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Underlying A_4 Symmetry for the Neutrino Mass Matrix and the Quark Mixing Matrix

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Abstract

The discrete non-Abelian symmetry A_4, valid at some high-energy scale, naturally leads to degenerate neutrino masses, without spoiling the hierarchy of charged-lepton masses. Realistic neutrino mass splittings and mixing angles (one of which is necessarily maximal and the other large) are then induced radiatively in the context of softly broken supersymmetry. The quark mixing matrix is also calculable in a similar way. The mixing parameter U_{e3} is predicted to be imaginary, leading to maximal CP violation in neutrino oscillations. Neutrinoless double beta decay and $\tau \to \mu\gamma$ should be in the experimentally accessible range.

It has often be said that the mixing pattern of neutrinos, which involves large angles, as evidenced by the atmospheric [1] and solar [2] neutrino data, is unexpected and difficult to understand, given that the quark charged-current mixing matrix V_{CKM} involves only small angles. However, as shown below, both can be explained in a simple and unified way as small radiative corrections of a fixed pattern, valid at some high-energy scale as the result of an underlying symmetry, which we identify here as A_4, the non-Abelian discrete symmetry group of the tetrahedron [3]. We show that at the high scale, neutrino masses are degenerate and V_{CKM} is the identity matrix. We then calculate the radiative corrections down at the electroweak scale in the framework of softly broken supersymmetry [4, 5] and obtain realistic versions of $\mathcal{M}_\nu$ and V_{CKM}. The reason that neutrino mixing involves large angles is a simple consequence of degenerate perturbation theory, where a small off-diagonal term induces maximal mixing between two states of equal energy, whereas in the quark sector with hierarchical masses, the same small off-diagonal element induces only a small mixing.

Our starting point is the model of Ref. [3], but with the following two important improvements. (I) Instead of breaking A_4 spontaneously at the electroweak scale, it is now broken at a very high scale. (II) Supersymmetry is added with explicit soft breaking terms which also break A_4. The resulting theory at the electroweak scale is a specific version of the MSSM (Minimal Supersymmetric Standard Model), where the scalar lepton and quark sectors are correlated with $\mathcal{M}_\nu$ and V_{CKM}. In this way we also provide a theoretical framework for realizing the neutrino unification idea suggested in the first paper of Ref. [4], but with different specific predictions.

The non-Abelian discrete finite group A_4 consists of 12 elements and has 4 irreducible representations. Three are one-dimensional, $\underline{1}$, $\underline{1}'$, $\underline{1}''$, and one is three-dimensional, $\underline{3}$, with the decomposition

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \tag{1}$$

The usual quark, lepton, and Higgs superfields transform under A_4 as follows:

$$\hat{Q}_i = (\hat{u}_i, \hat{d}_i), \; \hat{L}_i = (\hat{\nu}_i, \hat{e}_i) \sim \underline{3}, \quad \hat{\phi}_{1,2} \sim \underline{1}, \tag{2}$$

$$\hat{u}^c_1, \; \hat{d}^c_1, \; \hat{e}^c_1 \sim \underline{1}, \quad \hat{u}^c_2, \; \hat{d}^c_2, \; \hat{e}^c_2 \sim \underline{1}', \quad \hat{u}^c_3, \; \hat{d}^c_3, \; \hat{e}^c_3 \sim \underline{1}''. \tag{3}$$

We then add the following heavy quark, lepton, and Higgs superfields:

$$\hat{U}_i, \; \hat{U}^c_i, \; \hat{D}_i, \; \hat{D}^c_i, \; \hat{E}_i, \; \hat{E}^c_i, \; \hat{N}^c_i, \; \hat{\chi}_i \sim \underline{3}, \tag{4}$$

which are all $SU(2)$ singlets. The superpotential of this model is then given by

$$\begin{aligned} \hat{W} \;\; &= \;\; M_U \hat{U}_i \hat{U}^c_i + f_u \hat{Q}_i \hat{U}^c_i \hat{\phi}_2 + h^u_{ijk} \hat{U}_i \hat{u}^c_j \hat{\chi}_k \\ &+ \;\; M_D \hat{D}_i \hat{D}^c_i + f_d \hat{Q}_i \hat{D}^c_i \hat{\phi}_1 + h^d_{ijk} \hat{D}_i \hat{d}^c_j \hat{\chi}_k \\ &+ \;\; M_E \hat{E}_i \hat{E}^c_i + f_e \hat{L}_i \hat{E}^c_i \hat{\phi}_1 + h^e_{ijk} \hat{E}_i \hat{e}^c_j \hat{\chi}_k \\ &+ \;\; \frac{1}{2} M_N \hat{N}^c_i \hat{N}^c_i + f_N \hat{L}_i \hat{N}^c_i \hat{\phi}_2 + \mu \hat{\phi}_1 \hat{\phi}_2 \\ &+ \;\; \frac{1}{2} M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3, \end{aligned} \tag{5}$$

where we have adopted the usual assignment of R parity to distinguish between the Higgs superfields, i.e. $\hat{\phi}_{1,2}$ and $\hat{\chi}_i$, from the quark and lepton superfields. We have also forbidden the terms $\hat{\chi}_i \hat{N}^c_j \hat{N}^c_k$, etc. by assigning

$$\hat{\chi}_i \sim \omega, \quad \hat{u}^c_i, \; \hat{d}^c_i, \; \hat{e}^c_i \sim \omega^2, \tag{6}$$

and all others ~ 1 under a separate discrete symmetry Z_3 (with $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$). However, Z_3 is allowed to be broken explicitly but only softly, which is uniquely accomplished in the above by $M_\chi \neq 0$.

The scalar potential involving χ_i is given by

$$V = |M_\chi \chi_1 + h_\chi \chi_2 \chi_3|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1|^2 + |M_\chi \chi_3 + h_\chi \chi_1 \chi_2|^2, \tag{7}$$

which has the supersymmetric solution ($V = 0$)

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u = -M_\chi / h_\chi, \tag{8}$$

so that the breaking of A_4 at the high scale M_χ does not break the supersymmetry. [Note that Eq. (8) is only possible because A_4 allows the invariant symmetric product of $\underline{3} \times \underline{3} \times \underline{3}$, a highly nontrivial property not shared for example by the triplet representation of either SO(3) or SU(3).]

Consider now the 6×6 Dirac mass matrix linking (e_i, E_i) to (e^c_j, E^c_j).

$$\mathcal{M}_{eE} = \begin{bmatrix} 0 & 0 & 0 & f_e v_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_e v_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_e v_1 \\ h^e_1 u & h^e_2 u & h^e_3 u & M_E & 0 & 0 \\ h^e_1 u & h^e_2 \omega u & h^e_3 \omega^2 u & 0 & M_E & 0 \\ h^e_1 u & h^e_2 \omega^2 u & h^e_3 \omega u & 0 & 0 & M_E \end{bmatrix}, \tag{9}$$

where $v_1 = \langle \phi^0_1 \rangle$, and Eq. (17) of the first paper of Ref. [3] has been used, with similar forms for the quark mass matrices. The reduced 3×3 Dirac mass matrix for the charged leptons is then

$$\mathcal{M}_e = U_L \begin{bmatrix} h^{e\prime}_1 & 0 & 0 \\ 0 & h^{e\prime}_2 & 0 \\ 0 & 0 & h^{e\prime}_3 \end{bmatrix} \frac{\sqrt{3} f_e v_1 u}{M_E}, \tag{10}$$

where $h^{e\prime}_i = h^e_i [1 + (h^e_i u)^2 / M^2_E]^{-1/2}$ and

$$U_L = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}. \tag{11}$$

This shows that charged-lepton masses are allowed to be all different, despite the imposition of the A_4 symmetry, because there exist three inequivalent one-dimensional representations. [Note that the permutation symmetry groups S_3 and S_4 have only two inequivalent one-dimensional representations and S_3 has no three-dimensional representation.] Clearly, the *up* and *down* quark mass matrices are obtained in the same way, with the important conclusion that the charged-current mixing matrix V_{CKM} is automatically equal to the identity matrix, because both are diagonalized by U_L. Corrections to $V_{CKM} = 1$ may then be ascribed to the structure of the soft supersymmetry breaking sector [5, 6].

In the neutrino sector, the 6×6 Majorana mass matrix spanning $(\nu_e, \nu_\mu, \nu_\tau, N_1^c, N_2^c, N_3^c)$ is given by

$$\mathcal{M}_{\nu N} = \begin{bmatrix} 0 & U_L f_N v_2 \\ U_L^T f_N v_2 & M_N \end{bmatrix}, \tag{12}$$

where $v_2 = \langle \phi_2^0 \rangle$. Hence the 3×3 seesaw mass matrix for $(\nu_e, \nu_\mu, \nu_\tau)$ becomes

$$\mathcal{M}_\nu = \frac{f_N^2 v_2^2}{M_N} U_L^T U_L = \frac{f_N^2 v_2^2}{M_N} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{13}$$

This shows that neutrino masses are degenerate at this stage.

Consider now the above as coming from an effective dimension-five operator [7]

$$\frac{f_N^2}{M_N} \lambda_{ij} \nu_i \nu_j \phi_2^0 \phi_2^0, \tag{14}$$

where $\lambda_{ee} = \lambda_{\mu\tau} = \lambda_{\tau\mu} = 1$ and all other λ's are zero, which is valid at some high scale. As we come down to the electroweak scale, Eq. (14) is corrected [8] by the wavefunction renormalizations of ν_e, ν_μ, and ν_τ, as well as the corresponding vertex renormalizations. Even if only the standard model is considered, this will lift the degeneracy of Eq. (13) because of the different charged-lepton masses. The resulting pattern, i.e. $\lambda_{ee} = 1 + 2m_e^2 \epsilon$, $\lambda_{\mu\tau} = \lambda_{\tau\mu} = 1 + (m_\mu^2 + m_\tau^2)\epsilon$, where $\epsilon \sim 1/(16\pi^2 v^2) \ln(M_N/M_Z)$, is however not suitable for explaining the present data on neutrino oscillations. On the other hand, other radiative corrections exist in the context of softly broken supersymmetry with a general slepton mass matrix [4]. Given the structure of λ_{ij} at the high scale, its form at the low scale is necessarily fixed to first order as

$$\lambda_{ij} = \begin{bmatrix} 1+2\delta_{ee} & \delta_{e\mu}+\delta_{e\tau} & \delta_{e\mu}+\delta_{e\tau} \\ \delta_{e\mu}+\delta_{e\tau} & 2\delta_{\mu\tau} & 1+\delta_{\mu\mu}+\delta_{\tau\tau} \\ \delta_{e\mu}+\delta_{e\tau} & 1+\delta_{\mu\mu}+\delta_{\tau\tau} & 2\delta_{\mu\tau} \end{bmatrix}, \tag{15}$$

where we have assumed all parameters to be real as a first approximation. [The above matrix is obtained by multiplying that of Eq. (13) on the left and on the right by all possible $\nu_i \to \nu_j$ transitions.] Let us rewrite the above with $\delta_0 \equiv \delta_{\mu\mu} + \delta_{\tau\tau} - 2\delta_{\mu\tau}$, $\delta \equiv 2\delta_{\mu\tau}$, $\delta' \equiv \delta_{ee} - \delta_{\mu\mu}/2 - \delta_{\tau\tau}/2 - \delta_{\mu\tau}$, and $\delta'' \equiv \delta_{e\mu} + \delta_{e\tau}$. Then

$$\lambda_{ij} = \begin{bmatrix} 1+\delta_0+2\delta+2\delta' & \delta'' & \delta'' \\ \delta'' & \delta & 1+\delta_0+\delta \\ \delta'' & 1+\delta_0+\delta & \delta \end{bmatrix}, \tag{16}$$

and the *exact* eigenvectors and eigenvalues are easily obtained:

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta/\sqrt{2} & \sin\theta/\sqrt{2} \\ -\sin\theta & \cos\theta/\sqrt{2} & \cos\theta/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}, \tag{17}$$

and

$$\lambda_1 = 1+\delta_0+2\delta+\delta' - \sqrt{\delta'^2+2\delta''^2}, \tag{18}$$

$$\lambda_2 = 1+\delta_0+2\delta+\delta' + \sqrt{\delta'^2+2\delta''^2}, \tag{19}$$

$$\lambda_3 = -1-\delta_0. \tag{20}$$

With δ''^2/δ'^2 of order unity, this is then a very satisfactory description of present neutrino-oscillation data, i.e.

$$\sin^2 2\theta_{atm} = 1, \quad \tan^2\theta_{sol} = \frac{\delta''^2}{\delta''^2+\delta'^2-\delta'\sqrt{\delta'^2+2\delta''^2}} = 0.44, \tag{21}$$

if $\delta' < 0$ and $|\delta''/\delta'| = 1.7$. Note that for $\delta'' = \delta'$ Eq. (16) reproduces that proposed in the second paper of Ref. [3]. Whereas the latter was simply an ansatz, the form of Eq. (16) here is a *necessary* consequence of our assumption that radiative corrections are responsible for the splitting of the neutrino mass degeneracy enforced by the discrete A_4 symmetry. Assuming that $\delta', \delta'' << \delta$, we now have

$$\Delta m_{31}^2 \simeq \Delta m_{32}^2 \simeq 4\delta m_0^2, \quad \Delta m_{12}^2 \simeq 4\sqrt{\delta'^2 + 2\delta''^2} m_0^2, \tag{22}$$

where m_0 is the common mass of all 3 neutrinos.

Note that $U_{e3} = 0$ in Eq. (17), which would imply the absence of CP violation in neutrino oscillations. However, if we do not assume λ_{ij} to be real, then it has one complex phase which cannot be rotated away. Without loss of generality, we now rewrite Eq. (16) as

$$\lambda_{ij} = \begin{bmatrix} 1 + 2\delta + 2\delta' & \delta'' & \delta''^* \\ \delta'' & \delta & 1 + \delta \\ \delta''^* & 1 + \delta & \delta \end{bmatrix}, \tag{23}$$

where we have redefined $1 + \delta_0$ as 1, and δ, δ' are real. Although this mass matrix cannot be diagonalized exactly, if we assume that δ', $Re\delta''$ and $(Im\delta'')^2/\delta$ are all much smaller than δ in magnitude, then Eqs. (17) to (22) are again valid (but only approximately) with the following changes:

$$U_{e3} = \frac{iIm\delta''}{\sqrt{2}\delta}, \quad \delta' \to \delta' + \frac{(Im\delta'')^2}{2\delta}, \quad \delta'' \to Re\delta''. \tag{24}$$

Note the important result that U_{e3} is imaginary. Thus CP violation is *predicted* to be *maximal* in this model for neutrino oscillations. Using Eq. (22), we also have the relationship

$$\left[\frac{\Delta m_{12}^2}{\Delta m_{32}^2}\right]^2 \simeq \left[\frac{\delta'}{\delta} + |U_{e3}|^2\right]^2 + \left[\frac{Re\delta''}{\delta}\right]^2. \tag{25}$$

Note that $|U_{e3}|$ is naturally of the order $|\Delta m_{12}^2/\Delta m_{32}^2|^{1/2} \simeq 0.14$. This result depends only on the form of Eq. (23), which is itself derived in the most general way.

It remains to be shown in the rest of this paper that realistic values of δ, δ', and δ'' are possible from the soft breaking of supersymmetry, without running into conflict with present limits on neutrinoless double beta ($\beta\beta_{0\nu}$) decay and lepton flavor violating processes such as $\tau \to \mu\gamma$.

Let us calculate δ in the context of supersymmetry. We show in Figures 1 and 2 the wavefunction and vertex corrections respectively due to $\tilde{\mu}_L - \tilde{\tau}_L$ mixing. Let the two scalar mass eigenstates have masses $\tilde{m}_{1,2}$ and their mixing angle be θ. For illustration, let us take the approximation that $\tilde{m}_1^2 >> \mu^2 >> M_{1,2}^2 = \tilde{m}_2^2$, where μ is the superpotential Higgsino mixing term, while $M_{1,2}$ denote the soft supersymmetry breaking gaugino mass parameters. We then obtain

$$\delta \simeq \frac{\sin\theta\cos\theta}{16\pi^2}\left[(3g_2^2 - g_1^2)\ln\frac{\tilde{m}_1^2}{\mu^2} - \frac{1}{4}(3g_2^2 + g_1^2)\left(\ln\frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2}\right)\right]. \tag{26}$$

Using Eq. (22) and taking $\Delta m_{32}^2 = 2.5 \times 10^{-3}$ eV^2 from atmospheric neutrino oscillations, we find $\delta = 3.9 \times 10^{-3}(0.4 \text{ eV}/m_0)^2$. This implies that

$$\left[\ln\frac{\tilde{m}_1^2}{\mu^2} - 0.3\left(\ln\frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2}\right)\right]\sin\theta\cos\theta \simeq 0.535\left(\frac{0.4 \text{ eV}}{m_0}\right)^2. \tag{27}$$

To the extent that this factor cannot be much greater than one, the common mass m_0 as probed in neutrinoless double beta decay [9] cannot be much lower than the present upper bound of about 0.4 eV. This is in sharp contrast to the scenario proposed in the first paper of Ref. [4] where $\beta\beta_{0\nu}$ decay is strongly suppressed.

Similarly, $\delta'' = \delta_{e\mu} + \delta_{\tau e} = \delta_{e\mu} + \delta_{e\tau}^*$ is determined by $\tilde{e}_L - \tilde{\mu}_L$ and $\tilde{e}_L - \tilde{\tau}_L$ mixing. Using the experimental bound of $|U_{e3}| < 0.16$, we find $Im\delta'' < 8.8 \times 10^{-4}(0.4 \text{ eV}/m_0)^2$, and using $\Delta m_{12}^2 \simeq 5\times 10^{-5}$ eV^2 from solar neutrino oscillations, we find $Re\delta'' < 7.8\times 10^{-5}(0.4 \text{ eV}/m_0)^2$. These limits may be saturated mainly by $\tilde{e}_L - \tilde{\tau}_L$ mixing, allowing $\tilde{e}_L - \tilde{\mu}_L$ mixing to be much more suppressed. In other words, from the data on neutrino oscillations, we are able to make the direct connection in this model that flavor violation in the charged-lepton sector should be the greatest in the $\mu - \tau$ sector and smallest in the $e - \mu$ sector.

Using the same approximation which leads to Eq. (26), the $\tau \to \mu\gamma$ amplitude is calculated to be

$$\mathcal{A} = \frac{e(3g_2^2 + g_1^2)}{1536\pi^2}(\sin\theta\cos\theta)\frac{m_\tau}{\tilde{m}_2^2}\epsilon^\lambda q^\nu \bar{\mu}\sigma_{\lambda\nu}(1+\gamma_5)\tau. \tag{28}$$

The resulting branching fraction is then

$$B(\tau \to \mu\gamma) = 4.8 \times 10^{-6} \sin^2\theta \cos^2\theta \left(\frac{100 \text{ GeV}}{\tilde{m}_2}\right)^4. \tag{29}$$

Using the experimental upper limit of 1.1×10^{-6} on this number, we require thus $\tilde{m}_2 > 102$ GeV. Constraints from $\tau \to e\gamma$ and $\mu \to e\gamma$ are also similarly satisfied. Details will be given in a forthcoming comprehensive study of the correlation between the neutrino parameters and the pattern of soft supersymmetry breaking of this model.

Consider now the quark sector. Whereas the neutrino sector has only $L-L$ scalar mixings, we now also have $L-R$ and $R-R$ scalar mixings. In a previous study [5], $V_{CKM} = 1$ was obtained from proportional *up* and *down* quark mass matrices, and it was shown that a realistic V_{CKM} could then be generated with $L-R$ scalar quark mixings through gluino exchange. Here $V_{CKM} = 1$ is obtained from our A_4 symmetry for any set of arbitrary *up* and *down* quark masses, with the obvious implication that the above result also applies. [In the charged-lepton sector, the effect is smaller and does not significantly change the neutrino mixing angles except possibly for U_{e3}.] More details will be discussed in the forthcoming comprehensive study.

In conclusion, we have presented a concrete model based on the discrete symmetry A_4 where quark and charged-lepton masses can be all different and yet neutrino masses are degenerate at some high scale where $V_{CKM} = 1$ and the effective neutrino mass matrix in the ν_e, ν_μ, ν_τ basis is of the form

$$\mathcal{M}_\nu = \begin{bmatrix} m_0 & 0 & 0 \\ 0 & 0 & m_0 \\ 0 & m_0 & 0 \end{bmatrix}. \tag{30}$$

The parameter m_0 naturally lies in the range where it can be probed in cosmology, neutrinoless double beta decay and tritium beta decay. Radiative corrections lift the neutrino degeneracy leading to (A) $\sin^2 2\theta_{atm} = 1$, (B) U_{e3} small and imaginary, and (C) large (but not maximal) solar mixing angle. These corrections can be ascribed to the structure of the soft supersymmetry breaking terms in the scalar sector, which also break the A_4 symmetry explicitly and correlate the neutrino mass matrix with lepton flavor violating processes. Last but not least, a realistic quark mixing matrix V_{CKM} may be obtained in a totally analogous way.

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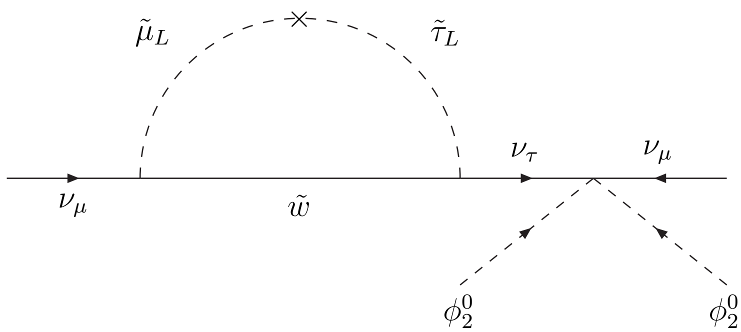

Figure 1: Wavefunction contribution to δ in supersymmetry.

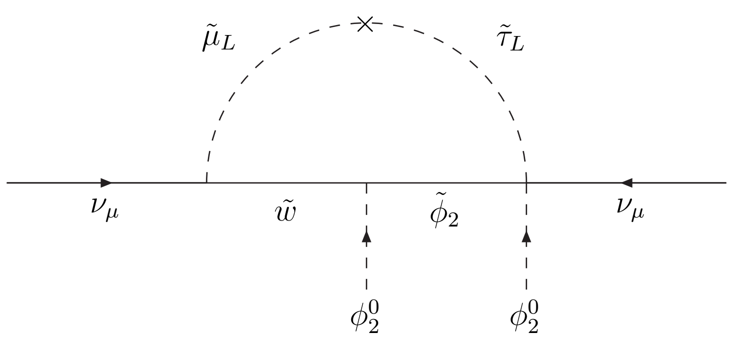

Figure 2: Vertex contribution to δ in supersymmetry.