Predictive Discrete Dark Matter Model

M. S. Boucenna,* S. Morisi,† E. Peinado,‡ and J. W. F. Valle§
AHEP Group, Instituto de Física Corpuscular – C.S.I.C./Universitat de València
Edificio de Institutos de Paterna, Apartado 22085, E–46071 València, Spain

Yusuke Shimizu¶
Department of Physics, Niigata University, Niigata 950-2181, Japan.

Dark Matter stability can be achieved through a partial breaking of a flavor symmetry. In this framework we propose a type-II seesaw model where left-handed matter transforms nontrivially under the flavor group $\Delta(54)$, providing correlations between neutrino oscillation parameters, consistent with the recent Daya-Bay and RENO reactor angle measurements, as well as lower bounds for neutrinoless double beta decay. The dark matter phenomenology is provided by a Higgs-portal.

Keywords: dark matter; neutrino masses and mixing; flavor symmetry

The discovery of neutrino oscillations [1] and the growing evidence for the existence of dark matter [2] provide strong indications for the need of physics beyond Standard Model (SM). However the detailed nature of the new physics remains elusive. On the one hand, the typology of mechanism responsible for neutrino mass generation and its flavor structure, as well as the nature of the associated messenger particle are unknown. Consequently the nature of neutrinos, their mass and mixing parameters are all unpredicted.

Likewise the nature of Dark Matter (DM) constitutes one of the most challenging questions in cosmology since decades, though recently some direct and indirect DM detection experiments are showing tantalizing hints favoring a light WIMP-like DM candidate [3–7] opening hopes for an imminent detection.

Linking neutrino mass generation to dark matter, two seemingly unrelated problems into a single framework is not only theoretically more appealing, but also may bring us new insights on both issues.

Among the requirements a viable DM candidate must pass, stability has traditionally been ensured through the *ad hoc* imposition of a stabilizing symmetry; usually a parity. Clearly a top-down approach where stability is naturally achieved is theoretically more appealing. This is what motivated attempts such gauged as $U(1)_{B-L}$ [8], gauged discrete symmetries [9] and the recently proposed discrete dark matter mechanism (DDM) [10–13], where stability arises as a remnant of a suitable flavor symmetry [1].

In its minimal realization, the DDM scenario provides a link between DM and neutrino phenomenology through the stability issue. Here we describe a DDM scenario which is able to connect the two sectors in a nontrivial way. The main idea behind DDM is outlined below.

Consider the group of the even permutations of four objects A_4. It has one triplet and three singlet irreducible representations, denoted $\mathbf{3}$ and $\mathbf{1}, \mathbf{1'}, \mathbf{1''}$ respectively. A_4 can be broken spontaneously to one of its Z_2 subgroups. Two of the components of any A_4 triplet are odd under such a parity, while the A_4 singlet representation is even. This residual Z_2 parity can be used to stabilize the DM which, in this case, must belong to an A_4 triplet representation, taken as an $SU(2)_L$ scalar Higgs doublet, $\eta \sim \mathbf{3}$ [10–13]. Assuming that the lepton doublets L_i are singlets of A_4 while right-handed neutrinos transform as A_4 triplets $N \sim \mathbf{3}$, the contraction rules imply that the DM couples only to Higgses and heavy right-handed neutrinos $\overline{L}_i\, N\, \tilde{\eta}$. In this case η and N have even as well as odd-components while L_i are even so that $\overline{L}_i\, N\, \tilde{\eta}$ interaction preserves the Z_2 parity. Invariance under Z_2 implies that N components odd under Z_2 are not mixed with the Z_2-even light neutrinos ν_i. This forbids the decay of the lightest Z_2-odd component of η to light neutrinos through the heavy right handed neutrinos, ensuring DM stability. However, simplest schemes of this type lead to $\theta_{13} = 0$ as a first-order prediction [10], at variance with recent reactor results [19, 20].

In contrast, assigning the three left-handed leptons to a flavor-triplet implies that the "would-be" DM candidate decays very fast into light leptons, through the contraction of the triplet representations, see general discussion in ref. [14]. This problem has been considered by Eby and Framptom [21] using a T' flavor symmetry. While the suggested model has the merit of incorporating quarks nontrivially, it requires an "external" Z_2 asymmetry in order to stabilize dark matter. In fact this observation lead ref. [22] to claim that a successful realization of the DDM scenario requires the lepton doublets to be in three inequivalent singlet representations of the flavour group.

Here we provide an explicit example of a model based on a $\Delta(54)$ flavour symmetry in which left-handed leptons are assigned to nontrivial representations of the flavour group, with a viable stable dark matter particle and a nontrivial inclusion of quarks. In contrast to the simplest "flavour-blind" inert dark matter scheme [23] our

---

*Electronic address: boucenna@ific.uv.es
†Electronic address: morisi@ific.uv.es
‡Electronic address: eduardo@ific.uv.es
§Electronic address: valle@ific.uv.es
¶Electronic address: shimizu@muse.sc.niigata-u.ac.jp

[1] For other flavor models with DM candidates see [11, 14–18]

model implies restrictions and/or correlations amongst the neutrino oscillation parameters, consistent with the recent reactor angle measurements [19, 20]. Needless to say the later will pave the way towards a new era for neutrino oscillations studies [25, 26], in which leptonic CP violation searches will play an important role, providing an additional motivation for our proposal.

We search for a group G that contains at least two irreducible representations of dimension larger than one, namely r_a and r_b with $\dim(r_{a,b}) > 1$. We also require that all the components of r_a transform trivially under an abelian subgroup of $G \supset Z_N$ (with $N = 2, 3$) while at least one component of r_b is charged with respect to Z_N. The stability of the lightest component of r_b is guaranteed by Z_N giving a potential [2] DM candidate.

The simplest group we have found with this feature is $\Delta(54)$, isomorphic to $(Z_3 \times Z_3) \rtimes S_3$. In addition to the irreducible triplet representations, $\Delta(54)$ contains four different doublets $\mathbf{2_{1,2,3,4}}$ and two irreducible singlet representations, $\mathbf{1_{\pm}}$. The product rules for the doublets are $\mathbf{2_k} \times \mathbf{2_k} = \mathbf{1_+} + \mathbf{1_-} + \mathbf{2_k}$ and $\mathbf{2_1} \times \mathbf{2_2} = \mathbf{2_3} + \mathbf{2_4}$. Of the four doublets $\mathbf{2_1}$ is invariant under the $P \equiv (Z_3 \times Z_3)$ subgroup of $\Delta(54)$, while the others transform nontrivially, for example $\mathbf{2_3} \sim (\chi_1, \chi_2)$, which transforms as $\chi_1\,(\omega^2, \omega)$ and $\chi_2\,(\omega, \omega^2)$ respectively, where $\omega^3 = 1$ [27, 28]. We can see that by taking $r_a = \mathbf{2_1}$ and $r_b = \mathbf{2_3}$ that $\Delta(54)$ is perfect for our purpose.

Let us now turn to the explicit model, described in table I, where $L_D \equiv (L_\mu, L_\tau)$ and $l_D \equiv (\mu_R, \tau_R)$. There are 5 $SU(2)_L$ doublets of Higgs scalars: the H is a singlet of $\Delta(54)$, while $\eta = (\eta_1, \eta_2) \sim \mathbf{2_3}$ and $\chi = (\chi_1, \chi_2) \sim \mathbf{2_1}$ are doublets. In order to preserve a remnant P symmetry, the doublet η is not allowed to take vacuum expectation value (vev). Such a prescription is not necessary for H, χ_1 and χ_2 since these are all invariant under P. We also need to introduce an $SU_L(2)$ Higgs triplet scalar field $\Delta \sim \mathbf{2_1}$ whose vev will induce neutrino masses through the type-II seesaw mechanism [29]. Regarding dark matter, note that the lightest P-charged particle in $\eta_{1,2}$ can play the role of "inert" DM [23], as it has no direct couplings to matter. The conceptual link between dark matter and neutrino phenomenology arises from the fact that the DM stabilizing symmetry is a remnant of the underlying flavor symmetry which accounts for the observed pattern of oscillations. See phenomenological implications below.

	$\overline{L}_e$	$\overline{L}_D$	e_R	l_D	H	χ	η	Δ
$SU(2)$	2	2	1	1	2	2	2	3
$\Delta(54)$	$\mathbf{1_+}$	$\mathbf{2_1}$	$\mathbf{1_+}$	$\mathbf{2_1}$	$\mathbf{1_+}$	$\mathbf{2_1}$	$\mathbf{2_3}$	$\mathbf{2_1}$

TABLE I: Lepton and higgs boson assignments of the model.

The lepton part of the Yukawa Lagrangian is given by

$$\begin{aligned}\mathcal{L}_\ell &= y_1\overline{L}_e e_R H + y_2\overline{L}_e l_D\chi + y_3\overline{L}_D e_R\,\chi + \\ &\quad + y_4\overline{L}_D l_D H + y_5\overline{L}_D l_D \chi \end{aligned} \tag{1}$$

$$\mathcal{L}_\nu = y_b\overline{L}_D\overline{L}_D\Delta + y_a\overline{L}_D L_e\Delta \tag{2}$$

After electroweak symmetry breaking the first term $\mathcal{L}_\ell$ gives the following charged lepton mass matrix

$$M_\ell = \begin{pmatrix} a & br & b \\ cr & d & e \\ c & e & dr \end{pmatrix} \tag{3}$$

where $a = y_1\langle H\rangle$, $b = y_2\langle\chi_1\rangle$, $c = y_3\langle\chi_1\rangle$, $d = y_5\langle\chi_1\rangle$, $e = y_4\langle H\rangle$, and

$$r = \langle\chi_2\rangle / \langle\chi_1\rangle\,.$$

On the other hand the $\mathcal{L}_\nu$ is the term responsible for generating the neutrino mass matrix. Choosing the solution $\langle\Delta\rangle \sim (1, 1)$ and $\langle\chi_1\rangle \neq \langle\chi_2\rangle$, consistent with the minimization of the scalar potential one finds that

$$M_\nu \propto \begin{pmatrix} 0 & \delta & \delta \\ \delta & \alpha & 0 \\ \delta & 0 & \alpha \end{pmatrix}, \tag{4}$$

where $\delta = y_a\langle\Delta\rangle$, $\alpha = y_b\langle\Delta\rangle$.

Our model corresponds to a flavour-restricted realization of the *inert dark matter* scenario proposed in [23]. As such, it has nontrivial consequences for neutrino phenomenology, which we now study in detail. As seen in eq. (4) the neutrino mass matrix depends only on two parameters, δ and α (taken to be real), which can be expressed as a function of the measured squared mass differences, as follows

$$m^\nu_{1,3} = \frac{\alpha \mp \sqrt{8\delta^2 + \alpha^2}}{2},\ m^\nu_2 = \alpha. \tag{5}$$

For simplicity, we fix the intrinsic neutrino CP–signs [24] as $\eta = diag(-, +, +)$, where η is defined as $U^\star = U\eta$, U being the lepton mixing matrix. It is easy to check that, in this case, only a normal hierarchy spectrum is allowed. In contrast, a different permutation of the eigenvalues corresponding to our η matrix, namely $(1, 2, 3) \to (1, 3, 2)$ in Eq. 5, gives only inverse hierarchy spectrum. Moreover, notice that the masses in eq. (5) obey a neutrino mass sum rule of the form $m^\nu_1 + m^\nu_2 = m^\nu_3$

---

[2] Of course, other requirements are necessary in order to have a viable DM candidate, such as neutrality, correct relic abundance, and consistency with constraints from DM search experiments.

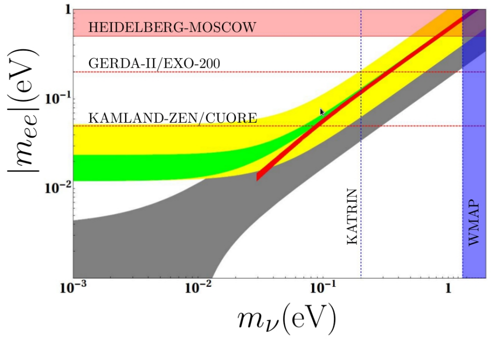

FIG. 1: Effective neutrinoless double beta decay parameter m_{ee} versus the lightest neutrino mass. The thick upper and lower branches correspond the "flavor-generic" inverse (yellow) and normal (gray) hierarchy neutrino spectra, respectively. The model predictions are indicated by the green and red (darker-shaded) regions, respectively. Only these sub-bands are allowed by the $\Delta(54)$ model. For comparison we give the current limit and future sensitivities on m_{ee} [32, 33] and m_ν [34, 35], respectively.

which has implications for the neutrinoless double beta decay process [31], as illustrated in Fig. (1).

We now turn to the second prediction. Although in our scheme neutrino mixing parameters in the lepton mixing matrix are not strictly predicted, there are correlations between the reactor and the atmospheric angle, as illustrated in Figs. 2 [3] and Fig. 3 for the cases of normal and inverse mass hierarchies, respectively. While the solar angle is clearly unconstrained and can take all the values within in the experimental limits, correlations exist with the reactor mixing angle, indicated by the curved yellow bands in Fig. 2 and 3. These correspond to 2σ regions of θ_{23} as determined in Ref. [30]. The horizontal lines give the best global fit value and the recent best fit values obtained in Daya–Bay and RENO reactors [19, 20] (see also recent result from T2K [36]). Now we turn to quarks. In Ref. [10–13] quarks were singlets of the flavor symmetry to guarantee the stability of the DM. Consequently the generation of quark mixing was difficult [37]. This problem has been recently considered in [21] using T' flavor symmetry.

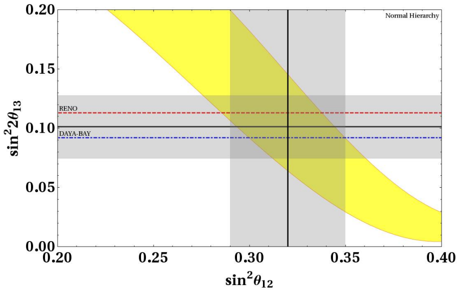

FIG. 2: The shaded (yellow) curved band gives the predicted correlation between solar and reactor angles when θ_{23} is varied within 2σ for the normal hierarchy spectrum. The solid (black) line gives the global best fit values for θ_{12} and θ_{13}, along with the corresponding two-sigma bands, from Ref. [30]. The dashed lines correspond to the central values of the recent reactor measurements [19, 20].

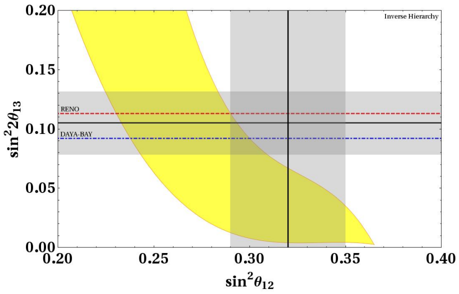

FIG. 3: Same as above for the inverse hierarchy case.

A nice feature of our current model is that with $\Delta(54)$ we can assign quarks to the singlet and doublet representations as shown in table II. This opens new possibilities to fit the CKM mixing parameters. Indeed, as shown in table II quarks transforming nontrivially under the flavor symmetry can be consistently added in our picture.

	$Q_{1,2}$	Q_3	(u_R, c_R)	t_R	d_R	s_R	b_R
$SU(2)$	2	2	1	1	1	1	1
$\Delta(54)$	$\mathbf{2_1}$	$\mathbf{1_+}$	$\mathbf{2_1}$	$\mathbf{1_+}$	$\mathbf{1_-}$	$\mathbf{1_+}$	$\mathbf{1_+}$

TABLE II: Quark gauge and flavour representation assignments.

The resulting up- and down-type quark mass matrices in our model are given by

$$M_d = \begin{pmatrix} ra_d & rb_d & rd_d \\ -a_d & b_d & d_d \\ 0 & c_d & e_d \end{pmatrix}, \quad M_u = \begin{pmatrix} ra_u & b_u & d_u \\ b_u & a_u & rd_u \\ c_u & rc_u & e_u \end{pmatrix}. \tag{6}$$

Note that the Higgs fields H and χ are common to the lepton and the quark sectors and in particular the parameter r. Assuming for simplicity real couplings we have 11 free parameters characterizing this sector, 10 Yukawa couplings plus the ratio of the the isodoublet vevs, r, introduced earlier in the neutrino sector. We have verified that we can make a fit of all quark masses and mixings provided r lies in the range of about $0.1 < r < 0.2$. We do not extend further the discussion on the quark interactions which can be easily obtained from table II (a full

---

[3] There is also a second band allowed in this case which is, however, experimentally ruled out by the measurents of θ_{12} and θ_{13}.

analysis of the quark phenomenology is beyond the scope of this paper and will be taken up elsewhere).

Notice that our scalar Dark matter candidate η_1 has quartic couplings with the Higgs scalars of the model such as $\eta^\dagger\eta H^\dagger H$ and $\eta^\dagger\eta\chi^\dagger\chi$. These weak strength couplings provide a Higgs portal production mechanism, and ensure an adequate cosmological relic abundance. Direct and indirect detection prospects are similar to a generic WIMP dark matter, as provided by multi-Higgs extensions of the SM.

In short we have described how spontaneous breaking of a $\Delta(54)$ flavor symmetry can stabilize the dark matter by means of a residual unbroken symmetry. In our scheme left-handed leptons as well as quarks transform nontrivially under the flavor group, with neutrino masses arising from a type-II seesaw mechanism. We have found lower bounds for neutrinoless double beta decay, even in the case of normal hierarchy, as seen in Fig. 1. In addition, we have correlations between solar and reactor angles consistent with the recent Daya-Bay and RENO reactor measurements, see Fig. 2 and Fig. 3.

Unfortunately, however, the DM particle is not directly involved as messenger in the neutrino mass generation mechanism. This issue will be considered elsewhere.

We thank Luis Dorame, Luis Lavoura and Martin Hirsch for discussions and interesting comments. This work was supported by the Spanish MEC under grants FPA2011-22975 and MULTIDARK CSD2009-00064 (Consolider-Ingenio 2010 Programme), by Prometeo/2009/091 (Generalitat Valenciana), by the EU ITN UNILHC PITN-GA-2009-237920. Y.S. is supported by Grand-in-Aid for Scientific Research, No.22.3014 in JSPS.

[1] A. McDonald, Talk at XIV International Workshop on Neutrino Telescopes, Venice, March, 2011; for a review see M. Maltoni et al, New J. Phys. **6** (2004) 122.

[2] G. Bertone, D. Hooper and J. Silk, Physics Reports **405**, 279 (2005).

[3] C. Aalseth *et al.*, Phys.Rev.Lett. **107**, 141301 (2011), [1106.0650].

[4] R. Bernabei *et al.* [DAMA Collaboration], Eur. Phys. J. C **56** (2008) 333 [arXiv:0804.2741 [astro-ph]].

[5] G. Angloher, M. Bauer, I. Bavykina, A. Bento, C. Bucci, C. Ciemniak, G. Deuter and F. von Feilitzsch *et al.*, arXiv:1109.0702 [astro-ph.CO].

[6] Z. Ahmed *et al.* [CDMS-II Collaboration], Phys. Rev. Lett. **106** (2011) 131302 [arXiv:1011.2482 [astro-ph.CO]].

[7] D. Hooper and L. Goodenough, Phys. Lett. B **697** (2011) 412 [arXiv:1010.2752 [hep-ph]].

[8] T. Hambye, PoS **IDM2010**, 098 (2011), [1012.4587].

[9] B. Batell, Phys.Rev. **D83**, 035006 (2011), [1007.0045].

[10] M. Hirsch, S. Morisi, E. Peinado and J. W. F. Valle, Phys.Rev. **D82**, 116003 (2010), [1007.0871].

[11] D. Meloni, S. Morisi and E. Peinado, Phys.Lett. **B703**, 281 (2011), [1104.0178].

[12] M. Boucenna *et al.*, JHEP **1105**, 037 (2011), [1101.2874].

[13] D. Meloni, S. Morisi and E. Peinado, Phys.Lett. **B697**, 339 (2011), [1011.1371].

[14] Y. Kajiyama, K. Kannike and M. Raidal, Phys.Rev. **D85**, 033008 (2012), [1111.1270], 8 pages, no figures.

[15] Y. Kajiyama, H. Okada and T. Toma, 1109.2722.

[16] Y. Daikoku, H. Okada and T. Toma, Prog.Theor.Phys. **126**, 855 (2011), [1106.4717].

[17] Y. Kajiyama and H. Okada, Nucl.Phys. **B848**, 303 (2011), [1011.5753].

[18] N. Haba, Y. Kajiyama, S. Matsumoto, H. Okada and K. Yoshioka, Phys.Lett. **B695**, 476 (2011), [1008.4777].

[19] Daya-Bay Collaboration, F. An *et al.*, 1203.1669, 5 figures.

[20] Soo-Bong Kim for RENO collaboration, 1204.0626.

[21] D. A. Eby and P. H. Frampton, 1111.4938.

[22] L. Lavoura, J.Phys.G **G39**, 025202 (2012), [1109.6854].

[23] R. Barbieri, L. J. Hall and V. S. Rychkov, Phys. Rev. **D74**, 015007 (2006), [hep-ph/0603188].

[24] J. Schechter and J. W. F. Valle, Phys. Rev. D **24** (1981) 1883 [Erratum-ibid. D **25** (1982) 283].

[25] H. Nunokawa, S. J. Parke and J. W. F. Valle, Prog. Part. Nucl. Phys. **60**, 338 (2008).

[26] ISS Physics Working Group, A. Bandyopadhyay *et al.*, Rept.Prog.Phys. **72**, 106201 (2009), [0710.4947].

[27] H. Ishimori *et al.*, Prog. Theor. Phys. Suppl. **183**, 1 (2010), [1003.3552].

[28] H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu and M. Tanimoto, JHEP **0904**, 011 (2009), [0811.4683].

[29] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980); Phys. Rev. **D25**, 774 (1982).

[30] M. Tortola, J. W. F. Valle, and D. Vanegas, arXiv:1205.4018; which updates previous analysis before the recent reactor results, e. g. T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. **13**, 063004 (2011); New J.Phys. **13**, 109401 (2011).

[31] L. Dorame, D. Meloni, S. Morisi, E. Peinado and J. Valle, 1111.5614.

[32] B. Schwingenheuer, arXiv:1201.4916 [hep-ex].

[33] W. Rodejohann, Int. J. Mod. Phys. E **20** (2011) 1833 [arXiv:1106.1334 [hep-ph]].

[34] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192** (2011) 18 [arXiv:1001.4538 [astro-ph.CO]].

[35] A. Osipowicz *et al.* [KATRIN Collaboration], hep-ex/0109033.

[36] T2K Collaboration, K. Abe *et al.*, Phys.Rev.Lett. **107**, 041801 (2011), [1106.2822].

[37] R. de Adelhart Toorop, F. Bazzocchi and S. Morisi, Nucl.Phys. **B856**, 670 (2012), [1104.5676].