

**$A_4$ -based neutrino masses with Majoron decaying dark matter**

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**Abstract**

We propose an  $A_4$  flavor-symmetric  $SU(3) \otimes SU(2) \otimes U(1)$  seesaw model where lepton number is broken spontaneously. A consistent two-zero texture pattern of neutrino masses and mixing emerges from the interplay of type-I and type-II seesaw contributions, with important phenomenological predictions. We show that, if the Majoron becomes massive, such seesaw scenario provides a viable candidate for decaying dark matter, consistent with cosmic microwave background lifetime constraints that follow from current WMAP observations. We also calculate the sub-leading one-loop-induced decay into photons which leads to a mono-energetic emission line that may be observed in future X-ray missions such as Xenia.

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## I. INTRODUCTION

The discovery of neutrino oscillations [1–5], now confirmed at reactors and accelerators [6–8], has brought neutrino physics to the center of particle physics research. Global analysis of current oscillation data indicate that the pattern of lepton mixing differs sharply from that characterizing quarks [9]. Understanding the origin of neutrino mass and the pattern of neutrino mixing angles from basic principles constitutes a major challenge [10, 11]. A paradigm framework to generate neutrino masses is provided by the seesaw mechanism, for which several realizations have been proposed [12]. The observed pattern of neutrino mixing may arise from suitable non-abelian flavour symmetries, as those based on the  $A_4$  group [13–16].

Elucidating the nature of dark matter constitutes another intriguing problem of modern physics which has so far defied all efforts. It is therefore crucial to build a fundamental particle physics theory of dark matter and, since the Standard Model of elementary particles (SM) fails to provide a dark matter candidate, such theory necessarily requires physics beyond the SM.

Here we suggest a version of the seesaw mechanism containing both type-I [17–24] and type-II contributions [23–28] in which we implement an  $A_4$  flavor symmetry with spontaneous violation of lepton number [22, 24]. We study the resulting pattern of vacuum expectation values (vevs) and show that the model reproduces the phenomenologically consistent and predictive two-zero texture proposed in Ref. [29].

In the presence of explicit global symmetry breaking effects, as might follow from gravitational interactions, the resulting pseudo-Goldstone boson - Majoron - may constitute a viable candidate for decaying dark matter if it acquires mass in the keV-MeV range. Indeed, this is not in conflict with the lifetime constraints which follow from current cosmic microwave background (CMB) observations provided by the Wilkinson Microwave Anisotropy Probe (WMAP) [30]. We also show how the corresponding mono-energetic emission line arising from the sub-leading one-loop induced electromagnetic decay of the Majoron may be observed in future X-ray missions [31].

The paper is organized as follows. In section II we describe our  $A_4$  model while in section III we discuss the symmetry breaking structure which is required to obtain the correct neutrino texture. In section IV, we update the neutrino parameter analysis and we study the implications of a decaying Majoron dark matter scenario in section V. Further discussion is presented in the concluding section VI.

## II. THE MODEL

Our model is described by the multiplet content specified in Table I where the transformation properties under the SM and  $A_4$  groups are shown (as well as the corresponding lepton number  $L$ ). The  $L_i$  and  $l_{Ri}$  fields are the usual SM lepton doublets and singlets and  $\nu_R$  the right-handed neutrinos. The scalar sector contains an  $SU(2)$  triplet  $\Delta$ , three Higgs doublets  $\Phi_i$  (which transform as a triplet of  $A_4$ ) and a scalar singlet  $\sigma$ . Three additional fermion singlets  $S_i$  are also included.

TABLE I: Lepton multiplet structure ( $Q = T_3 + Y/2$ )

	$L_1$	$L_2$	$L_3$	$l_{Ri}$	$\nu_{iR}$	$\Phi_i$	$\Delta$	$\sigma$	$S_i$
$SU(2)$	2	2	2	1	1	2	3	1	1
$U(1)_Y$	-1	-1	-1	-2	0	-1	2	0	0
$A_4$	1'	1	1''	3	3	3	1''	1''	3
$L$	1	1	1	1	1	0	-2	-2	1

Taking into account the information displayed in Table I, and imposing lepton number conservation, the Lagrangian responsible for neutrino masses reads

$$\begin{aligned}
-\mathcal{L}_L = & h_1 \bar{L}_1 (\nu_R \Phi)_1' + h_2 \bar{L}_2 (\nu_R \Phi)_1 + h_3 \bar{L}_3 (\nu_R \Phi)_1'' \\
& + \lambda L_1^T C \Delta L_2 + \lambda L_2^T C \Delta L_1 + \lambda' L_3^T C \Delta L_3 \\
& + M_R (\bar{S}_L \nu_R)_1 + h (S_L^T C S_L)_1' \sigma + \text{h.c.},
\end{aligned} \tag{1}$$

where  $h$  and  $\lambda$  are adimensional couplings,  $M_R$  is a mass scale and

$$\Delta = \begin{pmatrix} \Delta_0 & -\Delta^+/\sqrt{2} \\ -\Delta^+/\sqrt{2} & \Delta^{++} \end{pmatrix}, \quad \Phi_i = \begin{pmatrix} \phi_i^0 \\ \phi_i^- \end{pmatrix}. \tag{2}$$

Note that the term  $(\nu_R^T C \nu_R)_1' \sigma$  is allowed by the imposed symmetry. This term however does not contribute to the light neutrino masses to the leading order in the seesaw expansion and we omit it. Alternatively, such term may be forbidden by holomorphy in a supersymmetric framework with the following superpotential terms

$$\mathcal{W} = \dots + \lambda \epsilon_{ab} h_i^\nu \hat{L}_i^a \hat{\nu}^c \hat{H}_u^b + M_R \hat{\nu}^c \hat{S} + \frac{1}{2} h \hat{S} \hat{S} \hat{\sigma}$$

where the hats denote superfields and the last term replaces the corresponding bilinear employed in Ref. [32, 33]. Assuming that the Higgs bosons  $\Phi_i$ ,  $\Delta^0$  and  $\sigma$  acquire the

following vevs (see section III below)

$$\langle \phi_1^0 \rangle = \langle \phi_2^0 \rangle = \langle \phi_3^0 \rangle = \frac{v}{\sqrt{3}}, \quad \langle \Delta^0 \rangle = u_\Delta, \quad \langle \sigma \rangle = u_\sigma, \quad (3)$$

we obtain an extended seesaw neutrino mass matrix  $\mathcal{M}$  [32–34] in the  $(\nu_L, \nu^c, S)$  basis

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}, \quad m_D = v \text{diag}(h_1, h_2, h_3) U, \quad U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega \\ 1 & 1 & 1 \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad (4)$$

with  $\omega = e^{2\pi i/3}$ ,  $M = M_R \text{diag}(1, 1, 1)$  and  $\mu = u_\sigma h \text{diag}(1, w^2, w)$ . This leads to an effective light neutrino mass matrix  $\mathcal{M}_\nu^I$  given by

$$\mathcal{M}_\nu^I = m_D M^{T-1} \mu M^{-1} m_D^T = \frac{h v^2 u_\sigma}{M_R^2} \begin{pmatrix} h_1^2 & 0 & 0 \\ 0 & 0 & h_2 h_3 \\ 0 & h_2 h_3 & 0 \end{pmatrix}. \quad (5)$$

On the other hand the vev of the triplet,  $u_\Delta$ , will induce an effective mass matrix for the light neutrinos from type-II seesaw mechanism

$$\mathcal{M}_\nu^{\text{II}} = 2u_\Delta \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & \lambda' \end{pmatrix}, \quad (6)$$

and the total effective light neutrino mass matrix will then be

$$\mathcal{M}_\nu = \mathcal{M}_\nu^I + \mathcal{M}_\nu^{\text{II}}. \quad (7)$$

In Ref.[29] it was shown that the neutrino mass matrix given by Eq. (7) could explain the currently available neutrino data. In section IV we will present an update of that analysis taking into account the latest neutrino oscillation data.

### III. $A_4$ INVARIANT HIGGS POTENTIAL

We now address the question of the minimization of the neutral Higgs scalar potential, which is a necessary condition to reproduce the structure of the neutrino mass matrix presented in the previous section. With the assignments of Table I, the Higgs potential

consistent with gauge and  $A_4$  invariance and lepton number conservation reads,

$$V = V(\Phi) + V(\Phi, \Delta, \sigma), \quad (8)$$

where  $V(\Phi)$  is given as (the decomposition of the tensorial product of two triplets in  $A_4$  is shown in the Appendix):

$$\begin{aligned} V(\Phi) = & m_\Phi^2 (\Phi^\dagger \Phi)_1 + \lambda_1 (\Phi^\dagger \Phi)_1 (\Phi^\dagger \Phi)_1 + \lambda_2 (\Phi^\dagger \Phi)_{1'} (\Phi^\dagger \Phi)_{1''} \\ & + \lambda_3 (\Phi^\dagger \Phi)_{3s} \cdot (\Phi^\dagger \Phi)_{3s} + \lambda_4 (\Phi^\dagger \Phi)_{3s} \cdot (\Phi^\dagger \Phi)_{3a} + \lambda_5 (\Phi^\dagger \Phi)_{3a} \cdot (\Phi^\dagger \Phi)_{3a}, \end{aligned} \quad (9)$$

and  $V(\Phi, \Delta, \sigma)$  contains pure  $\Delta, \sigma$  terms, together with others involving mixed invariant combinations of the scalar fields. Assuming the so-called seesaw hierarchy  $u_\Delta \ll v \ll u_\sigma$  [24]<sup>1</sup>, the relevant terms in  $V(\Phi, \Delta, \sigma)$  are<sup>2</sup>

$$V(\Phi, \Delta, \sigma) = (M_\Delta^2 + \rho|\sigma|^2) \text{Tr}(\Delta^\dagger \Delta) + \lambda_\sigma |\sigma|^4 + [m_\sigma^2 + \xi (\Phi^\dagger \Phi)_1] |\sigma|^2 - (\delta \Phi^T \Delta \Phi \sigma^* + \text{h.c.}), \quad (10)$$

Taking the vacuum alignment for the Higgs doublets  $\Phi_a$  given in eq. (3) the minimization of the Higgs potential with respect to  $\Delta$  gives

$$\frac{\delta V}{\delta \Delta} = 0 \Rightarrow (M_\Delta^2 + \rho u_\sigma^2) u_\Delta - \delta v^2 u_\sigma = 0. \quad (11)$$

We stress that the  $A_4$  symmetry, together with the doublet vev alignment assumed in Eq. (3), requires that the product  $\Phi \otimes \Phi \sim \mathbf{1}$  under  $A_4$ . If  $\Phi \otimes \Phi \sim \mathbf{1}', \mathbf{1}''$ , then the second term in the above equation would reduce to  $2\delta(1 + \omega + \omega^2)u_\sigma = 0$  implying  $u_\Delta \sim 0$ . Moreover, as a direct consequence of the requirement  $\Phi \otimes \Phi \sim \mathbf{1}$  under  $A_4$ ,  $\Delta$  and  $\sigma$  must have the same (singlet) transformation properties under that group.

The above equation leads to the following solution for the triplet vev

$$u_\Delta = \frac{\delta v^2 u_\sigma}{M_\Delta^2 + \rho u_\sigma^2} \simeq \frac{\delta v^2}{\rho u_\sigma}, \quad (12)$$

where the last approximation holds for  $M_\Delta \ll u_\sigma$ . This result shows that the ‘‘vev-seesaw’’

<sup>1</sup> In contrast to the inverse seesaw models used in Refs. [33, 34] here we consider large values of  $u_\sigma$ ,  $u_\sigma > 10^7$  GeV or so.

<sup>2</sup> Notice that the scalar potential contains other invariant terms such as  $\Phi^\dagger \Phi \text{Tr}(\Delta^\dagger \Delta)$ ,  $\text{Tr}(\Delta^\dagger \Delta) |\sigma|^2$ ,  $[\text{Tr}(\Delta^\dagger \Delta)]^2$ , etc. Assuming the vev hierarchy  $u_\Delta \ll v \ll u_\sigma$  and that the adimensional coefficients of these terms are of the same order of the ones in  $V(\Phi, \Delta, \sigma)$ , then  $V(\Phi, \Delta, \sigma)$  is enough for our purposes.

relation  $u_\Delta u_\sigma \sim v^2$  is fulfilled. The minimization with respect to the  $\Phi_a$  gives

$$\frac{\delta V}{\delta \Phi_a} = 0 \Rightarrow \frac{\delta V(\Phi)}{\delta \Phi_a} + 2\xi v u_\sigma^2 - 4\delta v u_\Delta u_\sigma = 0. \quad (13)$$

Finally,

$$\frac{\delta V}{\delta \sigma} = 0 \Rightarrow 2\lambda_\sigma u_\sigma^3 + (m_\sigma^2 + \xi v^2 + \rho u_\Delta^2) u_\sigma - 2\delta v^2 u_\Delta = 0. \quad (14)$$

which, in the limit  $u_\Delta, v \ll u_\sigma$ , has the approximate solution

$$u_\sigma = \sqrt{-\frac{m_\sigma^2}{2\lambda_\sigma}}, \quad (15)$$

as it is typical from spontaneous symmetry breaking scenarios. In summary, we have shown that in our framework it is possible to achieve a consistent minimization of the scalar potential with non-zero vevs satisfying the “vev-seesaw” relation  $u_\Delta u_\sigma \sim v^2$ .

#### IV. NEUTRINO PARAMETER ANALYSIS

Given the two contributions to the light neutrino mass matrix discussed in Eqs. (5) and (6) one finds that the total neutrino mass matrix has the following structure:

$$\mathcal{M}_\nu = \begin{pmatrix} a & b & 0 \\ b & 0 & c \\ 0 & c & d \end{pmatrix}. \quad (16)$$

This matrix with two-zero texture has been classified as B1 in [35]. One can show that considering the  $(L_1, L_2, L_3)$  transformation properties under  $A_4$  as being  $(1', 1'', 1)$  or  $(1'', 1', 1)$  an effective neutrino mass matrix with  $\mathcal{M}_\nu(1, 2) = \mathcal{M}_\nu(3, 3) = 0$  is obtained (type B2 in [35]). Moreover, by choosing  $\Delta, \sigma \sim \mathbf{1}'$  and appropriate transformation properties of the  $L_i$  doublets, we could obtain the textures B1 and B2 as well. Still, the configuration  $\Delta, \sigma \sim \mathbf{1}$  would lead to textures which are incompatible with neutrino data since, in this case, both type I and type II contributions to the effective neutrino mass matrix would have the same form. Since the textures of the type B1 and B2 are very similar in what concerns to neutrino parameter predictions, we will restrict our analysis to B1, shown in (16).

In general, the neutrino mass matrix is described by nine parameters: three masses, three mixing angles and three phases (one Dirac + two Majorana). From neutrino oscillation experiments we have good determinations for two of the mass parameters (mass

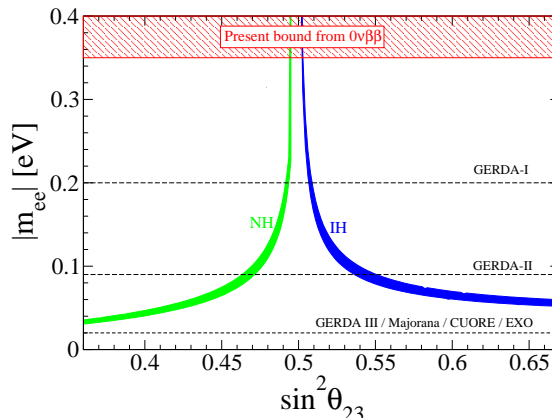


FIG. 1: Correlation between the neutrinoless double beta decay amplitude parameter  $|m_{ee}|$  and the atmospheric mixing parameter. Experimental sensitivities are also given for comparison.

squared differences) and for two of the mixing angles ( $\theta_{12}$  and  $\theta_{23}$ ) as well as an upper-bound on the third mixing angle  $\theta_{13}$ . Using the  $3\sigma$  allowed ranges for these five parameters and the structure of the mass matrix in Eq. (16) we can determine the remaining four parameters. The phenomenological implications of this kind of mass matrix have been analysed in Refs. [29] and [36]. Here we will update the results in light of the recently determined neutrino oscillation parameters [9].

The main results are shown in Figs. 1 and 2. In figure 1 we plot the correlation of the mass parameter characterizing the neutrinoless double beta decay amplitude:

$$|m_{ee}| = |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{2i\alpha} + s_{13}^2 m_3 e^{2i\beta}|, \quad (17)$$

with the atmospheric mixing angle  $\theta_{23}$ . Here  $c_{ij}$  and  $s_{ij}$  stand for  $\cos \theta_{ij}$  and  $\sin \theta_{ij}$  respectively. At the zeroth order approximation  $m_1/m_3 = \tan^2 \theta_{23}$ , and therefore  $\theta_{23} < 45^\circ$  for normal hierarchy (NH), while  $\theta_{23} > 45^\circ$  for inverted hierarchy (IH). The main result from this plot is a lower bound on the effective neutrino mass:  $|m_{ee}| > 0.03$  eV. For comparison the range of sensitivities of planned experiments as well as current bounds is also given. Note that the lower bound we obtain lies within reach of the future generation of neutrinoless double beta decay experiments.

The panels in Fig. 2 show the CP-violating phase  $\delta$  and the corresponding CP-violating invariant in neutrino oscillations:

$$J = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin \delta, \quad (18)$$

versus  $\sin^2 \theta_{13}$ . Note that these hold both for normal and inverted hierarchy spectra. In the middle panel one sees that  $\cos \delta < 0$  since, at first order in  $\sin^2 \theta_{13}$ ,  $m_1/m_2 = 1 + \frac{\cos \theta_{23}}{\cos \theta_{12} \sin \theta_{12} \sin^2 \theta_{23}} \sin \theta_{13} \cos \delta$ , and the ratio of masses should satisfy:  $m_1/m_2 < 1$ .

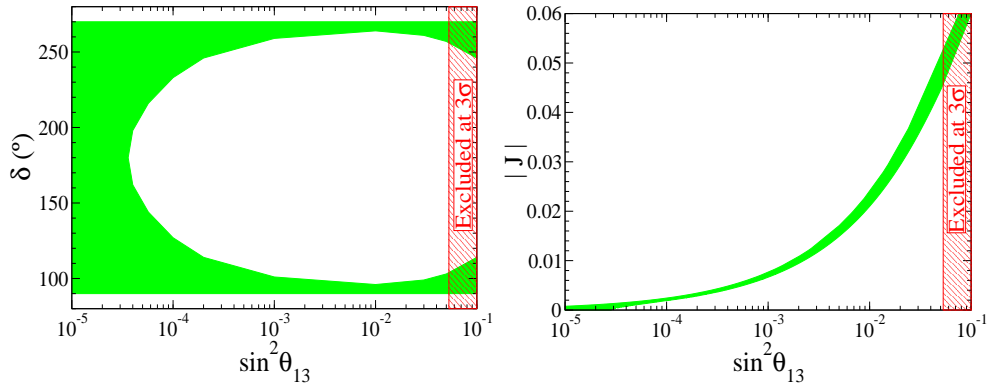


FIG. 2: CP violating phase  $\delta$  and CP-invariant  $J$  in terms of the reactor mixing parameter. The  $3\sigma$ -excluded range for  $\sin^2\theta_{ij}$  is given for comparison.

Moreover, for large  $\theta_{13}$  values, where CP violation is likely to be probed in neutrino oscillations, one can see that our model predicts maximal violation of CP. Quantitatively, from the right panel one sees that the  $3\sigma$  bound on  $\theta_{13}$ :  $\sin^2\theta_{13} < 0.053$  implies an upper bound:  $|J| \lesssim 0.06$  on the CP-invariant.

In addition, the two-zero texture structure of our neutrino mass matrix may have other implications, for example for the expected pattern of lepton flavor violating decays. In fact, thanks to the strong renormalization effects due to the presence of the triplet states, the latter are quite sizeable in supersymmetric models [37–39].

## V. MAJORON DARK MATTER

In models where neutrinos acquire mass through spontaneous breaking of an ungauged lepton number [22, 24] one expects that, due to non-perturbative effects, the Nambu-Goldstone boson (Majoron) may pick up a mass that we assume to lie in the kilovolt range [40]. This implies that the Majorons will decay, mainly in neutrinos. As the coupling  $g_{J\nu\nu}$  is proportional to  $\frac{m_\nu}{u_\sigma}$  [24], the corresponding mean lifetime can be extremely long, even longer than the age of the Universe. As a result the Majoron can, in principle, account for the observed cosmological dark matter (DM).

This possibility was explored in Refs. [41, 42] in a general context. Here, we just summarize the results. It was found that the relic Majorons can account for the observed cosmological dark matter abundance provided

$$\Gamma_{J\nu\nu} < 1.3 \times 10^{-19} \text{ s}^{-1}, \quad 0.12 \text{ keV} < \beta m_J < 0.17 \text{ keV}, \quad (19)$$

where  $\Gamma_{J\nu\nu}$  is the decay width of  $J \rightarrow \nu\nu$  and  $m_J$  is the Majoron mass. The parameter  $\beta$  encodes our ignorance about the number density of Majorons, being normalized to  $\beta = 1$



if the Majoron was in thermal equilibrium in the early Universe decoupling sufficiently early, when all other degrees of freedom of the standard model were excited [42]. In the following we will follow their choice and will take

$$10^{-5} < \beta < 1, \quad (20)$$

and calculate both the width into neutrinos as well as the subleading one-loop induced decay into photons.

### A. Decay into neutrinos

We now proceed with the computation of the Majoron decay width into neutrinos, which will be useful to obtain the allowed parameter space for which the Majoron can be a viable DM candidate. In order to calculate the decay amplitude we remind that the coupling  $g_{J\nu_i\nu_j}$  is defined through

$$\mathcal{L} = -\frac{1}{2}g_{J\nu_i\nu_j}J\nu_i\nu_j + \text{h.c.} \quad (21)$$

For the evaluation of  $g_{J\nu_i\nu_j}$ , we follow the steps developed in Ref. [24]. First we notice that with scalar potential defined in section III, the Majoron, in the basis  $[\text{Im}(\phi_i^0), \text{Im}(\Delta^0), \text{Im}(\sigma^0)]^T$ , is given by

$$J = N_J \left[ 2u_\Delta^2 \frac{v}{\sqrt{3}}, 2u_\Delta^2 \frac{v}{\sqrt{3}}, 2u_\Delta^2 \frac{v}{\sqrt{3}}, u_\Delta v^2, u_\sigma(4u_\Delta^2 + v^2) \right], \quad (22)$$

and

$$N_J = [4v^2u_\Delta^4 + v^4u_\Delta^2 + u_\sigma^2(4u_\Delta^2 + v^2)^2]^{-1/2} \simeq \frac{1}{v^2u_\sigma}, \quad (23)$$

where the last equality follows from the assumed hierarchy  $u_\Delta \ll v \ll u_\sigma$  implied by the vev-seesaw relation. Using this, one can obtain

$$g_{J\nu_i\nu_j} = -\frac{m_i^\nu \delta_{ij}}{\sqrt{2} u_\sigma}, \quad (24)$$

leading to the decay width

$$\Gamma_{J\nu\nu} = \frac{m_J}{32\pi} \frac{\sum_i (m_i^\nu)^2}{2u_\sigma^2}. \quad (25)$$

It is worth mentioning that the sum  $\sum_i (m_i^\nu)^2$  is in our framework constrained by the special form of the effective neutrino mass matrix shown in Eq. (16). In particular, there is a lower bound on the mass of the lightest neutrino:  $m \gtrsim 0.03$  eV.

## B. Decay into photons

The Majoron also couples with photons (at the quantum level) and therefore the radiative decay  $J \rightarrow \gamma\gamma$  is expected to occur with a photon energy  $E_\gamma \simeq m_J/2$ . Consequently, this decay exhibits a mono-energetic emission line which could be detected in a variety of X-ray observatories, see for example the discussion given in Refs. [31, 42].

The effective Majoron-photon interaction can be written as

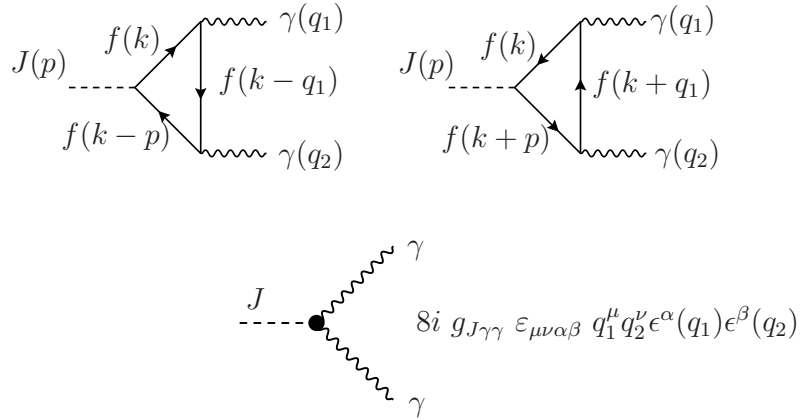


FIG. 3: Top: One loop diagrams for the decay  $J \rightarrow \gamma\gamma$ . Bottom: Effective  $J\gamma\gamma$  vertex.

$$\mathcal{L} = g_{J\gamma\gamma} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}, \quad (26)$$

resulting from the one-loop diagrams shown in Fig. 3 (top diagrams). The effective coupling  $g_{J\gamma\gamma}$  (bottom graph in Fig. 3) is

$$g_{J\gamma\gamma}^f \equiv \frac{N_f \alpha^2 g_{Jff} Q_f^2 X_f}{8\pi m_f}, \quad (27)$$

with  $X_f = -2m_f^2 C_0(0, 0, m_J^2, m_f^2, m_f^2, m_f^2) \simeq 1 + m_J^2/(12m_f^2)$  where  $C_0$  is the invariant Passarino-Veltman loop function [43]. The last approximation is valid for  $m_J \ll m_f$ .  $T_3^f$ ,  $Q_f$  and  $N_f$  denote the weak isospin, the electric charge and the colour factor of the corresponding charged fermion  $f$ , respectively. The coupling of the Majoron to the charged fermions  $g_{Jff}$  is given by [42]

$$g_{Jff} = -\frac{2u_\Delta^2}{v^2 u_\sigma} m_f (-2T_3^f). \quad (28)$$

We then get for the decay width,

$$\begin{aligned}\Gamma_{J\gamma\gamma} &= \frac{m_J^3}{\pi} \left| \sum_f g_{J\gamma\gamma}^f \right|^2 = \frac{\alpha^2 m_J^3}{64\pi^3} \left| \sum_f \frac{N_f g_{Jff} Q_f^2 X_f}{m_f} \right|^2 = \\ &= \frac{\alpha^2 m_J^3}{64\pi^3} \left| \sum_f N_f Q_f^2 \frac{2u_\Delta^2}{v^2 u_\sigma} (-2T_3^f) \frac{m_J^2}{12m_f^2} \right|^2,\end{aligned}\quad (29)$$

where the cancellation of the anomalous contribution has been taken into account.

### C. Numerical results

In this section we discuss some numerical results regarding the implementation of the decaying Majoron dark matter hypothesis in our scenario. In Ref. [42] it was shown that the experimental limit in the Majoron decay rate into photons is of the order of  $10^{-30} \text{ s}^{-1}$ . It was also shown that, in a generic seesaw model, a sizeable triplet vev plays a crucial role in bringing the decay rate close to this experimental bound. Here we have computed the width of the Majoron into neutrinos and photons in our extended seesaw model which incorporates the  $A_4$  flavor symmetry, generalizing the models of Ref. [29]. The results are

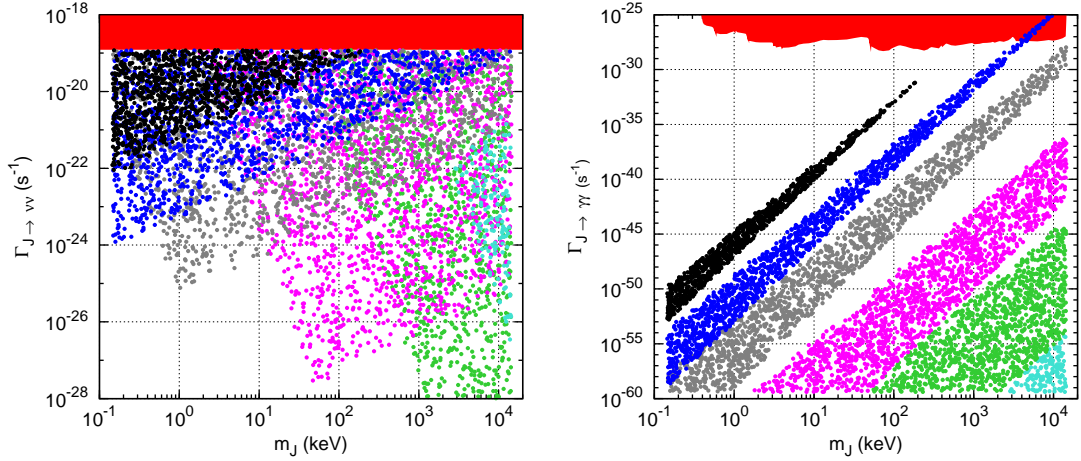


FIG. 4: Left panel:  $\Gamma_{J\nu\nu}$  as function of the Majoron mass respecting Eq. (19) for  $u_\Delta = 1 \text{ eV}$  (turquoise),  $100 \text{ eV}$  (dark green),  $10 \text{ keV}$  (magenta),  $1 \text{ MeV}$  (grey),  $10 \text{ MeV}$  (dark blue) and  $100 \text{ MeV}$  (black). Right panel:  $\Gamma_{J\gamma\gamma}$  as function of the Majoron mass for the same values of the triplet vev as in the left panel. The upper orange shaded region is the excluded region from X-ray observations taken from Ref.[31].

shown in Fig. 4. These take into account the current neutrino oscillation data, discussed in section IV. We chose five values for the triplet vev,  $u_\Delta = 1 \text{ eV}$  (turquoise),  $100 \text{ eV}$

(dark green), 10 keV (magenta), 1 MeV (grey) and 10 MeV (dark blue) and 100 MeV (black). For the right panel we consider only points that satisfy the WMAP constraint (19) indicated by the red horizontal band on the top of the left plot.

In order to be able to probe our decaying Majoron dark matter scenario through the mono-energetic emission line one must be close to the present experimental limits on the photon decay channel, discussed in Ref. [42] and references therein. As mentioned, this requires the triplet vev to be sizeable, as shown on the right panel of Fig. 4 for the same choices of  $u_\Delta$ . In principle there is an additional lower bound on the Majoron mass coming from the Tremaine-Gunn argument [44], which, for fermionic dark matter would be around 500 eV. Under certain assumptions this bound could be extended to bosons, and is expected to be somewhat weaker [45]. The upper orange shaded region is the excluded region from X-ray observations given in Ref. [31]. One should point out that, in this model, because of the vev seesaw relation  $u_\Delta u_\sigma \sim v^2$  one cannot arbitrarily take large values for  $u_\Delta$  to enhance  $\Gamma_{J\gamma\gamma}$  because then the singlet vev gets correspondingly smaller values, hence reducing the lifetime of the Majoron to values in conflict with the WMAP constraint. This interplay between the CMB bounds and the detectability of the gamma line is illustrated in Fig. 4, where the dark-blue points corresponding to  $u_\Delta = 10$  MeV illustrate the experimental sensitivity to our signal.

## VI. CONCLUSIONS

We have studied the possibility that the seesaw model with spontaneously broken ungauged lepton number may simultaneously account for the observed neutrino masses and mixing as well as the dark matter of the Universe. We have presented a two-texture structure for the neutrino mass which arises in a specific seesaw scheme implementing an  $A_4$  flavor symmetry. A predictive pattern of neutrino masses emerges from the interplay of type-I and type-II seesaw contributions, with a lower bound on the neutrinoless double beta decay rate, which correlates with the deviation from maximality of the atmospheric mixing angle  $\theta_{23}$ , as well as nearly maximal CP violation, correlated with the reactor angle  $\theta_{13}$ .

On the other hand, assuming that associated Majoron picks up a mass due to explicit lepton number violating effects that may arise, say, from quantum gravity, we showed how it can constitute a viable candidate for decaying dark matter, consistent with cosmic microwave background lifetime constraints that follow from current WMAP observations. We have also shown how the Higgs boson triplet, whose existence is required by the consistency of the model, plays a key role in providing a test of the decaying Majoron dark matter hypothesis, implying the existence of a mono-energetic emission line which arises from the sub-leading one-loop-induced decay of the Majoron into photons. We

also discussed the possibility of probing its existence in future X-ray observations such as expected in NASA’s Xenia mission [46]. The presence of the type-II seesaw Higgs triplet would also have other particle physics implications, such as lepton flavor violating decay rate enhancements due to the strong renormalization effects of the triplet, quite sizeable in a supersymmetric model.

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### Appendix A: Basic $A_4$ results

The group  $A_4$  consists of the even permutations of four elements and has three one-dimensional representations and one three-dimensional, see, e.g. [47]. Using the usual notation for transpositions and cyclic permutations (for instance,  $(123)4$  applied to  $abcd$  gives  $bcad$ ), the one-dimensional representations are shown in Table II, where  $\omega = e^{2\pi i/3}$

TABLE II: Unidimensional representations for  $A_4$ .

	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$
Class 1	1	1	1
Class 2	1	1	1
Class 3	1	$\omega^2$	$\omega$
Class 4	1	$\omega$	$\omega^2$

is the cubic root of unity, and the equivalence classes are defined as

Class 1: e

Class 2:  $(12)(34)$ ,  $(13)(24)$ ,  $(14)(23)$

Class 3:  $1(234)$ ,  $2(143)$ ,  $3(142)$ ,  $4(132)$

Class 4:  $1(243)$ ,  $2(134)$ ,  $3(124)$ ,  $4(123)$ .

It follows immediately that

$$\mathbf{1}' \times \mathbf{1}' = \mathbf{1}'', \quad \mathbf{1}' \times \mathbf{1}'' = \mathbf{1}, \quad \mathbf{1}'' \times \mathbf{1}'' = \mathbf{1}'.$$

As for the decomposition for the tensorial product of two triplets in  $A_4$  one has:

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_s \oplus \mathbf{3}_a, \quad (\text{A1})$$

where the triplet and singlet representations are

$$(u \otimes v)_{\mathbf{1}} = u_1 v_1 + u_2 v_2 + u_3 v_3 \quad (\text{A2})$$

$$(u \otimes v)_{\mathbf{1}'} = u_1 v_1 + \omega^2 u_2 v_2 + \omega u_3 v_3 \quad (\text{A3})$$

$$(u \otimes v)_{\mathbf{1}''} = u_1 v_1 + \omega u_2 v_2 + \omega^2 u_3 v_3 \quad (\text{A4})$$

$$(u \otimes v)_{\mathbf{3}_s} = (u_2 v_3 + v_3 u_2, u_3 v_1 + v_1 u_3, u_1 v_2 + u_2 v_1) \quad (\text{A5})$$

$$(u \otimes v)_{\mathbf{3}_a} = (u_2 v_3 - v_3 u_2, u_3 v_1 - v_1 u_3, u_1 v_2 - u_2 v_1). \quad (\text{A6})$$

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