) are positive, the corresponding running quantities $\widetilde{M}_i^2$, given by eqs. (III.1) – (III.4), may become negative at low energy scales provided that the corresponding Yukawa couplings are large enough. Those values for which this occurs at $\mu = 0$ we shall call the *critical* Yukawa couplings. For $\widetilde{M}_i^2$ to become negative at some scale $\mu^2 > 0$ the corresponding Yukawa couplings or combinations of them must be above their critical values. If this is to happen at scales $\mu \ll \Lambda$ the Yukawa couplings must be fine–tuned very closely to their critical values to ensure the proper cancelation between the large bare masses of the scalars and the Λ^2 corrections in eqs. (III.1)–(III.4). This is equivalent to the usual fine–tuning problem of gauge theories with elementary Higgs scalars [6][8].

From eqs. (III.1), (III.2), (IV.1) and (IV.2) one can find the critical values for the Yukawa couplings:

$$(Y_5^2)_{cr} \equiv 8\pi^2\left(\frac{M_0^2}{\Lambda^2}\right)\left[1 - \frac{3}{8}(3g_2^2 + g_1^2)\,l_0\right]^{-1} , \tag{III.5}$$

$$(\widetilde{Y}^2)_{cr} \equiv 8\pi^2\left(\frac{M_0^2}{\Lambda^2}\right)\left[1 - \frac{9}{4}g_2^2\,l_0\right]^{-1} , \tag{III.6}$$

where $\widetilde{Y}^2$ and l_0 are defined as

$$\widetilde{Y}^2 \equiv N_c(Y_1^2 + Y_2^2) + (Y_3^2 + Y_4^2) , \tag{III.7}$$

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[8]It has been claimed in [25] that taking into account the loops with the composite Higgs scalars results in the automatic cancelation of quadratic divergences and solves the gauge hierarchy problem of the Standard Model in the BHL approach. We do not discuss this possibility here.
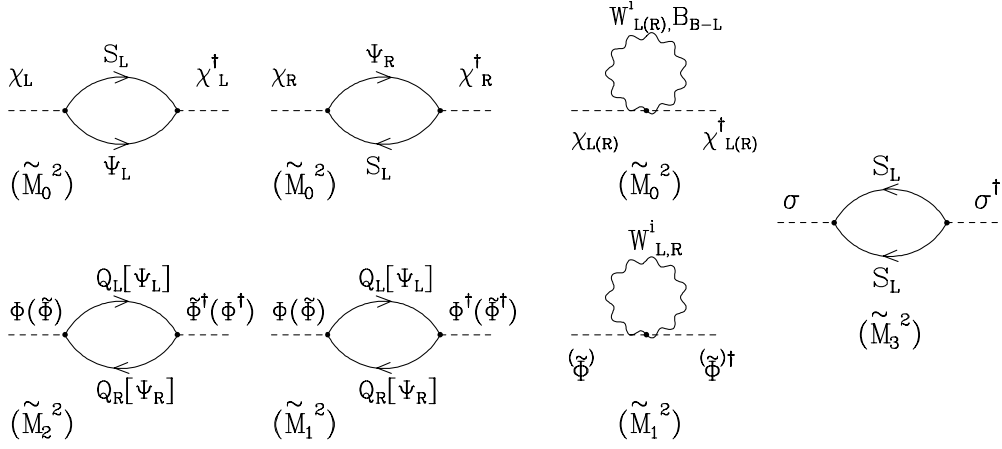
Figure 4: *Diagrams contributing to the effective scalar mass terms $\widetilde{M}_0^2 \ldots \widetilde{M}_3^2$ in Landau gauge.*
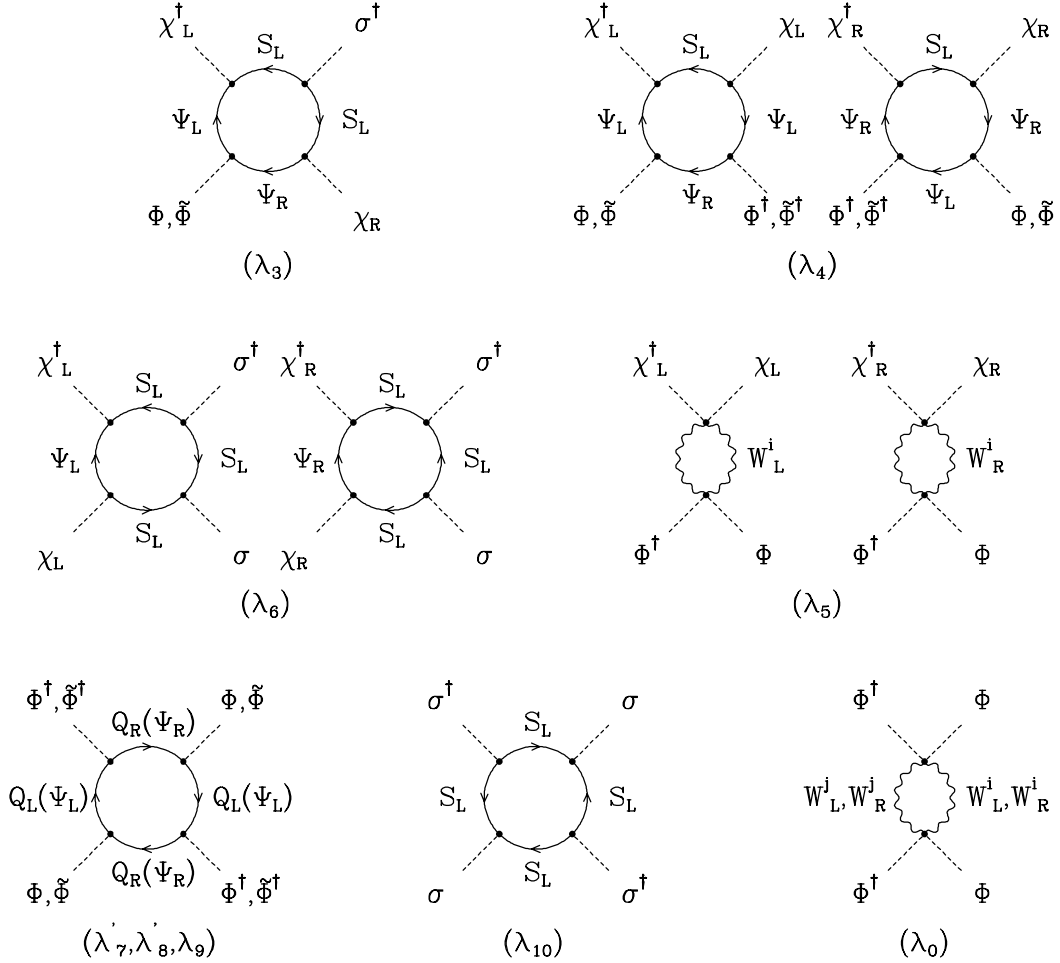

Figure 5: *Diagrams contributing to the quartic terms in the Higgs potential in Landau gauge.*

$$l_0 \equiv \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) . \tag{III.8}$$

For $Y_5^2 = (Y_5^2)_{cr}$, $\widetilde{M}_0^2$ becomes zero at $\mu^2 = 0$. Analogously, for $\widetilde{Y}^2 = (\widetilde{Y}^2)_{cr}$, $\widetilde{M}_1^2 = 0$ at $\mu^2 = 0$. Now let us introduce the scales μ_{R0} and μ_1 through the relations

$$\frac{\delta Y_5^2}{(Y_5^2)_{cr}} \equiv \frac{Y_5^2 - (Y_5^2)_{cr}}{(Y_5^2)_{cr}} = \frac{\mu_{R0}^2}{\Lambda^2 - \mu_{R0}^2} \simeq \frac{\mu_{R0}^2}{\Lambda^2} , \tag{III.9}$$

$$\frac{\delta\widetilde{Y}^2}{\widetilde{Y}_{cr}^2} \equiv \frac{\widetilde{Y}^2 - \widetilde{Y}_{cr}^2}{\widetilde{Y}_{cr}^2} = \frac{\mu_1^2}{\Lambda^2 - \mu_1^2} \simeq \frac{\mu_1^2}{\Lambda^2} , \tag{III.10}$$

where the last equalities in (III.9) and (III.10) hold for $\mu_{R0}^2, \mu_1^2 \ll \Lambda^2$. The meaning of the scales μ_{R0}^2 and μ_1^2 is very simple, they are the scales at which $\widetilde{M}_0^2$ and $\widetilde{M}_1^2$ become zero for given $\delta Y_5^2 > 0$ and $\delta\widetilde{Y}^2 > 0$. Consequently, a negative value of μ_1^2 or $\delta\widetilde{Y}^2$ corresponds to sub–critical $\widetilde{Y}^2$, and, as we shall see below, this will indeed be required in our scenario.

Using eqs. (III.1), (III.2), (III.9) and (III.10) one arrives at the following expressions for $\widetilde{M}_0^2(\mu^2)$ and $\widetilde{M}_1^2(\mu^2)$:

$$\widetilde{M}_0^2(\mu^2) = M_0^2\left(\frac{\mu^2 - \mu_{R0}^2}{\Lambda^2 - \mu_{R0}^2}\right) \simeq M_0^2\left(\frac{\mu^2 - \mu_{R0}^2}{\Lambda^2}\right) , \tag{III.11}$$

$$\widetilde{M}_1^2(\mu^2) = M_0^2\left(\frac{\mu^2 - \mu_1^2}{\Lambda^2 - \mu_1^2}\right) \simeq M_0^2\left(\frac{\mu^2 - \mu_1^2}{\Lambda^2}\right) . \tag{III.12}$$

For $\mu^2 \ll \mu_{R0}^2, \mu_1^2$ one finds $\widetilde{M}_0^2 \simeq -(M_0^2/\Lambda^2)\,\mu_{R0}^2 \sim -\mu_{R0}^2$ and $\widetilde{M}_1^2 \simeq -(M_0^2/\Lambda^2)\,\mu_1^2 \sim -\mu_1^2$.

III.3 Pattern of symmetry breaking

In order to determine the vacuum structure in our model, let us first consider the extremum conditions for the effective potential V, i.e. the conditions that the first derivatives of $V_{\rm eff}$ with respect to the VEVs of the scalar fields σ_0, v_R, v_L, κ and κ' vanish:

$$\left[\widetilde{M}_3^2 + \lambda_6(v_L^2 + v_R^2) + 2\lambda_{10}\sigma_0^2\right]\sigma_0 + (\lambda_3/2)\,m_D\,v_L\,v_R = 0 ; \tag{III.13}$$

$$\left[\widetilde{M}_0^2 + 2\lambda_1 v_R^2 + 2\lambda_2 v_L^2 + \lambda_4 m_D^2 + \lambda_5(\kappa^2 + \kappa'^2) + \lambda_6\,\sigma_0^2\right]v_R + (\lambda_3/2)\,m_D\,v_L\,\sigma_0 = 0 ; \tag{III.14}$$

$$\left[\widetilde{M}_0^2 + 2\lambda_1 v_L^2 + 2\lambda_2 v_R^2 + \lambda_4 m_D^2 + \lambda_5(\kappa^2 + \kappa'^2) + \lambda_6\,\sigma_0^2\right]v_L + (\lambda_3/2)\,m_D\,v_R\,\sigma_0 = 0 ; \tag{III.15}$$

$$\left[2\widetilde{M}_1^2 + 2(\lambda_4 Y_3^2 + \lambda_5)(v_L^2 + v_R^2) + 4\lambda_7\,\kappa^2 + 2\lambda_8\,\kappa'^2 + 3\lambda_9\,\kappa\kappa'\right]\kappa +$$
$$2\widetilde{M}_2^2\,\kappa' + \lambda_3\,Y_3\,v_L\,v_R\,\sigma_0 + 2\lambda_4\,Y_3\,Y_4\,\kappa'\,(v_L^2 + v_R^2) + \lambda_9\,\kappa'^3 = 0 ; \tag{III.16}$$

$$\left[2\widetilde{M}_1^2 + 2(\lambda_4 Y_4^2 + \lambda_5)(v_L^2 + v_R^2) + 4\lambda_7\,\kappa'^2 + 2\lambda_8\,\kappa^2 + 3\lambda_9\,\kappa\kappa'\right]\kappa' +$$
$$2\widetilde{M}_2^2\,\kappa + \lambda_3\,Y_4\,v_L\,v_R\,\sigma_0 + 2\lambda_4\,Y_3\,Y_4\,\kappa\,(v_L^2 + v_R^2) + \lambda_9\,\kappa^3 = 0 . \tag{III.17}$$

The quartic couplings λ_i are given in Appendix B along with the expression for the effective potential [see eqs. (B.9) and (B.8)]; m_D is defined in eq. (II.7).

Multiplying (III.14) by v_L and (III.15) by v_R and subtracting we get (for $v_R^2 \neq v_L^2$)

$$4\,(\lambda_1 - \lambda_2)\,v_L\,v_R = \lambda_3\,m_D\,\sigma_0 \;, \tag{III.18}$$

which is the analog of the "VEV seesaw" relation of Ref. [3]. Above the scale at which the bi–doublet develops non–vanishing VEVs $m_D = 0$ and hence either v_L or v_R is zero (or both).

We assume that the scale μ_R at which parity gets spontaneously broken (i.e. χ_R^0 develops a VEV) is higher than the electro–weak scale $\mu_{EW} \sim 100\ GeV$, i.e. that $\widetilde{M}_0^2$ changes its sign at a higher scale than $\widetilde{M}_1^2$. This means that the expression $Y_5^2 - (3/8)Z_\chi(3g_2^2 + g_1^2)$ should be bigger than the combination

$$N_c(Y_1^2 + Y_2^2) + (Y_3^2 + Y_4^2) - \frac{9}{4}Z_\phi g_2^2 \equiv \tilde{Y}^2 - \frac{9}{4}Z_\phi g_2^2 \;. \tag{III.19}$$

At scales $\mu > \mu_{EW}$ the VEVs of the bi–doublet are zero and it is sufficient to consider eqs. (III.13) and (III.14). It follows from eqs. (III.15) and (III.18) that v_L can be consistently set equal to zero in this energy range.

Let us now consider eqs. (III.13) and (III.14) with $\kappa = \kappa' = m_D = 0$. We have now essentially two possibilities. Either

$$Y_5^2 - \frac{3}{8}Z_\chi(3g_2^2 + g_1^2) > 2\,Y_6^2 \;, \tag{III.20}$$

or vice versa. If eq. (III.20) is satisfied, with decreasing scale μ, $\widetilde{M}_0^2$ will become negative earlier than $\widetilde{M}_3^2$, i.e. χ_R will develop a VEV earlier than σ. It is easy to see that in fact σ will never develop a VEV in this case. Indeed, solving (III.14) for v_R, substituting it into (III.13) and using eqs. (III.1) and (III.4) one can make sure that the combination $\widetilde{M}_3^2 + \lambda_6 v_R^2$, which is the effective "driving term" for the VEV of σ, never gets negative even if $\widetilde{M}_3^2$ does. This means that σ_0 never appears. Conversely, if the condition opposite to the one of eq. (III.20) holds, $\widetilde{M}_3^2$ will become negative first and the sum $\widetilde{M}_0^2 + \lambda_6 \sigma_0^2$, which is the effective driving term for v_R in eq. (III.14), never gets negative, i.e. $v_R = 0$. Clearly this situation is phenomenologically unacceptable, and from now on we therefore assume that eq. (III.20) is satisfied, which means that we choose the 4-f couplings G_7 and G_8 accordingly.

We have demonstrated that above the electro–weak scale the VEVs of χ_R and σ never coexist. The question remains whether below the electro–weak scale, when the VEVs of the bi–doublet ϕ come into play, σ and χ_L can acquire non–vanishing VEVs. This cannot be answered by just studying the first derivative conditions (III.13) – (III.17). However, as we will show in Appendix D, the condition that the matrices of second derivatives of the effective potential with respect to the fields be positive definite (i.e. the vacuum stability condition) results in $\sigma_0 = 0 = v_L$ even when $\kappa, \kappa' \neq 0$. The only exception may be the situation when the inequality in eq. (III.20) becomes equality. We do not consider such a possibility since it requires an extreme fine–tuning of the Yukawa couplings.

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Let us now consider the vacuum structure below the electro–weak breaking scale. The non–vanishing VEVs are v_R, κ and κ'. Since $m_t \gg m_b$, it follows from eq. (II.7) that κ should be much larger than κ' or vice versa provided no significant cancelation between $Y_1\kappa'$ and $Y_2\kappa$ occurs. This is also welcome because of the stringent upper limit on the $W_L - W_R$ mixing [26]: $\xi \approx 2\kappa\kappa'/(v_R^2 - v_L^2) < 0.0025$. To further simplify the discussion at this point, we shall therefore make the frequently used assumption [27] $\kappa' = 0$ which does not change any symmetry properties of the model. The general case $\kappa, \kappa' \neq 0$ is discussed in section V and Appendix D.

For $\kappa' = 0$ the relation $m_t \gg m_b$ translates into $Y_1 \gg Y_2$. In the conventional approach this assumption does not lead to any contradiction with phenomenology. However, as we shall see, in our case the condition $\kappa' = 0$ cannot be exact. Consistency of eqs. (III.16) and (III.17) with $\kappa' = 0$ requires $Y_1 Y_2 = 0$, $Y_3 Y_4 = 0$ and $M_2^2 = 0$ (this gives $\widetilde{M}_2^2 = \lambda_9 = 0$; as follows from eq. (B.9), all the terms in the effective potential which are linear in κ' become zero in this limit, as they should). The condition $Y_1 Y_2 = 0$ along with $\kappa' = 0$ implies that either $Y_1 = 0$, $m_t = 0$ or $Y_2 = 0$, $m_b = 0$. The first possibility is obviously phenomenologically unacceptable whereas the second one can be considered as a reasonable first approximation; we therefore assume $Y_2 = 0$, $Y_1 \neq 0$.

The situation is less clear for the lepton Yukawa couplings Y_3 and Y_4. Since $m_\tau \ll m_t$ and the Dirac mass m_D of ν_τ is unknown, one can choose either $Y_3 \neq 0$, $Y_4 = 0$ or $Y_3 = 0$, $Y_4 \neq 0$. In Appendix D we will show that the vacuum stability condition in our model requires $m_\tau^2 - m_D^2 > 0$, which implies $Y_4^2 > Y_3^2$, therefore we choose $Y_3 = 0$ and $Y_4 \neq 0$.

Now with $\sigma_0 = v_L = \kappa' = Y_2 = Y_3 = 0$ one can readily obtain the solutions of eqs. (III.14) and (III.16):

$$v_R^2 = \frac{\widetilde{M}_1^2 \lambda_5 - 2\widetilde{M}_0^2 \lambda_7}{4\lambda_1\lambda_7 - \lambda_5^2} \;, \tag{III.21}$$

$$\kappa^2 = \frac{\widetilde{M}_0^2 \lambda_5 - 2\widetilde{M}_1^2 \lambda_1}{4\lambda_1\lambda_7 - \lambda_5^2} \;. \tag{III.22}$$

To estimate the magnitudes of these VEVs we rewrite them using eqs. (III.11) and (III.12):

$$v_R^2(\mu^2) = \frac{1}{4\lambda_1\lambda_7 - \lambda_5^2} \left(\frac{M_0^2}{\Lambda^2}\right) \left\{ \left[2\lambda_7\,\mu_{R0}^2 + |\lambda_5|\,\mu_1^2 \right] - \left[2\lambda_7 + |\lambda_5| \right]\mu^2 \right\} \;, \tag{III.23}$$

$$\kappa^2(\mu^2) = \frac{1}{4\lambda_1\lambda_7 - \lambda_5^2} \left(\frac{M_0^2}{\Lambda^2}\right) \left\{ \left[|\lambda_5|\,\mu_{R0}^2 + 2\lambda_1\,\mu_1^2 \right] - \left[2\lambda_1 + |\lambda_5| \right]\mu^2 \right\} \;, \tag{III.24}$$

where we have taken into account that $\lambda_5 < 0$ [see (B.9)]. From these equations one can find the scales μ_R^2 and μ_{EW}^2 at which parity and electro–weak symmetry get broken, i.e. the VEVs v_R^2 and κ^2 become non–zero:

$$\mu_R^2 \;=\; \frac{2\lambda_7\,\mu_{R0}^2 + |\lambda_5|\,\mu_1^2}{2\lambda_7 + |\lambda_5|} \simeq \mu_{R0}^2 + \frac{|\lambda_5|}{2\lambda_7}\mu_1^2 \simeq \mu_{R0}^2 \;, \tag{III.25}$$

$$\mu_{EW}^2 \;=\; \frac{|\lambda_5|\,\mu_{R0}^2 + 2\lambda_1\,\mu_1^2}{2\lambda_1 + |\lambda_5|} \simeq \frac{|\lambda_5|}{2\lambda_1}\mu_{R0}^2 + \mu_1^2 \;, \tag{III.26}$$

where in the last equalities in eqs. (III.25) and (III.26) we have taken into account $|\lambda_5| \ll \lambda_1, \lambda_7$. Note that eqs. (III.23) and (III.24), as well as eqs. (III.21) and (III.22) above, are only valid for $\mu \leq \mu_R$ and $\mu \leq \mu_{EW}$, respectively.

Now one can rewrite eqs. (III.23) and (III.24) as

$$v_R^2(\mu^2) = \left(\frac{M_0^2}{\Lambda^2}\right) \frac{2\lambda_7 + |\lambda_5|}{4\lambda_1\lambda_7 - \lambda_5^2} \left(\mu_R^2 - \mu^2\right) \simeq \left(\frac{M_0^2}{\Lambda^2}\right) \frac{\mu_R^2 - \mu^2}{2\lambda_1} \,, \tag{III.27}$$

$$\kappa^2(\mu^2) = \left(\frac{M_0^2}{\Lambda^2}\right) \frac{2\lambda_1 + |\lambda_5|}{4\lambda_1\lambda_7 - \lambda_5^2} \left(\mu_{EW}^2 - \mu^2\right) \simeq \left(\frac{M_0^2}{\Lambda^2}\right) \frac{\mu_{EW}^2 - \mu^2}{2\lambda_7} \,. \tag{III.28}$$

We are interested in $v_R^2(0) \equiv v_R^2$ and $\kappa^2(0) \equiv \kappa^2$ which determine the masses of all the fermions and gauge bosons in the model:

$$v_R^2 \simeq \left(\frac{M_0^2}{\Lambda^2}\right) \frac{\mu_R^2}{2\lambda_1} \,, \tag{III.29}$$

$$\kappa^2 \simeq \left(\frac{M_0^2}{\Lambda^2}\right) \frac{\mu_{EW}^2}{2\lambda_7} \,. \tag{III.30}$$

This gives

$$\frac{\kappa^2}{v_R^2} \simeq \left(\frac{\lambda_1}{\lambda_7}\right) \frac{\mu_{EW}^2}{\mu_R^2} \sim \frac{\mu_{EW}^2}{\mu_R^2} \simeq \frac{|\lambda_5|}{2\lambda_1} + \frac{\mu_1^2}{\mu_R^2} \,. \tag{III.31}$$

Recall now that in conventional LR models with $\mu_{EW} \ll \mu_R \ll \Lambda_{GUT}$ (or Λ_{Planck}) one has to fine–tune two gauge hierarchies: Λ_{GUT}—μ_R and μ_R—μ_{EW}. We have a similar situation here: to achieve $\mu_{EW} \ll \mu_R \ll \Lambda$ one has to fine–tune two Yukawa couplings, Y_5^2 and $\widetilde{Y}^2$ [see (III.9) and (III.10)]. Tuning of Y_5^2 allows for the hierarchy $\mu_R^2 \ll \Lambda^2$; one then needs to adjust $\widetilde{Y}^2$ (or μ_1^2) to achieve $\mu_{EW}^2 \ll \mu_R^2$ through eq. (III.31).

Since λ_5 only contains relatively small gauge couplings while $Y_5 \sim \mathcal{O}(1)$, we typically have $|\lambda_5|/2\lambda_1 \sim 10^{-2}$. Thus, if there is no significant cancellation between the two terms in (III.31), one gets the right–handed scale of the order of a few TeV. Unfortunately, such a low LR scale scenario is not viable. As we shall show in Sec. IV, the squared masses of two Higgs bosons in our model become negative (i.e. the vacuum becomes unstable) unless $v_R \gtrsim 20\ TeV$. This requires some cancelation in eq. (III.31)[9], and then the right–handed scale $v_R \sim \mu_R$ can in principle lie anywhere between a few tens of TeV and Λ. However, if one prefers "minimal cancelation" in eq. (III.31), by about two orders of magnitude or so, one would arrive at a value of v_R around $20\ TeV$. It is interesting that the partial cancelation of the two terms in (III.31) implies $\mu_1^2 < 0$, i.e. $\widetilde{Y}^2$ must be below its critical value [see eq. (III.10)]. This means that $\widetilde{M}_1^2$ never becomes negative. In fact it is the $\widetilde{M}_0^2$ term, responsible for parity breakdown, that also drives the VEV of the bi–doublet. One can see from eq. (III.16) that the effective driving term for κ is $\widetilde{M}_1^2 + \lambda_5 v_R^2$; it may become negative for large enough v_R^2 even if $\widetilde{M}_1^2$ is positive since $\lambda_5 < 0$ [see eq. (B.9)]. Thus we have a tumbling scenario where the breaking of parity and $SU(2)_R$ occurring at the scale μ_R causes the breaking of the electro–weak symmetry at a lower scale μ_{EW}.

[9]Note that this does not increase the number of the parameters to be tuned but just shifts the value to which one of them should be adjusted.

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IV Predictions in Bubble Approximation

We can now obtain the predictions for fermion and Higgs boson masses in bubble approximation. As we have mentioned before, the static Higgs fields acquire gauge–invariant kinetic terms at low energies through radiative corrections, and the corresponding wave function renormalization constants can be derived from the 2–point Green functions. Direct calculation yields

$$Z_\phi \;=\; \frac{1}{16\pi^2} \left[N_c(Y_1^2 + Y_2^2) + Y_3^2 + Y_4^2 \right] \ln\left(\frac{\Lambda^2}{\mu^2}\right) \;, \tag{IV.1}$$

$$Z_\chi \;=\; \frac{1}{16\pi^2} Y_5^2 \ln\left(\frac{\Lambda^2}{\mu^2}\right) \;, \tag{IV.2}$$

$$Z_\sigma \;=\; \frac{1}{16\pi^2} 2Y_6^2 \ln\left(\frac{\Lambda^2}{\mu^2}\right) \;. \tag{IV.3}$$

To extract physical observables one should first rescale the Higgs fields so as to absorb the relevant Z factors into the definitions of the scalar fields and bring their kinetic terms into the canonical form. This amounts to dividing the squared mass terms by the corresponding Z factors, Yukawa couplings by $\sqrt{Z}$ and multiplying the scalar fields and their VEVs by $\sqrt{Z}$. Renormalization factors of the quartic couplings depend on the scalar fields involved and can be readily read off from the effective potential (see Appendix C for more details).

As we have already pointed out, the minimization of the effective Higgs potential gives $\sigma_0 = 0 = v_L$. This reduces the neutrino mass matrix (II.8) to

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & 0 \end{pmatrix} \;. \tag{IV.4}$$

Diagonalization of this matrix gives the following neutrino mass eigenstates and eigenvalues:

$$\begin{array}{llll}
\nu_1 &=& \cos\theta\,\nu_L - \sin\theta\,S_L & \qquad m_1 &=& 0 \\
\nu_2 &=& \frac{1}{\sqrt{2}}\left(\sin\theta\,\nu_L + \nu_L^c + \cos\theta\,S_L\right) & \qquad m_2 &=& \sqrt{M^2 + m_D^2} \\
\nu_3 &=& \frac{1}{\sqrt{2}}\left(\sin\theta\,\nu_L - \nu_L^c + \cos\theta\,S_L\right) & \qquad m_2 &=& -\sqrt{M^2 + m_D^2}
\end{array} \tag{IV.5}$$

with the mixing angle

$$\sin\theta = \frac{m_D}{\sqrt{M^2 + m_D^2}} \;. \tag{IV.6}$$

Thus, we have one massless left–handed neutrino ν_1 and two heavy Majorana neutrinos with degenerate masses $\sqrt{M^2 + m_D^2}$ and opposite CP–parities which combine to form a heavy Dirac neutrino. Since $m_D \ll M$ the electro–weak eigenstate $\nu_L \equiv \nu_\tau$ is predominantly the massless eigenstate whereas the right–handed neutrino ν_R and the singlet fermion S_L consist predominantly of the heavy eigenstates.

The gauge boson masses for our symmetry breaking scenario can be found in Appendix B. For $v_R \gg \kappa, \kappa' \gg v_L$, which implies strong parity violation at low energies and small LR–mixing, the masses reduce to

$$m_{W_L}^2 \approx \tfrac{g_2^2}{2}(\hat{\kappa}^2 + \hat{\kappa}'^2) \;, \qquad\qquad m_{W_R}^2 \approx \tfrac{g_2^2}{2}\hat{v}_R^2 \;,$$

$$m_{Z_L}^2 \approx \tfrac{g_2^2}{2}\sec\theta_W^2 (\hat{\kappa}^2 + \hat{\kappa}'^2) \;, \quad m_{Z_R}^2 \approx \tfrac{g_2^2 + g_1^2}{2}\hat{v}_R^2 \;, \tag{IV.7}$$

where the "hats" denote renormalized quantities. The usual VEV of the Standard Model should be identified with $(\hat{\kappa}^2 + \hat{\kappa}'^2)^{1/2}$.

IV.1 Relations between the scalar VEVs and fermion masses for $\kappa' = 0$

We will now assume that $\kappa' = 0$ which will simplify the discussion considerably. The general case $\kappa, \kappa' \neq 0$ will be considered at the end of this section and in Sec. V. It was shown above that $\kappa' = 0$ requires $Y_2 = 0 = Y_3$, which yields

$$m_t = Y_1 \kappa = \hat{Y}_1 \hat{\kappa} \;, \quad m_\tau = Y_4 \kappa = \hat{Y}_4 \hat{\kappa} \;, \quad m_b = m_D = 0 \;. \tag{IV.8}$$

Vanishing Dirac neutrino mass m_D implies $\sin\theta = 0$, and the heavy neutrino mass is now

$$M = Y_5 v_R = \hat{Y}_5 \hat{v}_R \;. \tag{IV.9}$$

From eqs. (IV.8) and (IV.1) and the definition of the renormalized Yukawa couplings one can readily find

$$\hat{\kappa}^2 = (174 \; GeV)^2 = N_c m_t^2 \left(1 + \frac{Y_4^2}{N_c Y_1^2}\right)\frac{1}{16\pi^2}\ln\left(\frac{\Lambda^2}{\mu^2}\right) \approx m_t^2 \frac{N_c}{16\pi^2}\ln\left(\frac{\Lambda^2}{\mu^2}\right) \;. \tag{IV.10}$$

This expression coincides with the one derived in bubble approximation by BHL [6] and we will see in Sec. V that this already corresponds to the renormalization group analysis in bubble approximation. Eq. (IV.10) gives the top quark mass in terms of the known electro–weak VEV and the scale of new physics Λ. For example, for $\Lambda = 10^{15} \; GeV$ one finds $m_t = 165 \; GeV$. However, this result is limited to the bubble approximation, and the renormalization group improved result will be substantially higher. Note that $m_t \approx 180 \; GeV$, which is the central value of the Fermilab results [28, 29], would mean $\hat{Y}_1 \approx 1$, or

$$l_0 \equiv \frac{1}{16\pi^2}\ln\left(\frac{\Lambda^2}{\mu^2}\right) \approx \frac{1}{3} \;. \tag{IV.11}$$

Similar considerations lead to the following relation between the right–handed VEV $\hat{v}_R$, heavy neutrino mass M and the scale Λ:

$$\hat{v}_R^2 = M^2 \frac{1}{16\pi^2}\ln\left(\frac{\Lambda^2}{\mu^2}\right) \;. \tag{IV.12}$$

Note that $\mu \approx m_t$ is understood in eqs. (IV.10) and (IV.11), whereas $\mu \approx M$ in eq. (IV.12). However, we assume $m_t, M \ll \Lambda$ and $M/m_t \ll \Lambda/M$ throughout this paper, therefore $\ln \frac{\Lambda^2}{m_t^2} \approx \ln \frac{\Lambda^2}{M^2}$, i.e. the logarithms are universal. From eqs. (IV.10) and (IV.12) one thus finds

$$\frac{\hat{v}_R^2}{M^2} \approx \frac{1}{3} \frac{\hat{\kappa}^2}{m_t^2} \ . \tag{IV.13}$$

The mass of the τ lepton is not predicted in our model since it is only weakly coupled to the bi–doublet; it is given by

$$m_\tau = \frac{Y_4}{Y_1} m_t \ , \tag{IV.14}$$

and can be adjusted to a desirable value by choosing the proper magnitude of the ratio Y_4/Y_1 (i.e. of G_3/G_1, see eq. (A.1) of Appendix A).

IV.2   Higgs Boson Masses

The Higgs boson masses can be obtained either as the poles of the corresponding propagators or, in the auxiliary field approach, by diagonalizing the matrices of second derivatives of the effective Higgs potential. In either case it is essential to use the (non–trivially satisfied) gap equation to cancel the large bare mass term in order to obtain a small composite Higgs mass. For example in the first approach, the inverse propagators of the composite scalar fields have a generic form

$$[iD(p^2)]^{-1} = M_0^2 + \Pi(p^2) \ , \tag{IV.15}$$

where $M_0^2 \sim \Lambda^2$ is the bare mass term and $\Pi(p^2)$ is the polarization operator. Using the gap equation one obtains $\Pi(p^2) = -M_0^2 - Z[p^2 - 4m_f^2 + \mathcal{O}(m_f^2/\ln(\Lambda^2/m_f^2)]$. Here m_f is the mass of the fermion whose bound state forms the composite Higgs boson. Thus the M_0^2 term in (IV.15) is exactly canceled, resulting in a light composite scalar with a mass of about $2m_f \ll \Lambda$. Without this cancelation the Higgs boson mass would have been of order Λ. Then the scalar field could either correspond to a real physical propagating state, or to a broad resonance, or the composite state might not exist at all, since at energy scales of $\mathcal{O}(\Lambda)$ the effective 4-f operators are not sufficient to reliably describe the theory. In any case this field would be decoupled from the low–energy spectrum of the model. In our model this is the case for the σ field since we assume the 4-f coupling G_8 to be sub–critical (see eq. (III.20) and the following discussion).

The situation is, however, quite different for the χ_L fields even though the corresponding gap equation (III.15) is only trivially satisfied ($v_L = 0$). First, one observes that $v_R \neq 0$ and so the gap equation for v_R *is* non–trivially satisfied. At the same time, the gap equations for v_R and v_L [eqs. (III.14) and (III.15)] have very similar structure; direct inspection shows that they differ from each other just by the interchange of λ_1 and λ_2, which is a consequence of the discrete parity symmetry. One can therefore use the information contained in the gap equation for v_R to cancel the large bare mass term in the propagator of χ_L. As a result, χ_L turns out to be a light physical propagating state. Direct calculation shows that its mass vanishes in the limit $(\lambda_2 - \lambda_1) = 0$, i.e. $g_2 = 0$, which corresponds to the fermion bubble approximation (see eq. (IV.27) below and eqs. (D.53) and (D.60) in Appendix D).

It should be possible to understand these vanishing scalar masses as a signal of some symmetry. Indeed, in the limit $\lambda_2 = \lambda_1$ the (χ_L, χ_R) sector of the effective Higgs potential [eq. (B.8)] depends on χ_L and χ_R only through the combination $(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)$. This means that the potential has a global $SU(4)$ symmetry which is larger than the initial $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry [$\{\chi_L^0, \chi_L^-, \chi_R^0, \chi_R^-\}$ forms a fundamental representation of $SU(4)$]. After χ_R^0 gets a non–vanishing VEV v_R, the symmetry is broken down to $SU(3)$, resulting in $15 - 8 = 7$ Goldstone Bosons. Three of them ($\chi_R^\pm$ and $\mathrm{Im}\,\chi_R^0$) are eaten by the $SU(2)_R$ gauge bosons $W_R^\pm$ and Z_R, and the remaining four ($\chi_L^\pm$, $\mathrm{Re}\,\chi_L^0$ and $\mathrm{Im}\,\chi_L^0$) are physical massless Goldstone Bosons. The $SU(4)$ symmetry is broken by the ϕ–dependent terms in the effective potential and by $SU(2)$ gauge interactions. As a result, $\chi_L^\pm$, $\mathrm{Re}\,\chi_L^0$ and $\mathrm{Im}\,\chi_L^0$ acquire small masses and become pseudo–Goldstone Bosons. In fact, the origin of this approximate $SU(4)$ symmetry can be traced back to the 4-f operators of eq. (II.4). It is an accidental symmetry resulting from the gauge invariance and parity symmetry of the G_7 term. Note that no such symmetry occurs in conventional LR models.

We now present our results for the composite Higgs bosons for the case $\sigma_0 = v_L = \kappa' = 0$. Further information and more general results are contained in Appendix D. We have the following Goldstone Bosons in our model[10]:

$$
\begin{aligned}
G_1^\pm &= \frac{1}{\sqrt{v_R^2 + \kappa^2}} \left(\kappa \phi_2^\pm + v_R \chi_R^\pm \right) \approx \chi_R^\pm , \qquad & G_1^0 &= \chi_{Ri}^0 , \\
G_2^\pm &= \phi_1^\pm , & G_2^0 &= \phi_{1i}^0 ,
\end{aligned}
\tag{IV.16}
$$

where $G_1^\pm, G_1^0$ are eaten by $W_R^\pm$, Z_R and $G_2^\pm, G_2^0$ by $W_L^\pm$, Z_L, respectively. The physical Higgs boson sector of the model contains two CP–even neutral scalars

$$
H_1^0 \approx \left(1 - \frac{\lambda_5^2}{8\lambda_1^2} \frac{\kappa^2}{v_R^2} \right) \chi_{Rr}^0 + \frac{\lambda_5}{2\lambda_1} \frac{\kappa}{v_R} \phi_{1r}^0 \approx \chi_{Rr}^0 ,
\tag{IV.17}
$$

$$
H_2^0 \approx -\frac{\lambda_5}{2\lambda_1} \frac{\kappa}{v_R} \chi_{Rr}^0 + \left(1 - \frac{\lambda_5^2}{8\lambda_1^2} \frac{\kappa^2}{v_R^2} \right) \phi_{1r}^0 \approx \phi_{1r}^0 ,
\tag{IV.18}
$$

with the masses

$$
M_{H_1^0}^2 \simeq 4M^2 \left[1 - \frac{3}{16} \left(3g_2^4 + 2g_2^2 g_1^2 + g_1^4 \right) l_0^2 \right] \approx 4M^2 ,
\tag{IV.19}
$$

$$
M_{H_2^0}^2 \simeq 4m_t^2 \left(1 - \frac{m_\tau^2}{3m_t^2} - \frac{9}{4} g_2^4 l_0^2 \right) \approx 4m_t^2 ,
\tag{IV.20}
$$

which are directly related to the two steps of symmetry breaking, $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ and $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. Further, there are the charged Higgs boson $H_3^\pm$ with its neutral CP–even and CP–odd partners H_{3r}^0 and H_{3i}^0,

$$
H_3^\pm = \frac{1}{\sqrt{v_R^2 + \kappa^2}} \left(-\kappa \chi_R^\pm + v_R \phi_2^\pm \right) \approx \phi_2^\pm ,
\tag{IV.21}
$$

$$
H_{3r}^0 = \phi_{2r}^0 , \qquad H_{3i}^0 = \phi_{2i}^0 ,
\tag{IV.22}
$$

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[10]Hereafter we omit the "hats" over the renormalized quantities.

and the previously mentioned pseudo–Goldstone Bosons χ_L:

$$H_4^\pm = \chi_L^\pm \ , \qquad H_{4r}^0 = \chi_{Lr}^0 \ , \qquad H_{4i}^0 = \chi_{Li}^0 \ , \tag{IV.23}$$

with the masses

$$M_{H_3^\pm}^2 \ \approx \ \frac{2}{3} M^2 \frac{m_\tau^2}{m_t^2} \ , \tag{IV.24}$$

$$M_{H_{3r}^0}^2 = M_{H_{3i}^0}^2 \ \approx \ \frac{2}{3} M^2 \frac{m_\tau^2}{m_t^2} - \frac{1}{2} M_{H_2^0}^2 \ , \tag{IV.25}$$

$$M_{H_4^\pm}^2 \ = \ \frac{3}{8} \left(3g_2^4 + 2g_2^2 g_1^2 \right) l_0^2 \, M^2 + 2m_\tau^2 \ , \tag{IV.26}$$

$$M_{H_{4r}^0}^2 = M_{H_{4i}^0}^2 \ = \ \frac{3}{8} \left(3g_2^4 + 2g_2^2 g_1^2 \right) l_0^2 \, M^2 \ . \tag{IV.27}$$

Note that the last expression for the mass of the neutral pseudo–Goldstone Bosons is proportional to $\lambda_2 - \lambda_1$.

Altogether we have 4 physical charged scalars, 4 CP–even and 2 CP–odd physical neutral scalars. The mass of the scalar H_2^0, which is the analog of the Standard Model Higgs boson [eq. (IV.20)], essentially coincides with the one obtained in bubble approximation by BHL [6]. This just reflects the fact that this boson is the $t\bar{t}$ bound state with a mass of $\approx 2m_t$. Analogously, the mass of the heavy CP–even scalar $H_1^0 \approx \chi_{Rr}^0$ is approximately $2M$ since it is a bound state of heavy neutrinos, $\chi_R^0 \sim \overline{S}_L \nu_R = [\overline{\nu}_{2L}(\nu_{2L})^c - \overline{\nu}_{3L}(\nu_{3L})^c]/2$.

In conventional LR models only one scalar, which is the analog of the Standard Model Higgs boson, is light (at the electro–weak scale), all the others have their masses of the order of the right–handed scale M [2, 3, 30]. In our case, the masses of those scalars are also proportional to M, but all of them except the mass of H_1^0 have some suppression factors. The masses of $H_4 = \chi_L$ are suppressed because of their pseudo–Goldstone nature and vanish in the limit $g_2 \to 0$, $m_\tau \to 0$. In fact, though the $SU(2)$ gauge coupling constant g_2 is smaller than the typical Yukawa constants in our model, it is not too small; taking the estimate of l_0 from eq. (IV.11), one arrives at $M_{\chi_L} \sim 10^{-1} M$.

The mass of the charged scalars $H_3^\pm \approx \phi_2^\pm$ is suppressed by the factor m_τ/m_t and is therefore of the order $10^{-2} M$. The masses of the neutral H_{3r}^0 and H_{3i}^0 are even smaller; they are related to the masses of charged $H_3^\pm$ and the Standard Model Higgs H_2^0 by eq. (IV.25). In fact this equation imposes an upper limit on the Standard Model Higgs boson mass $M_{H_2^0}$ (for a given M) or a lower limit on the right–handed mass M (for a given $M_{H_2^0}$). These limits follow from the requirement that $M_{H_3^0}^2$ be positive, i.e. from the vacuum stability condition. For example, for $M_{H_2^0} \approx 200 \ GeV$ we find $M \gtrsim 17 \ TeV$. This is the lower bound on the right–handed scale that we mentioned[11] in Sec. III.

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[11] The current experimental lower bound on the Standard Model Higgs boson mass is only $60 \ GeV$, which would yield $M \gtrsim 5 \ TeV$. It is interesting to notice that the same lower limit $M \gtrsim 5 \ TeV$ would result from eq. (IV.24) and the experimental lower bound on the charged Higgs boson mass $M^\pm > 45 \ GeV$ which follows from the LEP data. However, in our model the mass of the standard–model–like (composite) neutral Higgs boson must be at least of the order of the top quark mass, which yields the above–mentioned estimate $M \gtrsim 17 \ TeV$.

Thus, we have a number of intermediate scale Higgs bosons with relations between masses of various scalars [eqs. (IV.24)–(IV.27)] and between fermion and Higgs boson masses [eqs. (IV.19) and (IV.20)] which are in principle testable. If the right–handed scale v_R is of the order of a few tens of TeV, the neutral CP–even and CP–odd scalars H_{3r}^0 and H_{3i}^0 can be even lighter than the electro–weak Higgs boson. In fact, they can be as light as $\sim 60~GeV$ and so might be observable at LEP2.

Finally, we would like to comment on the approximation $\kappa' = 0$ which we used so far. Clearly, this is an oversimplification: In a realistic model with non–vanishing m_b and m_D both κ and κ' must be non–zero (note that the Yukawa couplings Y_2 and Y_3 will also be non–zero in this case). However, these masses are not predicted in our model and can merely be adjusted to desirable values. The Dirac neutrino mass m_D is unknown and so remains a free parameter; however, it must be smaller than m_τ in our model in order to satisfy the vacuum stability condition $(Y_4^2 - Y_3^2)(\kappa^2 - \kappa'^2) > 0$ [see eqs. (D.52) and (D.57)] which is equivalent to $m_\tau^2 - m_D^2 > 0$. The Higgs boson masses and mass eigenstates for the general case $\kappa, \kappa' \neq 0$ are given in Appendix D. In the next section we will obtain renormalization group improved predictions for the fermion masses and show that some interesting results (including a viable top quark mass) emerge for sizeable values of κ', $\kappa' \sim \kappa$.

V Renormalization Group Improved Predictions

In the preceding sections we studied our model and derived its predictions in bubble approximation, taking into account only fermion and $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge boson loops. However, important corrections arise from QCD effects and loops with composite Higgs scalars. Following the approach of BHL [6], we will incorporate these effects by solving the full one–loop renormalization group equations of the low energy effective LR–model with boundary conditions corresponding to compositeness.

These boundary conditions follow from the vanishing of the radiatively induced kinetic terms for the Higgs scalars at the scale Λ, where the composite particles break up into their constituents:

$$Z_\phi(\mu^2 \to \Lambda^2) = Z_\chi(\mu^2 \to \Lambda^2) = Z_\sigma(\mu^2 \to \Lambda^2) = 0 ~. \qquad (V.1)$$

After rescaling the scalar fields so as to bring their kinetic terms into the canonical form (see Sec. IV and Appendix C), eqs. (V.1) result in boundary conditions for the Yukawa and quartic couplings of the low energy effective Lagrangian. These conditions are similar to those obtained by BHL [eq. (I.5)] and have the following generic form:

$$\hat{Y}^2 = \frac{Y^2}{Z} \xrightarrow{\mu^2 \to \Lambda^2} \infty ~, ~~ \hat{\lambda} = \frac{\lambda}{Z^2} \xrightarrow{\mu^2 \to \Lambda^2} \infty ~, ~~ \frac{\hat{\lambda}}{\hat{Y}^4} = \frac{\lambda}{Y^4} \xrightarrow{\mu^2 \to \Lambda^2} 0 ~. \qquad (V.2)$$

The renormalized parameters of our model derived in bubble approximation already satisfy these compositeness conditions; for example, the renormalized Yukawa couplings are

$$\hat{Y}_1^2(\mu) ~=~ \frac{Y_1^2}{Z_\phi} \approx \left[\frac{3}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \right]^{-1} ~; ~~ (\hat{Y}_4 \ll \hat{Y}_1) ~, \qquad (V.3)$$

$$\hat{Y}_4^2(\mu) \approx \frac{Y_4^2}{Y_1^2} \hat{Y}_1^2(\mu) = \frac{G_3}{G_1} \hat{Y}_1^2(\mu) \ , \tag{V.4}$$

$$\hat{Y}_5^2(\mu) = \frac{Y_5^2}{Z_\chi} = \left[\frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \right]^{-1} \ , \tag{V.5}$$

$$\hat{Y}_6^2(\mu) = \frac{Y_6^2}{Z_\sigma} = \left[\frac{2}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \right]^{-1} \ . \tag{V.6}$$

Obviously they diverge as $\mu \to \Lambda$. Furthermore, their running coincides exactly with that described by the fermion loop contributions to one–loop β–functions of the corresponding LR–theory. These fermion loop contributions can be read off from the trace terms in eqs. (E.1)–(E.6) in Appendix E where the full set of gauge and Yukawa β–functions for our model is given.

The idea is now to identify the Landau poles in the *full* one–loop renormalization group evolution of couplings with the compositeness scale Λ and run the couplings down to low energy scales.

V.1   The case $\kappa' = 0$

We will first consider the simplified scenario with $\kappa' = 0$. The case $\kappa' \neq 0$ which leads to phenomenologically acceptable fermion masses will be discussed in the next subsection. In our one–generation scenario the renormalization group equations for the Yukawa couplings in the limit $\kappa' = 0$ (which requires $Y_2 = Y_3 = 0$, see Sec. III) reduce to[†]

$$16\pi^2 \frac{dY_1}{dt} = 5Y_1^3 + Y_1 Y_4^2 - \left(8g_3^2 + \frac{9}{2}g_2^2 + \frac{1}{6}g_1^2 \right) Y_1 \ , \tag{V.7}$$

$$16\pi^2 \frac{dY_4}{dt} = 3Y_4^3 + 3Y_4 Y_1^2 + Y_4 Y_5^2 - \left(\frac{9}{2}g_2^2 + \frac{3}{2}g_1^2 \right) Y_4 \ , \tag{V.8}$$

$$16\pi^2 \frac{dY_5}{dt} = \frac{7}{2}Y_5^3 + Y_5 \left(Y_4^2 + [2Y_6^2] \right) - \left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 \right) Y_5 \ , \tag{V.9}$$

$$16\pi^2 \frac{dY_6}{dt} = [6]Y_6^3 + 4Y_6 Y_5^2 \ . \tag{V.10}$$

For large values of Y_i the Y_i^3–contributions in the β–functions are dominant; they quickly drive the couplings down to values of order one as the scale μ decreases. In this regime gauge and other Yukawa coupling contributions become important, and the interplay of these contributions and Y_i^3 terms result in so–called infrared quasi–fixed points [14]. Thus a large range of initial values of the Yukawa couplings at the cutoff is focused into a small range at low energies. The masses of the fermions will then be given implicitly by conditions of the kind $Y(m) \cdot VEV = m$.

Strictly speaking it is not legitimate to evolve the Yukawa couplings with one–loop β–functions to their Landau poles, i.e. in the non–perturbative regime. However, it has been

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[†]In the following we will omit the hats over the renormalized quantities.

argued in [6] that this should not result in any significant errors since (1) the running time $t = \ln \mu$ in the non–perturbative domain is only a few percent of the total running time and, more importantly, (2) the infrared quasi–fixed point structure of the renormalization group equations makes the predictions fairly insensitive to the detailed behavior of the solutions in the large Yukawa coupling domain. Lattice gauge theory has generally confirmed the reliability of perturbation theory in this fixed point analysis [31].

Except for switching to the Standard Model renormalization group equations below the parity breaking scale the only relevant threshold effects in the evolution are due to the masses of the σ scalars. We have shown in bubble approximation that the VEVs of σ and χ_R do not coexist; phenomenology then dictates the choice $\sigma_0 = 0$, $v_R \neq 0$ which requires the 4-f coupling G_8 to be sub–critical, or at least satisfying the condition (III.20). We will assume that the same holds true beyond the bubble approximation (although we were unable to prove that) and consider σ to be non–propagating, or at least decoupled from the low–energy spectrum of the model. We will therefore switch off effects from propagating σ scalars directly at the cutoff by neglecting the contributions in square brackets of eqs. (V.9) and (V.10). In this limit the running of Y_6 does not influence the running of the other Yukawa couplings.

In numerical calculations a large number $Y_i(\Lambda)$ must be used as boundary condition for Yukawa couplings instead of infinity. Fortunately, the infrared quasi–fixed point structure of the renormalization group equation makes the solutions fairly insensitive to the actual values of $Y_i(\Lambda)$ provided that they are large enough [6, 14]. In fact, the infrared quasi–fixed–point behavior sets in already for $Y_i(\Lambda) \gtrsim 5$. In our calculations we have chosen $Y_5(\Lambda) = 10$; taking e.g. 10^3 instead of 10 results only in a correction of about 0.4% in the low–energy value of Y_5. The fermion–loop results of eqs. (V.3)–(V.6) imply a fixed ratio between the running coupling constants which one could, as a first approximation, also impose as the boundary condition at the cutoff for the full renormalization group evolution, e.g. $Y_1(\Lambda) = 3 Y_5(\Lambda) = 30$. Fortunately, once again, the numerical results depend very weakly on this scaling factor, and for this purpose it could just as well be taken to be unity. Fig. 6 shows $Y_5(\mu)$ obtained by numerically solving eqs. (V.7)–(V.9) for various values of the cutoff Λ. The heavy neutrino mass M is determined by the equation

$$M = Y_5(M) \cdot \hat{v}_R , \qquad (\text{V.11})$$

and for $v_R \sim \mu_R \ll \Lambda$ one finds values of $Y_5(M) \simeq Y_5(\mu_R)$ roughly between 1 and 2.

The evolution of Y_1 and Y_4 below the parity breaking scale is determined by the usual Standard Model β–functions [13]. It turns out that the numerically most important difference between the LR and Standard Model β–functions for Y_1 is a contribution of $1/2 \, Y_1^3$ coming from the self energy diagram with a ϕ_2^+ scalar exchange. Since the mass of ϕ_2^+ is not of the order of the right–handed scale but is suppressed by a factor $\approx 10^{-2}$ we switch to the Standard Model β–functions two orders of magnitude below the parity breaking scale μ_R.

In Fig. 7 we present our numerical solutions for $Y_1(\mu)$ for various values of Λ and μ_R. One can clearly see the infrared quasi–fixed point structure of the solutions. The values of Y_1 at $t = 0$ ($\mu = m_Z$) are to some extent sensitive to the magnitude of the cutoff but fairly insensitive to the scale where parity breaks. This is because in fact the previously mentioned contribution to the Y_1 β–function from the ϕ_2^+ exchange makes only a relatively
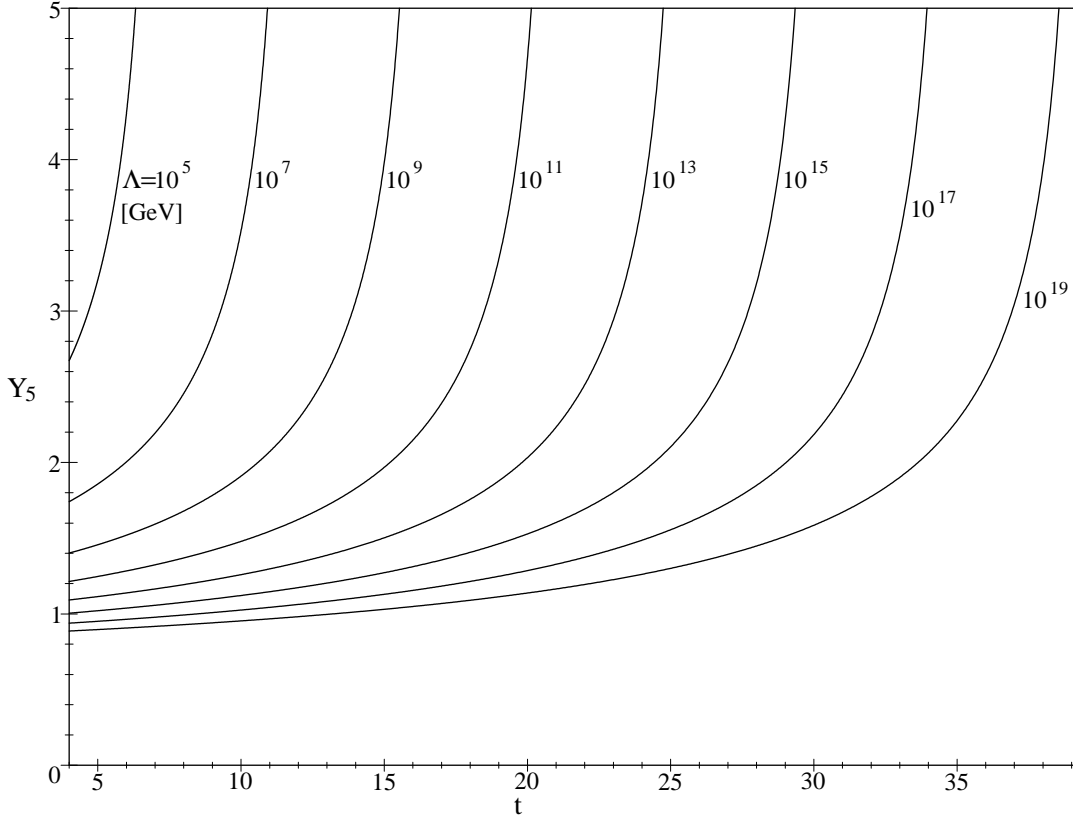
24

Figure 6: *Renormalization Group evolution of the Y_5 Yukawa coupling for various compositeness scales Λ, $t = \ln(\mu/m_Z)$.*

small difference between $(9/2)\,Y_1^3$ in the Standard Model and $5\,Y_1^3$ in the LR symmetric model with a bi–doublet. For the same reason our top quark mass prediction in the $\kappa' = 0$ case is very similar to the one of the BHL model [6], which is too high compared to the recent experimental results [28, 29]. We also conclude that the renormalization group analysis for the minimal left–right symmetric model in Ref. [32], which predicted a lower top quark mass, is wrong due to the incorrect trace terms in the β–functions. Even for a rather high cutoff $\Lambda \approx 10^{17}$ GeV one obtains a top quark mass about 229 GeV.

The evolution of the Yukawa coupling Y_4 which determines m_τ is not governed by a fixed point; the reason for this is that the τ lepton is too light and so does not contribute much to the composite Higgs bi–doublet, which is driven by large Y_1 and not by Y_4. In other words, our model exhibits a top condensate (along with a heavy–neutrino condensate) rather than a tau condensate. One can readily obtain a suitable low–energy value of Y_4 by choosing a proper value of $Y_4(\Lambda)$ (see Fig. 8). Thus, as we already mentioned in Sec. IV, although m_τ is not predicted in our model, it can be easily adjusted to the correct value.
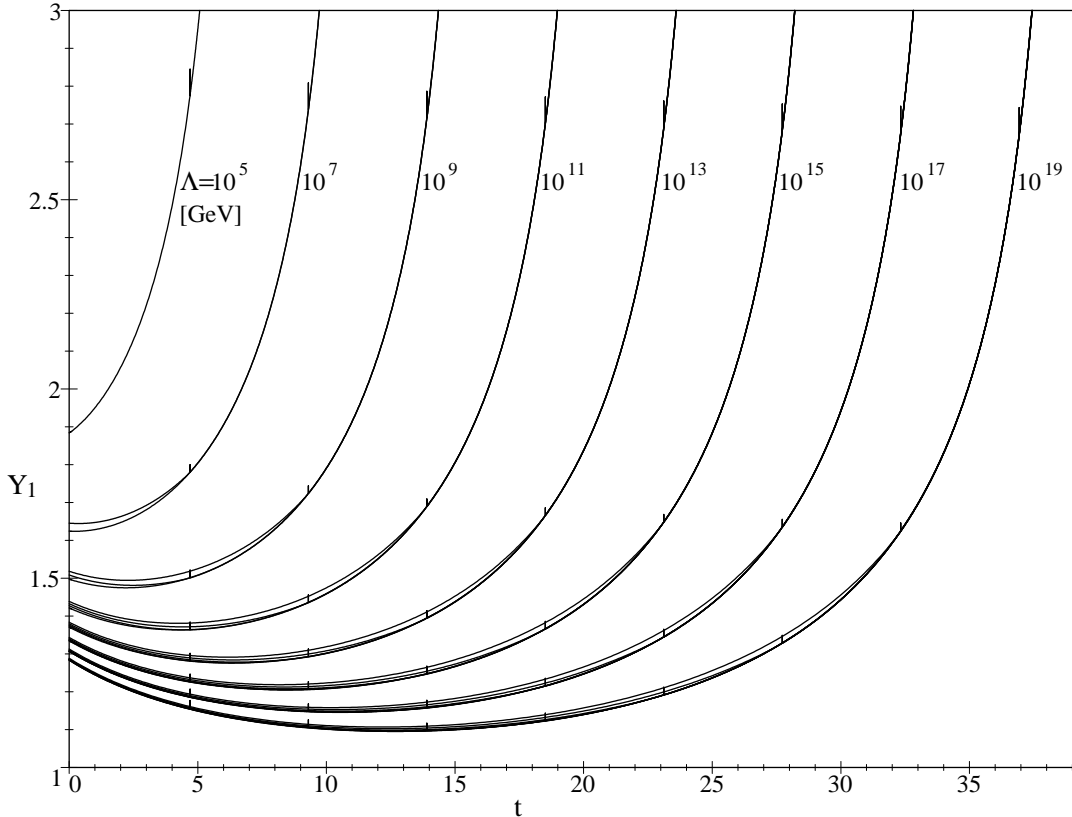
Figure 7: *Renormalization Group evolution of the Y_1 Yukawa coupling for various parity breaking scales μ_R (indicated by little ticks) and compositeness scales Λ; $t = \ln(\mu/m_Z)$.*

V.2   The general case $\kappa, \kappa' \neq 0$

In the preceding discussion we assumed for simplicity that only one of the two neutral components of the bi–doublet ϕ acquires a VEV ($\kappa \neq 0$, $\kappa' = 0$). Apparently the resulting fixed–point value of the top quark Yukawa coupling is in this limit outside the phenomenologically acceptable region [28, 29], and moreover the assumption $\kappa' = 0$ implies a zero bottom quark mass. Evidently a more realistic scenario with $m_b \neq 0$ would require either $\kappa' \neq 0$ or $Y_2 \neq 0$. In conventional LR models these two conditions are unrelated and can be satisfied separately, but in our model $Y_2 \neq 0$ automatically means $\kappa' \neq 0$ and vice versa. In this subsection we will show that it is indeed possible to obtain viable top and bottom quark masses for a range of values of $\kappa' \sim \kappa$.

From the minimization condition of the effective potential (III.13–III.17) one finds two approximate solutions, $\kappa'/\kappa \approx -Y_3/Y_4$ and $\kappa'/\kappa \approx Y_4/Y_3$. Without loss of generality we assume $|\kappa'| < |\kappa|$; then the latter solution is excluded by the vacuum stability condition [see eq. (D.48)], and only the solution

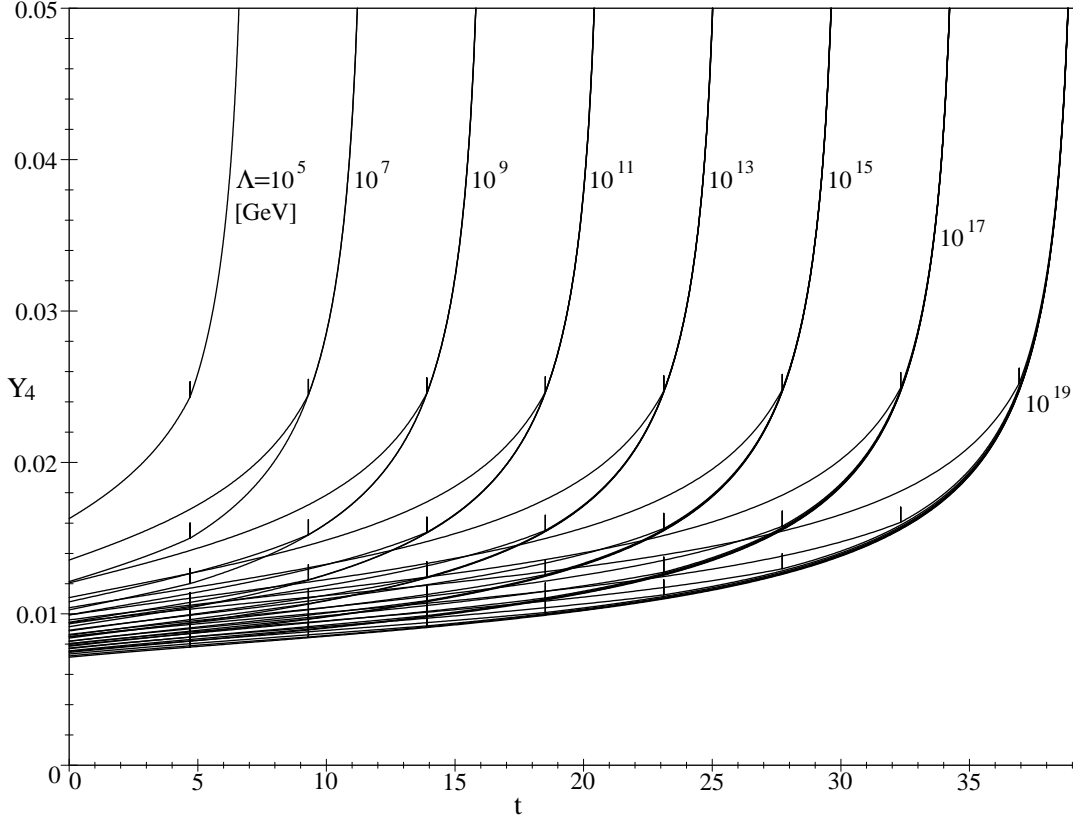$$\frac{\kappa'}{\kappa} = -\frac{Y_3}{Y_4} + \mathcal{O}\left(\frac{\kappa^2}{v_R^2}\right) \tag{V.12}$$

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Figure 8: *Renormalization Group evolution of the Y_4 Yukawa coupling with the boundary condition $Y_4 = 0.005Y_1$ at Λ.*

remains. On the other hand, to obtain $m_t \gg m_b$ for $\kappa \sim \kappa'$ one requires the condition

$$\frac{Y_2}{Y_1} \approx -\frac{\kappa'}{\kappa} \tag{V.13}$$

to be satisfied. From eq. (V.12) it is seen that the ratio of κ'/κ is determined by the low energy values of Y_3 and Y_4. Since these values are not governed by infrared quasi–fixed points and depend on the boundary conditions at the cutoff, the ratio

$$\tan \beta \equiv \frac{\kappa}{\kappa'} \tag{V.14}$$

is a free parameter in our model. Unfortunately, the renormalization group evolution of the Yukawa couplings does not automatically yield an infrared value of Y_2/Y_1 satisfying eq. (V.13) as a result of, e.g., an attractive fixed point. However, this ratio runs fairly slowly and by choosing its initial value appropriately one can always obtain the desirable value at low energies.

One can now study the renormalization group evolution of the full set of Yukawa couplings. It turns out that the combination $\sqrt{Y_1^2 + Y_2^2}$ exhibits a fixed point behavior which is similar to the one of Y_1 discussed in the previous subsection. Below the right–handed

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scale one should switch to the Standard Model β–functions of Y_t and Y_b which are obvious linear combinations of Y_1 and Y_2. Imposing the relation (V.13) we find those values of $\tan\beta$ which result in viable top quark masses, depending on the values of the cutoff Λ and the right–handed scale μ_R. This is in fact similar to supersymmetric or 2–Higgs doublet models in which the top mass depends on $\tan\beta$ which is essentially a free parameter.
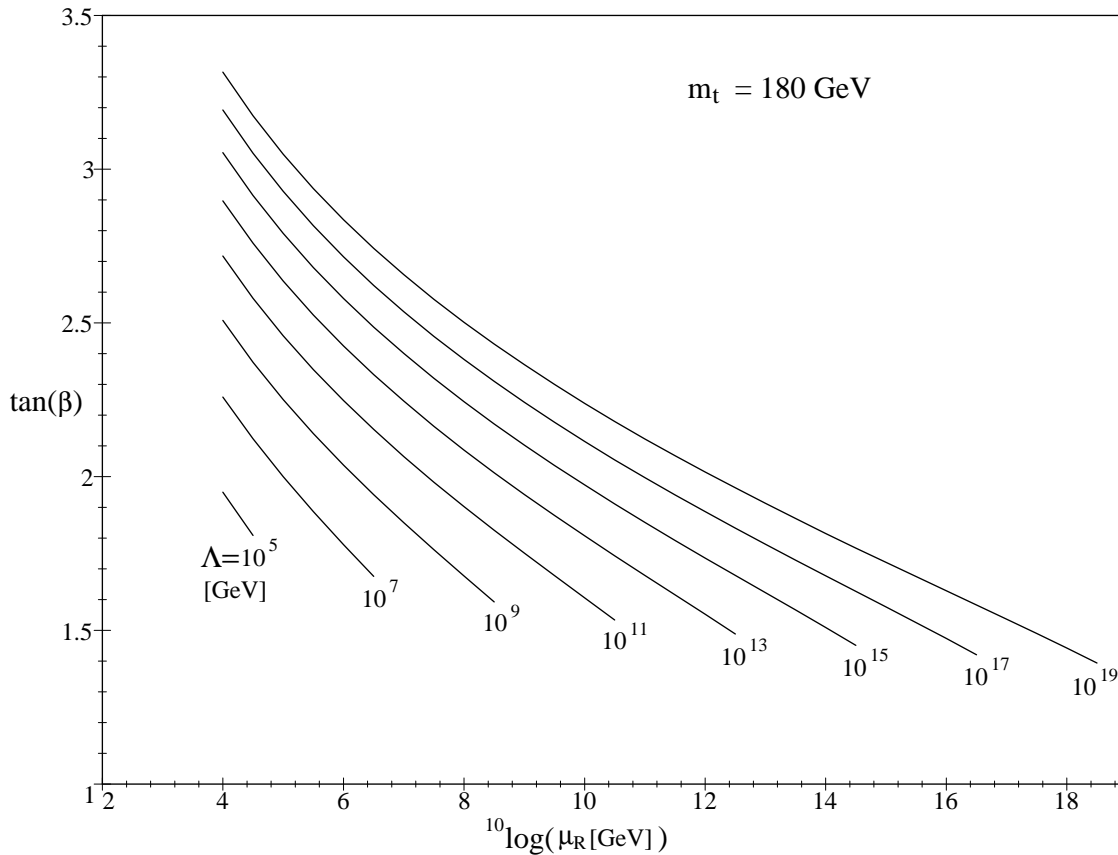

Figure 9: *Values of $\tan\beta$ for $m_t = 180$ GeV and various magnitudes of the cutoff Λ and right–handed scale μ_R.*

In Fig. 9 we show our results for $\tan\beta$ assuming a top mass of $m_t = 180$ GeV. One observes that a viable top mass can be obtained for a large range of possible values of the cutoff and for various parity breaking scales. This is in contrast with the top condensate approach to the Standard Model [6] where the lowest possible (but still too high) top mass arises for the largest possible cutoff. As can be seen from the figure, in our model we can have a viable top quark mass for values of the cut–off as low as 200 TeV and parity breaking scale about 20 TeV. This means that there is only a minimal amount of fine–tuning involved and the gauge hierarchy problem gets significantly ameliorated. If one considers different values for the top mass, the whole set of curves in Fig. 9 is slightly shifted vertically, e.g., for $m_t = (168 - 192)$ GeV [28, 29] one finds an overall range for $\tan\beta$ of $(1.3 - 4.0)$.

VI Phenomenological Considerations

A detailed study of all phenomenological aspects of our model is outside the scope of the present paper. We therefore discuss here only a few major points.

In the general version of our model with non–vanishing κ' we were able to obtain viable top and bottom quark masses. The renormalization group fixed point analysis leads to fermion mass predictions which are less constrained than in the BHL model [6] since the fermions acquire their masses through more than one VEV. The resulting masses depend on the fixed points and on the ratio of those VEVs, in our case $\tan\beta = \kappa/\kappa'$, which is a free parameter. The Fermilab Tevatron results for m_t can be reproduced for $\tan\beta \simeq 1.3$–4. In the limiting case of $\kappa' = 0$ ($\tan\beta \to \infty$) one arrives at a practically unique value of m_t, which turns out to be very similar to that of BHL, i.e. too high as compared to experiment.

In addition to the usual quarks and leptons, we have a neutral gauge singlet fermion S_L. Its existence along with the conditions $v_L = 0 = \sigma_0$ which follow from the minimization of the Higgs potential result in the tau neutrino ν_τ being massless in our model. This also applies to ν_e and ν_μ if the model is directly generalized to include the first two generations of fermions[12]. The reason for this is that the model possesses a global $U(1)$ symmetry $S_L \to e^{i\alpha} S_L$, $\chi_L \to e^{i\alpha}\chi_L$, $\chi_R \to e^{-i\alpha}\chi_R$, $\sigma \to e^{-2i\alpha}\sigma$. After χ_R^0 develops a VEV this symmetry gets redefined, but since $v_L = \sigma_0 = 0$, there still exists an unbroken global $U(1)$ symmetry such that ν_τ remains massless. The situation when this symmetry is spontaneously broken and the properties of the resulting Majoron were considered (within the standard elementary Higgs mechanism of symmetry breaking) in [33].

In the limiting case $\kappa' \to 0$ the Dirac neutrino mass term m_D vanishes and there is no neutrino mixing. However, as we already mentioned, realistic fermion masses require $\kappa, \kappa' \neq 0$. In this case $m_D \neq 0$ and one could expect interesting effects of lepton flavour violation mediated by S_L [34, 35]. The reason for this is that one can in principle have quite a sizeable Dirac neutrino mass and hence mixing in the lepton sector without generating inadmissible heavy physical neutrino states (in fact, the neutrinos taking part in the usual electro–weak interactions remain exactly massless however large m_D). Another interesting effect of S_L is the possibility of having CP non–conservation in the lepton sector even though the physical neutrinos are massless [36]. However, as follows from eqs. (II.7) and (V.12), due to the dynamical nature of the LR symmetry breaking, in our model m_D is suppressed by a factor $\mathcal{O}(\kappa^2/v_R^2)$ as compared to the charged lepton mass. This means that even for a right–handed scale as low as $\sim 20\ TeV$ the flavour violating and CP non–conservation effects in the lepton sector caused by the mixing of light and heavy neutrinos may not be observable. In this respect our dynamical model is more restrictive than the version of the same model based on the conventional Higgs mechanism [33] which allows for a large Dirac neutrino mass term m_D. However, as we shall discuss shortly, there still may exist observable effects of lepton flavour violation mediated by χ_L and χ_R scalar boson exchanges.

We will now discuss the implications of the Higgs sector of our model. Since χ_L does

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[12]We assume here that there is one singlet fermion S_{Li} for each quark–lepton family. One can also consider the situation when there is a unique S_L for all three families. In this case only one active neutrino would be massless and the remaining two would have masses of the order of m_D.

not mix with other Higgs multiplets, and the components of χ_R are either eaten by the right–handed gauge bosons or heavy, our low energy Higgs sector is very similar to that of the two–Higgs–doublet Standard Model (2HDSM). In our case there are however very specific relations between the masses of scalars (see Sec. IV) which would be a distinctive signature for our model.

Extra Higgs and gauge boson multiplets contribute to radiative corrections like e.g. in the $\Delta\rho$ parameter and so might be constrained by electro–weak precision data. The contribution of the mixing with $SU(2)_R$ gauge bosons is however negligible for $v_R \gtrsim 20 \, TeV$ [37]. So is the χ_L contribution to $\Delta\rho$; this follows from the fact that the charged and neutral components of χ_L are nearly degenerate [see eqs. (IV.26) and (IV.27)]. The bi–doublet contribution to $\Delta\rho$ coincides with the Higgs boson contributions in the 2HDSM. In general it depends on the masses of the relevant scalars and two mixing angles, α and β, where β coincides with our $\beta \equiv \tan^{-1}(\kappa/\kappa')$ and α is the mixing angle of neutral CP–even Higgs bosons [38]. In our case $\alpha \simeq \beta$, and the mass of the lightest CP–even scalar coincides with the mass of its CP–odd partner (see eq. (IV.25) and Appendix D). This reduces the Higgs boson contributions to $\Delta\rho$ to a simple expression

$$\Delta\rho_H \approx \frac{G_F}{8\sqrt{2}\pi^2} \left(M_{H_3^\pm}^2 + M_{H_{3i}^0}^2 - 2 \frac{M_{H_3^\pm}^2 M_{H_{3i}^0}^2}{M_{H_3^\pm}^2 - M_{H_{3i}^0}^2} \ln \frac{M_{H_3^\pm}^2}{M_{H_{3i}^0}^2} \right) \equiv \frac{G_F}{8\sqrt{2}\pi^2} \, f(M_{H_3^\pm}^2, M_{H_{3i}^0}^2) \,, \tag{VI.1}$$

where $M_{H_3^\pm}^2$ and $M_{H_{3i}^0}^2$ are the masses of the charged and neutral CP–odd Higgs bosons given by eqs. (IV.24) and (IV.25) respectively. Electro–weak precision measurements require $f(M_{H_3^\pm}^2, M_{H_{3i}^0}^2)$ to be $\lesssim 3 \times (100 \, GeV)^2$ [39]. This imposes an upper bound on the mass splitting between $M_{H_3^\pm}^2$ and $M_{H_{3i}^0}^2$. In our model $M_{H_3^\pm}^2 - M_{H_{3i}^0}^2 = M_{H_2^0}^2/2$ where $M_{H_2^0}$ is the Standard Model Higgs boson mass, which gives

$$f(M_{H_3^\pm}^2, M_{H_{3i}^0}^2) \approx \begin{cases} M_{H_2^0}^4/12M_{H_3^\pm}^2, & M_{H_2^0}^2 \ll 2M_{H_{3i}^0}^2 \\ M_{H_2^0}^2/2, & M_{H_2^0}^2 \gg 2M_{H_{3i}^0}^2 \end{cases}. \tag{VI.2}$$

For $M_{H_2^0}^2 \ll 2M_{H_{3i}^0}^2$ (i.e. for M not too close to its lower bound of about $20 \, TeV$) one then obtains the following upper limit on the Standard Model Higgs boson mass: $M_{H_2^0} \lesssim [36M_{H_3^\pm}^2 (100 \, GeV)^2]^{1/4} \approx 70\sqrt{M/TeV}$. For example, for $M \approx 100 \, TeV$ this gives $M_{H_2^0} \lesssim 700 \, GeV$. For $M_{H_2^0}^2 \gg 2M_{H_{3i}^0}^2$ (which means that M is close to its lower bound) one finds $M_{H_2^0} \lesssim 245 \, GeV$. These constraints are less restrictive than those following from the vacuum stability condition in our model.

In conventional LR models only one scalar particle, the neutral CP–even Higgs boson H_2^0 which is the analog of the Standard Model one, is light; all the others are at the right–handed scale v_R [2, 3, 30]. In our model, only one Higgs boson (H_1^0) is at the right–handed scale. In addition to H_2^0 which is at the electro–weak scale we have $\chi_L^\pm$, χ_{Lr}^0 and χ_{Li}^0 whose masses are typically one order of magnitude below the right–handed scale, and $\phi_2^\pm$ whose mass is about two orders of magnitude smaller than v_R. The masses of neutral ϕ_{2r}^0 and ϕ_{2i}^0 are even smaller [see eqs. (IV.24) and (IV.25)].

As mentioned, the vacuum stability conditions require the right–handed scale v_R to lie above $\sim 20 \, TeV$ in our model. This is an order of magnitude more stringent than the

bounds which follow from phenomenological considerations (see [26] for a recent analysis). For v_R close to its lower bound one can expect $\phi_2^\pm$ to be detectable at LHC whereas ϕ_{2r}^0 and ϕ_{2i}^0 may have a mass as small as $60 - 100\ GeV$ which might also be accessible at LEP2.

It is interesting that the Higgs sector of our model can naturally lead to a new positive contribution to $R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to hadrons)$ which may be desirable in order to reconcile the experimental result $R_b = 0.2205 \pm 0.0016$ [40] with theoretical predictions. It has been shown in [41] that this can be achieved in the two–Higgs–doublet Standard Model (2HDSM) with $\tan\beta \sim 50$ and degenerate or almost degenerate neutral CP–odd and CP–even scalars with mass $\sim 50\ GeV$. In our case the role of these scalars is naturally played by ϕ_{2r}^0 and ϕ_{2i}^0, and for $Y_2 \ll Y_1$ one has $\tan\beta \approx m_t/m_b \sim 50$. Note, however, that for such high values of $\tan\beta$ the top quark mass turns out to be too high as compared to the CDF/D0 results.

Low–lying neutral scalars usually pose a problem in LR models since they can mediate strong flavour–changing neutral currents. Typical lower bounds for the masses of such scalars, which come mainly from the $K_L - K_S$ mass difference, are of the order of a few TeV [26]. In the limit when the fermions of the first two generations are massless which we were studying in the present paper, there are however no flavour–changing neutral currents. The situation in a more realistic version of the model will depend very much on how the fermions of the first and the second generations acquire their masses. One possibility is a radiative mechanism induced by some horizontal interactions[13]. It has been argued recently that the smallness of the light quark masses might lead naturally to smallness of flavour–violating couplings of extra scalars to light quarks; as a result, the bounds of the masses of these scalars get significantly relaxed, and these masses may well be at the electro–weak scale [43]. Note that this does not require introducing any discrete symmetry to suppress flavour–changing neutral currents.

Light scalars can result in sizeable lepton–flavour violating decays such as $\mu \to e\gamma$. For example, the contribution of the one–loop diagrams with S_L and charged χ_L or χ_R in the loop to the branching ratio $BR(\mu \to e\gamma)$ is of the order

$$|(Y_5)_{\mu a}^\dagger (Y_5)_{ae}|^2 \frac{3\alpha}{8\pi} \frac{1}{G_F^2 M^4} \ . \tag{VI.3}$$

Assuming that the flavour–off–diagonal Y_5 couplings are of the order of the diagonal ones (i.e. ~ 1), one finds that the contribution of eq. (VI.3) becomes comparable with the present experimental upper bound $BR(\mu \to e\gamma) < 4.9 \times 10^{-11}$ for $M \simeq 20\ TeV$ which is just above the lower bound on M following from the vacuum stability condition. Contributions to $BR(\mu \to e\gamma)$ from the loops with ϕ_2 become important if the masses of its components are at the electro–weak scale [44]. Thus we may have observable $\mu \to e\gamma$ decay in our model.

In our phenomenological considerations we were mainly assuming that the right–handed scale v_R is not far from its lower bound. In this case one can expect several low or intermediate scale scalars in addition to the Standard Model Higgs boson to be present in the model, and also some interesting observable effects of lepton flavour violation mediated by the singlet fermion S_L and doublet Higgs scalars χ_L and χ_R. However, as was already mentioned

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[13]A scenario in which the masses of the bottom quark and τ lepton are generated radiatively in the top–condensate approach was suggested in [42].

in Sec. III, the right–handed scale can in principle be anywhere between $\sim 20 \ TeV$ and Λ in our model. The low values of v_R would result if one requires the "minimal cancelation" in eq. (III.31). Still, an unknown dynamics leading to our low–energy 4-f interactions may prefer a higher degree of cancelation, and so the right–handed scale may be very high. In this case, unfortunately, our model will be practically untestable.

VII   Discussion and Conclusions

The model we have presented here is to our knowledge the first successful attempt to break left–right symmetry dynamically. It is consistent with the currently available experimental data. A striking feature of the dynamical approach turns out to be the fact that whether or not parity can be spontaneously broken at low energies depends on the particle content of the model and not on the choice of the quartic couplings in the Higgs potential, as it is the case in the conventional approach. Our model exhibits a tumbling scenario where the breaking of parity and $SU(2)_R$ at a right handed scale μ_R eventually drives the breaking of the electro–weak symmetry at a lower scale $\mu_{EW} \sim 100 \ GeV$.

Our model has 9 input parameters (eight four–fermion couplings $G_1, ..., G_8$ and the scale of new physics Λ) in terms of which we calculate 16 physical observables (5 fermion masses, 8 Higgs boson masses and 3 VEVs κ, κ' and v_R), so there are $16 - 9 = 7$ predictions. First, the symmetry breaking is studied and the resulting predictions are derived in bubble approximation. The predictions for the fermion masses are then renormalization group improved thus including all the electro–weak and QCD corrections to one–loop order. Unlike in the minimal BHL model, the top quark is not predicted as an infrared quasi–fixed point value of the Yukawa coupling times the known electro–weak VEV. The mass formula is more complex here and in addition only the sum of squares of the bi–doublet VEV's is fixed to be $\kappa^2 + \kappa'^2 \simeq (174 \ GeV)^2$ while $\tan\beta = \kappa/\kappa'$ is essentially a free parameter. Our model gives a viable top quark mass value for $\tan\beta \simeq 1.3$–4 and exhibits a number of low and intermediate scale Higgs bosons. Furthermore it predicts relations between masses of various scalars and between fermion and Higgs boson masses which are in principle testable. If the right–handed scale μ_R is of the order of a few tens of TeV, the neutral CP–even and CP–odd scalars ϕ_{2r}^0 and ϕ_{2i}^0 can be even lighter than the electro–weak Higgs boson. In fact, they could be as light as $\sim 60 - 100 \ GeV$ and so might be observable at LEP2. Such light ϕ_{2r}^0 and ϕ_{2i}^0 might also provide a positive contribution to $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to hadrons)$ which could account for the discrepancy between the LEP observations and the Standard Model predictions.

Our model is formulated in terms of attractive 4-f interactions which are allowed by the symmetries and which trigger condensation if the couplings are strong enough. Since 4-f interactions are not renormalizable in the usual sense they should be regarded as effective low energy approximations of heavy degrees of freedom which have been integrated out. One could try to generate the interactions from renormalizable gauge theories as it has been done for the case of the BHL model. This goes however beyond the scope of the present paper and will be addressed elsewhere.

Appendix

A Auxiliary fields and 4-f terms

In the auxiliary field formalism the 4–fermion interaction terms of eqs. (I.12) and (II.4) are rewritten in terms of the static auxiliary scalar fields ϕ, χ_L, χ_R and σ with mass terms and Yukawa couplings but no kinetic terms (II.6). Since the Lagrangian is quadratic in the newly defined scalar fields they can always be integrated out in the functional integral. Equivalently, one can use the equations of motion of the auxiliary scalar fields to express them in terms of the fermionic degrees of freedom. After substituting the result into the auxiliary Lagrangian (II.6) one recovers the original 4-f structure. Direct comparison gives the following expressions for the 4-f couplings G_a in terms of Yukawa couplings and bare mass terms of the auxiliary scalar fields:

$$
\begin{aligned}
G_1 &= \frac{1}{M_1^4 - M_2^4}\left[M_1^2\,(Y_1^2 + Y_2^2) - 2M_2^2\,Y_1Y_2\right] \\[2mm]
G_2 &= \frac{1}{M_1^4 - M_2^4}\left[M_1^2\,Y_1Y_2 - \frac{1}{2}M_2^2\,(Y_1^2 + Y_2^2)\right] \\[2mm]
G_3 &= \frac{1}{M_1^4 - M_2^4}\left[M_1^2\,(Y_3^2 + Y_4^2) - 2M_2^2\,Y_3Y_4\right] \\[2mm]
G_4 &= \frac{1}{M_1^4 - M_2^4}\left[M_1^2\,Y_3Y_4 - \frac{1}{2}M_2^2\,(Y_3^2 + Y_4^2)\right] \\[2mm]
G_5 &= \frac{1}{M_1^4 - M_2^4}\left[M_1^2\,(Y_1Y_3 + Y_2Y_4) - M_2^2\,(Y_1Y_4 + Y_2Y_3)\right] \\[2mm]
G_6 &= \frac{1}{M_1^4 - M_2^4}\left[M_1^2\,(Y_1Y_4 + Y_2Y_3) - M_2^2\,(Y_1Y_3 + Y_2Y_4)\right] \\[2mm]
G_7 &= \frac{Y_5^2}{M_0^2}\,, \qquad G_8 = \frac{Y_6^2}{M_3^2}
\end{aligned}
\tag{A.1}
$$

Of 10 parameters $Y_1, ..., Y_6$, $M_0^2, ..., M_3^2$, three are redundant: they can be eliminated by rescaling the fields ϕ, σ and (χ_L, χ_R). We use this freedom to choose $M_1 = M_3 = M_0$, and in addition we could, for example, fix M_0 or one of the bare Yukawa couplings. Therefore we have only seven physical parameters to describe eight 4-f couplings $G_1, ..., G_8$. This means that the set of auxiliary fields which we introduced is insufficient to describe the most general set of 4-f terms in eqs. (I.12) and (II.4) (note however that new physics responsible for these 4-f terms need not produce the most general case). It is easy to see that the number of physical parameters in the (χ_L, χ_R) and σ sectors corresponds to the number of the 4-f terms, so the problem lies in the ϕ sector. In fact, with one bi–doublet only 4 of 6 couplings $G_1, ..., G_6$ are independent (using eqs. (A.1) one can make sure that, e.g., G_5 and G_6 can be expressed through $G_1, ... G_4$). To describe the most general case one needs at least two bi–doublets. However, in the limit $\kappa' = 0$ we have $Y_2 = Y_3 = 0$ and $M_2 = 0$, therefore

$G_2 = G_4 = G_5 = 0$, $G_6 = \sqrt{G_1 G_3}$, and the number of physical parameters in L_{aux} exactly coincides with the number of the independent 4-f couplings.

B   The effective potential

At energies below the cutoff Λ the auxiliary fields will acquire gauge invariant kinetic terms, quartic interactions and mass corrections via fermion and gauge boson loops. The kinetic terms and mass renormalization can be derived from the 2–point scalar Green functions, whereas the quartic couplings are given by the 4–point functions (see Figs. 4 and 5).

The effective Higgs potential V_{eff} of the model can be calculated directly from Figs. 4 and 5. However, a more convenient way to calculate V_{eff} is to consider the full one–loop Coleman–Weinberg effective potential [23]. If one substitutes the VEVs for the scalar fields and expresses the masses of all fermions and gauge bosons in terms of these VEVs, one arrives at a very simple expression for the Coleman–Weinberg potential,

$$V_{CW} = \sum_p \eta_p \cdot \frac{1}{32\pi^2} \int\limits_0^{\Lambda^2} dk^2 \, k^2 \ln\left[1 + \frac{m_p^2}{k^2}\right] \, , \tag{B.1}$$

where m_p is the mass of the pth particle and η_p is related to the number of degrees of freedom:

$$\eta_p = \begin{cases} 1 & \text{for scalar particles} \\ -4 & \text{for Dirac fermions, multiplied by } N_c \text{ for quarks} \\ -2 & \text{for Majorana fermions} \\ 3 & \text{for neutral gauge bosons} \\ 6 & \text{for charged gauge bosons} \end{cases} \tag{B.2}$$

In our calculation of V_{eff} no contributions with Higgs bosons in the loops will be included. The reason for this is that the scalars are composite particles and their very existence is (to the leading order) the result of the one–loop effective potential or, in 4-f language, due to the infinite bubble–sum (infinite loop order) of the constituent fermions. The loops with propagating Higgs scalars therefore correspond to a mixed loop order, and due to double counting problems it is difficult to self–consistently take into account the feedback of propagating Higgs bosons into the effective potential. However, the propagating Higgs effects can be consistently incorporated in the renormalization group approach which is discussed in Sec. V.

Instead of using the full one–loop Coleman–Weinberg potential one can truncate it so as to keep the terms up to and including quartic terms in the fields, since higher order contributions are finite in the limit $\Lambda \to \infty$ and so relatively unimportant. While the full Coleman–Weinberg potential of eq. (B.1) is infrared–finite, the truncated one is infrared–divergent, and so one has to introduce an infrared cutoff μ. Integrating over the momenta $\mu^2 \le k^2 \le \Lambda^2$ in (B.1) is equivalent to integrating out the field degrees of freedom with high momenta and so results in an effective Lagrangian at the scale μ [6, 22, 45, 46] in the sense of Wilson's renormalization group approach. For energy scales μ lower than the masses of the particles in the loops the scale μ should be replaced by the relevant particle masses.

Using this approach one arrives at the following formula for the effective potential:

$$
\begin{aligned}
V_{\text{eff}}\Big|_{VEV} = {} & M_0^2(v_L^2 + v_R^2) + M_1^2(\kappa^2 + \kappa'^2) + 2M_2^2\kappa\kappa' + M_3^2\sigma_0^2 \\
& - \frac{1}{32\pi^2}\Big[2(m_1^2 + m_2^2 + m_3^2) + 4(N_c m_t^2 + N_c m_b^2 + m_\tau^2) \\
& \qquad\qquad -6(m_W^2 + m_{W'}^2) - 3(m_Z^2 + m_{Z'}^2)\Big](\Lambda^2 - \mu^2) \\
& + \frac{1}{32\pi^2}\Big[m_1^4 + m_2^4 + m_3^4 + 2(N_c m_t^4 + N_c m_b^4 + m_\tau^4) \\
& \qquad\qquad -3(m_W^4 + m_{W'}^4) - \frac{3}{2}(m_Z^4 + m_{Z'}^4)\Big]\ln\left(\frac{\Lambda^2}{\mu^2}\right)
\end{aligned}
\tag{B.3}
$$

Here m_1, m_2 and m_3 are the eigenvalues of the neutrino mass matrix M_ν of eq. (II.8). The fermion masses can be expressed in terms of the scalar VEVs using eqs. (II.7) and (II.9). The sums of the squares and fourth powers of neutrino masses can be obtained by taking the traces of M_ν^2 and M_ν^4 respectively. This gives

$$
\begin{aligned}
m_1^2 + m_2^2 + m_3^2 = {} & 4Y_6^2\sigma_0^2 + 2(Y_3\kappa + Y_4\kappa')^2 + 2Y_5^2(v_L^2 + v_R^2) \\
m_1^4 + m_2^4 + m_3^4 = {} & 16\left[Y_6^4\sigma_0^4 + Y_6^2 Y_5^2\sigma_0^2(v_L^2 + v_R^2) + Y_6 Y_5^2(Y_3\kappa + Y_4\kappa')\sigma_0 v_L v_R\right] \\
& + 2\left[(Y_3\kappa + Y_4\kappa')^2 + Y_5^2(v_L^2 + v_R^2)\right]^2
\end{aligned}
\tag{B.4}
$$

The gauge boson masses in the LR model with a bi–doublet ϕ and doublets χ_L and χ_R are

$$
m_{W'/W}^2 = \frac{g_2^2}{2}(\kappa^2 + \kappa'^2) + \frac{g_2^2}{4}(v_L^2 + v_R^2) \pm \frac{g_2^2}{4}\sqrt{(v_L^2 - v_R^2)^2 + 16(\kappa\kappa')^2}\,,
\tag{B.5}
$$

$$
\begin{aligned}
m_{Z'/Z}^2 = {} & \frac{g_2^2}{2}(\kappa^2 + \kappa'^2) + \frac{g_2^2 + g_1^2}{4}(v_L^2 + v_R^2) \pm \frac{1}{4}\Big[(g_2^2 + g_1^2)^2(v_L^2 + v_R^2)^2 \\
& + 4g_2^4(\kappa^2 + \kappa'^2)^2 - 4g_2^2 g_1^2(\kappa^2 + \kappa'^2)(v_L^2 + v_R^2) - 4g_2^2(g_2^2 + 2g_1^2)v_L^2 v_R^2\Big]^{\frac{1}{2}}\,.
\end{aligned}
\tag{B.6}
$$

Note that the $U(1)_{B-L}$ gauge coupling g_1 is different from the standard–model $U(1)$ coupling g':

$$
g_1^2 = g'^2\frac{1 - s_W^2}{1 - 2s_W^2}\,, \qquad s_W^2 \equiv \sin^2\theta_W = \frac{g_1^2}{g_2^2 + 2g_1^2}\,,
\tag{B.7}
$$

where θ_W is the Weinberg angle.

The procedure outlined above yields the one–loop effective potential in terms of the scalar VEVs, from which the potential in terms of the fields can be recovered. This gives

$$
\begin{aligned}
V_{\text{eff}} = {} & \widetilde{M}_0^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R) + \widetilde{M}_1^2\,\text{tr}\,(\phi^\dagger\phi) + \frac{\widetilde{M}_2^2}{2}\,\text{tr}\,(\phi^\dagger\widetilde{\phi} + h.c.) + \widetilde{M}_3^2\sigma^\dagger\sigma \\
& + \lambda_1[(\chi_L^\dagger\chi_L)^2 + (\chi_R^\dagger\chi_R)^2] + 2\lambda_2(\chi_L^\dagger\chi_L)(\chi_R^\dagger\chi_R)
\end{aligned}
$$

$$+\frac{1}{2}\lambda_3[\chi_L^\dagger(Y_3\phi + Y_4\widetilde{\phi})\chi_R\sigma^\dagger + h.c.]$$

$$+\lambda_4[\chi_L^\dagger(Y_3\phi + Y_4\widetilde{\phi})(Y_3\phi^\dagger + Y_4\widetilde{\phi}^\dagger)\chi_L + \chi_R^\dagger(Y_3\phi^\dagger + Y_4\widetilde{\phi}^\dagger)(Y_3\phi + Y_4\widetilde{\phi})\chi_R]$$

$$+\lambda_5(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)\,\mathrm{tr}(\phi^\dagger\phi) + \lambda_6(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)\sigma^\dagger\sigma$$

$$+\lambda_7'\,\mathrm{tr}(\phi^\dagger\phi\phi^\dagger\phi) + \frac{1}{3}\lambda_8'\,\mathrm{tr}(\phi^\dagger\widetilde{\phi}\widetilde{\phi}^\dagger\phi)$$

$$+\frac{1}{12}\lambda_8'[\mathrm{tr}(\phi^\dagger\widetilde{\phi}\phi^\dagger\widetilde{\phi}) + h.c.] + \frac{1}{2}\lambda_9[\mathrm{tr}(\phi^\dagger\phi\phi^\dagger\widetilde{\phi}) + h.c.]$$

$$+\lambda_0[\mathrm{tr}(\phi^\dagger\phi)]^2 + \lambda_{10}(\sigma^\dagger\sigma)^2 \tag{B.8}$$

with the quartic couplings

$$\lambda_0 = \frac{1}{16\pi^2}\left[-\frac{3}{2}g_2^4 Z_\phi^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_1 = \frac{1}{16\pi^2}\left[Y_5^4 - \frac{3}{16}(3g_2^4 + 2g_2^2 g_1^2 + g_1^4)Z_\chi^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_2 = \frac{1}{16\pi^2}\left[Y_5^4 - \frac{3}{16}g_1^4 Z_\chi^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_3 = \frac{1}{16\pi^2}\left[8Y_5^2 Y_6\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_4 = \frac{1}{16\pi^2}\left[2Y_5^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_5 = \frac{1}{16\pi^2}\left[-\frac{9}{8}g_2^4 Z_\phi Z_\chi\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_6 = \frac{1}{16\pi^2}\left[8Y_5^2 Y_6^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_7' = \frac{1}{16\pi^2}\left[N_c(Y_1^4 + Y_2^4) + (Y_3^4 + Y_4^4)\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_8' = \frac{1}{16\pi^2}\left[12(N_c Y_1^2 Y_2^2 + Y_3^2 Y_4^2)\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_9 = \frac{1}{16\pi^2}\left[4[N_c Y_1 Y_2(Y_1^2 + Y_2^2) + Y_3 Y_4(Y_3^2 + Y_4^2)]\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_{10} = \frac{1}{16\pi^2}\left[8Y_6^4\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right), \tag{B.9}$$

and the effective mass terms are given by eqs. (III.1)–(III.4). Note that all the quartic couplings except λ_3 and λ_4 are proportional to the fourth power of Yukawa and/or gauge

couplings. The reason for the different structure of λ_3 and λ_4 is that they enter into eq. (B.8) being multiplied by Yukawa couplings.

To minimize the effective potential and find the vacuum of our model we will also need the effective potential in terms of the VEVs of the scalar fields:

$$
\begin{aligned}
V_{\text{eff}}\big|_{VEV} &= \widetilde{M}_0^2(v_L^2 + v_R^2) + \widetilde{M}_1^2(\kappa^2 + \kappa'^2) + 2\widetilde{M}_2^2 \kappa\kappa' + \widetilde{M}_3^2 \sigma_0^2 \\
&\quad + \lambda_1(v_L^4 + v_R^4) + \lambda_2\, 2v_L^2 v_R^2 + \lambda_3 v_L v_R \sigma_0(Y_3\kappa + Y_4\kappa') \\
&\quad + \lambda_4(Y_3\kappa + Y_4\kappa')^2(v_L^2 + v_R^2) + \lambda_5(\kappa^2 + \kappa'^2)(v_L^2 + v_R^2) + \lambda_6(v_L^2 + v_R^2)\sigma_0^2 \\
&\quad + \lambda_7(\kappa^4 + \kappa'^4) + \lambda_8\kappa^2\kappa'^2 + \lambda_9\kappa\kappa'(\kappa^2 + \kappa'^2) + \lambda_{10}\sigma_0^4 \ . \quad\quad (B.10)
\end{aligned}
$$

Here

$$
\lambda_7 = \lambda_7' + \lambda_0 \ , \qquad \lambda_8 = \lambda_8' + 2\lambda_0 \ . \quad\quad (B.11)
$$

Differentiating (B.10) with respect to the VEVs σ_0, v_R, v_L, κ and κ' gives the extremum conditions (III.13)–(III.17) for V_{eff}. They constitute the gap equations in our model in the auxiliary field approach. In order for an extremum of the potential to be a true minimum, the matrices of second derivatives of V_{eff} with respect to the fields must be positive definite. These matrices are studied in Appendix D.

C Renormalization

The wave function renormalization constants of the composite scalar fields in bubble approximation can be directly obtained from the 2–point Green functions given by the diagrams of Fig. 4. The results are presented in eqs. (IV.1)–(IV.3). To derive the physical predictions, it is convenient to absorb the Z–factors into the definitions of the scalar fields so as to bring their kinetic terms into the canonical form. This amounts to the following re–definition of the fields and the parameters of the effective Lagrangian:

$$
\hat{\phi} \equiv \sqrt{Z_\phi}\phi \ , \qquad \hat{\widetilde{\phi}} \equiv \sqrt{Z_\phi}\widetilde{\phi} \ , \qquad \hat{\kappa} \equiv \sqrt{Z_\phi}\kappa \ , \qquad \hat{\kappa}' \equiv \sqrt{Z_\phi}\kappa' \ , \quad\quad (C.1)
$$

$$
\hat{\chi} \equiv \sqrt{Z_\chi}\chi \ , \quad \hat{v}_{L,R} \equiv \sqrt{Z_\chi}v_{L,R} \ , \quad \hat{\sigma} \equiv \sqrt{Z_\sigma}\sigma \ , \quad \hat{\sigma}_0 \equiv \sqrt{Z_\sigma}\sigma_0 \ , \quad\quad (C.2)
$$

$$
\hat{\widetilde{M}}_0^{\,2} \equiv \frac{1}{Z_\chi}\widetilde{M}_0^2 \ , \quad \hat{\widetilde{M}}_{1,2}^{\,2} \equiv \frac{1}{Z_\phi}\widetilde{M}_{1,2}^2 \ , \quad \hat{\widetilde{M}}_3^{\,2} \equiv \frac{1}{Z_\sigma}\widetilde{M}_3^2 \ , \quad\quad (C.3)
$$

$$
\hat{Y}_i \equiv \frac{1}{\sqrt{Z_\phi}}Y_i \ \ (i = 1,\ldots,4) \ , \quad \hat{Y}_5 \equiv \frac{1}{\sqrt{Z_\chi}}Y_5 \ , \quad \hat{Y}_6 \equiv \frac{1}{\sqrt{Z_\sigma}}Y_6 \quad\quad (C.4)
$$

$$
\hat{\lambda}_{1,2} \equiv \frac{1}{Z_\chi^2}\lambda_{1,2} \ , \quad \hat{\lambda}_3 \equiv \frac{1}{Z_\chi\sqrt{Z_\sigma}}\lambda_3 \ , \quad \hat{\lambda}_4 \equiv \frac{1}{Z_\chi}\lambda_4 \ , \quad \hat{\lambda}_5 \equiv \frac{1}{Z_\chi Z_\phi}\lambda_5 \ , \quad\quad (C.5)
$$

$$
\hat{\lambda}_6 \equiv \frac{1}{Z_\chi Z_\sigma}\lambda_6 \ , \quad \hat{\lambda}_{0,7,8,9} \equiv \frac{1}{Z_\phi^2}\lambda_{0,7,8,9} \ , \quad \hat{\lambda}_{10} \equiv \frac{1}{Z_\sigma^2}\lambda_{10} \ . \quad\quad (C.6)
$$

D   Higgs boson mass matrices

The symmetric mass matrices of the scalar fields can be found as the matrices of second derivatives of V_{eff} with respect to the field components. We quote here the unrenormalized mass matrices; the renormalized ones can be obtained by applying the same rule as in eq. (C.3).

The mass matrix of the charged fields in the basis $(\phi_1^+, \chi_L^+, \phi_2^+, \chi_R^+)$ is

$$
\left(M_\pm^2\right)_{11} = \widetilde{M}_1^2 + v_R{}^2 Y_3{}^2 \lambda_4 + \lambda_4 v_L{}^2 Y_4{}^2 + \lambda_5 v_L{}^2 + v_R{}^2 \lambda_5 + 2\,\lambda_7 \kappa'^2
$$

$$
+ 2\,\lambda_7 \kappa^2 + \kappa\,\kappa'\,\lambda_9 \tag{D.1}
$$

$$
\left(M_\pm^2\right)_{12} = \frac{v_R\,\sigma_0\,Y_3 \lambda_3}{2} + \lambda_4 v_L\,Y_3{}^2 \kappa - \lambda_4 Y_4{}^2 \kappa\,v_L \tag{D.2}
$$

$$
\left(M_\pm^2\right)_{13} = -\widetilde{M}_2^2 - \lambda_4 v_L{}^2 Y_3 Y_4 - v_R{}^2 Y_3 Y_4 \lambda_4 + 2\,\lambda_7 \kappa\,\kappa' - \lambda_8 \kappa\,\kappa'
$$

$$
- \frac{\lambda_9 \kappa'^2}{2} - \frac{\kappa^2 \lambda_9}{2} \tag{D.3}
$$

$$
\left(M_\pm^2\right)_{14} = -\frac{v_L\,\sigma_0\,Y_4 \lambda_3}{2} - \lambda_4 v_R\,Y_4{}^2 \kappa' + \lambda_4 Y_3{}^2 \kappa'\,v_R \tag{D.4}
$$

$$
\left(M_\pm^2\right)_{22} = \widetilde{M}_0^2 + 2\,\lambda_1 v_L{}^2 + 2\,\lambda_2 v_R{}^2 + 2\,\lambda_4 Y_3 \kappa\,Y_4 \kappa' + \lambda_4 Y_3{}^2 \kappa'^2
$$

$$
+ \lambda_4 Y_4{}^2 \kappa^2 + \lambda_5 \kappa^2 + \lambda_5 \kappa'^2 + \lambda_6 \sigma_0{}^2 \tag{D.5}
$$

$$
\left(M_\pm^2\right)_{23} = -\frac{v_R\,\sigma_0\,Y_4 \lambda_3}{2} + \lambda_4 Y_3{}^2 \kappa'\,v_L - \lambda_4 v_L\,Y_4{}^2 \kappa' \tag{D.6}
$$

$$
\left(M_\pm^2\right)_{24} = \frac{\sigma_0\,(Y_4 \kappa + Y_3 \kappa')\,\lambda_3}{2} \tag{D.7}
$$

$$
\left(M_\pm^2\right)_{33} = \widetilde{M}_1^2 + \lambda_4 v_L{}^2 Y_3{}^2 + \lambda_4 v_R{}^2 Y_4{}^2 + \lambda_5 v_L{}^2 + v_R{}^2 \lambda_5
$$

$$
+ 2\,\lambda_7 \kappa'^2 + 2\,\lambda_7 \kappa^2 + \kappa\,\kappa'\,\lambda_9 \tag{D.8}
$$

$$
\left(M_\pm^2\right)_{34} = \frac{v_L\,\sigma_0\,Y_3 \lambda_3}{2} - \lambda_4 Y_4{}^2 \kappa\,v_R + \lambda_4 v_R\,Y_3{}^2 \kappa \tag{D.9}
$$

$$
\left(M_\pm^2\right)_{44} = \widetilde{M}_0^2 + 2\,\lambda_1 v_R{}^2 + 2\,\lambda_2 v_L{}^2 + 2\,\lambda_4 Y_3 \kappa\,Y_4 \kappa' + \lambda_4 Y_3{}^2 \kappa'^2
$$

$$
+ \lambda_4 Y_4{}^2 \kappa^2 + \lambda_5 \kappa^2 + \lambda_5 \kappa'^2 + \lambda_6 \sigma_0{}^2 \quad. \tag{D.10}
$$

The mass matrix of the neutral CP–odd Higgs bosons (imaginary components of the fields) in the basis $(\phi_{1i}^0, \phi_{2i}^0, \chi_{Li}^0, \chi_{Ri}^0, \sigma_i)$ is

$$
\left(M_{0i}^2\right)_{11} = \widetilde{M}_1^2 + \lambda_4\left(v_L{}^2 Y_3{}^2 + v_R{}^2 Y_3{}^2\right) + \left(v_L{}^2 + v_R{}^2\right)\lambda_5 + 2\,\lambda_7 \kappa^2
$$

$$
+ \frac{\lambda_8 \kappa'^2}{3} + \kappa\,\kappa'\,\lambda_9 \tag{D.11}
$$

$$\left(M_{0i}^2\right)_{12} = -\widetilde{M}_2^2 + \lambda_4\left(-v_L{}^2 Y_3 Y_4 - v_R{}^2 Y_3 Y_4\right) - \frac{2\,\lambda_8 \kappa\,\kappa'}{3} - \frac{\kappa^2 + \kappa'^2}{2}\lambda_9 \tag{D.12}$$

$$\left(M_{0i}^2\right)_{13} = \frac{v_R\,\sigma_0\,Y_3\lambda_3}{2} \tag{D.13}$$

$$\left(M_{0i}^2\right)_{14} = -\frac{v_L\,\sigma_0\,Y_3\lambda_3}{2} \tag{D.14}$$

$$\left(M_{0i}^2\right)_{15} = \frac{v_L\,v_R\,Y_3\lambda_3}{2} \tag{D.15}$$

$$\left(M_{0i}^2\right)_{22} = \widetilde{M}_1^2 + \lambda_4\left(v_L{}^2 Y_4{}^2 + v_R{}^2 Y_4{}^2\right) + \left(v_L{}^2 + v_R{}^2\right)\lambda_5 + 2\,\lambda_7\kappa'^2$$
$$+\frac{\lambda_8\kappa^2}{3} + \kappa\,\kappa'\,\lambda_9 \tag{D.16}$$

$$\left(M_{0i}^2\right)_{23} = -\frac{v_R\,\sigma_0\,Y_4\lambda_3}{2} \tag{D.17}$$

$$\left(M_{0i}^2\right)_{24} = \frac{v_L\,\sigma_0\,Y_4\lambda_3}{2} \tag{D.18}$$

$$\left(M_{0i}^2\right)_{25} = -\frac{v_L\,v_R\,Y_4\lambda_3}{2} \tag{D.19}$$

$$\left(M_{0i}^2\right)_{33} = \widetilde{M}_0^2 + 2\,\lambda_1 v_L{}^2 + 2\,\lambda_2 v_R{}^2 + \lambda_4\left(Y_3{}^2\kappa^2 + 2\,Y_3\kappa\,Y_4\kappa' + Y_4{}^2\kappa'^2\right)$$
$$+\left(\kappa^2 + \kappa'^2\right)\lambda_5 + \lambda_6\sigma_0{}^2 \tag{D.20}$$

$$\left(M_{0i}^2\right)_{34} = \left(\frac{\sigma_0\,Y_4\kappa'}{2} + \frac{\sigma_0\,Y_3\kappa}{2}\right)\lambda_3 \tag{D.21}$$

$$\left(M_{0i}^2\right)_{35} = -\left(\frac{v_R\,Y_4\kappa'}{2} + \frac{v_R\,Y_3\kappa}{2}\right)\lambda_3 \tag{D.22}$$

$$\left(M_{0i}^2\right)_{44} = \widetilde{M}_0^2 + 2\,\lambda_1 v_R{}^2 + 2\,\lambda_2 v_L{}^2 + \lambda_4\left(Y_3{}^2\kappa^2 + 2\,Y_3\kappa\,Y_4\kappa' + Y_4{}^2\kappa'^2\right)$$
$$+\left(\kappa^2 + \kappa'^2\right)\lambda_5 + \lambda_6\sigma_0{}^2 \tag{D.23}$$

$$\left(M_{0i}^2\right)_{45} = \left(\frac{v_L\,Y_4\kappa'}{2} + \frac{v_L\,Y_3\kappa}{2}\right)\lambda_3 \tag{D.24}$$

$$\left(M_{0i}^2\right)_{55} = \widetilde{M}_3^2 + \left(v_L{}^2 + v_R{}^2\right)\lambda_6 + 2\,\lambda_{10}\sigma_0{}^2 \tag{D.25}$$

The mass matrix of the neutral CP–even Higgs bosons (real components of the fields) in the basis $(\sigma_r,\ \chi_{Lr}^0,\ \phi_{2r}^0,\ \phi_{1r}^0,\ \chi_{Rr}^0)$ is

$$\left(M_{0r}^2\right)_{11} = \widetilde{M}_3^2 + \left(v_L{}^2 + v_R{}^2\right)\lambda_6 + 6\,\lambda_{10}\sigma_0{}^2 \tag{D.26}$$

$$\left(M_{0r}^2\right)_{12} = \left(\frac{v_R\,Y_4\kappa'}{2} + \frac{v_R\,Y_3\kappa}{2}\right)\lambda_3 + 2\,\sigma_0\,v_L\,\lambda_6 \tag{D.27}$$

$$\left(M_{0r}^2\right)_{13} = \frac{v_L\, v_R\, Y_4\lambda_3}{2} \tag{D.28}$$

$$\left(M_{0r}^2\right)_{14} = \frac{v_L\, v_R\, Y_3\lambda_3}{2} \tag{D.29}$$

$$\left(M_{0r}^2\right)_{15} = \left(\frac{v_L\, Y_4\kappa'}{2} + \frac{v_L\, Y_3\kappa}{2}\right)\lambda_3 + 2\,\sigma_0\, v_R\, \lambda_6 \tag{D.30}$$

$$\left(M_{0r}^2\right)_{22} = \widetilde{M}_0^2 + 6\,\lambda_1 v_L{}^2 + 2\,\lambda_2 v_R{}^2 + \lambda_4\left(Y_3{}^2\kappa^2 + 2\,Y_3\kappa\, Y_4\kappa' + Y_4{}^2\kappa'^2\right)$$
$$+ \left(\kappa^2 + \kappa'^2\right)\lambda_5 + \lambda_6\sigma_0{}^2 \tag{D.31}$$

$$\left(M_{0r}^2\right)_{23} = \frac{v_R\,\sigma_0\, Y_4\lambda_3}{2} + \lambda_4\left(2\,v_L\, Y_3\kappa\, Y_4 + 2\,v_L\, Y_4{}^2\kappa'\right) + 2\,v_L\,\kappa'\,\lambda_5 \tag{D.32}$$

$$\left(M_{0r}^2\right)_{24} = \frac{v_R\,\sigma_0\, Y_3\lambda_3}{2} + \lambda_4\left(2\,v_L\, Y_3 Y_4\kappa' + 2\,v_L\, Y_3{}^2\kappa\right) + 2\,v_L\,\kappa\,\lambda_5 \tag{D.33}$$

$$\left(M_{0r}^2\right)_{25} = 4\,\lambda_2 v_L\, v_R + \left(\frac{\sigma_0\, Y_4\kappa'}{2} + \frac{\sigma_0\, Y_3\kappa}{2}\right)\lambda_3 \tag{D.34}$$

$$\left(M_{0r}^2\right)_{33} = \widetilde{M}_1^2 + \lambda_4\left(v_L{}^2 Y_4{}^2 + v_R{}^2 Y_4{}^2\right) + \left(v_L{}^2 + v_R{}^2\right)\lambda_5 + 6\,\lambda_7\kappa'^2$$
$$+\lambda_8\kappa^2 + 3\,\kappa\,\kappa'\,\lambda_9 \tag{D.35}$$

$$\left(M_{0r}^2\right)_{34} = \widetilde{M}_2^2 + \lambda_4\left(v_R{}^2 Y_3 Y_4 + v_L{}^2 Y_3 Y_4\right) + 2\,\lambda_8\kappa\,\kappa' + \left(\frac{3\,\kappa^2}{2} + \frac{3\,\kappa'^2}{2}\right)\lambda_9 \tag{D.36}$$

$$\left(M_{0r}^2\right)_{35} = \frac{v_L\,\sigma_0\, Y_4\lambda_3}{2} + \lambda_4\left(2\,v_R\, Y_3\kappa\, Y_4 + 2\,v_R\, Y_4{}^2\kappa'\right) + 2\,v_R\,\kappa'\,\lambda_5 \tag{D.37}$$

$$\left(M_{0r}^2\right)_{44} = \widetilde{M}_1^2 + \lambda_4\left(v_L{}^2 Y_3{}^2 + v_R{}^2 Y_3{}^2\right) + \left(v_L{}^2 + v_R{}^2\right)\lambda_5 + 6\,\lambda_7\kappa^2$$
$$+\lambda_8\kappa'^2 + 3\,\kappa\,\kappa'\,\lambda_9 \tag{D.38}$$

$$\left(M_{0r}^2\right)_{45} = \frac{v_L\,\sigma_0\, Y_3\lambda_3}{2} + \lambda_4\left(2\,v_R\, Y_3{}^2\kappa + 2\,v_R\, Y_3 Y_4\kappa'\right) + 2\,v_R\,\kappa\,\lambda_5 \tag{D.39}$$

$$\left(M_{0r}^2\right)_{55} = \widetilde{M}_0^2 + 6\,\lambda_1 v_R{}^2 + 2\,\lambda_2 v_L{}^2 + \lambda_4\left(Y_3{}^2\kappa^2 + 2\,Y_3\kappa\, Y_4\kappa' + Y_4{}^2\kappa'^2\right)$$
$$+ \left(\kappa^2 + \kappa'^2\right)\lambda_5 + \lambda_6\sigma_0{}^2 \quad, \tag{D.40}$$

In the minimum of the effective potential all the Higgs boson mass matrices must be positive semi–definite, i.e. must have only positive or zero eigenvalues (vacuum stability conditions). This is equivalent to the requirement that all the physical Higgs boson masses be non–negative. Zero eigenvalues correspond to the Goldstone bosons; if all of them are eaten by gauge bosons, no zero–mass scalars will be present in the physical spectrum of the model.

A matrix is positive semi–definite if and only if all its principal minors are non–negative; this, in particular, means that all the diagonal elements of the scalar mass matrices must be non–negative. Let us now consider the principal minor Δ corresponding to the sub–matrix

of M_{0r}^2 acting in the basis (χ_{Rr}^0, σ_r). Assuming $\sigma_0 \neq 0$, $v_R \neq 0$ and using the first derivative conditions (III.13)–(III.17) one can rewrite the elements of this sub–matrix in the following form:

$$\left(M_{0r}^2\right)_{55} = 4\lambda_1 v_R^2 + 2(\lambda_2 - \lambda_1)v_L^2 \approx 4\lambda_1 v_R^2 \ , \tag{D.41}$$

$$\left(M_{0r}^2\right)_{11} = 4\lambda_{10}\sigma_0^2 + 2(\lambda_2 - \lambda_1)\frac{v_R^2}{\sigma_0^2}v_L^2 \ , \tag{D.42}$$

$$\left(M_{0r}^2\right)_{15} = 2\lambda_6 v_R \sigma_0 - 2(\lambda_2 - \lambda_1)\frac{v_R}{\sigma_0}v_L \ . \tag{D.43}$$

Using eq. (III.18) one can show that the second term in (D.43) is always small as compared to the first one (note that eq. (III.18) implies $v_L \ll \sigma_0$ when both v_L and σ_0 are non–zero). One therefore finds

$$\Delta \equiv \left(M_{0r}^2\right)_{55}\left(M_{0r}^2\right)_{11} - \left(M_{0r}^2\right)_{15}^2 \approx 4\frac{v_R^2}{\sigma_0^2}\left[2\lambda_1(\lambda_2 - \lambda_1)v_R^2 v_L^2 + (4\lambda_1\lambda_{10} - \lambda_6^2)\sigma_0^4\right] \ . \tag{D.44}$$

Since $(4\lambda_1\lambda_{10} - \lambda_6^2) \approx (-32Y_5^4 Y_6^4)[\ln(\Lambda^2/\mu^2)/16\pi^2]^2 < 0$, the minor Δ can only be positive for small enough σ_0. Using again eq. (III.18) one finds the following condition:

$$\sigma_0^2 < \frac{\lambda_3^2}{32(\lambda_2 - \lambda_1)\lambda_{10}}m_D^2 \ . \tag{D.45}$$

Solving eqs. (III.13)–(III.17) for σ_0 assuming non–vanishing v_L and σ_0 gives

$$\sigma_0^2 = \frac{\left(2\widetilde{M}_3^2\lambda_1 - \widetilde{M}_0^2\lambda_6\right) - \left[\frac{\lambda_3^2\lambda_1}{4(\lambda_2 - \lambda_1)} + \lambda_4\lambda_6\right]m_D^2 - \lambda_5\lambda_6(\kappa^2 + \kappa'^2)}{\lambda_6^2 - 4\lambda_1\lambda_{10}} \ . \tag{D.46}$$

Analysis of this expression shows that the condition (D.45) can only be satisfied if the inequality (III.20) is replaced by equality. Since this requires an extreme fine–tuning of the Yukawa couplings, we do not pursue such a possibility. Thus, for the solutions with $v_L \neq 0$, $\sigma_0 \neq 0$ we find $\Delta < 0$ which means that this solution is not a minimum of the effective potential. As we shall shortly see, the solution of eqs. (III.13)–(III.17) with $v_L = 0 = \sigma_0$ leads to non–negative masses of all the Higgs bosons, i.e. is a minimum of V_{eff}. Since this solution is unique[14] it describes the true vacuum of the model in bubble approximation.

Let us now rewrite the scalar mass matrices substituting the first–derivative conditions (i.e. the solutions of eqs. (III.13)–(III.17) with $v_L = 0 = \sigma_0$). We start with the charged scalar fields. The fields $\chi_L^\pm$ are no longer mixed with the rest of the charged Higgs bosons, i.e. they are mass eigenstates. Their mass is given by

$$\begin{aligned} M_{\chi_L^\pm}^2 &= 2(\lambda_2 - \lambda_1)v_R^2 + \lambda_4(Y_4^2 - Y_3^2)(\kappa^2 - \kappa'^2) \\ &= 2(\lambda_2 - \lambda_1)v_R^2 + \lambda_4(m_\tau^2 - m_D^2) \ . \end{aligned} \tag{D.47}$$

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[14]The solution is unique for given values of the parameters of the model, in particular, of Yukawa couplings related to the 4-f couplings by the formulas of Appendix A. Here we discuss the non–trivial solution which requires the conditions (III.20) and $Y_5^2 > (Y_5^2)_{cr}$ to be satisfied. The trivial solution corresponds to sub–critical values of the Yukawa couplings and so does not coexist with the non–trivial one.

The rest of the charged scalar mass matrix takes the form [in the basis $(\phi_2^+, \phi_1^+, \chi_R^+)$]

$$M_\pm^2 = \lambda_4(Y_4^2 - Y_3^2) \begin{pmatrix} \frac{\kappa^2}{\kappa^2 - \kappa'^2}v_R^2 & \frac{\kappa\kappa'}{\kappa^2 - \kappa'^2}v_R^2 & -v_R\kappa \\ \frac{\kappa\kappa'}{\kappa^2 - \kappa'^2}v_R^2 & \frac{\kappa'^2}{\kappa^2 - \kappa'^2}v_R^2 & -v_R\kappa' \\ -v_R\kappa & -v_R\kappa' & \kappa^2 - \kappa'^2 \end{pmatrix} \tag{D.48}$$

This matrix has two zero eigenvalues corresponding to the Goldstone bosons eaten by $W_{1,2}^\pm$:

$$G_1^\pm = \left[\left(\frac{\kappa^2 + \kappa'^2}{\kappa^2 - \kappa'^2}\right)^2 + \frac{\kappa^2 + \kappa'^2}{v_R^2}\right]^{-1/2} \left\{\frac{\kappa}{v_R}\phi_2^\pm + \frac{\kappa'}{v_R}\phi_1^\pm + \left(\frac{\kappa^2 + \kappa'^2}{\kappa^2 - \kappa'^2}\right)\chi_R^\pm\right\}, \tag{D.49}$$

$$G_2^\pm = \frac{\kappa}{\sqrt{\kappa^2 + \kappa'^2}}\phi_1^\pm - \frac{\kappa'}{\sqrt{\kappa^2 + \kappa'^2}}\phi_2^\pm. \tag{D.50}$$

The massive eigenstate of the matrix (D.48) is

$$H_3^\pm = \left[1 + \frac{\kappa'^2}{\kappa^2} + \frac{(\kappa^2 - \kappa'^2)^2}{v_R^2\kappa^2}\right]^{-1/2} \left(\phi_2^\pm + \frac{\kappa'}{\kappa}\phi_1^\pm - \frac{\kappa^2 - \kappa'^2}{v_R\kappa}\chi_R^\pm\right) \tag{D.51}$$

with the mass

$$M_{H_3^\pm}^2 = \lambda_4(Y_4^2 - Y_3^2)\left[\frac{\kappa^2 + \kappa'^2}{\kappa^2 - \kappa'^2}v_R^2 + (\kappa^2 - \kappa'^2)\right]. \tag{D.52}$$

In the limit $\kappa' \to 0$ there is no LR mixing in the gauge boson sector and so heavy and light $W_1^\pm$ and $W_2^\pm$ coincide with $W_R^\pm$ and $W_L^\pm$, respectively. In this case the expressions for the Higgs boson mass eigenstates and eigenvalues simplify significantly (see Sec. IV).

In the neutral CP–odd Higgs sector, $G_1^0 = \chi_{Ri}^0$ is the exact mass eigenstate with zero mass; it is the Goldstone boson eaten by $Z_1^0 \approx Z_R^0$. The rest of the mass matrix takes the form [in the basis $(\phi_{1i}^0, \phi_{2i}^0, \chi_{Li}^0, \sigma_i)$]

$$M_{0i}^2 = \begin{pmatrix} A_0\kappa'^2 & A_0\kappa\kappa' & 0 & 0 \\ A_0\kappa\kappa' & A_0\kappa^2 & 0 & 0 \\ 0 & 0 & 2(\lambda_2 - \lambda_1)v_R^2 & -m_D v_R/2 \\ 0 & 0 & -m_D v_R/2 & \widetilde{M}_3^2 + \lambda_6 v_R^2 \end{pmatrix}, \tag{D.53}$$

where

$$A_0 \equiv \lambda_4(Y_4^2 - Y_3^2)\frac{v_R^2}{\kappa^2 - \kappa'^2} - 2\lambda_7 + \lambda_8/3. \tag{D.54}$$

Since the gap equation for σ is only trivially satisfied, it decouples from the low–energy sector of the model, i.e. $\widetilde{M}_3^2$ is of the order of Λ^2 (see the discussion in Sec. IV). It follows from eq. (D.53) that χ_{Li}^0 is a mass eigenstate with the squared mass $2(\lambda_2 - \lambda_1)v_R^2$. The rest of the matrix has one zero mass eigenstate

$$G_2^0 = \frac{\kappa}{\sqrt{\kappa^2 + \kappa'^2}}\phi_1^0 - \frac{\kappa'}{\sqrt{\kappa^2 + \kappa'^2}}\phi_2^0, \tag{D.55}$$

which is the Goldstone boson eaten by $Z_2^0 \approx Z_L^0$, and one massive eigenstate

$$H_{3i}^0 = \frac{\kappa'}{\sqrt{\kappa^2 + \kappa'^2}}\phi_1^0 + \frac{\kappa}{\sqrt{\kappa^2 + \kappa'^2}}\phi_2^0 \tag{D.56}$$

with the mass

$$M_{H_{3i}}^2 = A_0(\kappa^2 + \kappa'^2) \approx \lambda_4(Y_4^2 - Y_3^2)v_R^2 . \tag{D.57}$$

The non–vanishing elements of the mass matrix of the CP–even neutral Higgs bosons take the following form:

$$\left(M_{0r}^2\right)_{11} = \widetilde{M}_3^2 + v_R{}^2\lambda_6 \tag{D.58}$$

$$\left(M_{0r}^2\right)_{12} = \lambda_3 m_D v_R/2 \tag{D.59}$$

$$\left(M_{0r}^2\right)_{22} = 2(\lambda_2 - \lambda_1)v_R^2 \tag{D.60}$$

$$\left(M_{0r}^2\right)_{33} = \lambda_4(Y_4^2 - Y_3^2)\frac{v_R^2}{\kappa^2 - \kappa'^2}\kappa^2 + 4\lambda_7\kappa'^2 + (\lambda_8 - 2\lambda_7)\kappa^2 + 2\lambda_9\kappa\kappa' \tag{D.61}$$

$$\left(M_{0r}^2\right)_{34} = -\lambda_4(Y_4^2 - Y_3^2)\frac{v_R^2}{\kappa^2 - \kappa'^2}\kappa\kappa' + (\lambda_8 + 2\lambda_7)\kappa\kappa' + 2\lambda_9(\kappa^2 + \kappa'^2) \tag{D.62}$$

$$\left(M_{0r}^2\right)_{35} = 2(\lambda_4 Y_4 m_D + \lambda_5\kappa')v_R \tag{D.63}$$

$$\left(M_{0r}^2\right)_{44} = \lambda_4(Y_4^2 - Y_3^2)\frac{v_R^2}{\kappa^2 - \kappa'^2}\kappa'^2 + 4\lambda_7\kappa^2 + (\lambda_8 - 2\lambda_7)\kappa'^2 + 2\lambda_9\kappa\kappa' \tag{D.64}$$

$$\left(M_{0r}^2\right)_{45} = 2(\lambda_4 Y_3 m_D + \lambda_5\kappa)v_R \tag{D.65}$$

$$\left(M_{0r}^2\right)_{55} = 4\lambda_1 v_R^2 \tag{D.66}$$

where, as before, the basis is $(\sigma_r, \chi_{Lr}^0, \phi_{2r}^0, \phi_{1r}^0, \chi_{Rr}^0)$. Since the σ field decouples, χ_{Lr}^0 is an eigenstate with squared mass $2(\lambda_2 - \lambda_1)v_R^2$. The remaining 3×3 matrix can be diagonalized exactly, but the resulting expressions for its eigenstates and eigenvalues are very cumbersome and so we do not present them here. The simplified formulas for the case $\kappa' = 0$ are given in Sec. IV.

E Renormalization Group Equations

Here we present the one–loop renormalization group equations obtained in the MS–scheme for Yukawa and gauge couplings in the the general left–right symmetric model with N_g generations of fermions and the Higgs sector consisting of a bi–doublet ϕ, two doublets χ_L and χ_R and a singlet σ. We assume that in addition to the usual fermions we have a gauge singlet fermion S_L in each generation. The presence of the singlet scalar field σ and singlet fermions S_L will only affect the renormalization group equations for the Yukawa couplings Y_5 and Y_6.

The relevant Yukawa interactions are given in eq. (II.6). All the Yukawa couplings are matrices in generation space (we suppress the generation indices for brevity). Note that the discrete left–right symmetry requires the Yukawa matrices $Y_1 \ldots Y_4$ and Y_6 to be hermitian. In addition, Y_6 is symmetric due to the Majorana nature of the coupling. There are no restrictions on the matrix Y_5 from symmetry arguments.

We find the following β–functions:

$$16\pi^2\beta(Y_1) = 2Y_1^3 - \left(Y_1Y_2^2 + Y_2^2Y_1\right) + Y_1\left[3\,\mathrm{tr}(Y_1^2 + Y_2^2) + \mathrm{tr}(Y_3^2 + Y_4^2)\right]$$

$$+2Y_2\left[3\,\mathrm{tr}(Y_1Y_2) + \mathrm{tr}(Y_3Y_4)\right] - Y_1\left(8g_3^2 + \frac{9}{2}g_2^2 + \frac{1}{6}g_1^2\right) \tag{E.1}$$

$$16\pi^2\beta(Y_2) = 2Y_2^3 - \left(Y_2Y_1^2 + Y_1^2Y_2\right) + Y_2\left[3\,\mathrm{tr}(Y_1^2 + Y_2^2) + \mathrm{tr}(Y_3^2 + Y_4^2)\right]$$

$$+2Y_1\left[3\,\mathrm{tr}(Y_1Y_2) + \mathrm{tr}(Y_3Y_4)\right] - Y_2\left(8g_3^2 + \frac{9}{2}g_2^2 + \frac{1}{6}g_1^2\right) \tag{E.2}$$

$$16\pi^2\beta(Y_3) = 2Y_3^3 - \left(Y_3Y_4^2 + Y_4^2Y_3\right) + \frac{1}{2}\left(Y_3Y_5Y_5^\dagger + Y_5Y_5^\dagger Y_3\right)$$

$$+Y_3\left[3\,\mathrm{tr}(Y_1^2 + Y_2^2) + \mathrm{tr}(Y_3^2 + Y_4^2)\right] + 2Y_4\left[3\,\mathrm{tr}(Y_1Y_2) + \mathrm{tr}(Y_3Y_4)\right]$$

$$-Y_3\left(\frac{9}{2}g_2^2 + \frac{3}{2}g_1^2\right) \tag{E.3}$$

$$16\pi^2\beta(Y_4) = 2Y_4^3 - \left(Y_4Y_3^2 + Y_3^2Y_4\right) + \frac{1}{2}\left(Y_4Y_5Y_5^\dagger + Y_5Y_5^\dagger Y_4\right)$$

$$+Y_4\left[3\,\mathrm{tr}(Y_1^2 + Y_2^2) + \mathrm{tr}(Y_3^2 + Y_4^2)\right] + 2Y_3\left[3\,\mathrm{tr}(Y_1Y_2) + \mathrm{tr}(Y_3Y_4)\right]$$

$$-Y_4\left(\frac{9}{2}g_2^2 + \frac{3}{2}g_1^2\right) \tag{E.4}$$

$$16\pi^2\beta(Y_5) = \frac{3}{2}Y_5Y_5^\dagger Y_5 + Y_5(Y_5^\dagger Y_5)^T + Y_5\,\mathrm{tr}(Y_5^\dagger Y_5)$$

$$+\left(Y_3^2 + Y_4^2\right)Y_5 + 2Y_5Y_6^2 - Y_5\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2\right) \tag{E.5}$$

$$16\pi^2\beta(Y_6) = 4Y_6^3 + 2Y_6\,\mathrm{tr}(Y_6^2) + \left(Y_6\left[Y_5^\dagger Y_5 + (Y_5^\dagger Y_5)^T\right] + \left[Y_5^\dagger Y_5 + (Y_5^\dagger Y_5)^T\right]Y_6\right) \tag{E.6}$$

The gauge couplings in the model evolve according to

$$16\pi^2\beta(g_3) = g_3^3\left(-11 + \frac{4}{3}N_g\right) \tag{E.7}$$

$$16\pi^2\beta(g_2) = g_2^3\left(-\frac{41}{6} + \frac{4}{3}N_g\right) \tag{E.8}$$

$$16\pi^2\beta(g_1) = g_1^3\left(\frac{8}{9}N_g + \frac{1}{3}\right) \tag{E.9}$$

where we set $N_g = 3$ in the analysis of Sec. V.

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