

Unraveling the pattern of the XYZ mesons

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(Dated: July 30, 2015)

Abstract

We present a plausible mechanism for the origin of the XYZ mesons in the heavy meson spectra within a standard quark-model picture. We discuss the conditions required for the existence of four-quark bound states or resonances contributing to the heavy meson spectra, being either compact or molecular. We concentrate on charmonium and bottomonium spectra, where several new states, difficult to understand as simple quark-antiquark pairs, have been reported by different experimental collaborations. The pivotal role played by entangled meson-meson thresholds is emphasized.

PACS numbers: 14.40.Gx, 21.30.Fe, 12.39.Mk

Keywords:

One of the most basic problems of QCD is to identify all the clusters of quarks, antiquarks and gluons that are sufficiently bound by QCD interactions that they are either stable particles or appropriately long-lived to be observed as resonances [1]. To this respect, charmonium spectroscopy has become a cornerstone during the ten years elapsed since the discovery of the first of the so-called XYZ states, the $X(3872)$ [2]. Before this discovery, and based on Gell-Mann conjecture [3], the hadronic experimental data were classified either as $q\bar{q}$ or qqq states according to $SU(3)$ irreducible representations. However, since 2003 more than twenty newly observed meson resonances reported by different experimental collaborations appeared, close to a two-meson threshold, presenting properties that make a simple quark-antiquark structure unlikely [4]. Although this observation could be coincidental due to the large number of thresholds in the energy region where the XYZ mesons have been reported, it could also point to a close relation between some particular thresholds and resonances contributing to the standard quark-antiquark heavy meson spectroscopy. Several alternatives have been proposed in the literature to address these XYZ states, being the most common ones conventional quarkonium [5], meson-meson molecules [6], quarkonium hybrids [7], four-quark states [8], and dynamically generated resonances [9]. However, none of the models that has been proposed provide a plausible pattern for all the XYZ mesons that have been observed.

In this letter we analyze heavy meson spectroscopy beyond open flavor thresholds considering higher order Fock space components looking for a general pattern of the XYZ states. We will highlight the pivotal role played by meson-meson thresholds in heavy meson spectroscopy and under which conditions, if any, they could cause a resonance to appear. This study is an interesting, and not settled, issue to address that may provide with a pattern to be compared with experiment and lattice QCD calculations. The outline is the following. We will first describe the main characteristics of the quark model and the four-body formalism chosen to illustrate our findings. Then, we will analyze the thresholds open for four-quark states contributing to heavy meson spectroscopy and their main model-independent properties. Finally, we will show how these thresholds may entangle so a bound four-quark state or a resonance may emerge and the conditions required for that.

Although the conclusions of this study aim to be independent of the formalism selected to solve the four-body Schrödinger equation and the particular details of the quark model used, we have to choose a particular method, that is going to be the Hyperspherical Harmonic

formalism (HH) [10], and model, the constituent quark cluster model (CQCM) [11], to illustrate them. Within the HH method, the four-quark wave function is written as a sum of outer products of color, isospin, spin and radial terms to obtain basis functions that have well-defined symmetry under permutations of the quark pairs. By using hyperspherical coordinates one can write the Laplace operator, where the hyperspherical, or grand angular momentum, is the 9-dimensional analogous of the angular momentum operator associated with the 3-dimensional Laplacian [12]. In the following we shall use the notations K for the total hyperangular momentum. The CQCM model was proposed in the early 90's in an attempt to obtain a simultaneous description of the nucleon-nucleon interaction and the baryon spectra [13]. It was later on generalized to all flavor sectors giving a reasonable description of the meson and baryon spectra. The model is based on the assumption that the light-quark constituent mass appears because of the spontaneous breaking of the original $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale. In this domain of momenta, quarks interact through Goldstone boson exchange potentials. QCD perturbative effects are taken into account through the one-gluon-exchange potential. Finally, it incorporates confinement as dictated by unquenched lattice calculations. A detailed discussion of the model can be found in Refs. [11, 13].

Standard mesons ($q\bar{q}$) and baryons (qqq) are the only clusters of quarks where it is not possible to construct a color singlet using a subset of their constituents. Thus, $q\bar{q}$ and qqq states are proper solutions of the two- and three-quark hamiltonian, respectively, corresponding in all cases to bound states. This, however, is not the case for multiquark combinations, and in particular for four-quark states addressing the meson spectra. Thus, when dealing with higher order Fock space contributions to meson spectroscopy, one has to discriminate between possible four-quark bound states or resonances and simple pieces of the meson-meson continuum. For this purpose, one has to analyze the two-meson states that constitute the threshold for each set of quantum numbers. These thresholds has to be determined assuming quantum number conservation within exactly the same scheme (parameters and interactions) used for the four-body calculation. If other models, parametrizations or experimental masses are used, then four-quark states might be misidentified as members of the meson spectra while being simple pieces of the continuum. Working with strongly interacting particles, two-meson states should have well-defined total angular momentum (J) and parity (P). If the two mesons are identical, the spin-statistics theorem imposes

a properly symmetrized wave function. Moreover, C -parity should be conserved in the final two-meson state for those four-quark states with well-defined C -parity. Finally, if noncentral forces are not considered, orbital angular momentum (L) and total spin (S) are also good quantum numbers.

Given a general four-quark state, $(q_1q_2\bar{q}_3\bar{q}_4)$, two different thresholds are allowed, $(q_1\bar{q}_3)(q_2\bar{q}_4)$ and $(q_1\bar{q}_4)(q_2\bar{q}_3)$. If the four-quark system contains identical quarks, like for instance $(QQ\bar{n}\bar{n})$ (in the following n stands for a light quark and Q for a heavy c or b quark), the two thresholds are identical, i.e., $(Q\bar{n})(Q\bar{n})$. The importance of this particular feature lies on the fact that a modification of the four-quark interaction would not necessarily translate into the mass of the two free-meson state. Therefore, the unique necessary condition required to have a four-quark bound state would be the existence of a sufficiently attractive interaction between quarks that do not coexist in the two free-meson states. This hypothesis was demonstrated by means of the Lippmann-Schwinger formalism in Ref. [14], concluding the existence of a single stable isoscalar doubly charmed meson with quantum numbers $J^P = 1^+$.

For those cases containing a heavy quark and its corresponding heavy antiquark $(Qn\bar{Q}\bar{n})$ the situation is remarkably different. Two different thresholds are allowed, namely $(Q\bar{Q})(n\bar{n})$ and $(Q\bar{n})(n\bar{Q})$. It has been proved [15] that ground state solutions of the Schrödinger $(q_1\bar{q}_2)$ two-body problem are concave in $m_{q_1}^{-1} + m_{q_2}^{-1}$ and hence $M_{Q\bar{n}} + M_{\bar{Q}n} \geq M_{Q\bar{Q}} + M_{n\bar{n}}$. This property is enforced both by nature¹ and by all models in the literature unless forced to do otherwise. Although this relation among ground-state masses makes the assumption of a strictly flavor independent potential, one should bear in mind that ground states of heavy mesons are perfectly reproduced by a Cornell-like potential [16], that it is flavor independent. The color-spin dependence of the potential would go in favor of this relation for ground states (spin zero) because the color-spin interaction is attractive for spin zero and comes suppressed as $1/(m_i m_j)$, making even lighter the mesons on the right hand side. Regarding the spin independent part, the binding of a coulombic system is proportional to the reduced mass of the interacting particles. Thus, for a two-meson threshold with a heavy-light light-heavy quark structure, the binding of any of the two mesons is proportional to the reduced mass of each meson, being close to the mass of the light quark. However, if

¹ $M_{D^*} + M_{\bar{D}^*} = 4014 \text{ MeV} \geq M_{J/\psi} + M_\omega = 3879 \text{ MeV}$

the two-meson state presents a heavy-heavy light-light quark structure, the binding of the heavy-heavy meson increases proportionally to the mass of the heavy particle while that of the light-light meson remains constant, becoming this threshold lighter than the heavy-light light-heavy two-meson structure. Thus, it implies that in all relevant cases the lowest two-meson threshold for any $(Qn\bar{Q}\bar{n})$ state will be the one made of quarkonium-light mesons, i.e., $(Q\bar{Q})(n\bar{n})$ (see Fig. 1 of Ref. [17]). The interaction between the heavy, $(Q\bar{Q})$, and light, $(n\bar{n})$, mesons forming the lowest threshold is almost negligible, due to the absence of a light pseudoscalar exchange mechanism between them [6]. Hence, any attractive effect in the four-quark system must have its origin in the interaction of the higher channel $(Q\bar{n})(n\bar{Q})$ or due to the coupled channel effect of the two thresholds, $(Q\bar{Q})(n\bar{n}) \leftrightarrow (Q\bar{n})(n\bar{Q})$ [18]. In Ref. [8] a comprehensive analysis of the $c\bar{c}n\bar{n}$ spectra was undertaken within the same working framework we use in this letter. All isoscalar states with total orbital angular momentum $L \leq 1$ were considered. No bound state was observed for any set of quantum numbers in any of the quark models considered. In all cases the four-quark system evolved for large values of K to a well separated $(Q\bar{Q})-(n\bar{n})$ two-meson state, corresponding to the lowest threshold. We have repeated the same calculation enlarging the number of states to include both the isovector and bottom sectors to no avail. No compact four-quark state was found for any isospin combination, heavy quark mass, or quark model studied.

In those energy regions where bound and unbound solutions of the four-quark hamiltonian coexist, methods based on infinite expansions become inefficient to hunt a bound state close to an unbound solution, because too many basis states would be required to disentangle them. The most interesting case where this may happen is in the vicinity of a two-meson threshold, because both the two free-meson state and a feasible slightly bound four-quark state are solutions of the same hamiltonian. Such cases have been studied by means of the Lippmann-Schwinger equation in Ref. [19], looking at the Fredholm determinant $D_F(E)$ at zero energy [20]. If there are no interactions then $D_F(0) = 1$, if the system is attractive then $D_F(0) < 1$, and if a bound state exists then $D_F(0) < 0$. All states made of S wave $(Q\bar{n})-(n\bar{Q})$ mesons up to $J = 2$ were scrutinized. A few channels were found to be slightly attractive, $D\bar{D}$ with $(I)J^{PC} = (0)0^{++}$, $D\bar{D}^*$ with $(0)1^{++}$ and $D^*\bar{D}^*$ with $(0)0^{++}$, $(0)2^{++}$, and $(1)2^{++}$, close to the results of Ref. [6]. However, the only bound state appeared in the $(I)J^{PC} = (0)1^{++}$ channel as a consequence of the coupling between $D\bar{D}^*$ and $J/\Psi\omega$ two-meson channels.

The conclusions of Refs. [8] and [19] point to a convoluted four-quark molecular structure with a dominant $D\bar{D}^*$ component for the $X(3872)$. However, this is not the only supernumerary state that has appeared in the charm and bottom sectors during the last years. A comprehensive list of such XYZ mesons and their properties can be found in Ref. [4]. 20 states have been reported by different experimental collaborations in the charmonium sector above the $D\bar{D}$ threshold, 15 of them neutral and 5 charged. In the bottom sector 2 charged and 1 neutral state have been reported. Of those 23 states only 8 have been observed independently by two different collaborations and with significance greater than 5σ : $X(3872)$, $X(3915)$, $\chi_{c2}(2P)$, $G(3900)$, $Y(4140)$, $Y(4260)$, $Y(4360)$, and $Z_c^+(3900)$ (see Table I of Ref. [4]). Therefore, although some of them might not resist cross-check examination by independent experimental collaborations, others are clearly established as real resonances and therefore they have to be accounted for in any description of the meson spectra. While some of them, like the $\chi_{c2}(2P)$, seem to fit nicely within a naive quark-antiquark scheme, others do not.

When four-quark components are considered in the wave function of charmonium, there are 72 $c\bar{c}n\bar{n}$ combinations of quantum numbers for total orbital angular momentum $L < 3$. Therefore, the question is not whether it is possible to design a model, or a formalism, able to match one of the newly observed XYZ states with a particular set of these quantum numbers but to understand where the attraction comes from and to explain the systematic that predicts where, if anywhere, experimentalists and theoreticians alike should look into. Since the lowest threshold interaction is rather weak, the possibility to obtain bound states may only stem from the vicinity of an attractive $(c\bar{n})(n\bar{c})$ threshold coupled sufficiently as to bind the system as it occurs with the $X(3872)$ [19]. Although for heavier mesons interactions are more attractive [17], the effect of channel coupling may not be enough to favor binding. Thus, to check the efficiency of this mechanism, let us solve with the HH formalism the bottom counterpart of the $X(3872)$, $b\bar{b}n\bar{n}$ with quantum numbers $L = 0$, $S = 1$, $I = 0$, $C = +1$, and $P = +1$. Within the CQCM the corresponding lowest thresholds, $B\bar{B}^*$ (10611 MeV) and $\Upsilon\omega$ (10155 MeV), are 456 MeV apart. We show in Fig. 1(upper panel) the convergence pattern of the energy of the four-quark system as a function of the hyperangular momenta K . It can be clearly seen how the energy of the four-quark system (red line) is converging to the lowest threshold $\Upsilon\omega$ (horizontal blue line), what is a sharp signal of an unbound state. One could however play around with the model

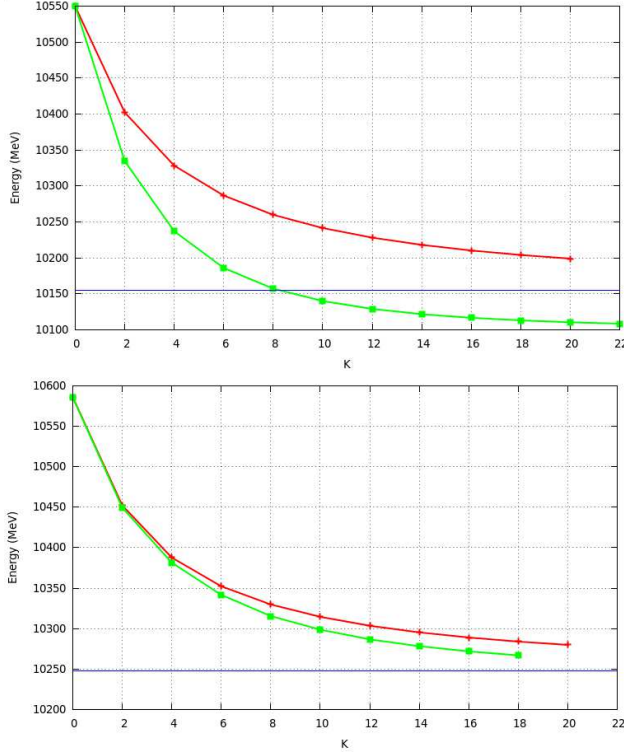


FIG. 1: Convergence of $b\bar{b}n\bar{n}$ with quantum numbers $L = 0$, $S = 1$, $C = +1$, $P = +1$, $I = 0$ (upper panel) and $I = 1$ (lower panel). Red lines correspond to the case where the thresholds are non-degenerate and green lines to the case where they are almost degenerate.

parameters to almost degenerate both thresholds by adding attraction in the heavy-light bn sector², what would also increase the coupled channel effect strengthening the $B\bar{B}^* \leftrightarrow \Upsilon\omega$ transition interaction. When this is done we note (green line) that the energy drops below threshold and a bound state emerges³. One may wonder if only the close-to-degeneracy of the thresholds is sufficient to bind this type of four-quark systems. If this would be the case, then the charged partner of this four-quark state ($I = 1$) should behave exactly in the same manner. However, amazingly this is not so. In Fig. 1(lower panel) we depict the convergence of the isovector state as a function of K for both cases, non-degenerate thresholds (red line) and almost degenerate ones (green line). In this case the lowest threshold would be $\Upsilon\rho$

² We have slightly increased the $\alpha_s(bn)$ strong coupling constant from 0.55 to 0.85, what would move the gap between thresholds without changing the lowest threshold.

³ Additional tests on the wave function, not included here for the sake of brevity, following the formalism outlined in Ref. [21] have been performed to verify that this state is a real bound state.

(10248 MeV). It can be observed that in both cases the four-quark state converges to the lowest threshold and does not form a bound state.

Thus, when the $(Q\bar{Q})(n\bar{n})$ and $(Q\bar{n})(n\bar{Q})$ thresholds are sufficiently far away, which means that the interaction in the higher $(Q\bar{n})(n\bar{Q})$ state is weak and therefore the coupled channel effect small, no bound states are found for any set of parameters. However, when the thresholds move closer, i.e., the attraction in the higher two-meson state and the coupled channel strength are simultaneously increased, bound states may appear for a subset of quantum numbers. Hence, threshold vicinity is a required but not sufficient condition to bind a four-quark state. An additional condition is required to allow the emergence of such bound states. Such condition is the existence of an attractive interaction in the higher $(Q\bar{n})(n\bar{Q})$ two-meson system that would also give rise to a strong $(Q\bar{Q})(n\bar{n}) \leftrightarrow (Q\bar{n})(n\bar{Q})$ coupling. To neatly illustrate this conclusion we return to the four-quark state $b\bar{b}n\bar{n}$ with quantum numbers $L = 0$, $S = 1$, $I = 0$, $C = +1$, and $P = +1$. In Refs. [17, 19] it was proved that the interaction provided by the CQCM model is attractive for these quantum numbers in the charm sector, although does not present a $D\bar{D}^*$ bound state. In the bottom sector the attraction is enhanced [17]. Thus, we have solved the four-body problem as a function of the threshold energy difference, $\Delta = E[(b\bar{n})(n\bar{b})] - E[(b\bar{b})(n\bar{n})]$, ranging from 500 MeV to -200 MeV. We show in Fig. 2 the energy normalized to the mass of the lowest two-meson threshold for each particular case, i.e., values smaller than 1 will point to a bound state and those larger to an unbound state. In this case, the results can be separated into two distinct categories: (i) $\Delta \gtrsim 50$ MeV and (ii) $\Delta \lesssim 50$ MeV. When the thresholds are separated by more than 50 MeV, the attractive interaction in the $B\bar{B}^*$ system and the coupled channel effect is not sufficient to overcome the threshold energy gap, and therefore the four-quark system evolves to an unbound two-meson state. However, when the thresholds get closer, even reversed for $\Delta < 0$, the system becomes a compact four-quark state. Of particular interest are those cases where $\Delta \simeq 50$ MeV in Fig. 2. In this case the attraction in the higher channel together with the coupled channel effect barely overcomes the threshold energy difference, and hence its wave function becomes strongly entangled. This would generate a molecular state just close to threshold. It should also be noted that Δ_0 , the Δ value for which $\text{Energy}(4q)/E(\text{lowest})$ is equal to one, will change depending on the particular set of quantum numbers considered. However, $\Delta_0 = 0$ implies that no binding energy is provided by the upper threshold and the off-diagonal terms and

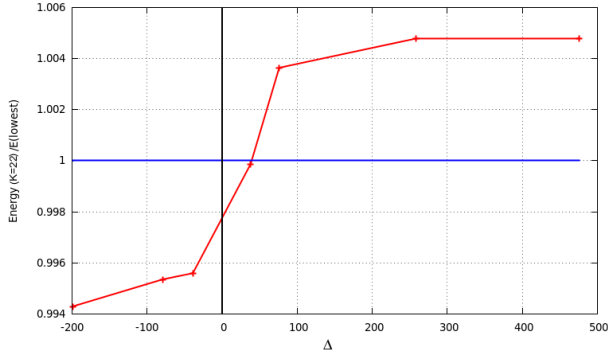


FIG. 2: Energy divided by the lowest two-meson threshold of $b\bar{b}n\bar{n}$ with quantum numbers $L = 0$, $S = 1$, $C = +1$, $P = +1$, $I = 0$ as a function of $\Delta = E[(b\bar{n})(n\bar{b})] - E[(b\bar{b})(n\bar{n})]$.

therefore such solution will not correspond to a bound state. In that case our calculation will be simply providing a piece of the meson-meson unbound continuum.

As one can see the mechanism proposed is restrictive enough as not predicting a proliferation of bound states when explaining the existence of an hypothetical molecular structure. This mechanism may also work in other two-hadron systems. For the sake of completeness, let us suggest a possible scenario where it could be relevant for an exotic meson-baryon system in the charm baryon spectroscopy. There is an unexplained structure recently reported by the BABAR Collaboration [23], with a mass of $3250 \text{ MeV}/c^2$ in the $\Sigma_c^{++}\pi^-\pi^-$ invariant mass that may be a consequence of the close-to-degeneracy of the lowest thresholds with $I = 2$ and $J^P = 5/2^-$, ΔD^* and $\Sigma_c^*\rho$ and the attractive interaction of the ΔD^* system [24]. Such state was identified by QCD sum rules analyses [25] as a pentaquark candidate, the $\Theta_c(3250)$.

To summarize, we have presented a plausible mechanism for the origin of the XYZ mesons in the heavy meson spectra within a standard quark-model picture. The existence of open flavor two-meson thresholds is a feature of the heavy meson spectra that needs to be considered as a relevant ingredient into any description of the plethora of new states reported in charmonium and bottomonium spectroscopy. They might be, at a first glance, identified with simple quark-antiquark states, however in some cases their energies and decay properties do not match such oversimplified picture. Our results prove the relevance of higher order Fock space components through the allowed two-meson thresholds. On the one hand, one has the lower $(Q\bar{Q})(n\bar{n})$ system, made by almost noninteracting mesons, that constitutes the natural breaking apart end-state. On the other hand, the higher $(Q\bar{n})(n\bar{Q})$

system appears. *When there is an attractive interaction characterizing the $(Q\bar{n})(n\bar{Q})$ upper system combined with a strong enough $(Q\bar{Q})(n\bar{n}) \leftrightarrow (Q\bar{n})(n\bar{Q})$ coupling, together with the vicinity of the two allowed thresholds, a four-quark bound state may emerge.* This is a rather restricting property that makes that no $c\bar{c}n\bar{n}$ or $b\bar{b}n\bar{n}$ bound state was reported in the analysis performed in Ref. [8]. Thus, an analysis of the threshold energy difference and the strength of the $(Q\bar{n})(n\bar{Q})$ and $(Q\bar{Q})(n\bar{n}) \leftrightarrow (Q\bar{n})(n\bar{Q})$ interactions using different state-of-the-art quark models for all allowed sets of quantum numbers is required to predict whose sets are candidates to lodge one of the XYZ states.

Once this is performed, the present experimental effort with ongoing experiments at BESIII, current analyses by the LHC collaboration and future experiments at Belle II and Panda together with the very impressive results that are being obtained by lattice gauge theory calculations [22] may confirm the theoretical expectations of our quark-model calculation pattern that will provide with a deep understanding of low-energy realizations of QCD.

Acknowledgments

This work has been partially funded by the Spanish Ministerio de Educación y Ciencia and EU FEDER under Contracts No. FPA2010-21750 and FPA2013-47443, by the Spanish Consolider-Ingenio 2010 Program CPAN (CSD2007-00042) and by Generalitat Valenciana Prometeo/2009/129. A.V. thanks financial support from the Programa Propio I of the University of Salamanca.

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