Supersymmetry Unification Predictions for m_{top}, V_{cb} and $\tan\beta$

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Abstract

We study the predictions for m_{top}, $\tan\beta$ and V_{cb} in a popular texture ansätze for the fermion mass matrices. We do this both for the Minimal Supersymmetric Standard Model (MSSM) and for the simplest model (MSSM–BRpV) where a bilinear R–Parity violating term is added to the superpotential. We find that taking the experimental values for m_{top} and V_{cb} at 99% c.l. and the GUT relations $h_b = h_\tau$ and $V_{cb}^2 = h_c/h_t$ within 5%, the large $\tan\beta$ solution, characteristic in the MSSM with bottom–tau unification, becomes disallowed. In contrast the corresponding allowed region for the MSSM–BRpV is slightly larger. We also find that important modifications occur if we relax the texture conditions at the GUT scale. For example, if the GUT relations are imposed at 40%, the large $\tan\beta$ branch in the MSSM becomes fully allowed. In addition, in MSSM–BRpV the whole $\tan\beta - m_{top}$ plane become allowed, finding unification at any value of $\tan\beta$.

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1 Introduction

Grand unified (GUT) symmetries [1] combined with flavour symmetries [2] constitute the most promising way of understanding the structure of flavour masses and mixings. These masses and mixings constitute the majority of the unknown parameters of the Standard Model (SM). On the other hand, supersymmetry allows the unification of gauge couplings to succeed where the SM fails [3, 4], implying the prediction of one of the three gauge coupling constants.

In some GUT models [for example SU(5)], the bottom quark and the tau lepton Yukawa couplings are equal at the unification scale, and the predicted ratio m_b/m_τ at the weak scale agrees with experiments. Several studies have been made about the effect of supersymmetry on gauge and Yukawa unification. In the Minimal Supersymmetric Standard Model (MSSM) bottom–tau unification is achieved at two disconnected and small regions of $\tan\beta$ (the ratio of the two vacuum expectation values), one at small and the other at large $\tan\beta$ [5, 6, 7].

Recently it was shown that if to the MSSM we add Bilinear R–Parity Violation (BRpV) [8, 9, 10, 11], the unification of the bottom and tau Yukawa couplings at the scale $M_{GUT} \approx 10^{16}$ GeV (where the gauge couplings unify) is dramatically different from the MSSM [12]. In the BRpV case, bottom–tau unification is achieved at any value of $\tan\beta$ provided the vacuum expectation value v_3 of the tau sneutrino is chosen appropriately. In addition, it was shown that the prediction of α_s, which in the MSSM is 2σ too high, in BRpV can be lowered by more than 1σ with respect to the MSSM prediction and therefore can lie closer to the experimental measurement [13].

The study of BRpV is motivated by the fact that it provides a simple and useful parametrization of many of the features of a class of models in which R-Parity is spontaneously broken [14]. One of the main features of R–Parity violating models is the appearance of masses for the neutrinos [14, 15], attracting a lot of attention [16] since the latest results from Super–Kamiokande [17]. It has in fact been demostrated that this model offers an attractive and predictive scheme for neutrino masses and mixing parameters which accounts for the observed data from atmospheric and solar neutrino observations [18].

In this paper we update the analysis of the relations between m_{top} and $\tan\beta$ within the MSSM for the case in which the bottom and tau Yukawa couplings unify and using the CKM matrix element V_{cb} that follows from the simplest Yukawa texture, adopting the most recent experimental values $0.036 < |V_{cb}| < 0.042$ at 90% c.l. prescribed by the Particle Data Group [19]. In addition, following closely the

method presented in ref. [12] we repeat the analysis for the MSSM–BRpV model [10, 11] and compare the results obtained with those found in the MSSM.

2   Zero Texture Ansätze

Flavour symmetries in two and three generations were first proposed in [20]. The validity of such mass matrix ansätze at the GUT scale was postulated by [21] and later the ansätze was modified in [22]. The final version of the mass matrix we are considering here is given in [23] and corresponds to

$$
\mathbf{h_U} = \begin{bmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{bmatrix}, \qquad
\mathbf{h_D} = \begin{bmatrix} 0 & Fe^{i\phi} & 0 \\ Fe^{-i\phi} & E & 0 \\ 0 & 0 & D \end{bmatrix}, \qquad
\mathbf{h_E} = \begin{bmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{bmatrix}
\tag{1}
$$

where $\mathbf{h_U}$, $\mathbf{h_D}$, and $\mathbf{h_E}$ are the up–type quark, down–type quark, and charged lepton Yukawa matrices respectively. The dimention-less parameters A, B, C, D, E, and F are real and ϕ is the only phase.

The fact that the third diagonal matrix element in the down–type quark and the charged lepton Yukawa matrices are the same indicates bottom–tau unification at the GUT scale. Another interesting prediction refers to the CKM matrix element V_{cb}. After defining running CKM matrix elements [5], the following relation holds at the GUT scale

$$
\left| V_{cb}(M_{GUT}) \right| = \sqrt{\frac{h_c(M_{GUT})}{h_t(M_{GUT})}}
\tag{2}
$$

In this way, together with the bottom–tau unification condition

$$
h_b(M_{GUT}) = h_\tau(M_{GUT}) \,,
\tag{3}
$$

The corresponding relations between m_{top}, V_{cb} and $\tan\beta$ have been derived in the literature [5, 24]. Here we closely followed the method developed in [5], updating the analysis of these relations for the case in which bottom–tau Yukawa couplings unify, as indicated by eq. (3), and with the CKM matrix element V_{cb} given by eq. (2) and satisfying the experimental constraint at the weak scale $0.036 < |V_{cb}| < 0.042$ at 90% c.l. [19]. This is done first for the MSSM case. In addition, following closely ref. [12] we do the same analysis for the MSSM–BRpV model [10, 11].

3   Bilinear R–Parity Violation

The MSSM–BRpV has one bilinear term in the superpotential for each generation. In this way, after including one-loop radiative corrections, neutrino masses and mixings can be predicted [18]. For our present purposes in this paper it will sufficient to consider lepton and Rp violation only in the tau sector. In this case, the superpotential has the following bilinear terms

$$W_{Bi} = \varepsilon_{ab} \left[-\mu \widehat{H}_d^a \widehat{H}_u^b + \epsilon_3 \widehat{L}_3^a \widehat{H}_u^b \right] , \qquad (4)$$

with μ and ϵ_3 having units of mass. The MSSM superpotential is recovered if we take $\epsilon_3 = 0$. The BRpV term can disappear from the superpotential if we make the rotation defined by $\mu' \widehat{H}_d' = \mu \widehat{H}_d - \epsilon_3 \widehat{L}_3$ and $\mu' \widehat{L}_3' = \epsilon_3 \widehat{H}_d + \mu \widehat{L}_3$, with $\mu'^2 = \mu^2 + \epsilon_3^2$. Nevertheless, BRpV effects are reintroduced through the soft terms in such a way that sneutrino vacuum expectation values are present in both basis: $\langle \widetilde{L}_3 \rangle = v_3/\sqrt{2}$ and $\langle \widetilde{L}_3' \rangle = v_3'/\sqrt{2}$. The VEV v_3 contributes to the W boson mass according to $m_W^2 = \frac{1}{4} g^2 (v_d^2 + v_u^2 + v_3^2)$. On the other hand, the relations of quark masses with Yukawa couplings are the same in BRpV–MSSM as in the MSSM, namely

$$h_{t,c}^2 = \frac{2m_{t,c}^2}{v_u^2} , \qquad h_b^2 = \frac{2m_b^2}{v_d^2} . \qquad (5)$$

except for the numerical value of v_d.

However, in the BRpV model the tau lepton mixes with the charginos, and in the *original* basis where $\psi^{+T} = (-i\lambda^+, \widetilde{H}_u^1, \tau_R^+)$ and $\psi^{-T} = (-i\lambda^-, \widetilde{H}_d^2, \tau_L^-)$, the charged fermion mass terms in the Lagrangian are $\mathcal{L}_m = -\psi^{-T} \boldsymbol{M}_C \psi^+$, with the mass matrix given by

$$\boldsymbol{M}_C = \begin{bmatrix} M & \frac{1}{\sqrt{2}} g v_u & 0 \\ \frac{1}{\sqrt{2}} g v_d & \mu & -\frac{1}{\sqrt{2}} h_\tau v_3 \\ \frac{1}{\sqrt{2}} g v_3 & -\epsilon_3 & \frac{1}{\sqrt{2}} h_\tau v_d \end{bmatrix} \qquad (6)$$

where M is the $SU(2)$ gaugino mass. In the limit $\epsilon_3 = v_3 = 0$ the MSSM chargino mass matrix is recovered in the upper–left 2×2 sub-matrix and at the same time the tau mass relation in the third diagonal element [analogous to the bottom mass relation in eq. (5)]. This tau mass relation is no longer valid in BRpV–MSSM and it is modified to

$$h_\tau^2 = \frac{2m_\tau^2}{v_d^2} \frac{1}{1+\delta} , \qquad \delta = \frac{v_3^2}{v_d^2} + \left[\frac{(A - m_\tau^2)\mu'^2}{Tm_\tau^2 - m_\tau^4 - \Delta} \right] \frac{v_3'^2}{v_d^2} \qquad (7)$$

where A, T, and Δ refer to the upper left 2×2 sub-matrix of the 3×3 matrix $\boldsymbol{M}_C'^T \boldsymbol{M}_C'$: A is its first diagonal element, T is its trace, and Δ is its determinant. The matrix $\boldsymbol{M}_C'$ is the chargino mass matrix analogous to eq. (6) but in the *rotated* basis. It is easy to see that $\boldsymbol{M}_C \to \boldsymbol{M}_C'$ when $(\mu, \epsilon_3, v_d, v_3) \to (\mu', 0, v_d', v_3')$.

4 RGE's and Matching Conditions

We use two-loop MSSM RGE's at scales $Q > M_{SUSY}$ and two loop SM RGE's at scales $Q < M_{SUSY}$. Therefore, we include leading and next–to–leading logarithmic supersymmetric threshold corrections in the approximation where all the SUSY particles decouple at the same scale $Q = M_{SUSY}$. In this way, the matching conditions at $Q = M_{SUSY}$ are defined by the continuity of the quark and lepton running masses at that scale, which translates into matching conditions on Yukawa couplings given in MSSM–BRpV as

$$\lambda_{t,c}(M_{SUSY}^-) = h_{t,c}(M_{SUSY}^+) \sin\beta \sin\theta \,,$$
$$\lambda_b(M_{SUSY}^-) = h_b(M_{SUSY}^+) \cos\beta \sin\theta \,, \qquad (8)$$
$$\lambda_\tau(M_{SUSY}^-) = h_\tau(M_{SUSY}^+) \cos\beta \sin\theta \sqrt{1+\delta} \,,$$

where we have defined the angles β and θ according to spherical coordinates

$$v_d = v \cos\beta \sin\theta \,, \qquad v_u = v \sin\beta \sin\theta \,, \qquad v_3 = v \cos\theta \,, \qquad (9)$$

with $v = 246$ GeV. Note that the MSSM relation $\tan\beta = v_u/v_d$ is preserved. In addition, the boundary condition for the quartic Higgs coupling is given by

$$\lambda(M_{SUSY}^-) = \tfrac{1}{4}\Big[(g^2(M_{SUSY}^+) + g'^2(M_{SUSY}^+)\Big](\cos 2\beta \sin^2\theta + \cos^2\theta)^2 \,. \qquad (10)$$

The corresponding MSSM boundary conditions are obtained by setting $\theta = \pi/2$.

Starting at the scale $Q = m_Z$ we randomly vary the parameters $\alpha_{em}^{-1}(m_Z) = 128.896 \pm 0.090$, $\sin^2\theta_w(m_Z) = 0.2322 \pm 0.0010$, and $\alpha_s(m_Z) = 0.118 \pm 0.003$ [26], looking for solutions with gauge unification at a scale M_{GUT} with a common gauge coupling α_{GUT}. These solutions are concentrated in a region of the plane $M_{GUT} - \alpha_{GUT}$ centered around $M_{GUT} \approx 2.3 \times 10^{16}$ GeV and $\alpha_{GUT}^{-1} \approx 24.5$. For simplicity, from now on, we fix the unification scale to that value. Since M_{GUT} depends on other input parameters, this simplification implies that we don't have "perfect" unification throughout our sampling. Nevertheless, we have checked that unification is good up to 0.4%.

Next, we evolve the Yukawa couplings using two-loop RGEs, starting from the experimental values of the quark and lepton masses at the weak scale and imposing unification of bottom–tau Yukawa couplings at M_{GUT} within 5%. Matching conditions at M_{SUSY} are well known in the MSSM. The main difference in our BRpV model lies in the fact that since the sneutrino vacuum expectation value v_3 contributes also to the W–boson mass, the Higgs VEVs will be in general smaller. This in turn makes the down-type Yukawa couplings larger than in the MSSM. In addition, tau mixing with charginos makes the tau Yukawa coupling h_τ a quantity which
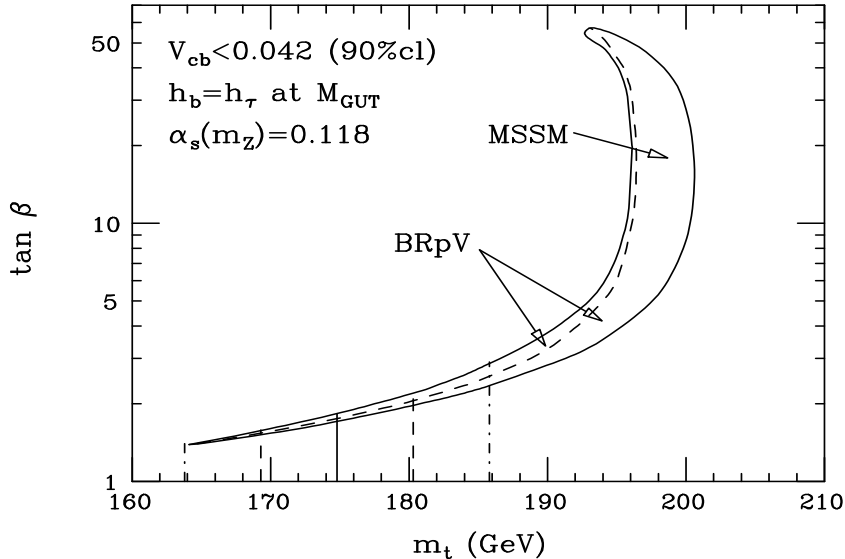
Figure 1: Allowed regions in the $\tan\beta - m_{top}$ plane where bottom–tau Yukawa unification is possible together with the texture prediction for V_{cb}. Accepted values of V_{cb} lie in the 90% c.l. The vertical lines correspond to the experimental measurement of the top quark mass, with its central value (solid), 1σ (dashes), and 2σ (dot–dash) regions.

not only depends on $\tan\beta$ but also on the other chargino (M, μ) and BRpV (ϵ_3, v_3) parameters [11].

Apart from imposing unification of bottom–tau Yukawa couplings we calculate the texture prediction for V_{cb} at the weak scale with the boundary condition given in eq. (2) within 5%. Regarding the MSSM part of the analysis, we have updated the analysis in refs. [5] and [24] by incorporating the most recent experimental values of the top quark mass and of V_{cb}. In contarst, in the case of the the BRpV model, the analysis is done for the first time.

5   Numerical Results

In Fig. 1 we display the regions in the $\tan\beta - m_{top}$ plane where bottom–tau Yukawa unification occurs together with the prediction for the Cabibbo–Kobayashi–Maskawa matrix element V_{cb}. This prediction lies in the region indicated by experiment, *i.e.*, $0.036 < V_{cb} < 0.042$, at 90% c.l. Nevertheless, it is worth mentioning that we do not find any point with $V_{cb} < 0.039$. The space between the solid curves is the allowed
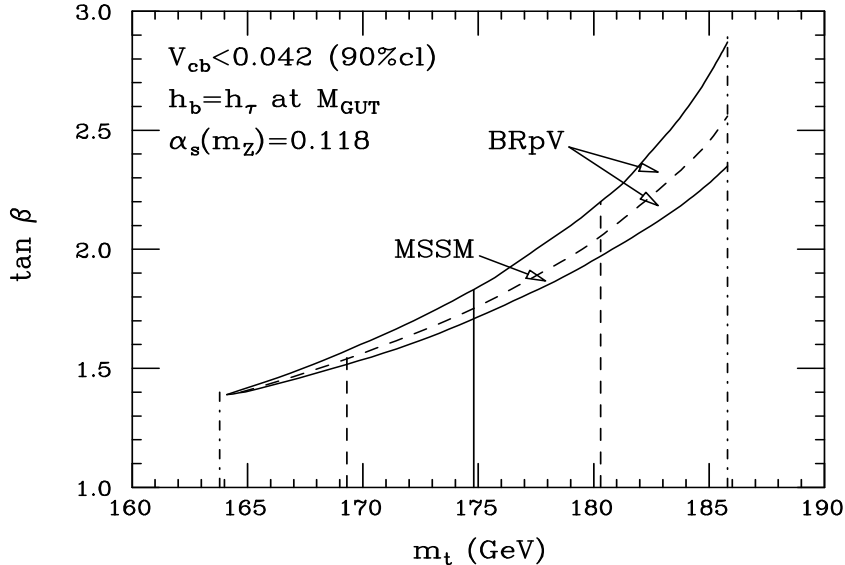
Figure 2: Allowed regions in the $\tan\beta - m_{top}$ plane where bottom–tau unification is possible together with the texture prediction for V_{cb}. This is a magnification of the low $\tan\beta$ region displayed in the previous figure. Accepted values of V_{cb} lie in the 90% c.l.

region in MSSM–BRpV. Similarly, the space between the right solid curve and the dashed curve is the allowed region in the MSSM. The solid curve at the right is common to both models and corresponds to the Landau pole of the quark Yukawa couplings (quasi–fixed point). The solid vertical line corresponds to the central value of the experimental measurement for m_{top}, and the dashed (dot–dashed) lines are the 1σ (2σ) limits.

It is known that bottom–tau unification in the MSSM is obtained in a region similar to the one in Fig. 1 but including an extra branch at high $\tan\beta$. By imposing the texture prediction for V_{cb} this branch disappears. It can be observed from the figure that the MSSM–BRpV region is only slightly larger than the MSSM region. Nevertheless, in the 2σ region for the top quark mass the MSSM–BRpV allowed region is about twice as large as the MSSM one. This can be seen in Fig. 2 which is a blow up of the previous figure. However in our scan we did not find any solution in the large $\tan\beta$ branch within the MSSM nor the MSSM–BRpV.

In Fig. 3 we have relaxed the allowed values of V_{cb} at the weak scale. In this figure we consider $V_{cb} < 0.0437$ which naively corresponds to the 99% c.l. region (here we don't find solutions with $V_{cb} < 0.039$ neither). Although the allowed regions are bigger, the large $\tan\beta$ branch is still not present. Nevertheless, the difference
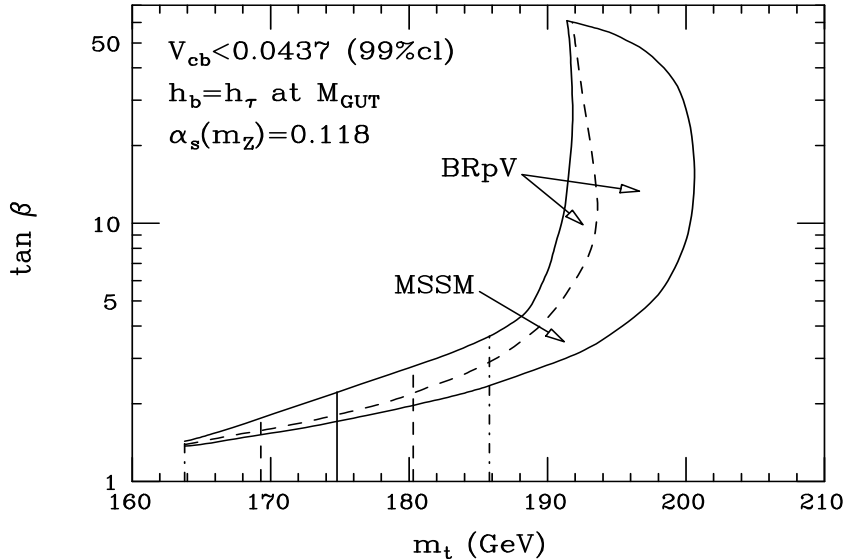
Figure 3: Allowed regions in the $\tan\beta - m_{top}$ plane where bottom–tau Yukawa unification is possible together with the texture prediction for V_{cb}. Accepted values of V_{cb} lie in the 99% c.l.

between the MSSM–BRpV and the MSSM is more pronounced in this case, as it can be seen from Fig. 4 where we blow up the region compatible with the top quark mass measurement. Note that preliminary results of Higgs searches by the ALEPH collaboration [25] which rule out low values of $\tan\beta$, pushing m_t to high values in the MSSM would not necessarily hold in our BRpV case, due to the importance of novel Higgs boson decay channels [8].

The previous four figures have been obtained imposing the validity of the bottom–tau Yukawa unification condition in eq. (3) and the V_{cb} texture condition in eq. (2) at the 5% level. In the next two figures we explore the effect of relaxing the 5%. As we can see, the effect is very interesting.

In Fig. 5 we have plotted the allowed regions in the $\tan\beta - m_{top}$ plane within the MSSM. There are five regions each one labelled by the maximum deviation in percent accepted for the conditions in eqs. (2) and (3). Clearly, the large $\tan\beta$ branch of the MSSM slowly reappears as we relax the GUT conditions and it is fully present in the 40% case. Therefore in this case two solutions are possible, one at large and one at small values of $\tan\beta$, in order to account for the measurement of the top quark mass. The situation is different in MSSM–BRpV. In this case the whole interval for $\tan\beta$ compatible with perturbativity of Yukawa couplings slowly reappears as we relax the GUT conditions. If we accept the GUT conditions within
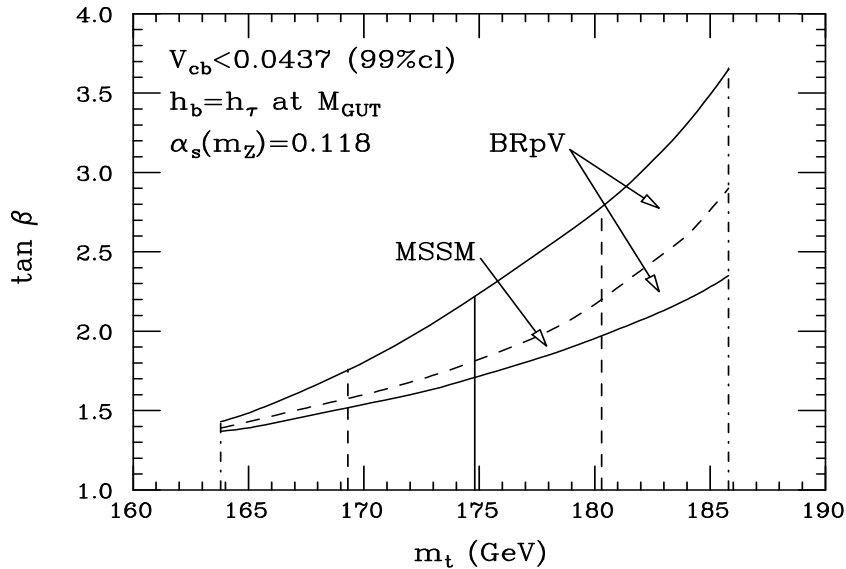
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Figure 4: Allowed regions in the $\tan\beta - m_{top}$ plane where bottom–tau Yukawa unification is possible together with the texture prediction for V_{cb}. This is a magnification of the low $\tan\beta$ region displayed in the previous figure. Accepted values of V_{cb} lie in the 99% c.l.
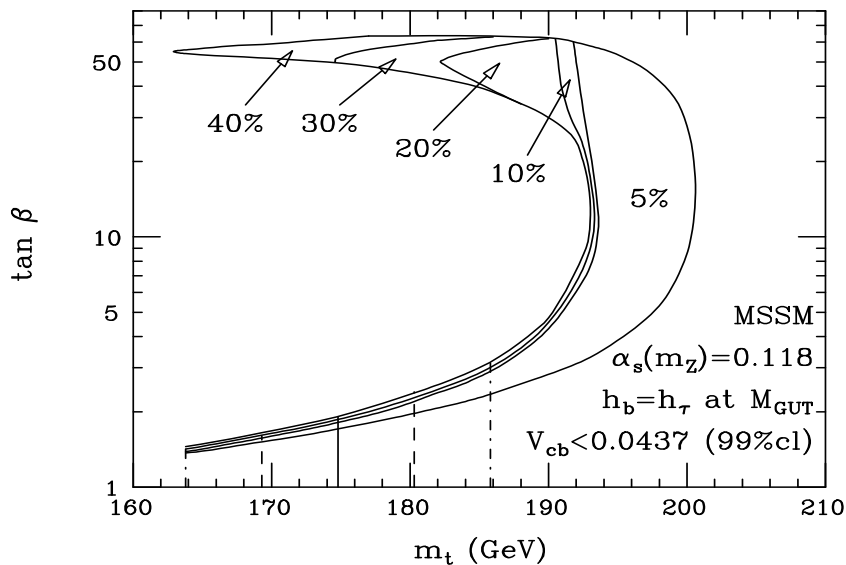

Figure 5: Allowed regions in the $\tan\beta - m_{top}$ plane for the MSSM. The texture conditions at the GUT scale are relaxed to lie within the indicated percent level. Accepted values of V_{cb} at the weak scale lie in the 99% c.l.
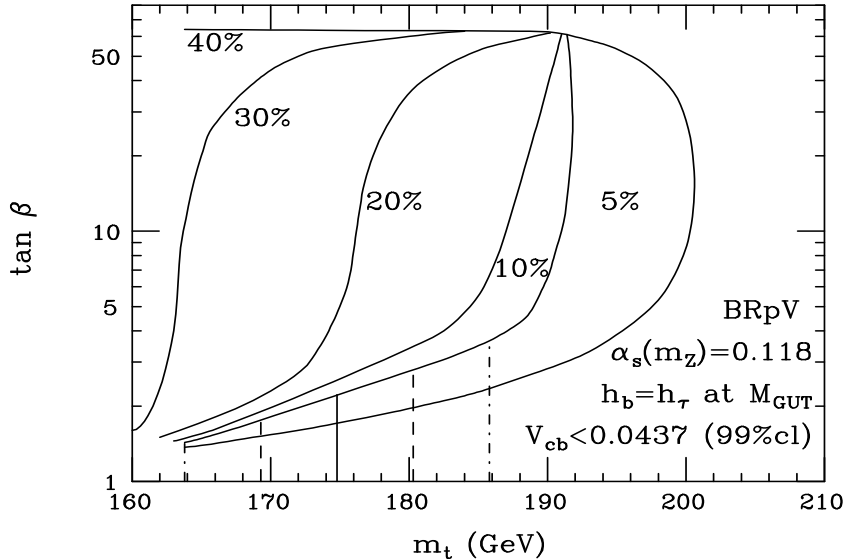
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Figure 6: Allowed regions in the $\tan\beta - m_{top}$ plane for the MSSM–BRpV. The texture conditions at the GUT scale are relaxed to lie within the indicated percent level. Accepted values of V_{cb} at the weak scale lie in the 99% c.l.

40%, then the allowed region is all the space at the left of the quasi–infrared fixed curve. In this case, the prediction for V_{cb}, m_{top} and $\tan\beta$ in BRpV is dramatically different from that in the MSSM. This was already pointed out for bottom–tau Yukawa unification in ref. [12].

6   Discussion

In this section we provide a way to understand of the results presented above in the figures 1 to 6. In to do this we make some approximations so that the relevant RGE's have simple analytical solutions. First of all, let us consider the question of why in BRpV bottom–tau Yukawa unification is achieved at any value of $\tan\beta$, as opposed to the MSSM, where only two disconnected regions of $\tan\beta$ are allowed [12]. We notice first that the quark and lepton masses are related to the different VEVs and Yukawa couplings in the following way

$$m_{top}^2 = \tfrac{1}{2}h_t^2 v_u^2\,, \qquad m_b^2 = \tfrac{1}{2}h_b^2 v_d^2\,, \qquad m_\tau^2 = \tfrac{1}{2}h_\tau^2 v_d^2(1+\delta)\,, \tag{11}$$

where δ depends on the parameters of the chargino/tau mass matrix and is positive [11, 12]. This implies that the ratio of the bottom and tau Yukawa couplings at the

weak scale is given by

$$\frac{h_b}{h_\tau}(m_{weak}) = \frac{m_b}{m_\tau}\sqrt{1+\delta} \tag{12}$$

and grows as $|v_3|$ is increased.

On the other hand, if h_b and h_τ unify at the GUT scale, then at the weak scale its ratio can be approximated by

$$\frac{h_b}{h_\tau}(m_{weak}) \approx exp\left[\frac{1}{16\pi^2}\left(\frac{16}{3}g_s^2 - 3h_b^2 - h_t^2\right)\ln\frac{M_{GUT}}{m_{weak}}\right] \tag{13}$$

implying that the combination $3h_b^2 + h_t^2$ should decrease when $|v_3|$ increases.

In the MSSM region of high $\tan\beta$ the bottom quark Yukawa coupling dominates over the top one, and the opposite happens in the region of low $\tan\beta$. Therefore, at high (low) values of $\tan\beta$, the Yukawa coupling h_b (h_t) will decrease if $|v_3|$ increases, which implies an increase of v_d (v_u) in order to keep constant the quark masses. Similarly, in order to keep constant the W mass, $m_W^2 = \frac{1}{4}g^2(v_d^2 + v_u^2 + v_3^2)$, the VEV v_u (v_d) decreases at the same time. This implies that unification occurs at lower (higher) values of $\tan\beta$ as $|v_3|$ increases. This explains why in BRpV intermediate values of $\tan\beta$ are compatible with bottom–tau unification.

Let us now understand why the high $\tan\beta$ branch is not allowed when we impose the $|V_{cb}|$ constraint at the unification scale. The RGE for the CKM angle $|V_{cb}|$ is [5]

$$\frac{d|V_{cb}|}{dt} = -\frac{|V_{cb}|}{16\pi^2}\left(h_t^2 + h_b^2\right) \tag{14}$$

where $t = \ln(Q)$. In addition, the RGE for the ratio between the charm and top quark Yukawa couplings $R_{c/t} \equiv h_c/h_t$ is

$$\frac{dR_{c/t}}{dt} = -\frac{R_{c/t}}{16\pi^2}\left(3h_t^2 + h_b^2\right). \tag{15}$$

Imposing now the relation in eq. (2) at the GUT scale, we obtain at the weak scale

$$\frac{R_{c/t}}{|V_{cb}|^2}(m_{weak}) \approx exp\left[\frac{1}{16\pi^2}\left(h_t^2 - h_b^2\right)\ln\frac{M_{GUT}}{m_{weak}}\right] \tag{16}$$

where we have approximated the RGE's to first order in perturbation series. Since the left hand side of eq. (16) is greater than one (approximately equal to 1.5), it is clear that the GUT condition $R_{c/t} = |V_{cb}|^2$ prefers the region of parameter space where the top Yukawa coupling is large while the bottom Yukawa coupling is small. This is obtained at small values of $\tan\beta$, since our definition of $\tan\beta = v_u/v_d$ retains the MSSM relation $h_b/h_t = m_b t_\beta/m_{top}$ *. This explains what it is seen in Figs. 5 and 6.

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* In refs. [13, 27] it was defined as $\tan\beta' = v_u/\sqrt{v_d^2 + v_3^2}$ which has the advantage of being invariant under rotations defined at the beginning of section 3, but spoils the relation between h_t and h_b described in the text.

If the GUT conditions are relaxed to more than 5%, eq. (16) should be modified by adding a numerical factor different from one in front of the exponential. The effect is to allow larger values of h_b that can only be achieved in BRpV by increasing v_3 without having to go to very large values of $\tan\beta$ as in the MSSM. Consequently, the plane $m_{top} - \tan\beta$ is filled up in BRpV and not in the MSSM.

Now we would like to understand why in BRpV larger values of $\tan\beta$ are acceptable compared with the MSSM when imposing the GUT conditions at 5%. This effect is observed in Figs. 1 to 4. We notice first that our numerical results with 5% of unification indicate that BRpV accepts values of h_t slightly smaller than the MSSM (the upper bound on h_t is the same in both models). Considering the base independent parameter $\cos\chi$ defined for example in refs. [13, 27] and whose expression in our basis is $\cos\chi = v_d/\sqrt{v_d^2 + v_3^2}$, we have for the top quark Yukawa coupling

$$h_t^2 = \frac{g^2 m_{top}^2}{2m_W^2}\left(1 + \frac{1}{t_\beta^2 c_\chi^2}\right). \tag{17}$$

This equation indicates that for a constant value of the top quark Yukawa coupling, larger values of $\tan\beta$ can be achieved in BRpV compared with the MSSM (in the MSSM $\cos\chi = 1$) when values of $\cos\chi$ smaller than one are considered (typically $0.87 \lesssim |c_\chi| \lesssim 1$). The widening of the allowed region $m_{top} - \tan\beta$ in BRpV is also observed, although less pronounced, if we use the alternative definition of $\tan\beta' = v_u/\sqrt{v_d^2 + v_3^2}$ where we have $t_\beta' = t_\beta c_\chi$. The reason is that $b - \tau$ unification in BRpV can be achieved at larger values of t_β', thus lowering h_t [12].

Now a word about the neutrino mass. The question is whether the values of $\cos\chi$ we find are compatible with small neutrino masses. The tau-neutrino neutrino mass is generated in BRpV via mixing with neutralinos and a weak-scale-type see–saw type mechanism and can be expressed as

$$m_{\nu_\tau} \approx \frac{m_Z^2 s_\zeta^2}{M_{1/2}(1 + t_\beta^2 c_\chi^2)} \tag{18}$$

where $s_\zeta \equiv \sin\zeta$ is another basis independent invariant which in our basis is equal to

$$\sin\zeta = \frac{(\mu v_3 + \epsilon_3 v_d)}{\sqrt{\mu^2 + \epsilon_3^2}\sqrt{v_d^2 + v_3^2}}. \tag{19}$$

This parameter, which is proportional to the tau–sneutrino VEV in the basis where the ϵ_3 term is absent from the superpotential, has to be small in order to have a small neutrino mass. In models with universality of soft mass parameters at the GUT scale, this parameter is naturally small and calculable, since it is generated by radiative corrections through the RGE's of the soft parameters. It can be shown

that

$$\sin \zeta \approx s_\chi c_\chi \frac{\mu' t_\beta c_\chi \Delta B \pm \Delta m^2}{m_{\tilde{\nu}}^2} \tag{20}$$

where $\mu'^2 = \mu^2 + \epsilon_3^2$, $\Delta m^2 = m_{H_d}^2 - M_{L_3}^2$, and $\Delta B = B_3 - B$ with B and B_3 the bilinear soft mass parameters associated to μ and ϵ_3, all at the weak scale. The fraction at the right hand side of eq. (20), which we denote as δ, is a good measure of the cancellation needed in order to have a small neutrino mass. It is approximately given by

$$\delta \approx \frac{\sqrt{m_{\nu_\tau} M_{1/2}(1 + t_\beta^2 c_\chi^2)}}{s_\chi c_\chi m_Z} . \tag{21}$$

Considering $M_{1/2} = 300$ GeV, $\tan \beta = 15$, and $\sin \chi = 0.3$, the amount of cancellation necessary to obtain a neutrino mass $m_{\nu_\tau} = 0.1$ eV is given by $\delta \approx 10^{-4}$ ($\sin \zeta \approx 3 \times 10^{-5}$). We do not think that this is a fine tuning. For example note that the same amount of cancellation between VEV's in the MSSM is necessary in $SO(10)$ models where $\tan \beta$ needs to be higher than 50.

7   Conclusions

In summary, in the context of supersymmetric models with universality of gauge and Yukawa couplings we have studied the predictions for m_{top}, V_{cb} and $\tan \beta$, implied by the Georgi–Jarlskog–Nanopoulos ansätze for fermion mass matrices. First, we have investigated the impact of the most recent experimental measurements of the top quark mass and the CKM matrix element V_{cb} in the MSSM analysis, which we have updated. As it is well-known, imposing bottom–tau unification at the GUT scale two solutions are found in the MSSM, characterized by large and low $\tan \beta$. Requiring in addition the texture constraint for V_{cb} at the GUT scale within 5%, the large $\tan \beta$ solution becomes disallowed even if we accept the experimental measurements for m_{top} and V_{cb} at 99% c.l. If we relax the level of validity of the latter condition, the large $\tan \beta$ solution starts to reappear and it is fully valid when the conditions at the GUT scale are imposed to within 40%. But no intermediate $\tan \beta$ solutions emerge.

We have also studied the same predictions in the MSSM–BRpV model, where a bilinear R–Parity violating term is added to the superpotential. This model is the simplest and most systematic way to include the effects of R-parity violation. Since no new interactions are added, its RGE are unchanged with respect to those of the MSSM. Nevertheless, boundary conditions for Yukawa couplings at the supersymmetric threshold are different. The allowed region in the $\tan \beta - m_{top}$ plane in the MSSM–BRpV is slightly larger than in the MSSM when we impose the GUT

conditions for V_{cb} and bottom–tau unification within 5%. This allowed region for the MSSM–BRpV grows as we relax the texture conditions on V_{cb} at the GUT scale. When these conditions are imposed within 40%, not only is the large $\tan\beta$ branch recovered as in the MSSM, but also the full $\tan\beta - m_{top}$ plane including every $\tan\beta$ value appears. These effects are compatible even with tau neutrino masses as small as 0.1 eV. Last but not least, such small ν_τ values are not really required by present phenomenology to the extent that the atmospheric neutrino data allow for alternative explanations involving sterile neutrinos [28], flavour changing interactions [29] or neutrino decay [30].

Acknowledgements

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