hep-ph/9906343
IFIC/99-35
FTUV/99-34
FSU-HEP-990529
June 1999

Unification of gauge couplings and the tau neutrino mass in Supergravity without R–parity

M. A. Díaz [1], J. Ferrandis [2], J. C. Romão [3], J. W. F. Valle [2]

[1] *High Energy Group, Physics Department, Florida State University Tallahassee, Florida 32306, USA*
[2] *Departament de Física Teòrica, IFIC–CSIC, Universitat de València 46100 Burjassot, València, Spain*
[3] *Departamento de Física, Instituto Superior Técnico Av. Rovisco Pais, 1, 1049-001 Lisboa, Portugal*

Abstract

Minimal R–parity violating supergravity predicts a value for $\alpha_s(\mathrm{M}_Z)$ smaller than in the case with conserved the R–parity, and therefore closer to the experimental world average. We show that the R–parity violating effect on the α_s prediction comes from the larger two-loop b-quark Yukawa contribution to the renormalization group evolution of the gauge couplings which characterizes R–parity violating supergravity. The effect is correlated to the tau neutrino mass and is sensitive to the initial conditions on the soft supersymmetry breaking parameters at the unification scale. We show how a few percent effect on $\alpha_s(\mathrm{M}_Z)$ may naturally occur even with ν_τ masses as small as indicated by the simplest neutrino oscillation interpretation of the atmospheric neutrino data from Super-Kamiokande.

email addresses:
diaz@hep.fsu.edu,
ferrandi@flamenco.ific.uv.es,
fromao@alfa.ist.utl.pt,
valle@flamenco.ific.uv.es

1 Introduction

The prediction for the strong gauge coupling constant $\alpha_s(\mathrm{M}_Z)$ is one of the milestones of unification models [1]. Recent studies of gauge coupling unification in the context of minimal R-parity conserving supergravity [2, 3, 4] agree that using the experimental values for the electro-magnetic coupling and the weak mixing angle the prediction obtained for $\alpha_s(\mathrm{M}_Z) \approx 0.129 \pm 0.010$ [2] is about 2σ larger than indicated by the most recent world average value $\alpha_s(\mathrm{M}_Z)^{W.A.} = 0.1189 \pm 0.0015$ [5].

Here we re-consider the α_s prediction in supergravity (SUGRA). In addition to the standard MSUGRA we consider simplest supergravity version with a bi-linear breaking of R parity [6, 7, 8, 9, 10]. This model is theoretically motivated by the fact that it provides parametrization of many of the features of a class of models in which R–parity breaks spontaneously due to a sneutrino vacuum expectation value (vev) [11]. Moreover, in the simplest case where R–Parity violation lies only in the third generation, the model coincides with the most general explicit R–parity violating model and provides its simplest description.

One of the main features of R–parity violating models is the appearance of masses for the neutrinos [11, 12]. As a result, these models have attracted a lot of attention [13, 14] since the latest round of Super-Kamiokande results [15].

In this paper we show that in the simplest SUGRA R–parity breaking model, with the same particle content as the MSSM and with no new interactions (such as trilinear R–parity breaking couplings), there appears an additional negative contribution to α_s, which can bring the theoretical prediction closer to the experimental world average. This additional contribution to α_s comes from two–loop b-quark Yukawa effects on the renormalization group equation (RGE) for α_s. Moreover, we show that this contribution is typically correlated to the tau–neutrino mass which is induced by R–parity breaking and which controls the R–parity violating effects. We also discuss this correlation within different models for the initial conditions on the soft supersymmetry breaking parameters at the unification scale. We show how to obtain a sizeable effect on $\alpha_s(\mathrm{M}_Z)$ even with ν_τ masses as small as indicated by the simplest neutrino oscillation interpretation of the atmospheric neutrino data from Super-Kamiokande.

2 The MSSM Renormalization Group Equations

The two loop renormalization group equations [16] for the gauge coupling constants in the MSSM have the form

$$\frac{dg_i}{dt} = \frac{g_i}{16\pi^2}\left(b_i g_i^2 + \frac{1}{16\pi^2}\left(\sum_{j=1}^{3} b_{ij}\, g_i^2 g_j^2 - \sum_{l=t,b,\tau} b'_{il}\, g_i^2 h_l^2\right)\right) \tag{1}$$

where g_i, $i = 1, 2, 3$, are the gauge couplings of the $U(1)$, $SU(2)$, and $SU(3)$ groups respectively, and h_l, $l = t, b, \tau$, are the quark and lepton Yukawa couplings of the third generation. The numerical coefficients b_i, b_{ij}, and b'_{il} are given in ref. [16].

It is useful to obtain an approximate analytical solution to the gauge coupling constants from eq. (1). This is done by neglecting the two loop Yukawa contribution in first approximation. The result is [17]

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_U(\mathrm{M}_U)} + b_i t + \frac{1}{4\pi}\sum_{j=1,2,3}\frac{b_{ij}}{b_i}\ln\left[1 + b_j\alpha_U(\mathrm{M}_U)t\right] - \Delta_i \tag{2}$$

where $t = \frac{1}{2\pi}\ln(\mathrm{M}_U/\mu)$, α_U is the unified gauge coupling constant, M_U is the unification scale, μ is an arbitrary scale, and Δ_i are corrections due to several effects, mainly threshold corrections. Although GUT-type threshold corrections are potentially sizeable, we neglect them here since they are in general model-dependent. For a discussion see ref. [2, 3, 18]. Leading logarithms from supersymmetric spectra threshold corrections to $\alpha_S(\mathrm{M}_Z)$ can be summarized in the following formula [3, 4]

$$\Delta\alpha_s^{SUSY} = -\frac{19\alpha_s^2}{28\pi}\ln\left(\frac{T_{SUSY}}{\mathrm{M}_t}\right) \tag{3}$$

where T_{SUSY} is an effective mass scale given by

$$T_{SUSY} = m_{\tilde H}\left(\frac{m_{\widetilde W}}{m_{\tilde g}}\right)^{\frac{28}{19}}\left[\left(\frac{m_{\tilde l}}{m_{\tilde q}}\right)^{\frac{3}{19}}\left(\frac{m_H}{m_{\widetilde H}}\right)^{\frac{3}{19}}\left(\frac{m_{\widetilde W}}{m_{\widetilde H}}\right)^{\frac{4}{19}}\right]. \tag{4}$$

This scale is not simply an average of SUSY masses since it can be smaller than all the masses of the supersymmetric particles [4, 3]. Large values of T_{SUSY} are experimentally preferred because in general they contribute negatively to $\Delta\alpha_s^{SUSY}$, bringing $\alpha_s(M_Z)$ closer to the experimental average by an estimated $|\Delta\alpha_s^{SUSY}| \le 0.003$ [2]. There is in addition, a finite contribution from supersymmetric threshold corrections which may be important if the supersymmetric spectrum is light [19]. Moreover there is also a small conversion factor from $\overline{\rm MS}$ to $\overline{\rm DR}$ [20], as well as possible contributions coming from non renormalizable operators which can be induced from physics between the Planck to the GUT-unification scale [21].

Let us now turn to the important issue of the two loop Yukawa contribution to the gauge coupling constants RGE. This contribution is not included in eq. (2) and is crucial for our purposes, providing a correction which is negative and can be important if h_t or h_b are large ($t_\beta \approx 1$ or $t_\beta \approx 50$ respectively). Making a one-step integration we obtain the approximate expression

$$\Delta\alpha_s^{YUK} \approx -\frac{\alpha_s^2}{32\pi^3} \ln\left(\frac{\mathrm{M}_U}{\mathrm{M}_t}\right) \left\{ b'_{3t} h_t^2 + b'_{3b} h_b^2 \right\} \tag{5}$$

In the small $\tan\beta$ region, the bottom Yukawa coupling is negligible compared to the top Yukawa, then we get $\Delta\alpha_s^{YUK} \approx -0.1\alpha_s^2 h_t^2$, giving us an estimate of the magnitude of this correction. Note that this correction is not bigger in the high $\tan\beta$ scenario, where both Yukawas are large, since they are not as large as the top Yukawa in the low $\tan\beta$ case.

In contrast, in the $R\!\!\!/_p$–MSSM model, the bottom Yukawa coupling can be non-negligible for any value of $\tan\beta$ [22]. As a result we cannot neglect the bottom quark Yukawa coupling, since it can be as large as the top quark Yukawa, especially if the R–parity violating parameters are large.

3 The $R\!\!\!/_p$-MSUGRA Model

In order to illustrate the essential features of the model, it is enough to consider a one generation $R\!\!\!/_p$–MSSM [6, 8, 9, 10], since it contains the main ingredients relevant for our present discussion. The superpotential W is

$$W = W_{MSSM} + W_{R\!\!\!/_p} \; , \tag{6}$$

where W_{MSSM} is the familiar superpotential of the MSSM

$$W_{MSSM} = \left[h_t \widehat{Q}_3 \widehat{H}_u \widehat{U}_3 + \lambda_0^D \widehat{L}_0 \widehat{Q}_3 \widehat{D}_3 + h_\tau \widehat{L}_0 \widehat{L}_3 \widehat{R}_3 - \mu_0 \widehat{L}_0 \widehat{H}_u \right] . \tag{7}$$

Here we are using the notation $\widehat{L}_0 \equiv \widehat{H}_d$, $\mu_0 \equiv \mu$, and $\lambda_0^D \equiv h_b$ in the superpotential, and $v_0 \equiv v_d$ for the $\widehat{H}_d$ vacuum expectation value. This notation is justified because $\widehat{H}_d$

and $\widehat{L}_3$ have the same quantum numbers. The piece of the superpotential which breaks R–parity is given by

$$W_{R\!\!\!/} = -\mu_3 \widehat{L}_3 \widehat{H}_u \tag{8}$$

where μ_3 is the bi-linear R–Parity violating term (BRpV), denoted ϵ_3 in ref. [6, 7].

Notice that we do not generate a tri-linear R–Parity violating (TRpV) term in models that arise from spontaneous breaking of R-parity. In fact, even if explicit tri-linear terms were present, for the simple one-generation case they can always be rotated away into the bi-linear term given in eq. (8). In other words, the most general one-generation explicit SUGRA R–Parity violation model is characterized by a single parameter, which may be chosen either as μ_3, or as λ_3^D or the sneutrino vev. The converse is not true, BRpV cannot be rotated away in favour of TRpV.

Although the above presentation would be in some sense the simplest and sufficient for our purposes, it will be useful for us in what follows to keep a redundant parametrization in which the bi-linear and tri-linear R parity violating terms coexist.

The scalar potential contains the following relevant soft terms

$$V_{soft} = \left(\begin{array}{c} \widetilde{L}_0 \\ \widetilde{L}_3 \end{array} \right)^{\dagger} \left(\begin{array}{cc} M^2_{L_0} & M^2_{L_{03}} \\ M^2_{L_{30}} & M^2_{L_3} \end{array} \right) \left(\begin{array}{c} \widetilde{L}_0 \\ \widetilde{L}_3 \end{array} \right) - \left(\mu_\alpha B_\alpha \widetilde{L}_\alpha H_u - A^D_\alpha \lambda^D_\alpha \widetilde{L}_3 \widetilde{Q}_3 \widetilde{D}_3 + h.c. \right) \tag{9}$$

where $M^2_{L_i}$ are the soft mass terms and mixing for the down type Higgs and slepton fields, B_α $\alpha = 0, 3$, are the bi-linear soft mass parameters (B_0 corresponds to the usual B term in the MSSM), while A^D_α are the tri-linear soft mass parameters (A^D_0 is the usual A_D term in the MSSM).

The equality of the quantum numbers of the down-type Higgs and tau lepton $SU(2) \otimes U(1)$ superfields opens the possibility to work in different basis [9, 23, 24, 25]. This field redefinition is defined by

$$\begin{pmatrix} \widehat{L}'_0 \\ \widehat{L}'_3 \end{pmatrix} = \begin{pmatrix} \cos\alpha_L & \sin\alpha_L \\ -\sin\alpha_L & \cos\alpha_L \end{pmatrix} \begin{pmatrix} \widehat{L}_0 \\ \widehat{L}_3 \end{pmatrix} \tag{10}$$

where α_L is the angle of rotation, which in turn induces a rotation of the μ –terms. Under this change of basis the Lagrangian parameters transform and it is impossible to eliminate completely the effects of the bi-linear terms [6, 7, 25]. Note that different basis may be convenient for different applications [24].

Here we are specially interested to express R–parity violating effects through basis independent parameters

$$v_d = \sqrt{v_0^2 + v_3^2} \tag{11}$$

$$\mu = \sqrt{\mu_0^2 + \mu_3^2} \tag{12}$$

$$\lambda^D = \sqrt{(\lambda_0^D)^2 + (\lambda_3^D)^2} \tag{13}$$

From the above we can deduce that the natural generalization of the MSSM definition of $\tan\beta$ is given by

$$\tan\beta = \frac{v_u}{v_d}, \tag{14}$$

which is also a basis invariant. This definition differs from the one used in ref. [6, 22], namely $\tan\beta = v_u/v_0$. There are other invariants which turn out to be very useful [26] and are defined as

$$\cos\zeta = \frac{\mu_\alpha v_\alpha}{\mu v_d} \tag{15}$$

$$\cos\gamma = \frac{\lambda_\alpha^D \mu_\alpha}{\lambda^D \mu} \tag{16}$$

$$\cos\chi = \frac{\lambda_\alpha^D v_\alpha}{\lambda^D v_d} \tag{17}$$

Note that these three parameters are not independent due to the trigonometric relation

$$\cos\chi = \cos(\gamma - \zeta) \tag{18}$$

The remaining R–parity violating variables $\sin\zeta$ and $\sin\gamma$ determine the ν_τ mass and the R–parity violating effects in general in the fermionic sector, while $\sin\chi$ characterizes the R–parity violating effects on α_s. As we will see below there is only one of these parameters which survives, owing to the minimization conditions of the theory.

In this model the top and bottom quark masses are given by

$$\mathrm{M}_t = \frac{h_t}{\sqrt{2}} v_u = s_\beta h_t \frac{\sqrt{2}\mathrm{M}_W}{g} \tag{19}$$

$$\mathrm{M}_b = \frac{1}{\sqrt{2}}\left(\lambda_0^D v_0 + \lambda_3^D v_3\right) = c_\beta c_\chi \lambda^D \frac{\sqrt{2}\mathrm{M}_W}{g} \tag{20}$$

This formula for the bottom quark mass is specially interesting, since it is expressed in terms of basis-independent R–parity violating effects parameters.

As in the MSSM case to connect the phenomenology at the electroweak scale with the SUGRA parameter space we need to use the renormalization group equations. A question immediately arises as to the number of additional parameters necessary to characterize the model. For a one-generation model with universality of soft parameters at the unification scale only one additional parameter is needed in addition to the MSUGRA parameters [6]. We have, however, some freedom in this choice. To compute the Lagrangian parameters at the electroweak scale we can follow two different approaches [24]

- the bi-linear or μ_3–approach, in which the parameters which fix the model are:

  $$\left(\mathrm{A}_0, \mathrm{M}_0, \mathrm{M}_{1/2}, \mathrm{t}_\beta, \mu_3^U\right)$$

  Because of the form of the RGE for λ_3^D, $d\lambda_3^D/dt \propto \lambda_3^D$, if λ_3^D is zero at the unification scale it will be zero at the electroweak scale

- The second possibility is the λ_3^D–approach, in this case the fundamental parameters of the model are

  $$\left(\mathrm{A}_0, \mathrm{M}_0, \mathrm{M}_{1/2}, \mathrm{t}_\beta, (\lambda_3^D)^U\right).$$

  In contrast to the previous case here one arrives at the electroweak scale to the coexistence of bi-linear and tri-linear R–parity breaking parameters.

It does not matter which approach we follow because both are equivalent. Notice that, while in the bi-linear approach one can ignore tri-linears without loss of generality, the converse is not true: one can not neglect bi-linears consistently due to the structure of the RGES. One can change from one basis to another and thus compare calculations which have been performed in different basis. These results have to be the same.

Now we are ready to understand how R–parity violation can affect the gauge coupling unification through the two loop Yukawa contribution to the RGES for α_s. One finds,

$$\Delta\alpha_s^{YUK} \approx -\frac{\alpha_s^2}{32\pi^3} \ln\left(\frac{\mathrm{M}_U}{\mathrm{M}_t}\right)\left\{b'_{3t}h_t^2 + b'_{3b}(\lambda_0^D)^2 + b'_{3b}(\lambda_3^D)^2\right\} \tag{21}$$

Where one notes the appearance of the R–parity violating coupling λ_3^D. Clearly this term combines with λ_0^D to form the basis invariant λ^D as follows,

$$\Delta\alpha_s^{YUK} \approx -\frac{\alpha_s^2}{32\pi^3} \ln\left(\frac{\mathrm{M}_U}{\mathrm{M}_t}\right)\left\{b'_{3t}h_t^2 + b'_{3b}(\lambda^D)^2\right\}$$

Using the formulas (20,19) for the top and bottom masses we obtain

$$\Delta\alpha_s^{YUK} \approx -\frac{\alpha_s^2}{32\pi^3}\ln\left(\frac{M_U}{M_t}\right)\frac{g^2}{2M_W^2}(1+t_\beta^2)\left\{b'_{3t}\frac{\mathrm{M}_t}{t_\beta}+b'_{3b}\frac{\mathrm{M}_b}{c_\chi}\right\} \tag{22}$$

We are now set to demonstrate the direct correlation between the last term in eq. (22) and the magnitude of R–parity violation which, as already mentioned, is characterized by a unique parameter in this model. To see this we must make use of the three minimization equations of the scalar potential of the theory, the two of the MSSM plus a third equation involving the vev for the tau sneutrino. Using this equation one can find a relation between $\sin\zeta$ and $\sin 2\gamma$ which finally reduces the extra number of parameters to simply one, when compared with the R-parity conserving supergravity model. At first order in μ_3/μ can be simplified to

$$\sin\zeta = \frac{\mu_0\mu_3}{\mu^2}\left(\delta_B t_\beta \pm \delta_M\right) = \frac{1}{2}\sin(2\gamma)\left(\delta_B t_\beta \pm \delta_M\right) \tag{23}$$

where

$$\delta_B = \frac{\mu\Delta B}{\left(M^2_{\tilde{\nu}_3} - \frac{\mu_3^2}{\mu^2}\Delta M^2\right)} \qquad \delta_M = \frac{\Delta M^2}{\left(M^2_{\tilde{\nu}_3} - \frac{\mu_3^2}{\mu^2}\Delta M^2\right)}\ ,$$

and we have defined

$$\Delta B = B_3 - B_0 \qquad \Delta M^2 = M_3^2 - M_0^2$$

We notice that the double sign in eq.(23) is the result of the solution to a quadratic equation in the minimization conditions of the scalar potential. In models with universality of soft terms, δ_M is positive but δ_B can take either sign.

Thus eq. (23) shows that, as anticipated, only one of the three parameters ζ,γ,χ is independent. Together with the above SUGRA parameters which fix the model it determines the Majorana mass for the tau neutrino. The latter is induced by the mixing of the original tau neutrino field with the neutralinos [11, 12] and is given mainly by the parameter $\sin\zeta$ through the approximate relation

$$\mathrm{M}_{\nu_\tau}^{\nu-\chi mixing} = \frac{\mathrm{M}_Z^2 M_{\tilde{\gamma}}\mu s_\zeta^2 c_\beta^2}{\left(M_Z^2 M_{\tilde{\gamma}} s_{2\beta} c_\zeta - M_1 M_2 \mu\right)} \tag{24}$$

which depends on the SUGRA parameters, where we have defined the parameter $M_{\tilde{\gamma}} \equiv c_W M_1 + s_W M_2$. From eq. (18), eq. (23) and (24) it is evident that we can get an expression for $\cos\chi$ whose exact form is unimportant for our present argument, except for the property that

$$\cos\chi \to 1 \ \text{as}\ M_{\nu_\tau} \to 0$$

Thus the maximum value $c_\chi = 1$ corresponds to the R-parity conserving case. From this it is clear that the larger the R parity violation the larger will be the additional contribution coming from the ratio M_b/c_χ in eq. (22). The above equation establishes a relationship that the basis-independent parameter c_χ bears with the tau neutrino mass.

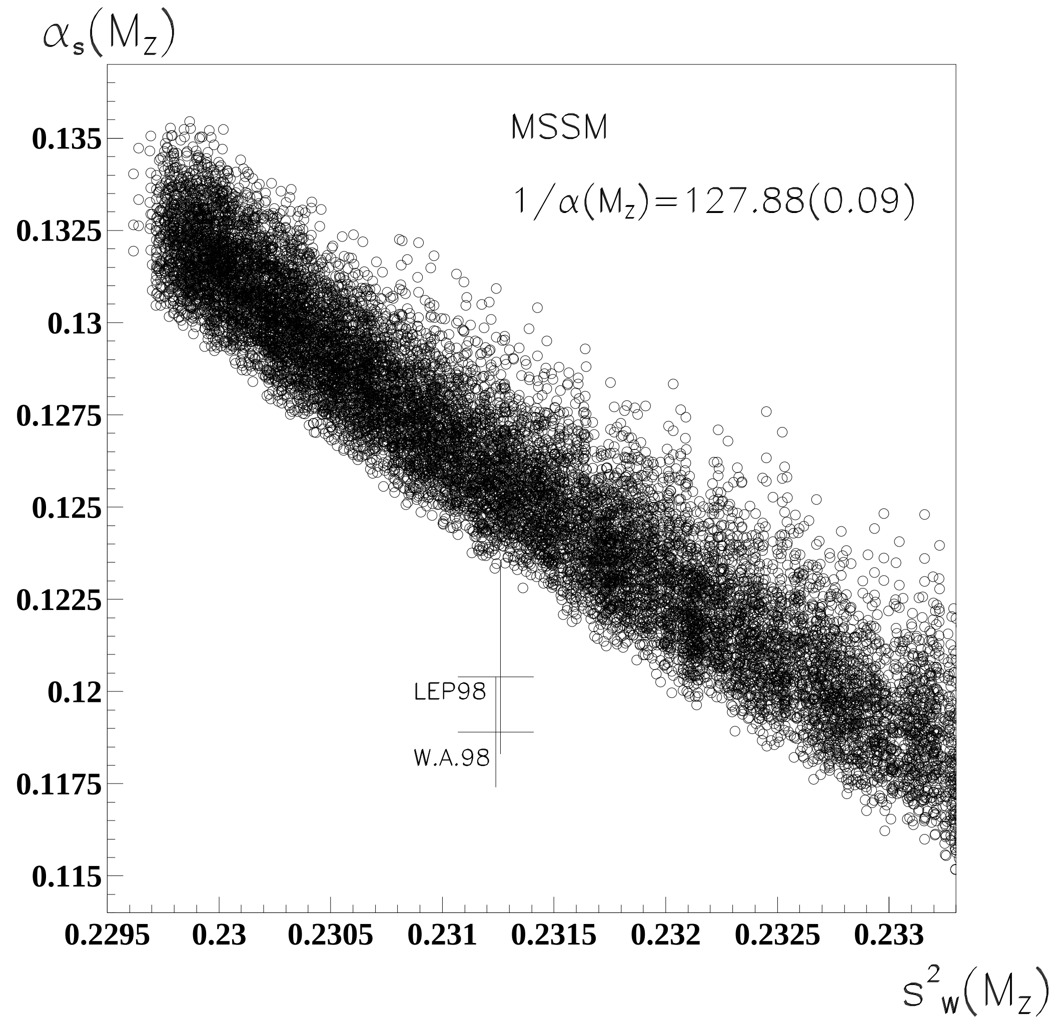

Figure 1: $\alpha_s(M_Z)$ *versus* $\hat{s}_Z$ *for the MSSM*

We now turn to the implications of R-parity violation on the α_s predictions derived from eq. (22) and to our numerical results. We have used the two-loop renormalization group equations for the gauge coupling constants and the Yukawa couplings and the one-loop RGE for μ–terms and for the rest of the soft parameters [27]. We will study the prediction for the gauge coupling constants at the M_Z scale in a model with universality of the soft terms at the unification scale [1]. We compare masses and couplings at the

[1] For the sake of generality and in order to simplify the discussion we will neglect possible GUT

M_Z scale with their experimental values (see appendix for a detailed description of the method we have used for the running of the effective masses to their pole values and the running of the gauge couplings to their $\overline{\rm MS}$ values at the M_Z scale).

As a first step in our study of the supersymmetric $\alpha_s(\mathrm{M}_Z)$ and $\widehat{s}_Z^2$ predictions we have updated the standard MSUGRA prediction taking into account the latest PDG experimental values for $\widehat{\alpha}(\mathrm{M}_Z)^{-1}$ [5]

$$\widehat{\alpha}(\mathrm{M}_Z)^{-1} = 127.88 \pm 0.09$$

On the other hand for the top, bottom and tau pole masses we have used [5]

$$\mathrm{M}_t^{pol} = 173 \pm 5.2 \quad \mathrm{GeV}$$

$$\mathrm{M}_b^{pol} = 4.1 \quad \mathrm{to} \quad 4.4 \quad \mathrm{GeV}$$

$$\mathrm{M}_\tau^{pol} = 1777.05 \begin{array}{c} +0.29 \\ -0.26 \end{array} \mathrm{MeV}$$

In figure (1) we display updated the MSUGRA prediction for $\alpha_s(\mathrm{M}_Z)$ and $\widehat{s}_Z^2$ given as a scatter plot, where each point corresponds to a different choice of SUGRA parameters, varying over a wide range, given as

$$\begin{array}{c} 0 < \mathrm{M}_0 < 500 \quad \mathrm{GeV} \\ 0 < \mathrm{M}_{1/2} < 500 \quad \mathrm{GeV} \\ -1000 < \mathrm{A}_0 < 1000 \quad \mathrm{GeV} \\ 2 \lesssim \mathrm{t}_\beta < 60 \end{array} \tag{25}$$

In the figure one can appreciate the difference between the present world average for $\alpha_s(\mathrm{M}_Z)$

$$\alpha_s(\mathrm{M}_Z)^{W.A.} = 0.1189 \pm 0.0015$$

and the 1998 average of the LEP measurements [5]

$$\alpha_s(\mathrm{M}_Z)^{LEP98} = 0.1214 \pm 0.0031$$

For a discussion on the question of the average of values of α_s deduced at different energy scales, see references [3, 28].

We notice that if we fix $\widehat{s}_Z^2$ inside its experimental range [2],

$$(\widehat{s}_Z^2)^{W.A.} = 0.23124 \pm 0.00024$$

threshold contributions, as well as non–renormalizable operator contributions.

[2]We have moved slightly the $\widehat{s}_Z^2$ for one of the measurements in order to observe clearly the difference in the$\widehat{s}_Z^2$ values.

the MSUGRA $\alpha_s(\mathrm{M}_Z)$ prediction lies in the range $\alpha_s(\mathrm{M}_Z) \approx 0.127 \pm 0.003$, which is a bit more than 2σ higher than the world average.

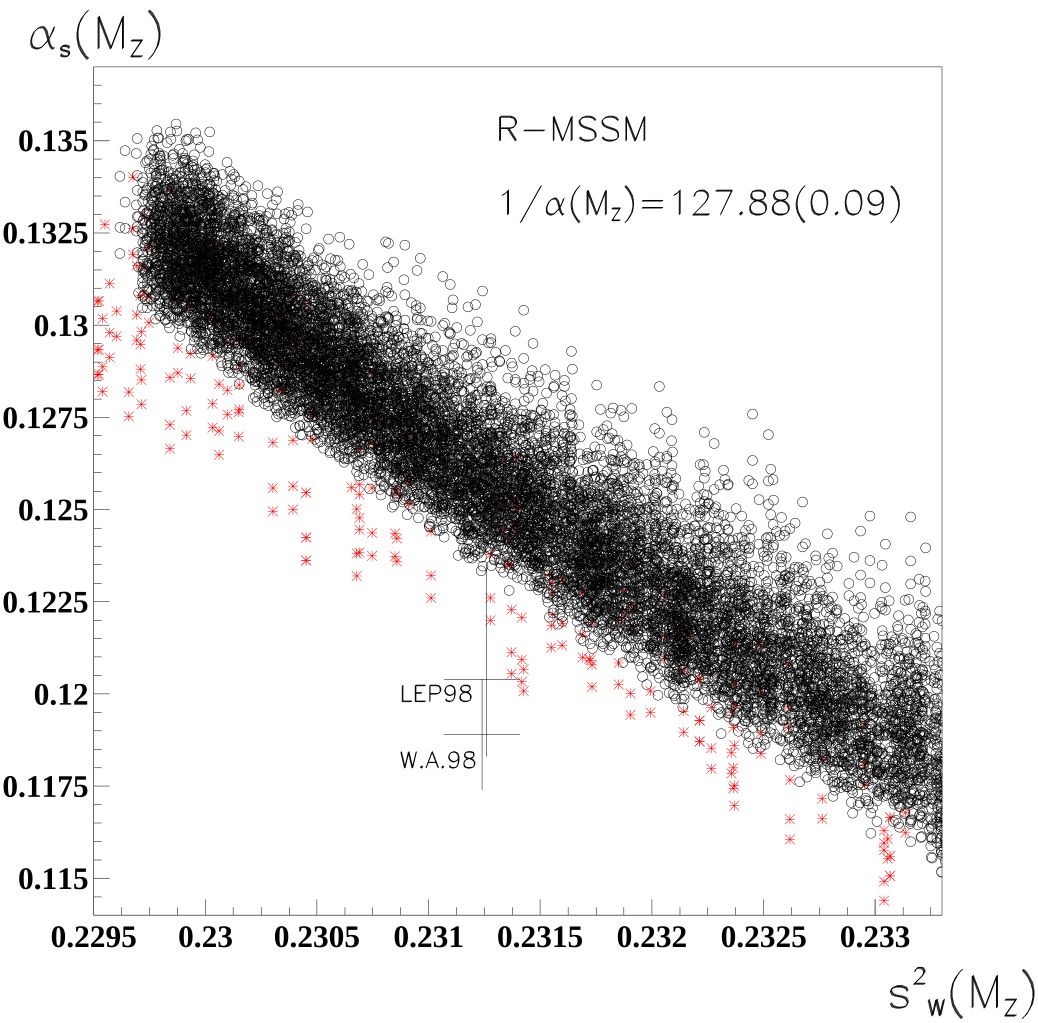

Figure 2: $\alpha_s(M_Z)$ *versus* $\hat{s}_Z$ *for the* $\not{R}$*–MSSM model*

Now we turn to discuss the results we obtain in our bi-linear R-parity breaking model, $\not{R}$–MSSM for short. The method we have used is similar to the previous procedure. In this case additional complications appear because of the mixing between charginos and the tau lepton and we need to ensure that the tau mass is the experimentally measured. On the other hand the mixing between the neutralinos and the neutrino implies the appearance of a mass for the tau neutrino which arises from the mixing with the neutralinos eq. (24) and we must ensure that it lies below the experimental bound [29]. As we have already seen, the non-zero tau sneutrino vev implies that we have to take into account the additional constraint given by the third minimization equation. Once we satisfy all these constraints we find that the $\not R$-MSUGRA predicts $\alpha_s(\mathrm{M}_Z)$ values nearer the experimental value than the R-parity conserving MSUGRA case. This comes from the enhanced negative two-loop bottom-quark Yukawa contribution to the RGE's. For example, taking the world average experimental value of $\tilde{s}^2_Z$, one can move $\alpha_s(\mathrm{M}_Z)$ from a minimum value of approximately 0.125 in the MSUGRA case down to a minimum value of 0.122 or so in the $\not R$–MSSM model, bringing closer to the W.A. and within one σ from the most recent average of LEP measurements given in ref. [5]. These results can be clearly seen from figure (2), where each point represents a different parameter choice in the $\not R$–MSUGRA model. Notice that the $\not R$–MSUGRA model is totally fixed if we know the ν_τ mass. We have varied the tau neutrino mass below the laboratory bound $\mathrm{M}_{\nu_\tau} < 18.2$ MeV [29].

4 Discussion: $\Delta\alpha_s$ versus m_{ν_τ}

As we have seen, the effect on $\alpha_s(\mathrm{M}_Z)$ in the $\not R$–MSUGRA model is is related with the ν_τ mass. What is the price to decrease $\alpha_s(\mathrm{M}_Z)$? Clearly points with large values for the R–parity violating parameter μ_3 will have small $\alpha_s(\mathrm{M}_Z)$. Typically these points also have large mas for the tau neutrino. In fact all the points with low $\alpha_s(\mathrm{M}_Z)$ in fig.2 correspond to large ν_τ masses close to the present laboratory bound. For these masses many R–parity violating phenomena have large rates. Among the latter is the decay of the lightest neutralino, which will typically occur inside the LEP and LHC detectors, for reasonable values of parameters.

Large tau neutrino masses would appear in conflict with the ν_μ to ν_τ oscillation interpretation of the recent atmospheric neutrino data from underground detectors [30]. First we point out that, as it stands, the present data allow for alternative explanations either in terms of flavour-changing neutrino matter interactions [31] or ν_μ deutrino decay to ν_τ [32] or ν_μ decau to ν_e [33] which might be relevant in the present model or in the presence of a majoron. Barring an enhanced statistics up-going muon event sample, it is hard to dismiss such alternative explanations of the atmospheric data on the basis of the present information on rates and zenith angle distributions both of sub as well as multi-GeV events.

Can our result on $\alpha_s(\mathrm{M}_Z)$ survive with ν_τ masses in the range indicated by the standard oscillation interpretation of the Superkamiokande results? In the one-generation approximation the ν_μ and the ν_e are massless, but it is known that they acquire non-zero masses due to loops. Barring the results of a detailed investigation of the loop-generated masses of the two low-lying neutrinos, one can not give a precise and complete answer to this question. However we can state that, neglecting these masses one can have a sizeable drop in $\alpha_s(\mathrm{M}_Z)$ for ν_τ mass in the range close to 3×10^{-2} eV, indicated by the best fit of the oscillation hypothesis [34].

To better understand this statement we make a few approximations. Consider first eq. (22). As we mentioned before, in BRpV the term proportional to m_t and the term proportional to m_b can be simultaneously large. In this case, with the two terms of similar magnitude we have

$$\cos\chi \approx \frac{M_b}{M_t} t_\beta \approx 0.017 t_\beta \tag{26}$$

which is a necessary condition for large Yukawa contributions to α_s in BRpV. On the other hand, it is convenient to rewrite the formula for the neutrino mass in eq. (24) by introducing the mass parameter Λ defined by the equation

$$\sin\zeta \equiv \frac{1}{c_\beta}\sqrt{\frac{M_\nu}{\Lambda}} \tag{27}$$

where the neutrino mass M_ν is in eq. (24) and $\Lambda = \mathcal{O}(M_Z^2/M_{1/2})$. Therefore, for a neutrino mass of the order of 0.1 eV we need $\sin\zeta \approx 10^{-5}\sqrt{\Lambda}/c_\beta$ with Λ in GeV, indicating that the parameter $\sin\zeta$ is very small. In this way, from eq. (18) we see that small neutrino mass implies $\cos\chi \approx \cos\gamma$. Using this last relation in eq. (23) we find a second expression for $\sin\zeta$:

$$\sin\zeta \approx s_\chi c_\chi (\delta_B t_\beta \pm \delta_M) \tag{28}$$

where the δ's are defined below eq. (23). The quantity in parenthesis is a good measure of the amount of cancelation necessary in order to have a sizable effect on α_s with small neutrino mass. The cancelation can occur with either sign since the sign of δ_B is not fixed. We have that:

$$\delta \equiv (\delta_B t_\beta \pm \delta_M) \approx \frac{1}{s_\chi c_\chi c_\beta}\sqrt{\frac{M_\nu}{\Lambda}}\,. \tag{29}$$

We note that in SUGRA with universality of soft SUSY breaking parameters at unification δ_B is usually smaller than δ_M. As a result, the cancellation necessary in order to obtain small neutrino mass favours large $\tan\beta$ values. For example, for $\tan\beta = 40$, $c_\chi \approx s_\chi \approx 0.7$, and a 0.1 eV neutrino mass we have that for $M_{1/2} = 200$ GeV the amount of cancelation is $\delta \approx 1 \times 10^{-4}$. If the gaugino mass parameter is increased to $M_{1/2} = 1000$ GeV, the cancellation is $\delta \approx 3 \times 10^{-4}$. Our approximation is conservative since we have assumed δ_M of order 1. However, δ_M can be smaller because it is zero at the unification scale and arises only from the RGE evolution from unification to the weak scale, typically, $\delta_M \lesssim \delta_M \lesssim$ few %. We do not think that this is a fine tuning. In fact we remind the reader that a similar amount of cancelation between vev's is already present in the MSSM at high values of $\tan\beta$.

In short, while our main result on $\alpha_s(\mathrm{M}_Z)$ does favour large ν_τ masses, there is a range of parameters, motivated by universality of the soft breaking terms, where the effect naturally survives even if the ν_τ mass is rather low. This guarantees also that the lightest neutralino would typically decay inside the detectors now under discussion, changing completely the phenomenology of supersymmetry from that expected in the MSSM.

5 Conclusion

In conclusion, we have shown how minimal R–parity violating supergravity can lower the $\alpha_s(\mathrm{M}_Z)$ prediction with respect to the case with conserved the R–parity, as suggested by the present experimental world average. We have identified the source of this effect on the α_s prediction as coming from the two-loop bottom Yukawa coupling contribution to the renormalization group evolution of the gauge couplings. This contribution can not be neglected if the R–parity violating parameters are sizeable. We have also shown how this effect on the α_s prediction is in general directly correlated to the value of the tau neutrino mass which is generated by the mixing of neutralinos and neutrinos. We have also discussed to which extent this correlation depends on the initial conditions for the soft supersymmetry breaking parameters at the unification scale. We showed how to obtain a sizeable effect on $\alpha_s(\mathrm{M}_Z)$ even in the case that the ν_τ mass lies in the range indicated by the simplest neutrino oscillation interpretation of the atmospheric neutrino data.

Acknowledgements

This work was supported by DGICYT under grants PB95-1077 and by the TMR network grant ERBFMRXCT960090 of the European Union. M. A. D. was supported by a DG-ICYT postdoctoral grant and partly by the U. S. Department of Energy under contract number DE-FG02-97ER41022. J. F. was supported by a Spanish MEC FPI fellowship.

A Appendix: Numerical procedure

In this appendix we describe with some detail the method we follow to predict the gauge coupling constants at the M_Z scale. We have used the 2–loop RGE's for the gauge coupling constants and for the Yukawa couplings including R–parity violating couplings [27]. We neglect the Yukawa couplings of the first two generations. For the rest of the parameters of the $\not R$–MSSM model we have used 1–loop RGE's [27]. We have imposed universality of soft parameters and gauge coupling unification at a scale M_U. We have explored two different values for the unification scale, M_U ($1.2 \times 10^{16} < \mathrm{M}_U < 3.6 \times 10^{16}$GeV), and for the gauge coupling constant at the unification scale α_U, ($23.5 < \alpha_U^{-1} < 24.5$). Using the RGE's we have found the gauge coupling constants at M_t and then we have evolved down to M_Z scale as explain below. On the other hand we have computed the pole masses from the running masses at M_t following the same procedure as the ref. [35]. First of all we have to explain how we compute the Yukawa couplings at M_t at the SM side. We have to use the right matching conditions at M_t which are easy to compute from the formulas (19) and (20) for the h_t, λ^D y h_τ Yukawas. In the $\not R$–MSSM model are [22]

$$h_t(\mathrm{M}_t)^{SM} = s_\beta h_t(\mathrm{M}_t)^{\not R}$$

$$h_b(\mathrm{M}_t)^{SM} = c_\chi c_\beta \lambda^D(\mathrm{M}_t)$$

$$h_\tau(\mathrm{M}_t)^{SM} = \frac{c_\beta}{\left(1 - s_\zeta^2 f(M_2, t_\beta, \mu, c_\zeta)\right)^{1/2}} h_\tau(\mathrm{M}_t)^{\not R}$$

These conditions reduce to the MSSM matching conditions in the limit c_ζ,$c_\chi \to 1$.

In order to run of masses and couplings to their experimental values we use known relations. First we have evolved α_1 and α_2 from scale M_t to scale M_Z to compute $\alpha(\mathrm{M}_Z)$ and $\widehat{s}^2_Z$. For α_s, given the value $\alpha_s(\mathrm{M}_t)$, which we get from the running of the RGE's from the unification to the M_t scale, we can compute Λ_{QCD} at M_t using the approximate solution for α_s in the SM [36] which includes 3–loop QCD contributions

$$\alpha_s(\mu) = \frac{\pi}{\beta_0 t}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln(t)}{t} + \frac{\beta_1^2}{\beta_0^4 t^2}\left(\left(\ln(t) - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{\beta_1^2} - \frac{5}{4}\right)\right],$$

where

$$t = \ln\left(\frac{\mu^2}{\Lambda^2}\right)$$

$$\beta_0 = \left(11 - \tfrac{2}{3}n_f\right)\tfrac{1}{4}$$
$$\beta_1 = \left(51 - \tfrac{19}{3}n_f\right)\tfrac{1}{8}$$
$$\beta_2 = \left(2857 - \tfrac{5033}{9}n_f - \tfrac{325}{27}n_f^2\right)\tfrac{1}{128}$$

Later using the same formula we can extrapolate α_s at M_Z. To compute the top quark pole mass we use [37]

$$\mathrm{M}_t^{pol} = \mathrm{M}_t(\mathrm{M}_t)\left[1 + \frac{4}{3\pi}\alpha_3(\mathrm{M}_t)\right]$$

On the other hand to compute the bottom quark pole mass we use the quark effective mass formula which includes 1–loop QED and 3–loop QCD contributions

$$\mathrm{M}_b(\mathrm{M}_t) = \mathrm{M}_b(\mathrm{M}_b)\left(\frac{\alpha(\mathrm{M}_t)}{\alpha(\mathrm{M}_b(\mathrm{M}_b))}\right)^{\frac{\gamma_0^{QED}}{b_0^{QED}}}\frac{F(\alpha_3(\mathrm{M}_t))}{F(\alpha_3(\mathrm{M}_b(\mathrm{M}_b)))},$$

where the QED beta function and the anomalous dimension, γ_0^{QED} and b_0^{QED}, are given by[17]

$$\gamma_0^{QED} = -3Q_f^2$$

$$b_0^{QED} = \frac{4}{3}\left(3\sum Q_u^2 + 3\sum Q_d^2 + \sum Q_e^2\right)$$

where the sum runs over all the active fermions at the relevant scale. The formula $F(\alpha_s(\mu))$ is given by [36]

$$F(\alpha_s(\mu)) = \left(\frac{2\beta_0\alpha_s(\mu)}{\pi}\right)^{\gamma_0/\beta_0}\left\{1 + \left(\frac{\gamma_1}{\beta_0} - \frac{\gamma_0\beta_1}{\beta_0^2}\right)\frac{\alpha_s(\mu)}{\pi} + \frac{1}{2}\left[\left(\frac{\gamma_1}{\beta_0} - \frac{\gamma_0\beta_1}{\beta_0^2}\right)^2 + \right.\right.$$

$$\left.\left.\left(\frac{\gamma_2}{\beta_0} + \frac{\gamma_0\beta_1^2}{\beta_0^3} - \frac{\beta_1\gamma_1 + \beta_2\gamma_0}{\beta_0^2}\right)\right]\left(\frac{\alpha_s(\mu)}{\pi}\right)^2 + O\left(\alpha_s^3(\mu)\right)\right\}$$

where

$$\gamma_0 = 1$$
$$\gamma_1 = \left(\tfrac{202}{3} - \tfrac{20}{9} n_f\right) \tfrac{1}{16}$$
$$\gamma_2 = \left(1249 - \left(\tfrac{2216}{27} + \tfrac{160}{3}\zeta(3)\right) n_f - \tfrac{140}{81} n_f^2\right) \tfrac{1}{64}$$

Finally to compute tau lepton pole mass from the tau running mass at M_t we use

$$\mathrm{m}_\tau^{\mathrm{pol}} = \mathrm{m}_\tau\left(\mu\right)\left[1 + \frac{\alpha\left(\mu\right)}{\pi}\left(1 + \frac{3}{4}\ln\left(\frac{\mu^2}{\mathrm{m}_\tau^2\left(\mu\right)}\right)\right)\right]$$

In summary, starting with the basic parameters M_0, A_0, $\mathrm{M}_{1/2}$, t_β, μ_3, M_U and α_G we have required that $\alpha(\mathrm{M}_Z)$ as well as the top, bottom and tau pole masses τ were inside their experimental measurements in order to obtain a prediction for the variables $\widehat{s}_Z^2$ and $\alpha_s(\mathrm{M}_Z)$ which can be seen in figures.

References

[1] P. Nath and R. Arnowitt, *Talk at International Symposium on Lepton and Baryon Number Violation, Trento, Italy, 20-25 Apr 1998.* [hep-ph/9808465]; R. Barbieri, *Talk given at 18th International Symposium on Lepton–Photon Interactions (LP 97), Hamburg, Germany, 28 Jul–1 Aug 1997.* [hep-ph/9711232]; K.R. Dienes, *Phys. Rept.* **287**, 447 (1997).

[2] P. Langacker and N. Polonsky, *Phys. Rev. D* **52**, 3081 (1995).

[3] P. Langacker and N. Polonsky, *Phys. Rev. D* **47**, 4028 (1993).

[4] M. Carena, S. Pokorski and C.E.M. Wagner, *Nucl. Phys. B* **406**, 59 (1993).

[5] C. Caso and et. al., *Eur. Phys. J.* **C3**, 1 (1998).

[6] M.A. Díaz, J.C. Romão, and J. W. F. Valle, *Nucl. Phys. B* **524**, 23 (1998).

[7] A. Akeroyd, M.A. Díaz, J. Ferrandis, M.A. Garcia–Jareño, J.W.F. Valle, *Nucl. Phys. B* **529**, 3 (1998); F. de Campos, M.A. Garcia-Jareno, Anjan S. Joshipura, J. Rosiek, J. W.F. Valle, *Nucl.Phys.* **B451** 3-15 (1995), [hep-ph 9502237]; A.G. Akeroyd, M.A. Díaz, J.W.F. Valle, *Phys. Lett. B* **441**, 224 (1998); M.A. Díaz, E. Torrente-Lujan, J.W.F. Valle, [hep-ph/9808412], to be published in *Nucl. Phys. B.*

[8] H.P. Nilles and N. Polonsky, *Nucl. Phys. B* **484**, 33 (1997); R. Hempfling, *Nucl. Phys. B* **478**, 3 (1996); A.S. Joshipura and M. Nowakowski, *Phys. Rev. D* **51**, 2421 (1995).

[9] M. Bisset, O.C.W. Kong, C. Macesanu, and L.H. Orr, *Phys. Lett. B* **430**, 274 (1998).

[10] M. Nowakowski and A. Pilaftsis, *Nucl. Phys. B* **461**, 19 (1996).

[11] A. Masiero, J.W.F. Valle, Phys.Lett.**B251**:273-278,1990; J. C. Romao, C.A. Santos, J.W.F. Valle, Phys.Lett.**B288**:311-320,1992; J. C. Romao, J.W.F. Valle, Nucl.Phys.**B381**:87-108,1992.

[12] G. G. Ross, J.W.F. Valle, *Phys. Lett.* **151B** 375 (1985); J. Ellis, G. Gelmini, C. Jarlskog, G.G. Ross, J.W.F. Valle, *Phys. Lett.* 150B:142,1985; C.S. Aulakh, R.N.

Mohapatra, *Phys. Lett.* **B119**, 136 (1982); L.J. Hall and M. Suzuki, *Nucl. Phys. B* **231**, 419 (1984); I.-H. Lee, *Phys. Lett. B* **138**, 121 (1984); *Nucl. Phys. B* **246**,120 (1984); A. Santamaria, J.W.F. Valle, *Phys. Rev. Lett.* 60 (1988) 397 and Phys. Lett. 195B:423,1987.

[13] O.C.W. Kong, [hep-ph/9808304]; B. Mukhopadhyaya, S. Roy, and F. Vissani, [hep-ph/9808265]; V. Bednyakov, A. Faessler, and S. Kovalenko, [hep-ph/9808224]; E.J. Chun, S.K. Kang, C.W. Kim, and U.W. Lee, [hep-ph/9807237]; A. Faessler, S. Kovalenko, and F. Simkovic, *Phys. Rev. D* **58**, 055004 (1998); A.S. Joshipura, S.K. Vempati, [hep-ph/9808232].

[14] M. Diaz et al, in preparation

[15] Y. Fukuda *et al.*, *Phys. Rev. Lett.* **81**, 1562 (1998); *Phys. Rev. Lett.* **81**, 1158 (1998).

[16] D.R.T. Jones, *Phys. Rev. D* **25**, 581 (1982); M.B. Einhorn and D.R.T. Jones, *Nucl. Phys. B* **196**, 475 (1982); M.E. Machacek and M.T. Vaughn, *Nucl. Phys. B* **222**, 83 (1983); *Nucl. Phys. B* **236**, 221 (1984); *Nucl. Phys. B* **249**, 70 (1985).

[17] L. Hall, *Nucl. Phys. B* **178**, 75 (1981).

[18] J. Bagger, K. Matchev, and D. Pierce, *Phys. Lett. B* **348**, 433 (1995).

[19] A. Faraggi and B. Grinstein, *Nucl. Phys. B* **422**, 3 (1994); R. Barbieri, P. Ciafaloni, and A. Strumia, *Nucl. Phys. B* **442**, 461 (1995); M. Bastero–Gil and J. Pérez–Mercader, *Nucl. Phys. B* **322**, 355 (1994).

[20] W. Siegel, *Phys. Lett. B* **84**, 193 (1979); D.M. Capper, *Nucl. Phys. B* **167**, 479 (1980); I. Antoniadis, *Phys. Lett. B* **119**, 377 (1982); S.P. Martin and M.T. Vaughn, *Phys. Lett. B* **318**, 311 (1993).

[21] C.T. Hill, *Phys. Lett. B* **135**, 47 (1984); Q. Shafi and C. Wetterich, *Phys. Rev. Lett.* **52**, 875 (1984).

[22] M. A. Diaz, J. Ferrandis, J.C. Romao, J. W. F. Valle. *Phys. Lett. B* **453**, 263 (1999), [hep-ph/9801391]

[23] E. Nardi, *Phys. Rev. D* **55**, 5772 (1997).

[24] J. Ferrandis, [hep-ph/9810371].

[25] M.A. Díaz, in *Beyond the Standard Model: From Theory to Experiment*, Proceedings of Valencia 97, ed. I. Antoniadis, L. E. Ibanez and J. W. F. Valle, W. Scientific, ISBN 981-02-3638-7, hep-ph/9802407; J.W.F. Valle, Supergravity Unification with Bi-linear R Parity Violation, Proceedings of PASCOS98, ed. P. Nath, W. Scientific, [hep-ph 9808292].

[26] S. Davidson, J. Ellis, *Phys.Lett.*B**390** (1997) 210-220; *Phys.Rev.***D56** (1997) 4182-4193, S. Davidson, hep-ph/9808425

[27] S. P. Martin and M. T. Vaughn, *Phys. Rev.*D**50** (1994) 2282; B. de Carlos and P.L. White, *Phys. Rev.*D**54** (1996) 3427; V. Barger, M.S. Berger, R.J.N. Phillips, T. Wohrmann *Phys. Rev.*D**53** (1996) 6407-6415; B.C. Allanach, A. Dedes, and H.K. Dreiner, hep-ph/9902251.

[28] P. Langacker, J. Erler, *Talk at Ringberg Workshop: The Higgs Puzzle– What can We Learn from LEP2, LHC, NLC, and FMC?, Ringberg, Germany, 8-13 Dec 1996.*[hep-ph/9703428]; M. Shifman, *Mod. Phys. Lett.*A**10** (1995) 605; W. J. Stirling, *Talk given at New Nonperturbative Methods and Quantization of the Light Cone, Les Houches, France, 24 Feb - 7 Mar 1997.* [hep-ph/9709429]

[29] Barate R. *et al.*, ALEPH Collaboration, *Eur.Phy.J.***B2** (1998) 395-406; Ackerstaff K. *et al.*, OPAL Collaboration, *Eur.Phy.J.***C5** (1998)2, 229-237.

[30] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], "Evidence for oscillation of atmospheric neutrinos," Phys. Rev. Lett. **81**, 1562 (1998) hep-ex/9807003.

[31] M.C. Gonzalez-Garcia *et al.*, Phys. Rev. Lett. **82** (1999) 3202 hep-ph/9809531.

[32] V. Barger, J.G. Learned, S. Pakvasa and T.J. Weiler, "Neutrino decay as an explanation of atmospheric neutrino observations," Phys. Rev. Lett. **82**, 2640 (1998), astro-ph/9810121.

[33] M.C. Gonzalez-Garcia *et al.*, talk presented at International Workshop on Particles in Astrophysics and Cosmology: From Theory to Observation, Valencia, Spain, May 3-8, 1999, and paper in preparation.

[34] M.C. Gonzalez-Garcia *et al.*, "Update on atmospheric neutrinos," Phys. Rev. **D58**, 033004 (1998) hep-ph/9801368; for the 535 days data sample update (Neutrino 98 data) see M.C. Gonzalez-Garcia, H. Nunokawa, O.L. Peres and J.W. Valle, "Active-active and active - sterile neutrino oscillation solutions to the atmospheric neutrino anomaly," Nucl. Phys. **B543**, 3 (1998) hep-ph/9807305.

[35] V. Barger and M. S. Berger and P. Ohmann *Phys. Rev.*D**47** (1993) 1093-1113

[36] O.V. Tarasov, A.A. Vladimirov, and A.Y. Zharkov, *Phys Lett B* **93**, (1980) 429; S.G. Gorishny, A.L. Kateav, and S.A. Larin, *Yad. Fiz.* **40**, (1984) 517[*Sov. J. Nucl. Phys.* **40**, (1984) 329]; S.G. Gorishny *et al.*, *Mod. Phys. Lett A***5**, (1990) 2703.

[37] R. Tarrach *Nucl.Phys.*B**183** (1981) 384