

# Constraining neutrinoless double beta decay

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A class of discrete flavor-symmetry-based models predicts constrained neutrino mass matrix schemes that lead to specific neutrino mass sum-rules (MSR). We show how these theories may constrain the absolute scale of neutrino mass, leading in most of the cases to a lower bound on the neutrinoless double beta decay effective amplitude.

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## I. INTRODUCTION

The discovery of oscillations [1–8] implies non-vanishing neutrino masses and mixing providing one of the most solid indications for physics beyond the Standard Model. The fact that neutrinos have very tiny masses, in contrast to charged leptons and quarks, and that two of the mixing angles are large, are among the deepest theoretical puzzles in particle physics. Since neutrinos carry no electric charge, they are expected on general grounds to be Majorana particles [9], leading to the existence of lepton number violating processes [10, 11]. This intriguing possibility will be hopefully confirmed by the observation of neutrinoless double beta decay ( $0\nu\beta\beta$ ) processes [12, 13]. Indeed, upcoming  $0\nu\beta\beta$  experiments are expected to improve the sensitivity by up to about one order of magnitude [14–17].

It seems unlikely that the observed pattern of lepton mixing angles is an accident: it probably indicates the existence of an underlying flavor symmetry of some sort, either an Abelian symmetry [18] or a non-Abelian one [19]. In the former case one typically obtains texture zeros for the mass matrices [20–22] but is unable to predict mixing angles. In contrast, non-Abelian flavor symmetries are potentially more powerful, allowing also in principle for mixing angle predictions. As an example, several realizations of non-Abelian discrete flavor symmetry schemes lead to an effective neutrino mass matrix which corresponds to a numerical (parameter-free) prediction for lepton mixing. A popular

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example of such neutrino mass matrix  $M^\nu$  is the tri-bimaximal (TBM) [32] type <sup>1</sup>, characterized by

$$M^\nu = M_{TBM} \equiv \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}, \quad (1)$$

which depends only on three complex parameters  $x$ ,  $y$  and  $z$ . In the mass eigenstate basis the three complex parameters  $x$ ,  $y$  and  $z$  would correspond to three neutrino mass parameters plus two Majorana CP phases [9, 70], as the Dirac phase disappears since  $\theta_{13} = 0$ . Many such schemes are characterized by a specific (complex) relation among the parameters  $x$ ,  $y$  and  $z$  [33–69], leaving only two free complex parameters, further reducing the number of independent model parameters describing the lepton sector. In the mass basis these correspond to only two independent neutrino mass eigenvalues (the other follows from the existence of a neutrino mass sum rule), plus two Majorana CP phases (as mentioned, the Dirac phase is unphysical).

In this paper we study the implications of these sum-rule schemes for the lower bound on the parameter  $|m_{ee}|$  characterizing the amplitude for neutrinoless double beta decay. We show that, given that two neutrino mass squared splittings are well-determined by neutrino oscillation data [4, 5], we are left, approximately, with a one-parameter family of neutrinoless double beta decay theories in which the corresponding amplitude is mainly determined just by the overall absolute neutrino mass scale. The paper is organized as follows: in Section II we present the mass relations; in section III, we obtain the lower limit on  $|m_{ee}|$  for all models considered here and briefly discuss their phenomenological implications, whereas in Section IV we present our conclusions.

## II. MASS RELATIONS

In this section we focus on a general sub-class of mass matrices leading to a numerical (parameter-free) prediction for the lepton mixing matrix, consistent with current neutrino oscillation data [4, 5] where the following types of mass relations hold:

$$A) \quad \chi m_2^\nu + \xi m_3^\nu = m_1^\nu, \quad (2)$$

$$B) \quad \frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}, \quad (3)$$

$$C) \quad \chi \sqrt{m_2^\nu} + \xi \sqrt{m_3^\nu} = \sqrt{m_1^\nu}. \quad (4)$$

Here  $m_i^\nu = m_i^0$  denote neutrino mass eigenvalues, up to a Majorana phase factor, while  $\chi$  and  $\xi$  are free parameters which specify the model, taken to be positive without loss of generality. For the sake of completeness, we also consider a fourth mass relation,

$$D) \quad \frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}. \quad (5)$$

As far as we can tell, this last relation has not yet been considered in the literature. In the following we show how this class of mass matrices arises in non-Abelian discrete flavor symmetry schemes. We first consider an effective

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<sup>1</sup> There are also non-Abelian discrete flavor symmetry schemes leading to the bi-maximal lepton mixing pattern [23–27], as well as golden-ratio schemes [28–31].

dimension-five operator description [71], and then discuss the cases where the neutrino mass matrix  $M^\nu$  arises from various seesaw mechanism realizations, such as type-I or type-II seesaw [9, 72–77] or, from alternative low-scale seesaw schemes, for example, the inverse seesaw [78–81].

### Effective dimension-five operator description

Consider first the dimension five operators ( $LLHH$ ), where the parentheses indicate all possible contractions among the irreducible representations of the underlying unspecified non-Abelian flavor symmetry group  $L$  and  $H$  belong to. Since we want a mass matrix with only two independent parameters, we assume that our effective Lagrangian contains only two terms, associated to two independent field contractions:

$$\mathcal{L} = \frac{y_a}{M} \epsilon_{ij}^a (L_i L_j)_a HH + \frac{y_b}{M} \epsilon_{ij}^b (L_i L_j)_b HH; \quad (6)$$

here  $M$  denotes the effective scale,  $a, b$  represent the two contractions, while  $\epsilon_{ij}^a$  and  $\epsilon_{ij}^b$  are Clebsch-Gordan (CG) coefficients involving relevant field components  $L_{i,j}$ . After electroweak symmetry breaking, the effective neutrino mass matrix elements are given as linear combinations involving only the two parameters  $a$  and  $b$ ,

$$M_{ij}^\nu = a \epsilon_{ij}^a + b \epsilon_{ij}^b, \quad (7)$$

where  $a = y_a \langle H \rangle^2 / M$  and  $b = y_b \langle H \rangle^2 / M$ .

A number of non-Abelian discrete flavor symmetry realizations lead to the TBM structure for the effective neutrino mass matrix  $M^\nu$  in Eq. (1) with some suitable relation among the  $x, y$  and  $z$  coefficients. It is clear that in this case the corresponding mass eigenvalues will always be expressed as linear a combination of  $a$  and  $b$  and hence will be related to each other through a relation of type (A) in Eq. (2).

### Type-I seesaw mechanism

For the type-I seesaw mechanism there are two simple ways to get a neutrino mass matrix similar to  $M_{TBM}$  depending only upon two free complex parameters. In the first case, the Dirac neutrino mass matrix  $m_D$  has the structure given in eq. (1) while the right-handed neutrino mass matrix  $M_R$  is proportional to a numerical  $\mu - \tau$  invariant matrix satisfying the relation  $(2, 2) + (2, 3) = (1, 1) + (1, 2)$  among its elements, like for instance in Eq. (8):

$$M_R \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 2 & 1 & 1 \\ 1 & 5 & -2 \\ 1 & -2 & 5 \end{pmatrix}, \dots \quad (8)$$

In what follows we call such matrix generically as ‘‘TBM-type’’. It is not difficult to verify that the light neutrino mass matrix arising from the type-I seesaw formula [82]  $M^\nu = -m_D M_R^{-1} m_D^T$  has mass eigenvalues of type  $m_i^\nu \propto (\alpha_i a + \beta_i b)^2$ , yielding mass relations of type (C). For instance, in Ref. [83] it has been found that  $m_1^\nu \propto (a + b)^2$ ,  $m_2^\nu \propto a^2$  and  $m_3^\nu \propto (a - b)^2$ , from which the relation  $\sqrt{m_1^\nu} + \sqrt{m_3^\nu} = 2\sqrt{m_2^\nu}$  holds.

The second possibility arises when  $M_R \sim M_{TBM}$  as in Eq. (1), while the Dirac neutrino mass matrix is a numerical ‘‘TBM-type’’ matrix, as in eq. (8). In this case it is simple to show that the eigenvalues of  $M^\nu$  are of the form  $m_i^\nu \propto 1/(\alpha_i a + \beta_i b)$ , where  $\alpha_i$  and  $\beta_i$  are numerical coefficients, giving a mass relation of type (B). For instance, in

the model of Ref. [34] the authors found  $m_1^\nu \propto 1/(a+b)$ ,  $m_2^\nu \propto 1/a$  and  $m_3^\nu \propto 1/(b-a)$  from which the relation  $1/m_1^\nu - 1/m_3^\nu = 2/m_2^\nu$  is satisfied.

### Other seesaw mechanisms

Similar conclusions can be obtained for different seesaw mechanisms, such as type-II. From the point of view of our classification, type-II seesaw is equivalent to the dimension five operator case (A).

We now move to the inverse seesaw mechanism [78, 79], which arises when introducing a fermion singlet  $S$  with opposite lepton number with respect to the right-handed neutrinos, so that the effective light neutrino mass matrix is  $m_\nu = m_D \frac{1}{M} \mu \frac{1}{M} m_D^T$ . Assuming  $m_D$  and  $\mu$  to be proportional to the identity matrix and  $M \sim M_{TBM}$ , it is straightforward to show that we can obtain the mass sum-rule of type (D).

A novel seesaw mechanism arises from left-right symmetry [81] or the full SO(10) [84] in the presence of gauge singlet fermions, and has been called the linear seesaw. In such scheme the effective light neutrino mass matrix is given in terms of three independent sub-matrices and scales linearly with respect to the usual Dirac neutrino Yukawa couplings, hence the name. In order to have a mass relation, we need two sub-matrices of numerical ‘‘TBM-type’’ like in eq.(8) and the third one similar to  $M_{TBM}$  (see also [83]), otherwise additional free parameters are introduced, beyond our assumed two. One can show that all four cases can be realized, depending on which matrix has the form  $M_{TBM}$ .

### III. LOWER BOUND FOR NEUTRINOLESS DOUBLE- $\beta$ DECAY

Let us first consider the amplitude for neutrinoless double- $\beta$  decay within a flavor-generic model. One can plot the effective neutrino mass parameter  $|m_{ee}|$  determining the  $0\nu\beta\beta$  decay amplitude, as a function of the lightest neutrino mass. As is well-known, by varying the neutrino oscillation parameters  $\Delta m_{atm}^2$ ,  $\Delta m_{sol}^2$ ,  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$  in their allowed ranges [4, 5] one obtains two types of relatively broad bands in the  $(|m_{ee}|, m_\nu^\nu)$  plane corresponding to normal and inverse hierarchy spectra, as shown in Fig. 1.

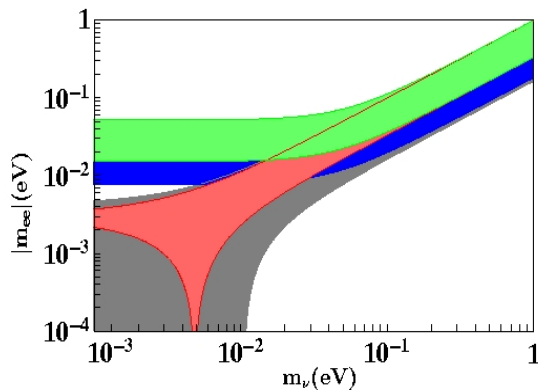


FIG. 1: Comparison on the allowed range of  $\langle |m_{ee}| \rangle$  as a function of the lightest neutrino mass. For the TBM mixing pattern (red and green bands for NH and IH respectively) and for the full allowed  $3\sigma$  C.L. ranges of oscillation parameters from [4, 5] (gray and blue bands for NH and IH respectively).

In this ‘‘generic’’ case there is a lower bound on the neutrinoless double- $\beta$  decay effective mass parameter  $|m_{ee}|$  only in the case of inverse mass hierarchy: due to the possibility of destructive interference among the light neutrinos from

the effect of having opposite CP signs or due to the effect of Majorana phases, no lower bound can be established for the case of normal hierarchy [85–87].

Let us now turn to the case where MSR relations like  $(A)$ ,  $(B)$ ,  $(C)$  and  $(D)$  hold. As discussed above these can be obtained in flavor models where the neutrino mass matrix only depends on two independent free parameters, so that the resulting mixing angles are fixed, like for example for the tri-bimaximal or bimaximal mixing patterns.

For definiteness here we focus on the case where the rotation in the neutrino sector is of tri-bimaximal form. Corrections from higher dimensional operators and/or from the charged lepton sector can yield  $\theta_{13} \neq 0$ , as suggested after the T2K [2] and Double-Chooz [3] first results [4].

Hence we retain the TBM approximation as a useful starting point to obtain our MSR relations. However, when evaluating a lower bound on the effective neutrino mass parameter  $|m_{ee}|$  determining the neutrinoless double- $\beta$  decay amplitude, we include explicitly the effects of non-vanishing  $\theta_{13}$ . We do this by taking the values at  $3\sigma$  determined in Ref. [4]. Such a lower bound can be obtained from the following procedure.

We first consider that the neutrino masses are complex parameters, where the two Majorana phases are encoded in  $m_2^\nu$  and  $m_3^\nu$ , i.e.

$$m_1^\nu = m_1^0, \quad (9)$$

$$m_2^\nu = m_2^0 e^{i\alpha}, \quad (10)$$

$$m_3^\nu = m_3^0 e^{i\beta}. \quad (11)$$

As shown in Fig. 2, the neutrino mass sum-rule can then be interpreted geometrically as a triangle in the complex plane, whose area provides a measure of Majorana CP violation <sup>2</sup>. Each of the above equations (9), (10) and (11)

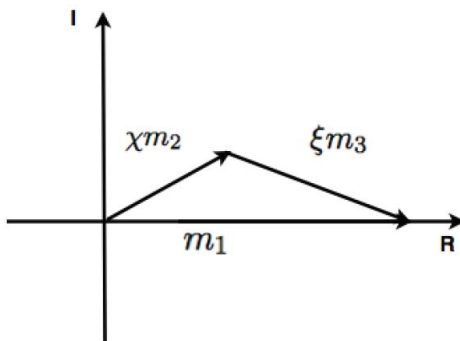


FIG. 2: Neutrino mass sum-rule triangle in the complex plane.

can be split into two independent equations for the real and imaginary parts.

For simplicity let us start from the idealized case where the neutrino oscillation parameters  $\Delta m_{atm}^2$ ,  $\Delta m_{sol}^2$ ,  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$  are perfectly well-measured quantities. In this case one can extract the two Majorana phases  $\alpha$  and  $\beta$  as functions of the base of the triangle, which is determined by  $m_1^0$  in case of normal hierarchy (NH) or by  $m_3^0$  in case of inverted hierarchy (IH), as well as the parameters  $\chi$  and  $\xi$  labeling the particular model under consideration.

<sup>2</sup> As the area shrinks to zero one obtains the CP-conserving limits corresponding to the four independent choices of CP sign [85, 86].

These relations obtained can then be inserted into the general expression of  $|m_{ee}|$ :

$$|m_{ee}| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3|. \quad (12)$$

For each  $(\chi, \xi)$  model this effective mass parameter depends on a single parameter, namely the length of the triangle base, which gives a measure of the absolute scale of neutrino mass.

For instance, for case (A) this procedure gives:

$$\xi \cos \alpha m_2^0 + \chi \cos \beta m_3^0 = m_1^0, \quad \xi \sin \alpha m_2^0 + \chi \sin \beta m_3^0 = 0, \quad (13)$$

so that the Majorana CP phases are determined as:

$$\begin{aligned} \cos \alpha &= \frac{m_1^2 - \chi^2 (\Delta m_{atm}^2 + m_1^2) + \xi^2 (\Delta m_{sol}^2 + m_1^2)}{2m_1 \xi \sqrt{\Delta m_{sol}^2 + m_1^2}} \\ \cos \beta &= \frac{m_1^2 + \chi^2 (m_1^2 + \Delta m_{atm}^2) - \xi^2 (m_1^2 + \Delta m_{sol}^2)}{2m_1 \chi \sqrt{\Delta m_{atm}^2 + m_1^2}}. \end{aligned} \quad (14)$$

The lower bound for the lightest neutrino mass can be obtained from our MSR, using the triangle inequality in the complex plane as suggested by Rodejohann and Barry in [88], see Fig.(2) for a schematic view.

We must first select the biggest side of the triangle; calling them  $x_1$ ,  $x_2$  and  $x_3$ , then the triangle inequality  $|x_i| \leq |x_j| + |x_k|$  must be fulfilled, where  $|x_i| \equiv \text{Max}(|x_1|, |x_2|, |x_3|)$  and  $i \neq j \neq k$ <sup>3</sup>. In case (A) and assuming NH for the neutrino mass spectrum, we always have  $(\chi m_2^0, \xi m_3^0) > m_1^0$  and the largest side of the triangle can be either  $\chi m_2^0$  or  $\xi m_3^0$ ; so we must consider separately these two cases. After rewriting two masses in terms of the two squared mass differences, we can obtain a lower limit for the lightest neutrino mass from the triangle inequality  $|x_i| \leq |x_j| + |x_k|$ . For the other cases we follow the same procedure. The lower bound on the lightest neutrino mass obtained in this way is then used to estimate the lower bound for  $|m_{ee}|$  from the general expression in eq. (12). Notice that, although we focus here on TBM schemes, some of the MSR considered in our analysis can also be derived using bimaximal mixing as a starting point: for instance, in Refs. [26, 27] a relation of type (A) with  $(\chi, \xi) = (1, 2)$  has been derived and the phenomenological consequences studied.

### Classification

The results obtained from the procedure discussed above are summarized in Tab.I, where we report the lower limits of  $|m_{ee}|$  corresponding to different integer choices of  $(\chi, \xi)$  between 1 and 3 and for each of the four MSR considered in eqs. (2)-(5), for both normal and inverted hierarchies (compare also with Figs. 4-7). The cases already discussed in the literature are indicated by giving the corresponding reference. The entries denoted with the symbol ( $\star$ ) represent situations that do not satisfy the inequality for any value of the lightest neutrino mass  $m_{light}$ . Cases marked by a (-) correspond to models which, as far as we can tell, have not been considered. Some comments are in order. First let us consider the effect of a possible non-zero effect of  $\theta_{13}$  as indicated by recent experiments [2, 3] as well as global neutrino oscillation fits [4-6]. In Fig 3 we show the prediction for  $|m_{ee}|$  as function of  $m_{light}$  obtained from the MSR

<sup>3</sup> Notice that there are three inequalities of the type  $|x_i| \leq |x_j| + |x_k|$  obtained by permuting the three indices  $i, j$  and  $k$ , but only one of these constrains the lightest neutrino mass.

$\chi, \xi$	A-NH	A-IH	Ref.	B-NH	B-IH	Ref.	C-NH	C-IH	Ref.	D-NH	D-IH
1,1	0.010	0.044	[33–49]	0.008	0.036	[50–52]	0.006	0.029	-	.005	.008
1,2	*	0.046	-	0.008	0.027	-	*	0.014	-	.004	.026
1,3	*	0.011	-	0.030	0.005	-	*	0.014	-	.018	.025
2,1	0.006	*	[33–40], [41, 53]	0.006	0.007	[34, 35, 43], [49, 54–68]	0.000	*	[83]	*	.007
2,2	0.019	0.026	-	0.023	0.008	-	0.017	*	-	.003	.015
2,3	*	0.046	-	0.007	0.008	-	*	0.031	-	.005	.026
3,1	0.004	*	-	0.004	0.008	-	*	*	-	*	*
3,2	0.011	*	-	0.004	0.021	-	0.000	*	-	*	.007
3,3	0.023	0.061	-	0.029	0.031	-	0.011	0.019	-	.018	.016

TABLE I: Minimal values for the effective  $0\nu\beta\beta$  decay mass parameter  $|m_{ee}|$ , in eV, see text for details.

$3\sqrt{m_2} + 3\sqrt{m_3} = \sqrt{m_1}$  (right panel) and  $2\sqrt{m_2} + \sqrt{m_3} = \sqrt{m_1}$  (left panel). For the red bands we assumed the TBM values of the oscillation parameters (implying  $\theta_{13} = 0$ ) while in the yellow bands corresponds to the same MSR, but now varying the values of  $\theta_{13}$ ,  $\theta_{23}$  and  $\theta_{12}$  within their  $3\sigma$  C.L. interval. By looking at the left panel in Fig. 3 one

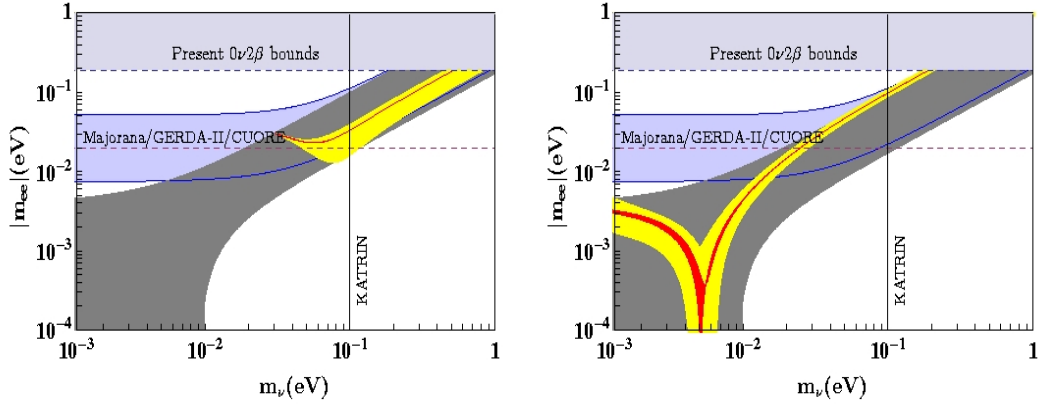


FIG. 3:  $\langle |m_{ee}| \rangle$  as a function of the lightest neutrino mass corresponding to the mass sum-rule  $2\sqrt{m_2} + \sqrt{m_3} = \sqrt{m_1}$  [83] (left) and  $3\sqrt{m_2} + 3\sqrt{m_3} = \sqrt{m_1}$  (right). The red bands correspond to the TBM mixing pattern, while the yellow bands correspond to the same MSR, but now varying the values of  $\theta_{13}$ ,  $\theta_{23}$  and  $\theta_{12}$  to  $3\sigma$  C.L. range.

sees that, indeed, the  $0\nu\beta\beta$  lower bound is sensitive to the value of  $\theta_{13}$ .

One also finds that, as expected on general grounds, all inverse hierarchy schemes corresponding to various choices of  $(\chi, \xi)$  within sum-rules A-D have a lower bound for the parameter  $|m_{ee}|$ . However, the numerical value obtained depends on the MSR scheme, signaling that not all values within the corresponding band in Fig. 1 are covered.

On the other hand, even though normal hierarchy models do not lead to a lower bound on the  $0\nu\beta\beta$  amplitude due to the possibility of destructive interference amongst the light neutrinos, one finds that the possibility of full cancellation is precluded for all schemes in the table, except for the (2,1) case considered in Ref. [83] and the (3,2) scheme, both of which correspond to MSR of type (C). All other NH MSR schemes considered here imply a minimum value for the  $0\nu\beta\beta$  decay amplitude<sup>4</sup>. One finds that the most favorable cases are given by:

<sup>4</sup> Of course some of the bounds are phenomenologically less interesting since they fall outside realistic sensitivities of coming experiments.

$$\begin{aligned}
(\chi, \xi) &= (3, 3) \text{ for the case (A) NH,} \\
(\chi, \xi) &= (3, 3) \text{ for the case (A) IH,} \\
(\chi, \xi) &= (1, 3) \text{ for the case (B) NH,} \\
(\chi, \xi) &= (3, 3) \text{ for the case (B) IH,} \\
(\chi, \xi) &= (3, 3) \text{ for the case (C) NH,} \\
(\chi, \xi) &= (2, 3) \text{ for the case (C) IH,} \\
(\chi, \xi) &= (3, 3) \text{ for the case (D) NH,} \\
(\chi, \xi) &= (1, 2) \text{ for the case (D) IH.}
\end{aligned}$$

In particular, the maximal value we have found for the lower bound on  $|m_{ee}|$  is  $|m_{ee}| = 0.061 \text{ eV}$ , obtained in correspondence with the set of values  $(\chi, \xi) = (3, 3)$  for the case (A) in IH. Such a value for  $|m_{ee}|$  lies within the sensitivity of upcoming experiments; hence it would be interesting, from the model building point of view, to find from first principles a flavor-symmetry-based model predicting such a mass relation; we will return to this problem elsewhere.

The same phenomenologically interesting cases are now studied more in detail, showing the behavior of  $|m_{ee}|$  as function of the lightest neutrino mass in Figs. 4,5,6 and 7. In all plots, the two bands are the most generic ones compatible with both normal and inverted hierarchies, derived considering the  $3\sigma$  allowed ranges on the neutrino oscillation parameters as obtained in Ref.[4] and consistent with latest T2K and Double-Chooz experiments, see Fig. 1.

In Fig. 4 we give the allowed  $\langle |m_{ee}| \rangle$  values as a function of the lightest neutrino mass. The figures correspond to case (A). In the left panel the yellow band corresponds to the model which predicts the mass sum rule  $3m_2 + 3m_3 = m_1$  in case of NH. On the right the red band corresponds to the same sum rule in the case of IH. Other MSR  $0\nu\beta\beta$  amplitude lower bounds are illustrated in subsequent figures.

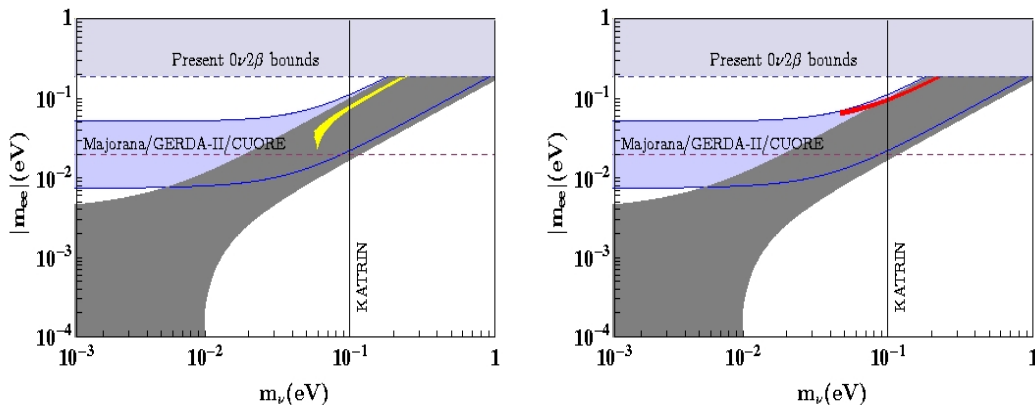


FIG. 4: Effective  $0\nu\beta\beta$  mass parameter  $\langle |m_{ee}| \rangle$  as a function of the lightest neutrino mass for MSR type-A schemes. On the left panel the yellow band corresponds to the model which predicts the sum mass rule  $3m_2 + 3m_3 = m_1$  for the case of NH. On the right the red band corresponds to the same sum rule in the case of IH.

#### IV. CONCLUSIONS

In this paper we have analyzed the implications for the lower bound on the effective  $0\nu\beta\beta$  neutrino mass parameter  $|m_{ee}|$  arising from possible mass sum-rules obtained in the context of flavor models. Mass sum rules are classified in four different categories, some have already been considered in the literature. For each case, we have first extracted the allowed numerical values of  $|m_{ee}|$ , for both mass orderings of the neutrino mass eigenstates and we have then given the behavior of  $|m_{ee}|$  as a function of the lightest neutrino mass. Although our MSR schemes were obtained within



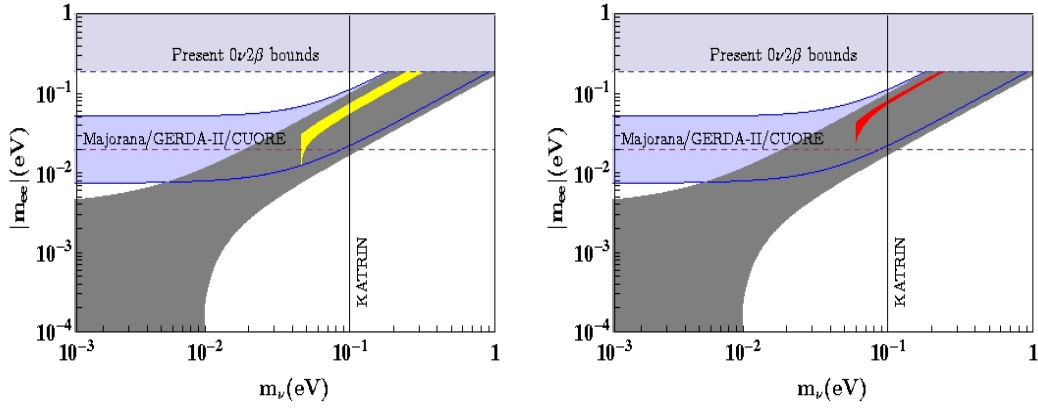


FIG. 5:  $\langle |m_{ee}| \rangle$  as a function of the lightest mass. The figures correspond to the case (B). On the left panel the yellow band corresponds to the model which predicts the sum mass rule  $1/m_2 + 3/m_3 = 1/m_1$  for the NH case. On the right, the red band corresponds to the prediction of the sum rule  $3/m_2 + 3/m_3 = 1/m_1$  in the case of IH.

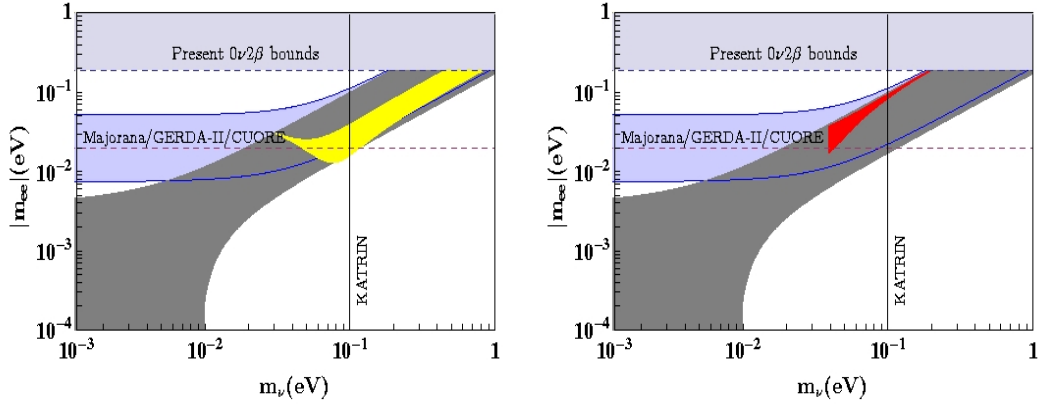


FIG. 6:  $\langle |m_{ee}| \rangle$  as a function of the lightest neutrino mass. The figures correspond to the case (C). On the left the yellow band corresponds to the model which predicts the sum mass rule  $3\sqrt{m_2} + 3\sqrt{m_3} = \sqrt{m_1}$  in case of NH. On the right, the red band corresponds to the prediction of the sum mass rule  $2\sqrt{m_2} + 3\sqrt{m_3} = \sqrt{m_1}$  in the case of IH.

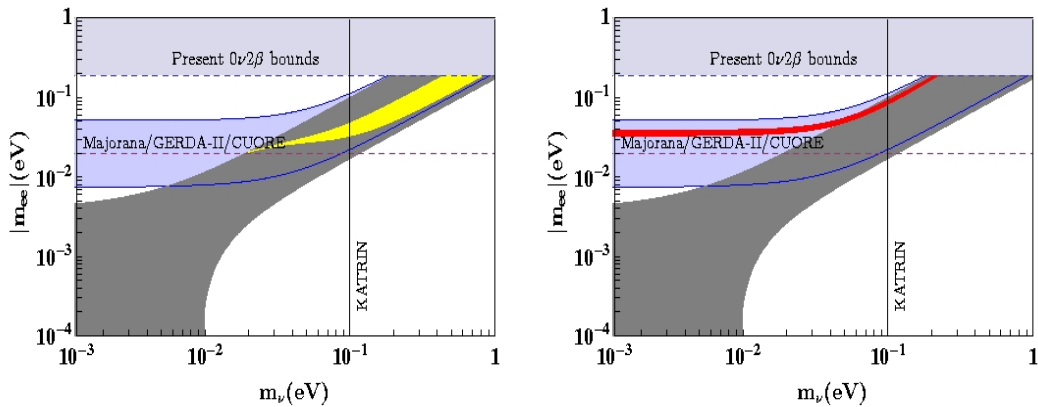


FIG. 7:  $\langle |m_{ee}| \rangle$  as a function of the lightest neutrino mass, case (D). On the left the yellow band corresponds to the MSR scheme which predicts  $3/\sqrt{m_2} + 3/\sqrt{m_3} = 1/\sqrt{m_1}$  for NH. On the right panel, the red band corresponds to the prediction of the mass sum rule  $1/\sqrt{m_2} + 2/\sqrt{m_3} = 1/\sqrt{m_1}$  in the case of IH.

the TBM ansatz, we have computed the possible values of  $|m_{ee}|$  considering all the neutrino parameters (including

a non-vanishing  $\theta_{13}$ ) within their  $3\sigma$  allowed ranges. In most MSR schemes one finds a lower bound for the  $0\nu\beta\beta$  amplitude, even for NH spectra. We find that the most favorable case (large lower bound) corresponds to a sum-rule of type (A) obtained in correspondence of the set of values  $(\chi, \xi) = (3, 3)$ ,  $|m_{ee}| = 0.061$  eV. Such a mass relation has not been considered so far, and the searching of a flavor model able to predict it at leading order is now in progress.

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