

Chern-Simons anomaly as polarization effect

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Abstract. The parity violating Chern-Simons term in the epoch before the electroweak phase transition can be interpreted as a polarization effect associated to massless right-handed electrons (positrons) in the presence of a large-scale seed hypermagnetic field. We reconfirm the viability of a unified seed field scenario relating the cosmological baryon asymmetry and the origin of the protogalactic large-scale magnetic fields observed in astronomy.

Keywords: Chern-Simons anomaly, plasma polarization, hypermagnetic field, dynamo

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1 Introduction

Magnetic fields are known to play an important role in the physics of a variety of astrophysical objects, from stars to galaxies and galaxy clusters. The nature of the initial weak seed fields for the following dynamo or turbulent amplification is largely unknown [1, 2]. It might be that the seed fields are produced during the epoch of galaxy formation from frozen-in magnetic fields of proto-galaxies experiencing gravitational collapse, or ejected by the first supernovae or active galactic nuclei.

Primordial hypermagnetic fields may alternatively arise from phase transitions in the very early universe, before electroweak phase transition, such as during the inflationary epoch [3].

The clue for choosing between these possibilities may lie in measurements of the initial seed fields. However, up to recently there was little hope that extremely weak fields outside galaxies and galaxy clusters would be ever be detected. Nevertheless, the extragalactic magnetic fields originated from a seed magnetic field in the early Universe could, in principle, be observed in γ -astronomy with help of satellite instruments like FERMI [4].

Here we adopt the second scenario with a cosmological seed field present before the epoch of electroweak phase transition. Starting from the effective Lagrangian for the hypercharge gauge field in the presence of the seed field, we derive the parity violating Chern-Simons term as resulting from a polarization effect of the seed field upon the primeval plasma. This way we provide a clear physical interpretation for the Chern-Simons term, as resulting from the right-handed electron chemical potential and the associated asymmetry.

Solving Faraday equation one finds exponential amplification of the seed hypermagnetic field [5] through the α^2 -dynamo mechanism. Such a seed hypermagnetic field is subsequently converted into a seed Maxwellian field. It has been shown in Ref. [6] that, thanks to the anomaly term [7, 8], such seed field may induce a sizeable baryon asymmetry of the Universe, providing an alternative to conventional leptogenesis [9] which directly involves nonzero neutrino masses [10], induced by the seesaw mechanism [11].

We revisit briefly such “magneto-baryogenesis” scenario and confirm that, indeed, the required field strength estimates needed to account for the cosmological baryon asymmetry match those inferred by current galactic magnetic field observations, providing a remarkable connection between astronomical and cosmological observations.

2 Chern-Simons as Polarization

Despite the various conserved charges of the standard model, the early electroweak plasma at temperatures above the characteristic chiral-flip temperature, $T_{RL} \sim \text{few } 10 \text{ TeV}$, is described by just one non-zero chemical potential associated to right-handed electrons, μ_{eR} , with the corresponding number perturbatively conserved [12]. Above T_{RL} the statistically averaged standard model Lagrangian density for the hypercharge field Y_μ contains a term

$$f_R(g') < \bar{e}_R \gamma_\mu e_R > Y^\mu \quad (2.1)$$

involving $\mu_{eR} \neq 0$. Here g' is the $U_Y(1)$ gauge coupling, and $f_R(g') = g' y_R / 2$ plays the role of an "electric" charge associated to $U_Y(1)$, $y_R = -2$ being hypercharge of the right-handed electron.

For simplicity we take an external large-scale hypermagnetic field $\mathbf{B}_Y = (0, 0, B_Y)$ directed along an arbitrary "z" axis. In the presence such field one can take the statistical average using the equilibrium density matrix for right-handed electrons (positrons),

$$f_{\lambda'\lambda}^{(e_R, \bar{e}_R)}(\varepsilon(p_z, n, \lambda)) = \frac{\delta_{\lambda'\lambda}}{\exp[(\varepsilon(p_z, n, \lambda) \mp \mu_{eR})/T] + 1}. \quad (2.2)$$

The resulting Landau spectrum of the massless right-handed electrons (positrons),

$$\varepsilon(p_z, n, \lambda) = \sqrt{p_z^2 + |f_R(g')| B_Y (2n + 1 \mp \lambda)}, \quad (2.3)$$

depends on the Landau number $n = 0, 1, 2, \dots$, and the spin projection on the hypermagnetic field $\lambda = \pm 1$, $(\sigma_z)_{\lambda'\lambda} = \lambda \delta_{\lambda'\lambda}$. In Eqs. (2.2), (2.3) the upper sign applies to particles, the lower one to antiparticles. Note that together with chirality $\gamma_5 \Psi_{eR} = +\Psi_{eR}$ the spin projection λ is a good quantum number since $[\gamma_5, \Sigma_z] = 0$.

Thus, the corresponding macroscopic right-handed electron four-current in eq. (2.1) includes the pseudovector term for which the 3-vector component \mathbf{J}_5^Y is given by

$$\begin{aligned} J_{j5}^Y &= \frac{f_R(g')}{2} < \bar{e} \gamma_j \gamma_5 e > = \frac{f_R(g')}{2} \sum_{n=0}^{\infty} \frac{|f_R(g')| B_Y}{(2\pi)^2} \int_{-\infty}^{+\infty} dp_z \times \\ &\times \text{Tr} \left[\sigma_j \left(f^{(e_R)}(\varepsilon(p_z, n, \lambda)) - f^{(\bar{e}_R)}(\varepsilon(p_z, n, \lambda)) \right) \right] = \\ &= \frac{f_R(g')}{2} \sum_{n=0}^{\infty} \frac{|f_R(g')| B_Y \delta_{jz}}{(2\pi)^2} \int_{-\infty}^{+\infty} dp_z \sum_{\lambda} \lambda \left[f_{\lambda\lambda}^{(e_R)}(\varepsilon(p_z, n, \lambda)) - f_{\lambda\lambda}^{(\bar{e}_R)}(\varepsilon(p_z, n, \lambda)) \right] = \\ &= \frac{f_R(g') |f_R(g')| B_Y \delta_{jz}}{(2\pi)^2} \int_0^{\infty} \left[\frac{1}{\exp[(p - \mu_{eR})/T] + 1} - \frac{1}{\exp[(p + \mu_{eR})/T] + 1} \right] dp. \end{aligned} \quad (2.4)$$

Here summing over λ and n in the second line in Eq. (2.4) we used the cancellation of all degenerate terms $n = 1, 2, \dots$. This happens separately for particles and for antiparticles due to $\varepsilon_{n+1,1} = \varepsilon_{n,-1}$ and $\varepsilon_{n+1,-1} = \varepsilon_{n,+1}$. The asymmetry at the main Landau level in the last line gives exactly μ_{eR} for integral, so that one obtains the mean pseudovector current as

$$\mathbf{J}_5^Y = -g'^2 \mu_{eR} \mathbf{B}_Y / 4\pi^2, \quad (2.5)$$

while its time component vanishes, $J_{05}^Y = 0$, due to the zero global hypercharge condition $< Y > = 0$.

As result of statistical averaging the effective standard model Lagrangian density at finite fermion density $\mu_{eR} \neq 0$ in the early hot plasma bath ($T > T_{EW}$) takes the form [12]:

$$L = -\frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - J_\mu Y^\mu - \frac{g'^2\mu_{eR}}{4\pi^2}\mathbf{B}_Y\mathbf{Y}, \quad (2.6)$$

where J_μ is the vector (ohmic) current with zero time component due to the electro-neutrality of the plasma as a whole, $J_0 = \langle Q \rangle = 0$.

This way the physical meaning of the mean pseudovector current eq. (2.5) emerges, in terms of comoving right-handed electrons and positrons at the main Landau level in the external hypermagnetic field, in a way similar to the discussion given in Ref. [5]¹. Thanks to the Coulomb force electrons and positrons move in the same direction with respect to the hypermagnetic field, with a net electric current \mathbf{J}_5^Y resulting from a slight difference in their densities if $\mu_{eR} \neq 0$. In other words, the origin of the Chern-Simons interaction term $\mathbf{J}_5\mathbf{Y}$ as a polarization effect is a direct consequence of the spin paramagnetism of electrons and positrons at the main Landau level which leads to a magnetization in opposite directions, $\lambda = \mp 1$, weighted with different populations due to the asymmetry density, $\mu_{eR} \neq 0$. According to Faraday equation the current in eq. (2.5) induces a "wrong" *transversal* component of hypermagnetic field $\nabla \times \alpha_Y \mathbf{B}_Y$, where $\alpha_Y = -g'^2\mu_{eR}/4\pi^2\sigma_{cond}$ is the hypermagnetic helicity parameter, $\sigma_{cond} \simeq 100T$ is the plasma conductivity. This hypermagnetic field component winds around the pseudovector current \mathbf{J}_5^Y parallel to the self-consistent \mathbf{B}_Y . Note that such term violates parity, or the total hypermagnetic field has both a vector and an axial vector components.

3 Magneto-baryogenesis revisited

We now briefly revisit the "magneto-baryogenesis" scenario proposed in Ref. [6]. Its basic ingredient is a primordial hypercharge field that induces a nonzero lepton asymmetry of the early universe plasma through the Abelian anomaly for right electrons,

$$\partial_\mu j_{eR}^\mu = -\frac{g'^2 y_R^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu}. \quad (3.1)$$

Irrespective of such anomaly one can re-derive Faraday equation from the effective Lagrangian we have obtained through the statistical averaging of the standard model Lagrangian in vacuum quantum field theory. Following Ref. [5], we take the rest frame of the Universe as a whole, and re-obtain the α^2 -dynamo amplification of the primordial seed hypermagnetic field, instead of $\alpha\Omega$ -dynamo mechanism of standard magnetohydrodynamics [15], namely

$$\begin{aligned} B_Y(t) &= B_0^Y \exp \left[\left(\frac{1}{\kappa} - \frac{1}{\kappa^2} \right) \int_{t_0}^t \frac{\alpha_Y^2(t')}{\eta_Y(t')} \right] = \\ &= B_0^Y \exp \left[83 \left(\frac{1}{\kappa} - \frac{1}{\kappa^2} \right) \int_x^{x_0} \frac{dx'}{x'^2} \left(\frac{\xi_{eR}(x')}{0.0001} \right)^2 \right]. \end{aligned} \quad (3.2)$$

¹By contrast, we recall that the conventional derivation of the Chern-Simons term [13, 14] involves the use of alternative one-loop diagrammatic calculations in finite temperature field theory.

Here Λ denotes an arbitrary scale of the hypermagnetic field $\Lambda = \kappa\eta_Y/\alpha_Y$, $\kappa > 1$, where $\eta_Y = 1/\sigma_{cond}$ is the magnetic diffusion coefficient; $\xi_{eR} = \mu_{eR}/T$ is the dimensionless right electron asymmetry parameter; $x = T/T_{EW} \geq 1$; the moment $x_0 \gg 1$ corresponds to the initial time $t_0/t_{EW} = x_0^{-2} = (T_{EW}/T_0)^2$ when a tiny seed hypermagnetic field B_0^Y starts to polarize hot plasma at the initial temperature $T_0 \gg T_{EW}$.

Note that the ‘‘slope’’ of hypermagnetic field enhancement in the α^2 -dynamo mechanism given in eq. (3.2) differs from that obtained in Ref. [5] using the net neutrino asymmetry instead of the right-handed electrons asymmetry used above, and dictated by the correct equilibrium conditions found in Ref. [12].

Turning to the baryon asymmetry of the Universe, η_B , in Ref. [6] it was noted that, due to the global charge conservation

$$2dL_{eL}/dt = -dL_{eR}/dt + \dot{B}/3$$

and the presence of the Abelian anomaly term (3.1), at the electroweak phase transition epoch η_B ‘‘sits’’ in the hypermagnetic field [12],

$$\eta_B(t_{EW}) = \frac{3g'^2}{4\pi^2 s} \int_{t_0}^{t_{EW}} [\alpha_Y B_Y^2 - \eta_Y (\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y] dt, \quad (3.3)$$

where $s = 2\pi^2 g^* T^3/45$ is the entropy density. Now substituting α_Y for right-handed electrons, instead of that for neutrinos, and neglecting the diffusion term $\sim \eta_Y$ one notes that today’s observed baryon asymmetry

$$\eta_B \simeq 10^{-10}$$

is reproduced if

$$\left(\frac{|\xi_{eR}|}{0.0001} \right) \frac{B_Y^2(t_{EW})}{T_{EW}^4} \simeq 7 \times 10^{-14}, \quad (3.4)$$

at the electroweak phase transition epoch. For example if we take $|\xi_{eR}| \leq 10^{-5}$ we get from Eq. (3.4) $B_0 \geq 10^{18}$ G for the initial Maxwellian seed field $B_0 = \cos\theta_W B_Y$ at t_{EW} .

Let us discuss our reference value choice for the right-electron asymmetry $|\xi_{eR}| \simeq 0.0001$ used above. In the adiabatic approximation $\dot{s} = \dot{T} = 0$ for the right-electron asymmetry density $n_{eR} = \mu_{eR}T^2/6$ one gets, from eq. (3.1),

$$\frac{d\xi_{eR}}{dt} = -\frac{6g'^2}{4\pi^2 T^3} \mathbf{E}_Y \mathbf{B}_Y,$$

where \mathbf{E}_Y ,²

$$\mathbf{E}_Y = \eta_Y \nabla \times \mathbf{B}_Y - \mathbf{V} \times \mathbf{B}_Y - \alpha_Y \mathbf{B}_Y,$$

is the hyper-electric field. With this one gets the ordinary differential equation

$$\frac{d\xi_{eR}}{dt} + [P(t) + \Gamma(t)]\xi_{eR} = Q(t). \quad (3.5)$$

²Such field comes from the Ohm law $\mathbf{J} = \sigma_{cond}[\mathbf{E}_Y + \mathbf{V} \times \mathbf{B}_Y]$, substituted into the generalized Maxwell equation $-\partial_t \mathbf{E}_Y + \nabla \times \mathbf{B}_Y = \mathbf{J} + \mathbf{J}_5^Y$, derived from the effective Lagrangian (2.6) with the pseudovector current \mathbf{J}_5^Y given by eq. (2.5) when neglecting in MHD approach the displacement current $\partial_t \mathbf{E}_Y$.

Here the coefficients P, Q given by

$$P(t) = \left(\frac{6g'^2}{4\pi^2 T^3 \sigma_{cond}} \right) \frac{g'^2 B_Y^2(t) T}{4\pi^2},$$

$$Q(t) = - \left(\frac{6g'^2}{4\pi^2 T^3 \sigma_{cond}} \right) k_0 B_Y^2(t)$$

depend on $\xi_{eR}(t)$ through the hypermagnetic field amplitude in eq. (3.2). In getting the last coefficient Q we used the Chern-Simons wave $Y_0 = Y_z = 0$, $Y_x = Y_0(t) \sin k_0 z$, $Y_y = Y_x = Y_0(t) \cos k_0 z$ as the simplest configuration for the hypercharge field which allows to get $(\nabla \times \mathbf{B}_Y) \cdot \mathbf{B}_Y = k_0 B_Y^2(t)$. In Eq. (3.5) $\Gamma = 2\Gamma_{RL}$ denotes the rate for chirality flip processes (inverse Higgs decays $e_R \bar{e}_L \rightarrow \varphi^{(0)}$, $e_R \bar{\nu}_{eL} \rightarrow \varphi^{(-)}$ with equivalent rates), and we neglect for simplicity left lepton asymmetries.

The formal solution of the nonlinear integro-differential equation (3.5) takes the form

$$\xi_{eR}(t) = e^{-\int_{t_0}^t (P+\Gamma) dt} \left[\xi_{eR}^{(0)} + \int_{t_0}^t Q(t') e^{\int_{t_0}^{t'} [P(t'')+\Gamma(t'')] dt''} dt' \right].$$

Here we give the asymptotic solution for slowly changing hypermagnetic fields when $P \approx const$, $Q \approx const$ assuming also $\Gamma \approx const$

$$\xi_{eR}(t) = \left[\xi_{eR}^{(0)} - \frac{Q}{P+\Gamma} \right] e^{-[P+\Gamma](t-t_0)} + \frac{Q}{P+\Gamma} \approx \frac{Q}{P+\Gamma} \approx -\frac{4\pi^2 k_0}{g'^2 T}. \quad (3.6)$$

In the last step in Eq. (3.6) we have neglected chirality flip rates in strong hypermagnetic field, $\Gamma \ll P$. Indeed one has [16],

$$\Gamma_{RL} = 0.88 \times 10^{-14} \left[1 - \left(\frac{T_{EW}}{T} \right)^2 \right] T, \quad (3.7)$$

which is much less than the coefficient $P \sim B_Y^2$ above,

$$P = 5.6 \times 10^{-7} \left[\frac{B_Y}{T^2} \right]^2 T.$$

Note that for hypermagnetic field $B_Y(t)$ frozen-in ideal plasma the ratio $B_Y/T^2 = const$ during the cooling of the Universe. Hence for hypermagnetic field values, say, $B_Y = 10^{-3} T^2$, our approximation $P \gg \Gamma$ holds ³.

One can easily check that the exponential term in (3.6) vanishes at the EWPT time, since $P t_{EW} \gg 1$ for $B_Y < T^2$. Then taking into account for the survival condition of the Chern-Simons wave versus ohmic diffusion, $k_0 < 10^{-7} T$, substituting weak coupling $g'^2 = 0.12$ we get the estimate of the lepton asymmetry in a strong hypermagnetic field,

$$|\xi_{eR}| \leq 3.3 \times 10^{-5},$$

³Note that this inequality holds also for moderate fields at $T \sim T_{EW}$ where the rate (3.7) vanishes. However, such rate can modify the evolution of $\xi_{eR}(t)$ above T_{EW} and below chirality flip temperature T_{RL} , $T_{RL} > T > T_{EW}$, when left leptons enter equilibrium with the right ones, see below in Section 4.

close to what we used above. This value can be enhanced for a more complicated configuration of a helical hypercharge field accounting for the linkage integer number n (number of knots, $n = \pm 1, \pm 2, \dots$) entering the coefficient $Q \sim \mathbf{B}_Y \cdot (\nabla \times \mathbf{B}_Y \sim n)$.

The subsequent evolution of the initial Maxwellian field, $B_0 = B_Y \cos \theta_W$ given by Eq. (3.4) was studied in Ref. [6] and illustrated in Fig. 1 of that paper. Although the “slope” of hypermagnetic field enhancement found here differs from that obtained in Ref. [5] we note that this does not alter in any significant way the “magneto-baryogenesis” mechanism proposed in Ref. [6]. A very similar generic evolution picture emerges. Of course the causal approach we adopt cannot provide a complete scenario for generating the proto-galactic large-scale fields observed in astronomy. One needs to assume the presence of a large-scale field somewhere after the electroweak phase transition. For example a noncausal spectator field model, or a similar α -helicity mechanism induced by a nonvanishing neutrino asymmetry as in Ref. [17]. In both cases the magnetic field scale can exceed the horizon size at temperatures $T \leq T_{EW}$, $\Lambda_B \geq l_H(T)$, which later enters the horizon and becomes frozen-in plasma decreasing in amplitude in the standard way. Simultaneously its scale increases slower than the horizon, let us say, starting from the temperature $T \leq 1$ GeV, $z = 10^{13}$, its evolution is given by $\Lambda_B(z) = l_H(1 \text{ GeV})[(1+z)/(10^{13})]^{-1} < l_H \sim z^{-2}$.

As a result at the stage of a galaxy formation $z \simeq 10$ the magnetic field scale is too small, only $\Lambda_B \simeq 1.44 \times 10^{16} \text{ cm} \ll 1 \text{ pc}$. Thus, there are $N = (1 \text{ pc}/\Lambda_B)^3 = 8 \times 10^6$ expanding domains of randomly oriented fields for which the mean field $B_{mean} = B(z=10)/\sqrt{N}$ has the large initial galactic scale $\Lambda = 1 \text{ pc}$ and a small amplitude of the order $B_{mean} \simeq 3.5 \times 10^{-13} \text{ G}$. The corresponding dynamo mechanism then provides amplification up to $B_{gal} \sim 10^{-6} \text{ G}$. This way we reconfirm the viability of a unified scenario proposed in Ref. [6].

Let us comment on the difficulty to amplify large-scale hypermagnetic fields in any causal scenario. This holds in our dynamo mechanism as well as for the case where additional pseudoscalar coupling with hypercharge fields are assumed as in [18]. Therefore a preliminary amplification (of a seed field B_0^Y) like in the inflationary scenario seems to be important for large-scale fields.

4 Leptogenesis at $T_{EW} < T < T_{RL}$

We now turn to the issue of the possible washout of the electron asymmetry in the electroweak plasma in the early universe at temperatures below the chirality flip temperature T_{RL} . The evolution of the asymmetries is determined by processes involving the Higgs scalar, as well as a pseudoscalar term proportional to $\mathbf{E}_Y \mathbf{B}_Y$ and related to the anomaly.

In Quantum Electrodynamics the electron lepton number is conserved and the Abelian anomaly terms in an external electromagnetic field $F_{\mu\nu}$ [19]⁴,

$$\frac{\partial j_L^\mu}{\partial x^\mu} = + \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \frac{\partial j_R^\mu}{\partial x^\mu} = - \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (4.1)$$

do not contradict to this law, $\partial_\mu j^\mu = \partial_\mu (j_L^\mu + j_R^\mu) = 0$. Conversely, in the present case, $\partial_\mu (j_L^\mu + j_R^\mu) \neq 0$, since the coupling constants $e \rightarrow g' y_{R,L}/2$ are different for right singlet e_R and left doublet $L = (\nu_{eL} e_L)^T$. In other words, the coupling of the hypercharge field is chiral, while Maxwellian field has a vector-like coupling to fermions. This difference ensures a non-zero electron asymmetry in the electroweak plasma also below T_{RL} .

⁴We change here sign of γ_5 matrix, $\gamma_5 \rightarrow -\gamma_5$, with respect to the notation used in [19]. This is in the agreement with the notation and sign for Abelian anomaly in Ref. [12].

Note that in the kinetic equations for lepton asymmetries we must include the Abelian anomalies for right electrons (3.1) as well as for the left doublet:

$$\frac{\partial j_L^\mu}{\partial x^\mu} = \frac{g'^2 y_L^2}{64\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} = \frac{g'^2}{16\pi^2} \mathbf{E}_Y \mathbf{B}_Y, \quad y_L = -1. \quad (4.2)$$

Here we are concerned with the stage of chirality flip processes when left electrons and left neutrinos enter the equilibrium with the right electrons through the Higgs decays (inverse decays). These left leptons have the same densities at temperatures below $T < T_{RL}$, $n_L = n_{eL} = n_{\nu_{eL}}$, and the same chemical potentials $\mu_{eL} = \mu_{\nu_{eL}}$.

Taking into account the Abelian anomalies for e_R and left doublet $L = (\nu_{eL} e_L)^T$ one gets the following system of kinetic equations for the lepton asymmetry densities $n_R = n_{eR} - n_{\bar{e}R}$, $n_L = n_{eL} - n_{\bar{e}L}$, or for the corresponding lepton numbers $L_{eL} = L_{\nu_{eL}} = n_L/s$, $L_{eR} = n_R/s$:

$$\begin{aligned} \frac{dL_{eR}}{dt} &= -\frac{g'^2}{4\pi^2 s} \mathbf{E}_Y \mathbf{B}_Y + 2\Gamma_{RL}(L_{eL} - L_{eR}), \quad \text{for } e_R \bar{e}_L \rightarrow \varphi^{(0)}, e_R \bar{\nu}_{eL} \rightarrow \varphi^{(-)}, \\ \frac{dL_{eL}}{dt} &= \frac{g'^2}{16\pi^2 s} \mathbf{E}_Y \mathbf{B}_Y + \Gamma_{RL}(L_{eR} - L_{eL}), \quad \text{for } \bar{e}_R e_L \rightarrow \bar{\varphi}^{(0)}, \\ \frac{dL_{\nu L}}{dt} &= \frac{g'^2}{16\pi^2 s} \mathbf{E}_Y \mathbf{B}_Y + \Gamma_{RL}(L_{eR} - L_{eL}), \quad \text{for } \bar{e}_R \nu_{eL} \rightarrow \varphi^{(+)}. \end{aligned} \quad (4.3)$$

Here the factor 2 in the first line takes into account the equivalent reaction branches for inverse Higgs scalar decays and for simplicity we neglected Higgs boson asymmetries, $n_\varphi = n_{\bar{\varphi}}$, so Higgs decays into leptons do not contribute in the kinetic equations (4.3).

Summing the equations (4.3) one can easily see that the inverse Higgs processes $\sim \Gamma_{RL}$ do not contribute in the Hooft's rule $d\eta_B/dt = 3[dL_{eR}/dt + dL_{eL}/dt + dL_{\nu_{eL}}/dt]$ directly. In other words, we find that leptogenesis exists at the stage $T < T_{RL}$ and the baryon asymmetry is generated through hypermagnetic fields.

Note that for $T > T_{RL}$, before left leptons enter equilibrium with right electrons, the anomaly (4.2) was not efficient since the left electron (neutrino) asymmetry was zero, $\mu_{eL} = \mu_{\nu_{eL}} = 0$, while a non-zero primordial right electron asymmetry, $\mu_{eR} \neq 0$, kept the baryon asymmetry at the necessary level. In other words, for $T > T_{RL}$ the anomaly (4.2) was present at the stochastic level, with $\langle \delta j_L^\mu \rangle = 0 = \langle \mathbf{E}_Y \mathbf{B}_Y \rangle$ valid only on large scales.

We stress that $L_{eL}(T_0) = 0 = \mu_{eL}$ initially, at $T_0 = T_{RL}$, while L_{eR} grew before T_{RL} . Therefore $L_{eR} - L_{eL} > 0$ remains non-zero till T_{EW} . We have checked quantitatively that this inequality holds as a result of the kinetic equations, though a detailed study of this issue is beyond scope of the present work. In contrast, in Ref. [12] $\mu_{eL} = 0$ below T_{RL} all the way down to T_{EW} , while we assume this only as an initial condition, $\mu_{eL}(T_0) = 0$ valid at $T_0 = T_{RL}$ and take into account the corresponding chirality flip reactions associated to Higgs boson decays (inverse decays). As a result, the wash-out of the lepton and baryon asymmetries at $T < T_{RL}$ does not occur in our scenario.

5 Summary

In short, by statistically averaging the standard model effective Lagrangian in the presence of a seed hypermagnetic field we have provided a novel physical interpretation of the parity violating Chern-Simons term as a polarization effect associated to massless right-handed electrons (positrons) moving in the plasma. The hyper-magnetic field will grow exponentially

via the dynamo effect and may induce today's observed cosmological baryon asymmetry, re-confirming the viability of a unified scenario proposed in Ref. [6] relating the cosmological baryon asymmetry with the origin of the protogalactic large-scale magnetic fields observed in astronomy.

As a final comment we mention the existence of alternative ways to connect the baryon asymmetry with hypermagnetic fields, as suggested in Ref. [20] using physics beyond the Standard Model.

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