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Inverse tri-bimaximal type-III seesaw and lepton flavor violation

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We present a type-III version of inverse seesaw or, equivalently an inverse version of type-III seesaw. Naturally small neutrino masses arise at low-scale from the exchange of neutral fermions transforming as hyperchargeless SU(2) triplets. In order to implement tri-bimaximal lepton mixing we supplement the minimal $SU(3) \times SU(2) \times U(1)$ gauge symmetry with an A_4-based flavor symmetry. Our scenario induces lepton flavour violating (LFV) $l_i \to l_j \bar{l}_k l_m$ decays that can proceed at the tree level, while radiative $l_i \to l_j \gamma$ decays and mu-e conversion in nuclei are also expected to be sizeable. LFV decays are related by the underlying flavor symmetry and the new fermions are also expected to be accessible for study at the Large Hadron Collider (LHC).

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I. PRELIMINARIES

Experiments [1, 2, 3, 4, 5] have now confirmed that leptonic flavour is not conserved in nature: the historical observation of neutrino oscillations has changed our picture of fundamental physics. In contrast to the quark sector, neutrino oscillations are characterized by two large mixing angles [6]. It is natural to expect that lepton flavour violation (LFV) effects also take place among the electrically charged partners of neutrinos under the weak interaction $SU(2)$. The simplest and well-motivated way to induce neutrino LFV effects is through the exchange of neutral leptons involved in generating neutrino masses via various variants [7] of the simplest type-I seesaw [8, 9, 10, 11]. The basic feature of such seesaw picture is that neutrino masses arise only as a result of the exchange of heavy gauge singlet fermions through

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \tag{1}$$

leading to an effective neutrino mass matrix

$$m_\nu = M_D^T M_R^{-1} M_D \tag{2}$$

in the (ν, ν^c) basis, where ν^c denote the heavy $SU(3) \times SU(2) \times U(1)$ singlet right-handed neutrino states which are sequentially added to the Standard Model. The smallness of neutrino mass follows naturally from the heaviness of ν^c.

As an alternative to the simplest type-I seesaw, it has long been proposed that, thanks to the protecting $U(1)_L$ global lepton number symmetry, the exchange of heavy neutral Dirac fermions implied by the matrix

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & 0 \end{pmatrix}, \tag{3}$$

(in the basis ν, ν^c, S) will keep the neutrinos massless and yet allow for LFV effects. This is the idea behind the so–called inverse seesaw model [12, 13] (for other extended seesaw schemes see, e.g. [14, 15, 16]). Note that, to each of the isodoublet neutrinos ν two $SU(3) \times SU(2) \times U(1)$ isosinglets ν^c, S are added [1]. Neutrinos get masses only when $U(1)_L$ is broken, for example through a nonzero μSS mass term. Thanks to the lepton number symmetry which arises as $\mu \to 0$ the magnitude of μ can be chosen to be small in a natural way, in the sense of 't Hooft [17]. Moreover, in specific models, the smallness of μ may be dynamically preferred [18]. After $U(1)_L$ breaking the effective light neutrino mass matrix is given as

$$M_\nu = M_D M^{T^{-1}} \mu M^{-1} M_D^T. \tag{4}$$

so that, when μ is small, M_ν is also small, even when M lies at the electroweak scale. In other words, the

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[1] For simplicity we add the isosinglet pairs sequentially, though more economical variants may be possible.

smallness of neutrino masses does not require superheavy physics.

II. TYPE-III SEESAW VARIANTS

We now turn to simple variants of the above schemes where the $SU(3) \times SU(2) \times U(1)$ singlet fermions ν^c are replaced by SU(2) triplets Σ [19].

A. Normal type III seesaw

The minimal type III seesaw model is described by the Lagrangian

$$\mathcal{L}_{III} = M_{l_{ij}} L_i l_j^c H + Y_{D_{ij}} L_i \Sigma_j \tilde{H} - \frac{1}{2} M_{\Sigma_{ij}} \mathrm{Tr}(\Sigma_i \Sigma_j) + \mathrm{h.c.} \tag{5}$$

where

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix} \tag{6}$$

denotes the hyperchargeless isotriplet fermion, $Y(\Sigma) = 0$ and $H = (\phi^+, \phi^0)^T$ is the Standard Model Higgs scalar doublet. The effective neutrino mass matrix is fully analogous to Eq. (2) and its smallness requires a very large isotriplet fermion mass.

The charged lepton mass matrix is a 6×6 matrix,

$$M_{lep} = \begin{pmatrix} M_l & M_D \\ 0 & M_\Sigma \end{pmatrix} \tag{7}$$

which is brought to diagonal form by a 6×6 unitary matrix $V_{\alpha\beta}$ of the same dimension, $\alpha, \beta = 1, .., 6$

$$V^\dagger M_{lep} M_{lep}^\dagger V = (M_{lep}^{diag})^2,$$

leading to three light fermions, namely e, μ and τ, and three heavy charged fermions C_i with $i = 1, 2, 3$.

In analogy with the matrix describing neutrino NC interactions in general type-I and type-II seesaw schemes introduced in Ref. [10] we define the $\mathcal{P}$ matrix as below

$$\mathcal{P} = \begin{pmatrix} \mathcal{P}_{LL} & \mathcal{P}_{LH} \\ \mathcal{P}_{HL} & \mathcal{P}_{HH} \end{pmatrix}. \tag{8}$$

The piece

$$\mathcal{P}_{LL} = 1 - M_D^\dagger M_\Sigma^{-2} M_D \tag{9}$$

characterizes the NC Lagrangian of charged leptons in the mass basis

$$\mathcal{L}_{NC} = \frac{g'}{c_W} \mathcal{P}_{LL_{i\alpha}} \overline{L}_i \gamma_\mu (g_V - g_A \gamma_5) L_\alpha Z^\mu. \tag{10}$$

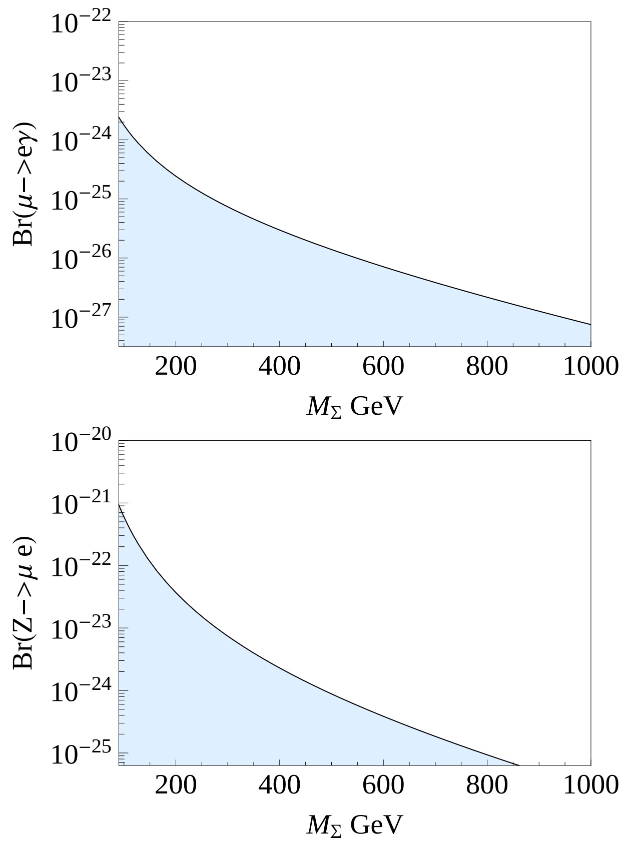

FIG. 1: Maximun attainable $\mu \to e\gamma$ and $Z \to e\mu$ decay branching ratios in normal type-III seesaw.

For finite M_Σ values there are non-diagonal elements $\mathcal{P}_{LL}$ that induce tree level FCNC among the charged leptons e, μ, τ. In other words the mixing between different isospins implies the violation of the GIM mechanism [20] with amplitude of order ϵ^2 where $\epsilon^2 \sim m_\nu M_\Sigma$. The smallness of neutrino masses implies that, for M_Σ values accessible at the LHC, and barring fine-tuned parameter choices, the expected LFV rates are expected to be too small to be of phenomenological interest. We have estimated the maximum attainable values for (i) the tree level LFV Z vertex, which also induces the $l_i \to l_j \bar{l}_k l_m$ decays, and (ii) for the electromagnetic penguin vertices, which induce the radiative LFV decays and $\mu - e$ conversion. Barring fine-tuning, one finds that they are far from the sensitivities expected in the upcoming LFV searches [21, 22, 23]. As an example Fig. 1 illustrates the expected rates for LFV Z-decay process. Similarly LFV processes involving taus are too small.

B. Inverse type-III seesaw

Having discarded normal type-III seesaw [2] as an interesting model for lepton flavor violation, we turn instead to an inverse type-III seesaw variant, characterized generically by the Lagrangian

$$\begin{aligned}\mathcal{L}_{inv} = & Y_{Dij}\, L_i\, \Sigma_j \tilde{H} + Y_{M\,ij}\, \mathrm{Tr}(\Sigma_i \Delta) S_j + \\ & + \mu_{ij} S_i S_j + Y_{l_{ij}}\, L_i\, l_j^c H \; - \tfrac{1}{2} M_{\Sigma_{ij}} \mathrm{Tr}(\Sigma_i \Sigma_j),\end{aligned} \tag{11}$$

where, as before, $H = (\phi^+, \phi^0)^T$ denotes the Standard Model Higgs scalar doublet and now Δ is a hyperchargeless scalar $SU(2)$-triplet, $Y(\Delta) = 0$

$$\Delta = \begin{pmatrix} \Delta^0/\sqrt{2} & \Delta^+ \\ \Delta^- & -\Delta^0/\sqrt{2} \end{pmatrix}, \tag{12}$$

leading to

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_\Sigma & M \\ 0 & M^T & \mu \end{pmatrix}. \tag{13}$$

This leads to six heavy states N_j with $j = 4,..,9$ and an effective three light Majorana eigenstates ν_i with $i = 1, 2, 3$. The light effective neutrino mass matrix is similar to that of the inverse seesaw model with isosinglet instead of isotriplets [3].

$$M_\nu \approx M_D M^{T^{-1}} \mu M^{-1} M_D^T. \tag{14}$$

The smallness of the parameter μ may also arise dynamically [18] and/or spontaneously in a Majoron-like scheme with $\mu \sim \langle\sigma\rangle$ where σ is a $SU(3) \times SU(2) \times U(1)$ singlet [25]. In the latter case, for sufficiently low values of $\langle\sigma\rangle$ there may be Majoron emission effects in neutrinoless double beta decay [26].

Note that now the ratio $\epsilon \sim M_D M_\Sigma^{-1}$ need not be too small to reproduce acceptably small neutrino masses, since the latter vanish in the limit where the parameter μ goes to zero [12]. The smallness of μ is not only natural [17] but also dynamically preferred in some cases [18]. The smaller the μ values the larger can be the ϵ. This implies that when the mass of Σ is accesible at LHC, say of the order of TeV, one expects relatively large LFV decay rates. In fact the situation is completely novel with respect to what one is used to, in the sense that LFV as well as CP violation effects survive even in the limit when neutrinos become massless [13] [27]. Clearly now FCNC effects can be naturally enhanced without conflict with the smallness of neutrino masses.

[2] We consider here a non-supersymmetric model. Supersymmetry adds new sources of LFV.

[3] We neglect loop contributions which exist due to the nonzero value of M_Σ. For an alternative inverse seesaw model with two lepton triplets see Ref. [24].

III. TRI-BIMAXIMAL INVERSE TYPE-III SEESAW

The neutrino mixing angles [6] indicated by neutrino oscillation experiments [1, 2, 3, 4, 5] should be explained from first principles. Here we consider the possibility of doing so in the framework of the inverse $SU(3) \times SU(2) \times U(1)$ seesaw mechanism. To this end we adopt the attractive tribimaximal (TBM) ansatz for lepton mixing [28]

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \tag{15}$$

which provides a good first approximation to the values indicated by current neutrino oscillation data.

Here we propose a simple A_4 flavor symmetry realization of the TBM lepton mixing pattern within the inverse type-III seesaw scheme. An A_4 realization of the TBM in inverse seesaw has already been studied in [29].

Recall that A_4 is the group of the even permutations of four objects. Such a symmetry was introduced to yield $\tan^2\theta_{\mathrm{atm}} = 1$ and $\sin^2\theta_{\mathrm{Chooz}} = 0$ [30, 31, 32]. Most recently A_4 has also been used to derive $\tan^2\theta_{\mathrm{sol}} = 0.5$ [33]. The group A_4 has 12 elements and is isomorphic to the group of the symmetries of the tetrahedron, with four irreducible representations, three distinct singlets 1, $1'$ and $1''$ and one triplet 3. For their multiplications see for instance Ref. [33]. The matter fields are assigned as in table I.

	L	l^c	Σ	S	ξ, ϕ	$\xi'\phi'$	Δ
$SU_L(2)$	2	1	3	1	2	2	3
Z_3	ω	ω	1	1	ω^2	ω	1
A_4	3	3	3	3	1,3	1,3	1

TABLE I: Matter assignment for inverse seesaw model.

The renormalizable [4] Lagrangian invariant under the $A_4 \times Z_3$ symmetry is

$$\mathcal{L} = {Y_D}_{ij}^k \, L_i \, \Sigma_j \phi_k + Y_D \, L_i \, \Sigma_i \xi + {Y_M}_{ij} \, \Sigma_i^0 S_j \Delta + \mu_{ij} S_i S_j + Y_{l_{ij}}^k \, L_i \, l_j^c \phi_k' + Y_l L_i \, l_i^c \xi' - \frac{1}{2} M_\Sigma \mathrm{Tr}(\Sigma_i \Sigma_j) \tag{16}$$

where from A_4-contractions one finds $\mu_{ij} \equiv \mu I_{ij}$, $M_{ij} = M I_{ij}$. When ξ takes a vacuum expectation value (vev) and ϕ takes a vev along the A_4 direction

$$\langle \phi \rangle \sim (1, 0, 0), \tag{17}$$

we generate the Dirac mass entry, given as

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b_1 \\ 0 & b_2 & a \end{pmatrix}, \tag{18}$$

where we will also assume $b_1 = b_2 = b$. Such a relation can be obtained in the context of $SO(10)$. Moreover, when ξ' and ϕ' take on nonzero vevs, the latter along the A_4 direction

$$\langle \phi' \rangle \sim (1, 1, 1), \tag{19}$$

we induce the charged lepton mass matrix as

$$M_l = \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix} = U_\omega \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\nu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} U_\omega^\dagger. \tag{20}$$

where

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}.$$

The light neutrino mass matrix is diagonalized by

$$V_\nu = \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix} \tag{21}$$

and the corresponding eigenvalues are

$$\{m_1, m_2, m_3\} = \frac{v_\mu}{v_M^2} \{(a+b)^2, a^2, -(a-b)^2\}. \tag{22}$$

It follows that the lepton mixing matrix $U_\omega^\dagger \cdot V_\nu$ is the tri-bimaximal matrix.

[4] Here we have introduced several Higgs doublets. We can equivalently avoid having many Higgs doublets by introducing corresponding scalar electroweak singlet *flavon* fields.

As seen in Eq. (7) the couplings $L\Sigma\phi$, $L\Sigma\xi$ give us an off-diagonal block to the following 6 by 6 charged lepton mass matrix for L and Σ. As a result the GIM mechamism is violated and there are FCNC among the charged leptons e, μ, τ at the tree level.

Note that when the Higgs doublets ϕ and ϕ' take nonzero vevs, the A_4 symmetry breaks spontaneously into its Z_2 and Z_3 subgroups, respectively. Such a *mis-alignment* implies a large mixing in the neutrino sector. The implemention of such alignment has been studied in many contexts [33, 34, 35, 36, 37, 38, 39, 40].

IV. LFV IN INVERSE TYPE-III SEESAW

A characteristic feature of our seesaw scheme based on the use of isotriplet instead of isosinglet lepton exchange is the existence of tree level FCNC among the charged leptons. While typically small in high-scale type-I seesaw, LFV effects are well known to be potentially large in low-scale seesaw schemes, such as the inverse [12, 18] or the linear seesaw [41]. In fact, in such schemes LFV rates are restricted only by weak universality limits [13] [42, 43, 44, 45] evading all constraints from the observed smallness of neutrino masses.

We now consider an inverse seesaw scheme based on an underlying A_4 flavor symmetry. In contrast to Ref. [29] we consider now a type-III seesaw variant. For simplicity we neglect contributions from Higgs boson exchange, which is a reasonable approximation. We divide the LFV decay processes into three classes: A) $Z \to l_i \bar{l}_j$, B) $l_i \to l_j \bar{l}_k l_m$ which proceed at the tree level, and C) the loop-calculable $l_i \to l_j \gamma$ decays.

Note that in our model we have only two parameters in the Dirac mass matrix plus a relative phase, and two extra TeV-scale parameters M, M_Σ, in addition to the small parameter μ characterizing the low-scale violation of lepton number. Two of these parameters are determined by solar and atmospheric splittings [6].

Note also that the two parameters M, M_Σ may be traded for the heavy lepton mass M_N, and the mixing $\cos\theta_{\Sigma S}$ which will specify its production cross section at the LHC, through the following rotation (Σ_α, S_β)

$$\begin{pmatrix} \cos\theta_{\Sigma S} I & \sin\theta_{\Sigma S} I \\ -\sin\theta_{\Sigma S} I & \cos\theta_{\Sigma S} I \end{pmatrix} \tag{23}$$

As we will see, the mass matrices are expressed in terms of very few parameters, with a strong impact in the expected pattern of LFV decays.

A. $Z \to l_i \bar{l}_j$

In our model the charged lepton mass matrix is a 6×6 matrix, which is brought to left-diagonal form by corresponding unitary matrix $V_{\alpha\beta}$ of the same dimension, $\alpha, \beta = 1, .., 6$, leading to three light fermion masses, namely e, μ and τ, and three heavy charged fermions C_i with $i = 1, 2, 3$.

Defining the $\mathcal{P}$ matrix as in Eq. (8) one expresses the NC Lagrangian in the mass basis as in Eq. (10) where

$$\mathcal{P}_{LL} = 1 - U_\omega^\dagger M_D^\dagger M_\Sigma^{-2} M_D U_\omega \tag{24}$$

This implies that for $i \neq j$ we have

$$\Gamma(Z \to l_i \bar{l}_j) = \frac{G_F M_Z^3}{6\sqrt{2}\pi}(g_V^l + g_A^l)^2 |\mathcal{P}_{LL_{ij}}|^2, \tag{25}$$

where g_A and g_V are respectively the axial and vector couplings of the charged leptons. This way one gets an effective GIM-mechanism-violating vertex which possesses a well-defined structure that follows from the flavor symmetry. This relates ratios of branching ratios of FCNC decays. However, none of these decays is allowed to be large in view of the stringent bounds on LFV muon violating decays, see below.

B. $l_i \to l_j \bar{l}_k l_m$

This process occurs through the exchange of a virtual Z boson, due to the basic $Z l_i \bar{l}_j$ vertex. The resulting branching ratio is

$$\Gamma(l_i \to l_j \bar{l}_k l_m) = \frac{G_F m_i^5}{192\pi^3} Q_i Q_k |\mathcal{P}_{LL_{ij}} \mathcal{P}_{LL_{km}}|^2$$

where Q_i are the electroweak charges defined as $g_V^{l_i} + g_A^{l_i}$ for left-handed fields and as $g_V^{l_i} - g_A^{l_i}$ for right-handed fields. Note that in contrast to the case of the Z-decay which is proportional to ϵ^4, the three body decay with double LFV is proportional to ϵ^8 and hence irrelevant. As we will show in Table II even the tau decays that fo as ϵ^4 will turn out to be small once the muon decay constraints are implemented.

In Fig. 2 we present the dependence of the $\mu \to eee$ branching ratio on the μ parameter that characterizes the lepton number violation scale, for a fixed value of the M_N. Although LFV exists in the limit where neutrinos go massless, there is an indirect dependence on the value of μ reflecting the need to account for neutrino oscillation data.

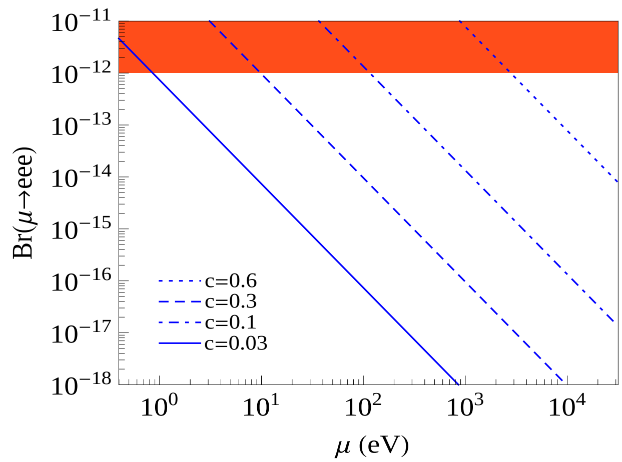

FIG. 2: Branching of μ decay into $3e$ as a function of the μ parameter for different values of c equivalent to $\cos\theta_{\Sigma S}$, 0.6 (dotted), 0.3 (dot-dashed), 0.1 (dashed) and 0.03 (contineous). Here M_N is fixed at 1 TeV.

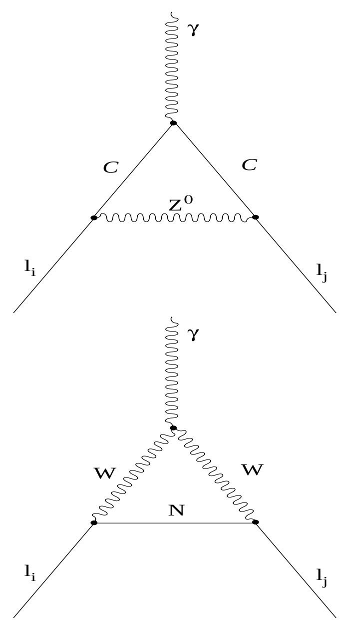

FIG. 3: Feynman graphs for $\mu \to e\gamma$ decay in type-III seesaw models.

C. $l_i \to l_j \gamma$

The decay $l_i \to l_j \gamma$ arises in our model at one loop both from charged as well as neutral current contributions, see Fig. 3. The neutrino mass matrix in Eq. (13) is a 9×9 symmetric matrix, diagonalized by a unitary matrix $U_{\alpha\beta}$. The effective charged current weak interaction is characterized by a rectangular lepton mixing matrix

$K_{i\alpha}$ [10]

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} K_{i\alpha} \overline{L}_i \gamma_\mu (1+\gamma_5) N_\alpha \, W^\mu \ , \tag{26}$$

where $i = 1,2,3$ denote the left-handed charged leptons and α the label the neutral states, $\alpha,\ \beta = 1...9$.

Similarly the effective neutral current weak interaction of the left-handed charged leptons with the heavy charged fermions is characterized by

$$\mathcal{L}_{NC} = \frac{g}{\sqrt{2}} \mathcal{P}_{LH\,i\alpha} \overline{L}_i \gamma_\mu (1+\gamma_5) C_\alpha \, Z^\mu. \tag{27}$$

where $i = 1,2,3$ and $\alpha = 4,5,6$.

The $l_i \to l_j\gamma$ decays occur mainly through the exchange of the six neutral heavy leptons N_j subdominantly coupled to the charged leptons [13, 43, 44] and that of the three heavy charged fermion triplets which couple to the charged leptons through the exchange of neutral Z^0 gauge boson (see, for instance [46]).

The resulting branching ratio is given by

$$Br(l_i \to l_j \gamma) = \frac{\alpha^3 s_W^2}{256\pi^2} \frac{m_{l_i}^5}{M_W^4} \frac{1}{\Gamma_{l_i}} |G_{ij}^W + G_{ij}^Z|^2 \tag{28}$$

where

$$\begin{array}{l} G_{ij}^W = \sum_{k=4}^{9} K_{ik}^* K_{jk} G_\gamma^W \left(\frac{m_{N_k}^2}{M_W^2}\right) \\ G_{ij}^Z = \sum_{k=4}^{6} V_{ik}^* V_{jk} G_\gamma^Z \left(\frac{m_{\Sigma_k}^2}{M_Z^2}\right) \\ G_\gamma^W(x) = -\frac{2x^3+5x^2-x}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4}\ln x \\ G_\gamma^Z(x) = -\frac{-5x^3+9x^2-30x+8}{(1-x)^3} - \frac{18x^2}{(1-x)^3}\ln x. \end{array} \tag{29}$$

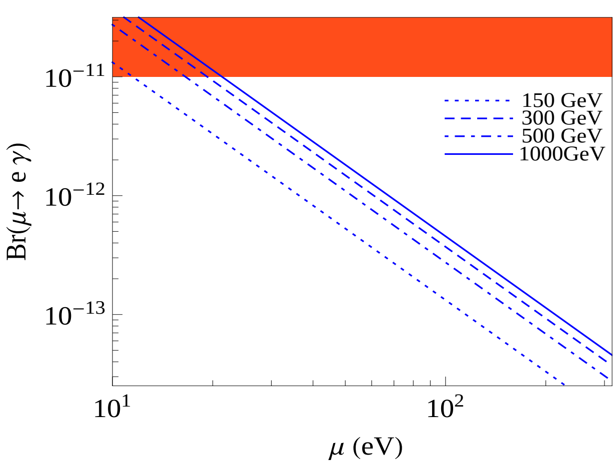

FIG. 4: Branching of μ decay into $e\gamma$ as a function of the μ parameter for different values of M_N, 150 GeV (dotted), 300 GeV (dot-dashed), 500 GeV (dashed) and 1 TeV (contineous) fixing $\cos\theta_{\Sigma S} = 0.1$

In Fig. 4 we study the dependence of the $\mu \to e\gamma$ decay branching ratio on the parameter μ which characterizes lepton number violation. The same comment made in the discussion of $\mu \to eee$ applies also here. Note that the branching $\mu \to e\gamma$ depends somewhat on the physical mass M_N of the neutral heavy states, reflecting the fact that is a one loop process.

D. Relating different LFV decays

Note that, thanks to the admixture of the neutral and charged TeV states in the weak interaction currents, the LFV branching ratios in our inverse type-III seesaw model can be sizeable even in the absence of supersymmetry. Moreover, the assummed A_4 based flavor symmetry implies that the structure of the matrices K and $\mathcal{P}$ describing these processes is special, leading to relationships among the LFV branching ratios (see Table II below). As a result the G^W, G^Z loop factor matrices of Eq. (29) and the $\mathcal{P}_{LL}$ matrix in Eq. (10) are determined by just two model parameters,

$$G^W \sim G^Z \sim \mathcal{P}_{LL} \sim \begin{pmatrix} a^2+\frac{4ab}{3}+\frac{2b^2}{3} & -\frac{1}{3}b(2a+b) & -\frac{1}{3}b(2a+b) \\ -\frac{1}{3}b(2a+b) & a^2-\frac{2ab}{3}+\frac{2b^2}{3} & \frac{1}{3}b(4a-b) \\ -\frac{1}{3}b(2a+b) & \frac{1}{3}b(4a-b) & a^2-\frac{2ab}{3}+\frac{2b^2}{3} \end{pmatrix}. \tag{30}$$

Taking ratios of branching ratios, prefactors cancel and one finds for example that

$$\frac{Br(\tau\to\mu\gamma)}{Br(\tau\to e\gamma)} = \left(\frac{4+t}{2-t}\right)^2 , \tag{31}$$

where $t \equiv -b/a$ is the solution of the eq.

$$\alpha = \frac{1-(1-t)^4}{(1+t)^4-1}. \tag{32}$$

where the ratio

$$\alpha = \Delta m_{\rm sol}^2 / \Delta m_{\rm atm}^2$$

is well determined by neutrino oscillation data [6].

The symmetry predictions are listed in Table II. They show that, as long as the flavor symmetry holds, all LFV decay braching ratios can be expressed in terms of the branching ratios for the processes $\mu^- \to e^-e^+e^-$ and $\mu \to e\gamma$. Thanks to the tree-level violation of lepton flavor in the neutral current, the relative ratio between $\mu^- \to e^-e^+e^-$ and $\mu \to e\gamma$ is also unusual, and allows the rate for $\mu^- \to e^-e^+e^-$ to be larger than that for $\mu \to e\gamma$.

$\frac{Br(\mu^- \to e^- e^+ e^-)}{Br(\tau^- \to e^- e^+ e^-)}$	$\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\Gamma(\tau \to all)}{\Gamma(\mu \to all)}$
$\frac{Br(\mu^- \to e^- e^+ e^-)}{Br(\tau^- \to e^- \mu^+ \mu^-)}$	$\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\Gamma(\tau \to all)}{\Gamma(\mu \to all)}$
$\frac{Br(\mu^- \to e^- e^+ e^-)}{Br(\tau^- \to \mu^- e^+ e^-)}$	$\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\Gamma(\tau \to all)}{\Gamma(\mu \to all)} \left(\frac{2-t}{4+t}\right)^2$
$\frac{Br(\mu^- \to e^- e^+ e^-)}{Br(\tau^- \to \mu^- \mu^+ \mu^-)}$	$\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\Gamma(\tau \to all)}{\Gamma(\mu \to all)} \left(\frac{2-t}{4+t}\right)^2$
$\frac{Br(\tau^- \to \mu^- \mu^- e^+)}{Br(\tau^- \to e^- e^- \mu^+)}$	$\left(\frac{4+t}{2-t}\right)^2$
$\frac{Br(Z^0 \to \mu^- e^+)}{Br(Z^0 \to \tau^- e^+)}$	1
$\frac{Br(Z^0 \to \mu^- e^+)}{Br(Z^0 \to \tau^- \mu^+)}$	$\left(\frac{2-t}{4+t}\right)^2$
$\frac{Br(\mu \to e\gamma)}{Br(\tau \to e\gamma)}$	$\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\Gamma(\tau \to all)}{\Gamma(\mu \to all)}$
$\frac{Br(\tau \to \mu\gamma)}{Br(\tau \to e\gamma)}$	$\left(\frac{4+t}{2-t}\right)^2$

TABLE II: Predictions for ratio of LFV branching, where t is defined in the text and $(m_\mu/m_\tau)^5\Gamma_\tau/\Gamma_\mu = 0.18$.

Regarding the rates for mu-e conversion in nuclei, as already noted in Ref. [45], in the limit where we neglect Higgs boson contributions, these rates are strongly correlated with $\mu \to e\gamma$. This means that for a given target nucleus they are relatively well determined from the $\mu \to e\gamma$ rate. We refer the reader to Fig. 5 in Ref. [45]. Finally, tau LFV decay rates are expected to be small, even those that scale as ϵ^4 like those corresponding to semi-leptonic modes, which are not displayed in the Table.

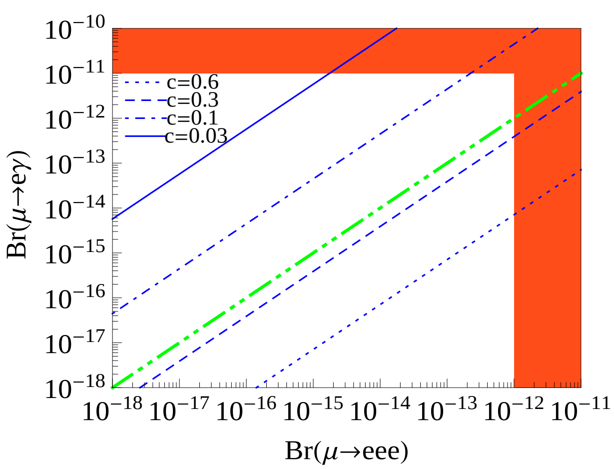

FIG. 5: Branching of μ decay into $e\gamma$ *vs* the branching of μ decay into $3e$ for different values of $\cos\theta_{\Sigma S}$, 0.6 (dotted), 0.3 (dot-dashed), 0.1 (dashed) and 0.03 (continous) and M_N = 1 TeV.

Discussion

We have proposed a new inverted version of type-III seesaw or equivalently, a new type-III version of the inverse seesaw mechanism. This way the physics responsible for neutrino masses can lie at low-scale and can be accessible at the Large Hadron Collider (LHC), due to: (i) the TeV–scale neutral fermions having large cross sections at the LHC and (ii) the TeV–scale neutral fermions inducing large LFV processes due to the low-scale violation of the Glashow-Iliopoulos-Maiani mechanism, which implies potentially large tree-level FCNC involving charged leptons. By assuming an A_4-based underlying flavor symmetry we have implemented a tri-bimaximal lepton mixing pattern to account for the observed neutrino oscillation parameters. We have studied the phenomenology of the resulting LFV decays and given the typical expectations for their magnitude, in addition to discussing the predictions for their relative rates. In Fig. 5 we give the correlation between the branching of $\mu \to e\gamma$ vs the branching of $\mu \to eee$ fixing M_N and for different values of $\cos\theta_{\Sigma S}$. Clearly neutral heavy fermion states can lie at the TeV scale and their production cross section at the LHC is enhanced with respect to that expected in type-I inverse seesaw [47]. Indeed the much larger production cross sections expected for the type-III models should encourage detailed dedicated MonteCarlo simulations [48] in order to scrutinize the viability of detecting the associated signals. Last but not least, given the underlying flavor symmetry predictions these should also take into account the details of flavor physics which will determine the expected decay pattern of the heavy leptons.

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