

Bi-large neutrino mixing and the Cabibbo angle

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Recent measurements of the neutrino mixing angles cast doubt on the validity of the so-far popular tri-bimaximal mixing ansatz. We propose a parametrization for the neutrino mixing matrix where the reactor angle seeds the large solar and atmospheric mixing angles, equal to each other in first approximation. We suggest such *bi-large* mixing pattern as a model building standard, realized when the leading order value of θ_{13} equals the Cabibbo angle λ_C .

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INTRODUCTION

Understanding the structure of neutrino mixing from first principles is part of the flavor problem of the Standard Model, one of the deepest in particle physics. A partial useful strategy is to formulate attractive mixing patterns that may help to seek for possible underlying flavor symmetries. The tri-bimaximal mixing (TBM) ansatz [1] for describing the neutrino mixing matrix [2] assumes a maximal atmospheric angle and zero reactor angle, as was suggested by the experimental data. Such striking features point towards an underlying symmetry and indeed, many models based on (mostly discrete) non-abelian flavour symmetries were successful in reproducing this ansatz [3, 4]. Small deviations from the TBM are expected on theoretical grounds.

However the recent results published by the Double-Chooz [5], Daya Bay [6], RENO [7], T2K [8] and MINOS [9] collaborations, indicate that the reactor angle is relatively large so that corrections to the TBM pattern should be, in fact, quite large, casting doubt on its validity as a good first approximation reproducing the neutrino mixing pattern. To be more precise, on theoretical grounds a small deviation of order of the square of the Cabibbo angle was expected for the reactor angle, while recent observations indicate a much larger value of about the order of the Cabibbo angle. To evade this problem, different ansatz have been considered like the bimaximal mixing [10] or the Golden ratio, see [11] for a review. However, most of these models assume a $\mu - \tau$ -invariant structure in order to predict a maximal atmospheric mixing angle. On the other hand, at the Neutrino 2012 conference the MINOS collaboration also gave hints for a non-maximal atmospheric mixing angle.

Here we propose a different approach where we take the reactor mixing angle as the fundamental parameter. As will be shown below, the resulting parametrization does not reproduce the TBM pattern as a limiting case, though a maximal atmospheric angle can be obtained. The main idea is that, since the reactor angle is the only small mixing parameter for the leptons, we can use it to

seed both the solar and atmospheric mixing angles, as follows,

$$\begin{aligned}\sin \theta_{13} &= \lambda; \\ \sin \theta_{12} &= s \lambda; \\ \sin \theta_{23} &= a \lambda,\end{aligned}\tag{1}$$

where the small parameter λ is the reactor angle, while $s \simeq a$ are free parameters of order few. Solar and atmospheric mixings are expressed in terms of a linear dependence on the reactor angle. In the limit where $\lambda \rightarrow 0$ neutrinos are unmixed.

Using the general symmetric parametrization of the neutrino mixing matrix [2] one can trivially obtain a simple approximate description by expanding only in the small parameter λ . For example, the Jarlskog-like invariant describing CP violation in neutrino oscillations is then given by:

$$J_{CP} \approx a s \lambda^3 \sqrt{1 - a^2 \lambda^2} \sqrt{1 - s^2 \lambda^2} \sin(\phi_{13} - \phi_{12} - \phi_{23})\tag{2}$$

given explicitly in terms of the rephase-invariant Dirac combination. Likewise, the effective mass parameter describing the amplitude for neutrinoless double-beta decay is given in terms of the two Majorana CP phases.

In what follows, for simplicity, we take all parameters to be real. In order to fix the values of the large parameters s and a we consider the latest experimental results on neutrino oscillation parameters. At the Neutrino 2012 conference in Kyoto the MINOS Collaboration has reported a non-maximal value for the atmospheric mixing angle [9]:

$$\sin^2 2\theta_{23} = 0.94_{-0.05}^{+0.04}\tag{3}$$

with maximal mixing disfavoured at 88% C.L. This result comes from the analysis of ν_μ disappearance in the

¹ In a three-neutrino analysis this quantity will be equal to $4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) = 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23})$.

MINOS accelerator beam and corresponds to two degenerate points for $\sin^2 \theta_{23}$, namely

$$\sin^2 \theta_{23} = 0.38 \quad \text{and} \quad \sin^2 \theta_{23} = 0.62. \quad (4)$$

This comes from the fact that the disappearance channel is octant-symmetric and therefore MINOS data by themselves can not show a preference for a given octant of θ_{23} . However, one may expect that in combination with the searches for electron-neutrino appearance at the long-baseline experiments MINOS and T2K, and with the recent measurements of θ_{13} at reactor experiments the degeneracy in Eq. (3) will be broken and one octant will be preferred over the other. Global analysis so far have not been able to give a completely clear picture about the true octant of θ_{23} . The analysis given in [12] indicates small deviations of maximality, with the octant preference correlated with the mass hierarchy. This correlation is also seen in the recent analysis of atmospheric neutrino data in Super-Kamiokande [13]. However, the analyses given in [14, 15] have shown a preference for $\theta_{23} < \pi/4$ with different levels of significance. All previous neutrino oscillation global fits agree in getting θ_{23} in the first octant for normal mass hierarchy, so for the purpose of this article we will assume that this is the case.

BI-LARGE MIXING

Here we discuss the recent neutrino oscillation results in terms of our proposed ansatz. As already pointed out above, the recent experimental data provide a robust measurement of a relatively large θ_{13} .

Using the best fit values of the mixing angles in Ref. [12] or Ref. [14], we can fix the three parameters in Eq. (1): $\lambda \sim 0.16$ (0.15), $a \sim 4.13$ (4.21) and $s \sim 3.53$ (3.65). Then, from the data we can directly read that

$$\sin \theta_{12} = \mathcal{O}(\sin \theta_{23}), \quad (5)$$

Now we go a step further and assume the following

$$\sin \theta_{12} = \sin \theta_{23}, \quad (6)$$

which in our parametrization means

$$s = a. \quad (7)$$

Since both solar and atmospheric angles are large we call this case *bi-large mixing ansatz*.

Suppose now that we are given some model predicting bi-large mixing $a = s$ at leading order. Next-to-leading order operators in the Lagrangian in general induce deviations from the reference values in Eq. (1) which may

be reliably determined within a given model. Here we present a simple model-independent estimate of such corrections, obtained as follows. Typically it is expected that the corrections to the three mixing angles from next to leading order terms are of the same order, that is $\sin \theta_{ij} \rightarrow \sin \theta_{ij} \pm \epsilon$ where we have introduced a new parameter ϵ to characterize the magnitude of the correction. In this case our bi-large mixing gets corrections of the same order for the three mixing angles (given by ϵ) and which may either increase or decrease the starting bi-large values of the mixing angles. For definiteness let us consider an example where bi-large mixing is corrected as

$$\begin{aligned} \sin \theta_{13} &= \lambda - \epsilon; \\ \sin \theta_{12} &= s\lambda - \epsilon; \\ \sin \theta_{23} &= a\lambda + \epsilon. \end{aligned} \quad (8)$$

where we take $s = a$ as in Eq. (7). Since we have three free parameters, we can fix them using the best fit values reported by global analysis of neutrino oscillation data in Refs. [12, 14]. The results are given in Table I.

Ref.	λ	s	ϵ
Forero <i>et al.</i> [12]	0.23 ± 0.04	$2.8^{+0.5}_{-0.4}$	$0.067^{+0.035}_{-0.025}$
Fogli <i>et al.</i> [14]	$0.19^{+0.03}_{-0.02}$	$3.0^{+0.5}_{-0.3}$	$0.038^{+0.019}_{-0.018}$

TABLE I: Best fit values and 1σ ranges for the parameters λ , s and ϵ in Eq. (8) according to the global fits to neutrino oscillations.

In order to quantitatively clarify the role of the relation in Eq. (7) with respect to the reactor mixing angle, we consider here the most general case given by Eq. (8) where the three angles are given in terms of four parameters instead of three. Three of these parameters can be fixed from the three measured mixing angles, leaving one free parameter that we choose to be λ . In order to quantify the deviation from our exact bi-large mixing ansatz defined in Eq. (7) we plot the combination $(a-s)/(a+s)$ as a function of the expansion parameter λ in Fig. 1. The colored/shaded bands are calculated from the two and three sigma allowed ranges for the neutrino oscillation parameters obtained in the current global fits. The solid and dashed lines indicate the best fits of Refs. [12] and [14], respectively. It is remarkable that the strict bi-large ansatz in Eq. (7) holds when $\lambda \simeq \lambda_C$ where $\lambda_C \approx 0.22$. This means that λ_C is the leading order value of $\sin \theta_{13}$.

In Fig. 2 we show the average value of a and s , that is $(a+s)/2$, as a function of λ . As displayed in the figure, the correlation is such that $(a+s)/2 \sim 3$ when $\lambda \sim \lambda_C$. It follows that one possible form of our bi-large ansatz,

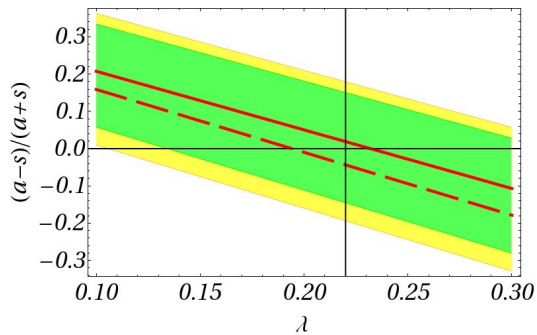


FIG. 1: Deviation from the bi-large ansatz versus the expansion parameter λ at two and three sigma in the neutrino oscillation parameters. The solid and dashed lines indicate the best fits of Refs. [12] and [14], respectively. The strict bi-large ansatz holds when $\lambda \simeq \lambda_C$ (vertical line).

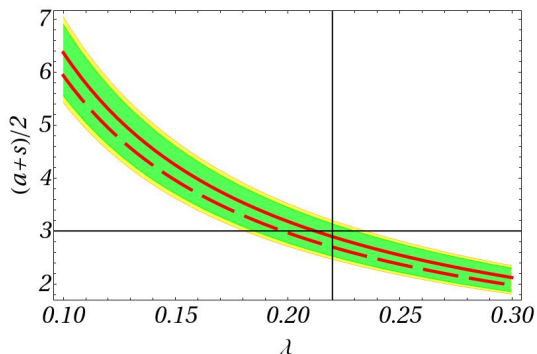


FIG. 2: Average of solar and atmospheric angles versus the expansion parameter λ at two and three sigma in the neutrino oscillation parameters. The solid and dashed lines indicate the best fits of Refs. [12] and [14], respectively.

which can be useful for model building, is

$$\begin{aligned} \sin \theta_{13} &= \lambda; \\ \sin \theta_{12} &= 3\lambda; \\ \sin \theta_{23} &= 3\lambda. \end{aligned} \quad (9)$$

It is remarkable to see how such a simple form is nearly consistent with current global neutrino oscillation data.

Now a few words on model building. Given a particular mixing matrix U , the structure of the neutrino mass matrix is fixed as

$$m_\nu = U \cdot D \cdot U^T$$

where D is a diagonal matrix. For the sake of illustration, we consider in our mixing ansatz in Eq. (9) a normal and strongly hierarchical spectrum ($m_{\nu_1} = 0$) for neutrino masses and fix the square mass differences at their best fit values, as given in Ref. [12]. We find that the resulting

weak-basis neutrino mass matrix m_ν has the form

$$m_\nu \sim \begin{pmatrix} 0.20 & 0.32 & 0.15 \\ & 0.75 & 0.70 \\ & & 1 \end{pmatrix} \sim \begin{pmatrix} \lambda_C & \lambda_C & \lambda_C \\ & 1 & 1 \\ & & 1 \end{pmatrix} \quad (10)$$

where in the last step the Cabibbo angle is used as the expansion parameter and we do not specify numerical coefficients of order one.

From the form obtained in Eq. (10) one sees that the parameter λ_C appears only in the first row. On the other hand in the “atmospheric sector” there seems to be “democracy” in the choice of the neutrino mass entries. Altogether the above indicates two general features regarding the neutrino mass generation mechanism.

- a Froggatt-Nielsen-like flavour symmetry [16] that could perhaps generate the required pattern given in Eq. (10).
- some gravity-like [17] “flavor-blind” mechanism operating within the “atmospheric sector”. The problem in this case is that, for reasonable values of the coefficients of the dimension five operator, the induced neutrino masses are too small. However there may be other well-motivated “anarchy”-type schemes [18, 19].

It is beyond the scope of this paper to provide a definite model realization, but rather to stress the simplicity of the ansatz in Eq. (10) which may provide a fresh model-building guideline that may perhaps replace the tri-bimaximal ansatz.

SUMMARY AND OUTLOOK

We have argued that recent experimental results on neutrino oscillations are very well-described by the ansatz in Eq. (1) with small deviations, as in Eqs. (8). It is truly remarkable that with the expansion parameter λ taken as $\lambda \simeq \lambda_C$, the Cabibbo angle characterizing the quark mixing matrix, we obtain the simplest bi-large limit, $s = a$, given in Eq. (7). This appears to be an important numerical “coincidence” which may drastically change our theoretical approach for constructing neutrino mass models, by moving from a *geometrical* interpretation of the neutrino mixing angles to one in which these are no longer associated to Clebsch-Gordan coefficients of any symmetry, in sharp contrast to the previous paradigm². The simple correlations between ansatz deviations and

² We should stress, however, that the TBM pattern may still be tenable if the underlying theory is capable of providing sufficiently large corrections to θ_{13} without affecting too much the solar angle, which is in principle possible.

λ are illustrated in Figs. 1 and 2. The bi-large ansatz holds for $\lambda \simeq \lambda_C$ (vertical line). It remains to be seen whether nature is perhaps telling us something profound regarding the ultimate theory of flavour.

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