

# Heavy-baryon quark model picture from lattice QCD

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## Abstract

The ground state and excited spectra of baryons containing three identical heavy quarks,  $b$  or  $c$ , have been recently calculated in nonperturbative lattice QCD. The energy of positive and negative parity excitations has been determined with high precision. Lattice results constitute a unique opportunity to learn about the quark-confinement mechanism as well as elucidating our knowledge about the nature of the strong force. We analyze the nonperturbative lattice QCD results by means of heavy-quark static potentials derived using SU(3) lattice QCD. We make use of different numerical techniques for the three-body problem.

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## I. INTRODUCTION

A precise calculation of the mass of the ground state triply-bottom baryon,  $\Omega_{bbb}$ , and ten excited positive and negative parity states with angular momentum up to  $J = 7/2$  has been recently performed in nonperturbative lattice quantum chromodynamics (QCD) with  $2 + 1$  flavors of dynamical light quarks in Refs. [1, 2]. The  $b$  quark was implemented very accurately with improved lattice nonrelativistic QCD as suggested in Ref. [3]. There are also recent nonperturbative calculations of the ground and excited states with angular momentum up to  $J = 7/2$  of triply-charm baryons using anisotropic lattice QCD with a background of  $2 + 1$  dynamical light quark fields [4]. Within the same framework of Refs. [1, 2], dynamical  $2 + 1$  flavor lattice QCD and improved nonrelativistic lattice QCD for the  $b$  quarks, the bottomonium spectrum was computed in Ref. [5] obtaining an excellent agreement with the experiment. A calculation of the charmonium spectrum with the same lattice action and the same lattice spacing used in Ref. [4] can be found in Ref. [6], where one can get an idea of the typical size of the systematic uncertainties. Such studies grant the lattice QCD results for the bound state problem of three-heavy quarks with the category of experimental data and thus provide us with a unique opportunity to test phenomenological quark models for baryons in the energy regime where the description using potential models is expected to work best.

As pointed out by Bjorken time ago [7], bound states of three-heavy quarks,  $QQQ$ , may provide a new window for the understanding of baryon structure. Baryons made of three-heavy quarks reveal a pure baryonic spectrum without light-quark complications and provide valuable insight into the quark-confinement mechanism as well as to elucidate our knowledge about the nature of the strong force. On the theoretical side one would expect the potential models would be able to describe triply-heavy baryons to a similar level of precision as their success in charmonium and bottomonium. In the same way the  $Q\bar{Q}$  interactions are examined in heavy mesons, the study of triply-heavy baryons will probe the  $QQ$  interactions in the heavy quark sector. Triply-heavy baryons have been studied by different methods, including quark models (see Refs. [8, 9] for a comprehensive review of quark-model theoretical approaches), QCD sum rules [10–13] and potential nonrelativistic QCD with static potentials from perturbation theory at leading order [14] and next-to-next-to-leading order [15, 16]. However, no experimental results are available so far for triply-heavy baryons (see Ref. [17] for a recent calculation of production cross section at the LHC), and thus, the predictions of their properties cannot yet be compared to the real world.

Our poor knowledge of the three-quark,  $3Q$ , potential stems on the difficulty to produce  $QQQ$  states and, as said above, the consequent lack of experimental data. The precise calculation of the ground and excited states of triply-bottom baryons [1, 2], together with the ground and excited states of triply-charm baryons [4], provide us with the ideal testbench to improve our understanding of phenomenological quark models for baryons in the heavy-quark sector. The quark-model dependent calculations could be tested by comparing them to nonperturbative first-principles calculations in lattice QCD of the  $bbb$  and  $ccc$  systems. An adequate starting point may be the static  $3Q$  potential derived in  $SU(3)$  lattice QCD that will be described in the next section. In Sect. III we will present our results making emphasis in the comparison between nonperturbative lattice QCD data and the results obtained with the static  $3Q$  potential derived in  $SU(3)$  lattice QCD. For this purpose we will make use of different numerical techniques for the three-body problem, generalized Gaussians variational approaches, hyperspherical harmonics and Faddeev equations [18–20], to test quark-quark

potential descriptions in the heavy quark limit. Finally in Sect. IV we will briefly summarize the main conclusions of this study.

## II. THE STATIC THREE-QUARK POTENTIAL IN SU(3) LATTICE QCD

Since the early days of QCD the interaction among heavy quarks has been explored as an important tool to learn about the behavior of QCD at low-energies. The  $Q\bar{Q}$  static potential has been studied extensively by lattice gauge theories [21], being nowadays a very well-known quantity. The typical shape of the color-singlet  $Q\bar{Q}$  static potential, which is characterized by a short-range Coulomb behavior and a long-range linear rise, well represents the double nature of QCD as an asymptotically free and infrared confined theory. The excitation spectrum of the gluon field around a static quark-antiquark pair has also been explored by lattice calculations [22]. On the large length scale the spectrum agrees with that expected for stringlike excitations while in the short range it shows a Coulomb-like behavior as it was first noted within the context of the static bag picture of gluon excitations [23].

In QCD the three-quark potential is of prime importance reflecting the SU(3) gauge symmetry and being directly responsible for the structure and properties of baryons, in the same way the  $Q\bar{Q}$  potential is responsible for the meson properties. Furthermore, the  $3Q$  potential is a key quantity to clarify the quark confinement mechanism in baryons. However, up to now lattice QCD studies have paid relatively less attention to the potential that describes the interaction of three-heavy quarks, as a consequence of their still missing experimental evidence. Thus, the  $3Q$  potential is much less known than the heavy  $Q\bar{Q}$  potential for which many lattice studies exist [21]. Even for the ground-state, its accurate measurement using lattice QCD is relatively difficult and has been performed recently [24–27]. The gluonic excitation (excitations of the gluon field that are not thus of quark origin) of the three-quark system has also been studied in SU(3) lattice QCD [28, 29], concluding that the lowest excitation would be almost 1 GeV above the ground state. This is rather large in comparison with the low-lying excitation energy of the quark origin what makes the gluonic excitation mode invisible in the low-lying excitations of baryons. Thus, quark-degrees of freedom play the dominant role in low-lying baryons with excitation energy below 1 GeV. This large gluonic excitation energy is conjectured to give a physical reason of the success of the quark model for low-lying baryons even without explicit gluonic modes [29].

Most of the existing lattice studies of the three-quark static potential have mainly explored the region of large interquark distances [24–36]. As for the  $Q\bar{Q}$  case, the characteristic signature of the long-range non-Abelian dynamics is believed to be a linear rising of the static interaction. Moreover, the general expectation for the baryonic case is that, at least classically, the strings meet at the so-called Fermat (or Torricelli) point, which has minimum distance from the three sources ( $Y$ -shape configuration). If this is the case, one should see a genuine three-body interaction among the static quarks. The confining short-range  $3Q$  potential could be also described as the sum of two-body potentials ( $\Delta$ -shape configuration) [27, 29, 32, 34]. Many of the lattice calculations have focused on distinguishing the  $Y$ - from the  $\Delta$ -configuration, despite the difference between a  $\Delta$  and a  $Y$ -shape potential being rather small and difficult to detect.

In recent years, according to the remarkable progress of the computational power, the lattice QCD Monte Carlo calculations have become a reliable and useful method for the analysis of nonperturbative QCD. In particular, the  $Q\bar{Q}$  potential, responsible for the meson properties, has been extensively studied using lattice QCD. The data of the  $Q\bar{Q}$  ground state

TABLE I: Standard string tension,  $\sigma$ , Coulomb coefficient,  $A$ , and constant term,  $C$ , of the static heavy quark potential obtained from SU(3) lattice QCD taken from Ref. [25] for a lattice spacing  $a = 0.19$  fm.

	$\sigma$ ( $a^{-2}$ )	$\sigma$ ( $\text{GeV}^2$ )	$A$	$C$ ( $a^{-1}$ )	$C$ (GeV)
$Q\bar{Q}$	0.1629(47)	0.1757	0.2793(116)	0.6203(161)	0.6442
$3Q(Y)$	0.1524(28)	0.1644	0.1331(66)	0.9182(213)	0.9536
$3Q(\Delta)$	0.0858(16)	0.0925	0.1410(64)	0.9334(210)	0.9694

potential are well reproduced by a sum of a Coulomb term, due to the perturbative one-gluon exchange (OGE), and a linear confinement [25, 29],

$$V_{Q\bar{Q}}(r) = -\frac{A_{Q\bar{Q}}}{r} + \sigma_{Q\bar{Q}} r + C_{Q\bar{Q}}. \quad (1)$$

We give in Table I the parameters of the  $Q\bar{Q}$  potential taken from Ref. [25]. The value of the  $Q\bar{Q}$  confinement strength was determined to reproduce the value obtained from the linear Regge trajectories of the pseudoscalar  $\pi$  and  $K$  mesons,  $\sqrt{\sigma} = (429 \pm 2)$  MeV [21].

However, there is almost no reliable formula to describe the  $3Q$  potential directly based on QCD, in spite of its importance for the study of baryon properties. There have been recent advances on the determination of the ground state  $3Q$  potential that is expected to take the form [25],

$$V_{3Q}(r) = -A_{3Q} \sum_{i < j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma_{3Q}^Y L_{\min} + C_{3Q}, \quad (2)$$

where  $L_{\min}$  is the minimal value of the total length of color flux tubes linking the three quarks, and

$$A_{3Q} \simeq \frac{1}{2} A_{Q\bar{Q}}, \quad \sigma_{3Q}^Y \simeq \sigma_{Q\bar{Q}}. \quad (3)$$

The short-distance behavior of  $V_{3Q}(r)$  is expected to be described by the two-body Coulomb potential as the one-gluon exchange result in perturbative QCD. Fit analysis in terms of a Yukawa potential considering a possible gluon mass  $m_B$  have concluded  $m_B = 0$ , what reduces the Yukawa to the Coulomb potential [25]. The one-gluon exchange result indicates also the simple relation on the Coulomb coefficients in the  $Q\bar{Q}$  and the  $QQQ$  potentials as  $A_{3Q} \simeq \frac{1}{2} A_{Q\bar{Q}}$ . For the long-distance behavior, the confining baryonic static potential rises like the  $Y$ -ansatz [29, 32].

Calculations both in full and quenched QCD demonstrates that the confining baryonic static potential approaches the  $\Delta$ -ansatz at short distances [27, 29, 32]. The  $\Delta$ -ansatz behavior at short distances is of great importance for phenomenological models since the calculation of orbital excited states with the  $\Delta$ -ansatz are much more simpler. For an equilateral  $QQQ$  arrangement, as expected for a system of three-heavy identical quarks, departure from the  $\Delta$ -ansatz is not significant until  $d_{QQ} \sim 0.7$  fm [27, 32], so that the  $\Delta$ -ansatz may be the more relevant one for quarks confined inside a baryon and then, for the case of  $QQQ$  bound states whose root-mean square radii are much smaller than such

distance (see Table II). Thus, the static three-quark potential could be described by a simple sum of the effective two-body  $QQ$  potentials with a reduced string tension [25–27],

$$V_{3Q}(r) = -A_{3Q} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma_{3Q}^{\Delta} \sum_{i<j} |\vec{r}_i - \vec{r}_j| + C_{3Q}, \quad (4)$$

where

$$A_{3Q} \simeq \frac{1}{2} A_{Q\bar{Q}}, \quad \sigma_{3Q}^{\Delta} \simeq 0.53 \sigma_{Q\bar{Q}}. \quad (5)$$

The reduction factor in the string tension can be naturally understood as a geometrical factor rather than the color factor, due to the ratio between  $L_{\min}$  and the perimeter length of the  $3Q$  triangle, suggesting  $\sigma_{3Q}^{\Delta} = (0.50 \sim 0.58) \sigma_{Q\bar{Q}}$  [25]. For the particular case of quarks in an equilateral triangle  $\sigma_{3Q}^{\Delta} = \frac{1}{\sqrt{3}} \sigma_{Q\bar{Q}} = 0.58 \sigma_{Q\bar{Q}}$  [32]. When the  $\Delta$ -ansatz is adopted for the two-body linear potential, still the same relation holds for the strength of the Coulomb potential  $A_{3Q} \simeq \frac{1}{2} A_{Q\bar{Q}}$ . This  $\Delta$ -ansatz has been widely adopted in the nonrelativistic quark model because of its simplicity [8, 9, 18, 19, 37–39]

### III. RESULTS AND DISCUSSION

In order to check if the static three-quark potential of Eq. (4) (or Eq. (2)) with the parameters in Table I determined from lattice QCD [25] can reproduce the  $bbb$  and  $ccc$  baryon spectra measured also in lattice QCD [1, 2, 4] we will make use of three different numerical methods: generalized Gaussians variational approach [20], hyperspherical harmonics [19] and Faddeev equations [18]. The three-methods have been used and the difference in results is negligible. In all cases we solve the nonrelativistic Schrödinger equation

$$\{H_0 + V_{3Q}(r)\} \Psi(\vec{r}) = E \Psi(\vec{r}),$$

where  $H_0$  is the free part of three-heavy quarks without center of mass motion

$$H_0 = \sum_{i=1}^3 \left( M_Q + \frac{\vec{p}_i^2}{2M_Q} \right) - T_{CM}$$

and  $M_Q$  is the mass of the heavy quark. The mass of the heavy baryon will be finally given by  $M_{3Q} = 3M_Q + E$ .

We have checked the validity of the  $\Delta$ -ansatz for the spectroscopy of triply-heavy baryons calculating their masses and root mean square radii by means of a simple confining interaction given either by a  $Y$ -shape or a  $\Delta$ -shape interaction. The  $Y$ -shape potential would be given by [25],

$$V_{3Q}(r) = \sigma_{3Q}^Y L_{\min}, \quad (6)$$

where  $L_{\min}$ ,

$$L_{\min} = \left[ \frac{1}{2} (a^2 + b^2 + c^2) + \frac{\sqrt{3}}{2} \sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)} \right]^{1/2}, \quad (7)$$

being  $a$ ,  $b$  and  $c$  the three sides of the  $3Q$  triangle, when the angles of the  $3Q$  triangle do not exceed  $2\pi/3$ . When an angle of the  $3Q$  triangle exceeds  $2\pi/3$ , one gets,

$$L_{\min} = a + b + c - \max(a, b, c). \quad (8)$$

TABLE II: Mass, in GeV, and root-mean square radius, in fm, of the ground state,  $E_1(\frac{3}{2}^+)$ , and the first positive parity excited state,  $E_2(\frac{3}{2}^+)$ , of  $QQQ$  baryons for different values of the mass of the heavy quark  $M_Q$ , in GeV, and for the  $Y$ -shape and  $\Delta$ -shape confining potentials of Eqs. (6) and (9), respectively.

$M_Q$	$Y$ -shape		$\Delta$ -shape			
	$E_1(\frac{3}{2}^+)$	$E_2(\frac{3}{2}^+)$	$E_1(\frac{3}{2}^+)$	$\sqrt{\langle r_1^2 \rangle}$	$E_2(\frac{3}{2}^+)$	$\sqrt{\langle r_2^2 \rangle}$
4.0	12.814	13.121	12.795	0.236	13.094	0.334
4.4	13.989	14.286	13.970	0.228	14.260	0.324
4.8	15.166	15.455	15.148	0.222	15.430	0.315
5.2	16.346	16.627	16.328	0.216	16.603	0.306
5.6	17.528	17.801	17.509	0.211	17.778	0.299

We have taken for the string tension a standard value of  $\sigma_{3Q}^Y = 0.1648 \text{ GeV}^2$  [24–29]. The  $\Delta$ -shape potential would have the form,

$$V_{3Q}(r) = \sigma_{3Q}^\Delta \sum_{i < j} |\vec{r}_i - \vec{r}_j|, \quad (9)$$

with  $\sigma_{3Q}^\Delta = 0.53\sigma_{3Q}^Y = 0.0874 \text{ GeV}^2$ . The results are shown in Table II for different values of the heavy quark mass. As can be seen the two geometrical configurations give the same result with a small difference of 0.15%. If the geometrical factor in front of the  $\Delta$ -shape interaction is slightly modified, 0.55 instead of 0.53, the  $Y$ -shape and the  $\Delta$ -shape results would be exactly the same. This result was already noted in Ref. [40]. Besides, one observes how the root-mean square radii are much smaller than the quark-quark distance for which the  $Y$ - and  $\Delta$ -shapes start to slightly differ,  $\sim 0.7 \text{ fm}$  [27].

Looking for the general pattern of the results of nonperturbative lattice QCD for the  $bbb$  system we have compared against the result of a single  $\Delta$ -shape confining potential or a single Coulomb interaction. The results are shown in Fig. 1. In both cases we have fixed the strength of the potential to reproduce the  $E_2(3/2^+) - E_1(3/2^+)$  mass difference correctly, obtaining  $\sigma_{3Q}^\Delta = 0.2076 \text{ GeV}^2$  for the case of the single  $\Delta$ -shape confining interaction and  $A_{3Q} = 0.4231$  for the single Coulomb potential case. We clearly note the large strength of the two interactions as compared to the predictions of SU(3) lattice QCD (see Table I), where both terms would simultaneously contribute. The  $\Delta$ -shape interaction (upper panel) produces a splitting between the positive (i.e., the first excited state of  $J^P = 3/2^+$ ) and negative (i.e., the first excited state of  $J^P = 3/2^-$ ) parity excited states too large and a rather small excitation energy for the positive parity excitations. On the contrary, the Coulomb interaction (lower panel), being an almost hypercentral potential, drives to the expected degeneracy between the positive and negative parity excited states, not observed in the nonperturbative lattice QCD results. Thus, as the strength of the potential has been fixed to reproduce the  $E_2(3/2^+) - E_1(3/2^+)$  mass difference correctly, the negative parity excited states are obtained close to the positive parity excited states in clear disagreement with the nonperturbative lattice QCD results. Besides, the positive parity states (except the first excited state of  $J^P = 3/2^+$  that has been fitted) are predicted slightly above the nonperturbative lattice QCD results.

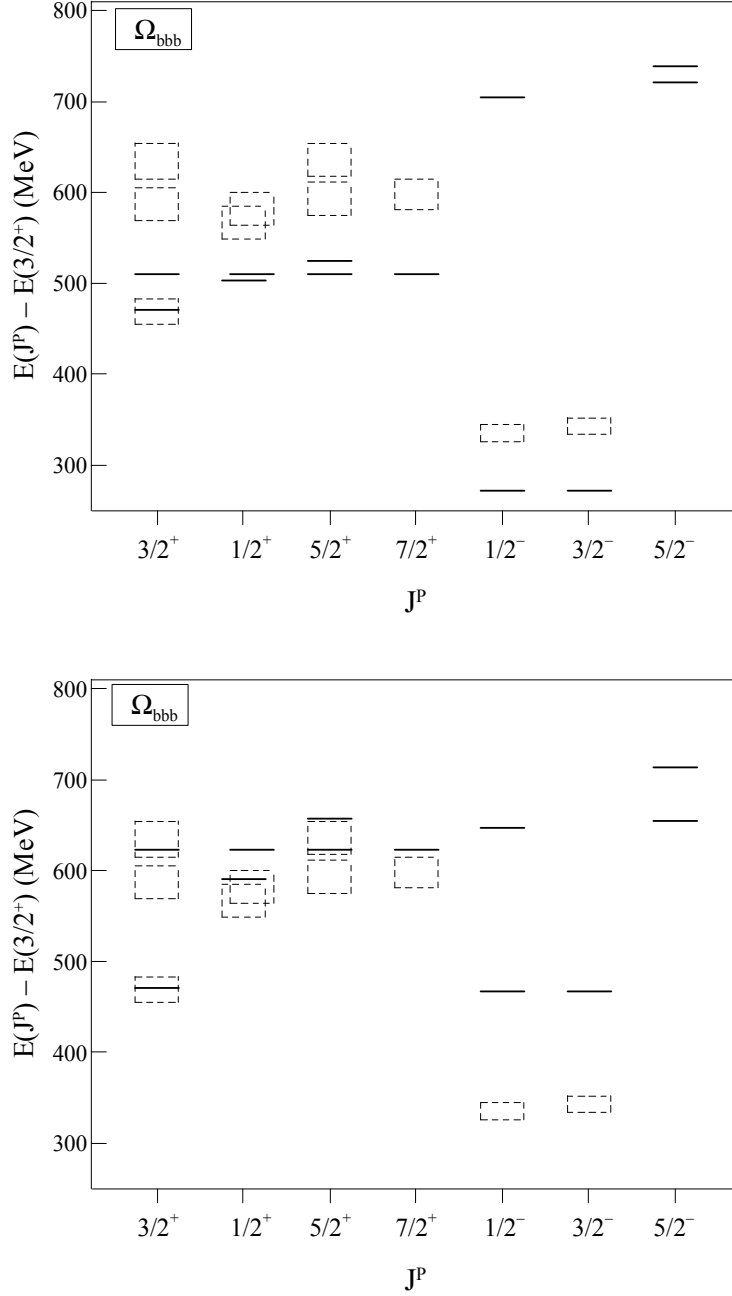


FIG. 1:  $bbb$  excited state spectra, solid lines, for a single  $\Delta$ -shape confining potential (upper panel) or a single Coulomb interaction (lower panel). In both cases we have fixed the strength of the potential to reproduce the  $E_2(3/2^+) - E_1(3/2^+)$  mass difference correctly, obtaining  $\sigma_{3Q}^\Delta = 0.2076 \text{ GeV}^2$  for the case of the single  $\Delta$ -shape confining interaction and  $A_{3Q} = 0.4231$  for the case of the single Coulomb potential. The boxes stand for the nonperturbative lattice QCD results of Ref. [2].

Thus,  $bbb$  nonperturbative lattice QCD calculations point to a static  $3Q$  potential given by a mixture of a  $\Delta$ -shape confinement and a Coulomb interaction. We have therefore make use of the standard  $3Q$  static potential derived from SU(3) lattice QCD and shown in Eq. (4) with the parameters reported in Ref. [25] and quoted in Table I,  $A_{3Q} = 0.1410$  and  $\sigma_{3Q}^\Delta = 0.0925 \text{ GeV}^2$  to calculate the  $bbb$  and  $ccc$  spectra. The results are shown in Fig. 2. We

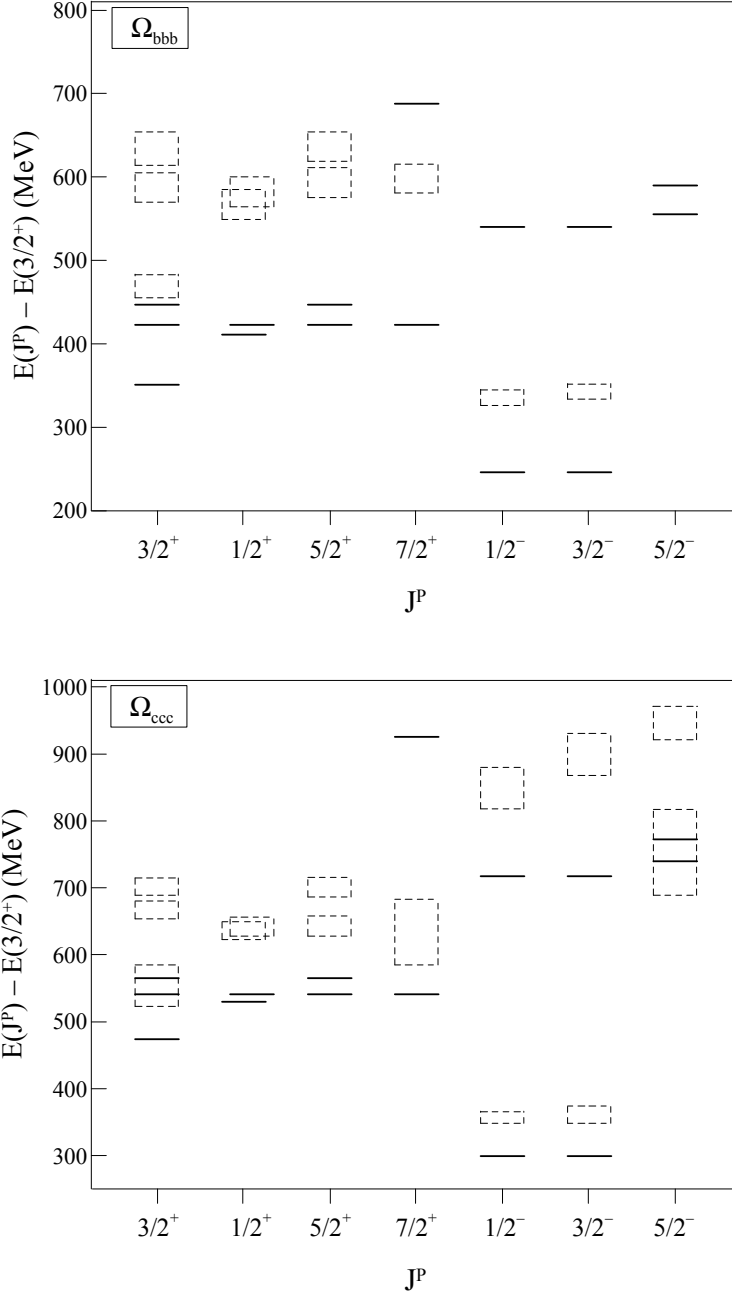


FIG. 2:  $bbb$  (upper panel) and  $ccc$  (lower panel) excited state spectra, solid lines, for the  $3Q$  static  $SU(3)$  lattice QCD potential of Eq. (4) with the parameters of Table I. The boxes stand for the nonperturbative lattice QCD results of Ref. [2] for the  $bbb$  system and Refs. [4, 41] for the  $ccc$  system.

have fixed the mass of the heavy quark to reproduce the  $J^P = 3/2^+$  ground state of the  $bbb$  and  $ccc$  systems, 14.372 GeV [1, 2] and 4.758 GeV [4], respectively, obtaining  $m_b = 4.655$  GeV and  $m_c = 1.269$  GeV. As we can see in Fig. 2, there is a large difference in the excited states with the nonperturbative lattice QCD results, predicting a small splitting between positive and negative parity excited states and also a small excitation energy for the positive parity states. These results clearly point to a lack of strength either in the confining or in the Coulomb potential.



One may think about several possible reasons for the disagreement:

- The parameters of Eq. (4) obtained in quenched QCD in Ref. [25] might be different from those in 2+1 flavor QCD employed in Refs. [1, 2, 4]. To check this possibility, one has to calculate parameters of Eq. (4) in 2+1 flavor QCD but such calculations do not exist so far.
- Results from both Ref. [25] and Refs. [1, 2, 4] contain systematic errors such as lattice artifact and chiral extrapolation as well as statistical errors. This may cause the disagreement.
- The fitting form Eq. (4), or Eq. (2), might not be appropriate to describe the static three-quark potential in lattice QCD.
- The static quark description might be inaccurate for  $bbb$  and, in particular, for  $ccc$ . Higher order terms might be necessary.
- The quark model description with "3-quark potential" might be inappropriate for  $bbb$  and  $ccc$  systems.

#### IV. SUMMARY

To summarize, the spectra of baryons containing three identical heavy quarks,  $b$  or  $c$ , have been recently calculated in nonperturbative lattice QCD. The energy of the lowest positive and negative parity excited states has been determined with high precision. These achievements constitute a unique opportunity to test phenomenological potential models in the regime where they are expected to work best. We have analyzed these results by means of static three-quark potentials derived using SU(3) lattice QCD using different numerical techniques for the three-body problem. Our results confirm the expectations of SU(3) lattice QCD of an almost indistinguishable confining  $Y$ - or  $\Delta$ -type at short distances for heavy baryons. The static three-quark potential with parameters from lattice QCD does not reproduce  $bbb$  and  $ccc$  spectra.

At the light of the present results a further effort to obtain a constituent quark-model potential description of the nonperturbative lattice QCD results, as has been done in the past for the heavy-meson systems [42–44], may help in understanding the connection between static three-quark potential parameters and simple Cornell-like potential descriptions. This work is in progress [45].

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