# The composite nature of the $\Lambda(1520)$ resonance

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## Abstract

Recently, the Weinberg compositeness condition of a bound state was generalized to account for resonant states and higher partial waves. We apply this extension to the case of the $\Lambda(1520)$ resonance and quantify the weight of the meson-baryon components in contrast to other possible genuine building blocks. This resonance was theoretically obtained from a coupled channels analysis using the s-waves $\pi\Sigma^*$, $K\Xi^*$ and the d-waves $\bar{K}N$ and $\pi\Sigma$ channels applying the techniques of the chiral unitary approach. We obtain that this resonance is essentially dynamically generated from these meson-baryon channels, leaving room for only 15% weight of other kind of components into its wave function.

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## I. INTRODUCTION

One of the most important issues in hadron spectroscopy is the determination of the nature of different hadronic states, mesons and baryons, found in different facilities and reported in the PDG [1], and the description of the spectrum of excited hadrons.

It has become clear that the traditional idea that considers mesons and baryons as pure $q\bar{q}$ and $qqq$ quark states, respectively, has to be replaced in some cases by more complex pictures involving more than two or three quarks. In the last years, the application of Chiral Perturbation Theory ($\chi PT$) to the study of the interactions of hadrons [2, 3] had remarkable success in describing hadron structures. However, this effective field theory, in which the ground states of mesons and baryons are considered as the relevant degrees of freedom, is not suitable to deal with the problem of spectroscopy, due to its very limited range of convergence.

The method has been improved constructing a non-perturbative unitary extension of the theory, called chiral unitary approach [4–16], which allows to explain many mesons and baryons as composite states of hadrons. This kind of resonances are commonly known as "dynamically generated".

One of the most challenging issues in this field is to understand whether a resonance can be considered as composite of other hadrons or something different, eventually a genuine resonance. The first breakthrough in the investigation of the composite nature of a system of particles was made in 1965 by Weinberg in the well-known paper in which he determined that the deuteron was a bound state of a proton and a neutron [17]. The same issue was studied later in [18–20]. However, the method was only suitable for $s$-waves and small binding energies.

A generalization to more heavily bound systems and using many coupled channels was made in [21] and extended to the case of resonances in [22], while in [23] the analysis was modified to include any partial waves. The method contained in [23] was successfully applied to the $\rho$ meson in that work, confirming the commonly accepted idea that it is not a $\pi\pi$ composite state but a genuine resonance, and in [24] to the $K^*$ meson, finding a very small weight for the $K\pi$ component in its wave function.

This generalization to any partial waves and resonant states of the Weinberg compositeness condition found in [23], was applied to baryons in [25] for the first time in order to determine the weight of the meson-baryon component in the members of the $J^P = \frac{3}{2}^+$ baryons decuplet. An amount of 60% has been found for the $\pi N$ cloud in the $\Delta(1232)$, while the higher-energy members of the decuplet seem to be better represented by a genuine component.

In the present work we investigate the structure of another baryonic resonance, the $\Lambda(1520)$. This resonance belongs to the negative parity $J^P = \frac{3}{2}^-$ resonances that, in the last few years, have been interpreted as dynamically generated from the interaction of the octet of the pseudoscalar mesons with the decuplet of the baryons [26, 27]. It has been studied theoretically in [26, 27] and considered as generated from the interaction of the coupled channels $\pi\Sigma(1385)$ and $K\Xi(1530)$ in s-wave. In this picture, it couples mostly to the first channel, qualifying as a quasi-bound state of $\pi\Sigma^*$, with a nominal mass of few MeV below the $\pi\Sigma^*$ threshold. However, the large branching ratios to $\bar{K}N$ and $\pi\Sigma$ indicate that these two channels must play a remarkable role in the building up of the resonance in spite of the fact that they couple in d-wave.

In [28] a coupled channels analysis of the $\Lambda(1520)$ data using $\pi\Sigma^*$, $K\Xi^*$, $\bar{K}N$ and $\pi\Sigma$

has been performed. In this work, the $\pi\Sigma^*$ channel is still the one with the largest coupling, but its strength is reduced with respect to the quasibound $\pi\Sigma^*$ picture. At the same time, the couplings to $\bar{K}N$ and $\pi\Sigma$ are remarkable, making these two channels relevant for the interpretation of different reactions involving the $\Lambda(1520)$. The model provided in [28] has been tested in [29] through the study of the two reactions $pp \to pK^+K^-p$ and $pp \to pK^+\pi^0\pi^0\Lambda$ close to the $\Lambda(1520)$ threshold, giving important information about the couplings of the $\Lambda(1520)$ to $\bar{K}N$ and $\pi\Sigma^*$.

In this work, by means of the extension of the Weinberg sum rule to resonant states in any partial waves, we make an estimation of the relevance of the different channels in the wave function of the $\Lambda(1520)$, starting from the coupled channel study of Ref. [28] and [29].

## II. SUMMARY OF THE FORMALISM AND MEANING OF THE SUM RULE FOR RESONANCES

For the sake of completeness, let us first briefly summarize the approach used in Ref. [23] to study the composite nature of a resonance in any partial waves. In order to create a resonance from the interaction of many channels at a certain energy, two particles must collide in a channel which is open at this energy. The process is described by the set of coupled Schrödinger equations,

$$
\begin{aligned}
|\Psi\rangle &= |\Phi\rangle + \frac{1}{E - H_0} V |\Psi\rangle \\
&= |\Phi\rangle + \frac{1}{E - M_i - \frac{\vec{p}^{\,2}}{2\mu_i}} V |\Psi\rangle \ ,
\end{aligned}
\tag{1}
$$

where

$$
|\Psi\rangle = \left\{
\begin{array}{c}
|\Psi_1\rangle \\
|\Psi_2\rangle \\
\vdots \\
|\Psi_N\rangle
\end{array}
\right\} , \qquad
|\Phi\rangle = \left\{
\begin{array}{c}
|\Phi_1\rangle \\
0 \\
\vdots \\
0
\end{array}
\right\} ,
\tag{2}
$$

$H_0$ is the free Hamiltonian and $\mu_i$ is the reduced mass of the system of total mass $M_i = m_{1i} + m_{2i}$. The state $|\Phi_1\rangle$ is an asymptotic scattering state used to create a resonance which will decay into other channels.

The potential $V$ has the form

$$
\langle \vec{p} | V | \vec{p}\,' \rangle \equiv (2l+1)\, v\, \Theta(\Lambda - p)\Theta(\Lambda - p')|\vec{p}|^l|\vec{p}\,'|^l P_l(\cos\theta) \ ,
\tag{3}
$$

where $\Lambda$ is a cutoff in the momentum space and $v$ is a $N \times N$ matrix, with $N$ the number of channels. The form of the potential is such that the generic $l$-wave character of the process is factorized in $|\vec{p}|^l$ and $|\vec{p}\,'|^l$, and in the Legendre polynomial $P_l(\cos\theta)$. Thus, the matrix $v$ is a constant matrix.

The $N \times N$ scattering matrix such that $T\Phi = V\Psi$, can be written as

$$
T = (2l+1)P_l(\hat{p}, \hat{p}')\Theta(\Lambda - p)\Theta(\Lambda - p')|\vec{p}|^l|\vec{p}\,'|^l t \ ,
\tag{4}
$$

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and the Schrödinger equation leads to the Lippmann-Schwinger equation for $T$ ($T = V + VGT$), by means of which one obtains

$$t = \frac{v}{(1 - vG)} = \frac{1}{v^{-1} - G} \ .$$ (5)

The matrix $G$ in Eq. (5) is a diagonal matrix accounting for the two hadron loop functions in the intermediate state (see Eq. (6)). It is important to stress that there is no explicit $\Lambda(1520)$ pole included into the formalism, and thus the resonant shape (see Fig. 1) and pole comes from the non-linear dynamics involved in the unitarization.

On the other hand, note that the definition $T\Phi = V\Psi$ makes $T$ independent of the phase convention of the wave function. This derivation leads to a $t$ matrix which does not contain the factor $|\vec{p}|^l$, due to the constant $v$ matrix. Other approaches for $p$-waves, like the ones in [30, 31], factorize on shell $|\vec{p}|^l$ and associate it to the potential $v$. In this new approach, this factor is absorbed in a new loop function

$$G_{ii} = \int_{|\vec{p}|<\Lambda} d^3p \, \frac{|\vec{p}\,|^{2l}}{E - m_{1i} - m_{2i} - \frac{\vec{p}^{\,2}}{2\mu_i} + i\epsilon} \ ,$$ (6)

which is different from the one normally used in the chiral unitary approach [32].

This choice is necessary for the generalization of the sum rule for the couplings to any partial wave found in [23], which holds both for resonances and bound states dynamically generated by the interaction in coupled channels of two hadrons,

$$\sum_i g_i^2 \left[\frac{dG_i}{dE}\right]_{E=E_R} = -1 \ .$$ (7)

In Eq. (7), $E_R$ is the position of the complex pole of the scattering matrix in the second Riemann sheet (see definition below) representing the resonance and $g_i$ is the coupling to the channel $i$ defined as

$$g_i g_j = \lim_{E \to E_R} (E - E_R) t_{ij} \ .$$ (8)

Note that this definition leads to complex couplings. This means that the terms of Eq. (7) are complex, which implies that the imaginary parts cancel and one is left with

$$\sum_i Re \left(g_i^2 \left[\frac{dG_i}{dE}\right]_{E=E_R}\right) = -1 \ .$$ (9)

Each term in Eq. (7) represents the integral of the wave function squared (not the modulus squared as it was in the case of bound states [21]) of each component,

$$\int d^3p \, (\Psi_i(p))^2 = -g_i^2 \frac{\partial G_i^{II}}{\partial E} \ ,$$ (10)

where the superindex $II$ stands for second Riemann sheet, but this occurs only with the phase convention in which the wave function in momentum space is given by

$$\Psi_i(p) = g_i \frac{\Theta(\Lambda - |\vec{p}|)p^l}{E - m_{1i} - m_{2i} - p^2/2\mu_i + i\epsilon} \ .$$ (11)

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This is a most appropriate choice, in which the radial wave function is real for a bound state.

We can consider each one of the terms in Eq. (9) as a measure of the relevance, or the weight, of a channel in the wave function of the state, but not a probability, which for open channels is not a useful concept since it will diverge, as explained in detail in [25].

Sometimes, we only have information on hadron-hadron scattering. There can be a genuine component different to the hadron-hadron one and, in order to take into account its weight, Eq. (9) must be rewritten as

$$ -\sum_i Re\left( g_i^2 \left[\frac{dG_i}{dE}\right]_{E=E_R} \right) = 1 - Z , \qquad Z = Re\int d^3p \left(\Psi_\beta(p)\right)^2 , \qquad (12) $$

where $\Psi_\beta(p)$ is the genuine component in the wave function of the state, when it is omitted from the coupled channels [33].

The left-hand side of Eq. (12) is the measure of the relevance of the hadron-hadron component, while its diversion from unity, $Z$, measures the weight of something different in the wave function.


## III.   APPLICATION TO THE $\Lambda(1520)$ RESONANCE

As mentioned in the introduction, the method of Sec. II has already been applied to the $\rho$ and $K^*$ mesons in [23] and [24] respectively, and, for the first time, the structure of baryonic resonances has been investigated in [25], making an estimation of the meson-baryon component in the wave function of the baryons of the decuplet of the $\Delta(1232)$.

We apply now the method to another baryonic resonance, the $\Lambda(1520)$, with quantum numbers $J^P = \frac{3}{2}^-$.


### A.   The chiral unitary model for the $\Lambda(1520)$

Our starting point is the analysis of Ref. [28]. The $\Lambda(1520)$ is studied in the framework of a coupled channels formalism including the channels $\pi\Sigma(1385)$ and $K\Xi(1530)$ in $s$-waves and $\bar{K}N$ and $\pi\Sigma$ in $d$-waves.

The matrix containing the tree-level amplitudes can be written as

$$ V = \begin{pmatrix} \mathcal{C}_{11}(k_1^0 + k_1^0) & \mathcal{C}_{12}(k_1^0 + k_2^0) & \gamma_{13}\, q_3^2 & \gamma_{14}\, q_4^2 \\ \mathcal{C}_{21}(k_1^0 + k_2^0) & \mathcal{C}_{22}(k_2^0 + k_2^0) & 0 & 0 \\ \gamma_{13}\, q_3^2 & 0 & \gamma_{33}\, q_3^4 & \gamma_{34}\, q_3^2\, q_4^2 \\ \gamma_{14}\, q_4^2 & 0 & \gamma_{34}\, q_3^2\, q_4^2 & \gamma_{44}\, q_4^4 \end{pmatrix} , \qquad (13) $$

where $q_i = \sqrt{(s-(m_i-M_i)^2)(s-(m_i+M_i)^2)}/2\sqrt{s}$ and $k_i^0 = (s - M_i^2 + m_i^2)/2\sqrt{s}$, with $m_i$ and $M_i$ the masses of the meson and baryon in channel $i$ $(i = 1, 4)$, respectively. The $s$- and $d$-waves character of the transitions is taken into account by means of the dependence of the potentials on the incoming and outgoing squared momenta. The s-wave transition elements are obtained from the lowest order chiral Lagrangian involving the interaction of the decuplet of baryons and the octet of pseudoscalar mesons, which gives $\mathcal{C}_{11} = -1/f^2$, $\mathcal{C}_{12} = \mathcal{C}_{21} = -\sqrt{6}/4f^2$ and $\mathcal{C}_{22} = -3/4f^2$, with $f = 1.15\, f_\pi$ and $f_\pi = 93$ MeV. The factor 1.15 in $f$ represents the average between $f_\pi$ and $f_K$ [9].

The scattering amplitudes are then evaluated by means of the Bethe-Salpeter equation, which reads

$$T = [1 - V\,G]^{-1}\,V\;.\tag{14}$$

As explained in detail in [23], the method summarized in Sec. II requires a potential independent of the momenta of the particles, which is not the case of Eq. (14). For this reason we define a new potential

$$V' = \begin{pmatrix} \mathcal{C}_{11}(k_1^0 + k_1^0) & \mathcal{C}_{12}(k_1^0 + k_2^0) & \gamma_{13} & \gamma_{14} \\ \mathcal{C}_{21}(k_1^0 + k_2^0) & \mathcal{C}_{22}(k_2^0 + k_2^0) & 0 & 0 \\ \gamma_{13} & 0 & \gamma_{33} & \gamma_{34} \\ \gamma_{14} & 0 & \gamma_{34} & \gamma_{44} \end{pmatrix}\tag{15}$$

independent of the $q^2$ factors from the d-waves and, according to Sec. II, we include this dependence in the new loop functions, that will now have the form

$$G_i^{(s)} = 2M_i \int \frac{d^3p}{(2\pi)^3} \frac{\omega_i(p) + E_i(p)}{2\omega_i(p)E_i(p)} \frac{1}{P^{02} - (\omega_i(p) + E_i(p))^2 + i\epsilon}\;,\tag{16}$$

$$G_i^{(d)} = 2M_i \int \frac{d^3p}{(2\pi)^3} \frac{\omega_i(p) + E_i(p)}{2\omega_i(p)E_i(p)} \frac{p^4}{P^{02} - (\omega_i(p) + E_i(p))^2 + i\epsilon}\;,\tag{17}$$

where $\omega_i(p) = \sqrt{p^2 + m_i^2}$ and $E_i(p) = \sqrt{p^2 + M_i^2}$ are the energies of the meson and the baryon involved in the loop, respectively. This modification concerns only the loop function for the d-wave channels, $G_i^{(d)}$ ($i = 3, 4$), which now contains the factor $p^{2l} = p^4$. Note that we improve on Eq. (6) by taking the relativistic propagator [6].

Note that the s-wave elements of the $V'-$matrix ($V_{11}'$, $V_{12}'$, $V_{21}'$ and $V_{22}'$) still contain an energy dependence through the $k_i^0$ factors. However this dependence is very smooth and it will play a role in the diversion from unity in the sum rule of Eq. (12), as will be further explained in the results section.

The loop functions in Eqs. (16) and (17) are regularized by means of two different cutoffs in momentum space, $p_{max}^{(s)}$ and $p_{max}^{(d)}$ for the s- and d-waves channels respectively. These two cutoffs, together with the coefficients $\gamma_{13}$, $\gamma_{14}$, $\gamma_{44}$, $\gamma_{44}$ and $\gamma_{34}$ of Eq. (15), constitute the set of free parameters in the theory.

However, this procedure presents a problem due to the different dimensions of the magnitudes involved. The elements of $V'$ in Eq. (15) concerning the transitions involving the d-waves channels, after removing the dependence on the momenta, will have different dimension with respect to the other ones. The same happens to the loop function, that now have different dimensions in the cases of s- or d-waves.

In order to render the dimensions homogeneous and evaluate the scattering amplitudes by means of the Bethe-Salpeter equation (Eq. (14)), which is a matrix equation, we define

$$\tilde{V} = \begin{pmatrix} \mathcal{C}_{11}(k_1^0 + k_1^0) & \mathcal{C}_{12}(k_1^0 + k_2^0) & \gamma_{13}\,q_3^2(m_{\Lambda^*}) & \gamma_{14}\,q_4^2(m_{\Lambda^*}) \\ \mathcal{C}_{21}(k_1^0 + k_2^0) & \mathcal{C}_{22}(k_2^0 + k_2^0) & 0 & 0 \\ \gamma_{13}\,q_3^2(m_{\Lambda^*}) & 0 & \gamma_{33}\,q_3^4(m_{\Lambda^*}) & \gamma_{34}\,q_3^2(m_{\Lambda^*})\,q_4^2(m_{\Lambda^*}) \\ \gamma_{14}\,q_4^2(m_{\Lambda^*}) & 0 & \gamma_{34}\,q_3^2(m_{\Lambda^*})\,q_4^2(m_{\Lambda^*}) & \gamma_{44}\,q_4^4(m_{\Lambda^*}) \end{pmatrix}\;,\tag{18}$$

with $q_i(m_{\Lambda^*}) = \sqrt{(m_{\Lambda^*}^2 - (m_i - M_i)^2)(m_{\Lambda^*}^2 - (m_i + M_i)^2)}/2m_{\Lambda^*}$, where we choose for $m_{\Lambda^*}$ the $\Lambda(1520)$ mass. Nevertheless this specific choice is obviously irrelevant for the final results.

| Set | $p_{max}^{(s)}$ [MeV] | $p_{max}^{(d)}$ [MeV] | $\gamma_{13}$ [MeV$^{-3}$] | $\gamma_{14}$ [MeV$^{-3}$] | $\gamma_{33}$ [MeV$^{-5}$] | $\gamma_{44}$ [MeV$^{-5}$] | $\gamma_{34}$ [MeV$^{-5}$] |
|---|---|---|---|---|---|---|---|
| 1) | 1797.960 | 868.265 | $-0.875 \cdot 10^{-7}$ | $1.169 \cdot 10^{-7}$ | $-0.030 \cdot 10^{-11}$ | $-0.055 \cdot 10^{-11}$ | $0.003 \cdot 10^{-11}$ |
| 2) | 1427.119 | 865.693 | $3.938 \cdot 10^{-7}$ | $-5.028 \cdot 10^{-7}$ | $-0.748 \cdot 10^{-11}$ | $-1.345 \cdot 10^{-11}$ | $0.966 \cdot 10^{-11}$ |
| 3) | 1324.125 | 904.062 | $-0.695 \cdot 10^{-7}$ | $0.840 \cdot 10^{-7}$ | $-0.025 \cdot 10^{-11}$ | $-0.033 \cdot 10^{-11}$ | $-.002 \cdot 10^{-11}$ |
| 4) | 1438.782 | 897.246 | $2.799 \cdot 10^{-7}$ | $-3.502 \cdot 10^{-7}$ | $-0.037 \cdot 10^{-11}$ | $0.048 \cdot 10^{-11}$ | $-0.028 \cdot 10^{-11}$ |
| 5) | 1747.956 | 911.004 | $2.248 \cdot 10^{-7}$ | $-2.873 \cdot 10^{-7}$ | $0.126 \cdot 10^{-11}$ | $0.162 \cdot 10^{-11}$ | $-0.174 \cdot 10^{-11}$ |

TABLE I: Values of the parameters of the theory resulting from a sample of five best fits to the scattering data for the $\bar{K}N$ and $\pi\Sigma$ amplitudes.

Now all the elements of the matrix $\tilde{V}$ have the same dimensions. Moreover, we can now write

$$\tilde{G}_i^{(d)} = 2M_i \int \frac{d^3p}{(2\pi)^3} \frac{\omega_i(p) + E_i(p)}{2\omega_i(p)E_i(p)} \frac{p^4/q_i^4(m_{\Lambda^*})}{P^{02} - (\omega_i(p) + E_i(p))^2 + i\epsilon} . \tag{19}$$

This new loop function for the $d$-waves cases has the same dimension as $G_i^{(s)}$, and now the Bethe-Salpeter equation reads

$$\tilde{T} = \frac{1}{\tilde{V}^{-1} - \tilde{G}} , \tag{20}$$

with

$$\tilde{G} = \begin{pmatrix} G_1^{(s)} & 0 & 0 & 0 \\ 0 & G_2^{(s)} & 0 & 0 \\ 0 & 0 & \tilde{G}_3^{(d)} & 0 \\ 0 & 0 & 0 & \tilde{G}_4^{(d)} \end{pmatrix} . \tag{21}$$

## IV. RESULTS

The scattering amplitudes derived using Eq. (20) contain, as we already mentioned, seven free parameters: $p_{max}^{(s)}$, $p_{max}^{(d)}$, $\gamma_{13}$, $\gamma_{14}$, $\gamma_{44}$, $\gamma_{44}$ and $\gamma_{34}$. In order to obtain their values, we fit the model to the experimental scattering amplitudes for $\bar{K}N$ and $\pi\Sigma$ in $d$-wave and for $I = 0$ [34, 35]. The relation between the experimental and the theoretical amplitudes is given by

$$T_{ij}^{exp}(\sqrt{s}) = -\sqrt{\frac{M_i q_i}{4\pi\sqrt{s}}} \sqrt{\frac{M_j q_j}{4\pi\sqrt{s}}} T_{ij}(\sqrt{s}) , \tag{22}$$

with $i$ and $j$ the channels involved in the transition.

We obtain several equivalent best fits to the experimental data and, in Tab. I the values of the parameters obtained from a sample of five sets are listed. In Fig. 1 we show the results of the fit for the first set of Tab. I, but, in any case, the results are consistent for all the sets. The fact that we get approximately the same solutions with different sets of parameters indicates that there are strong correlations between them. The final results are also very similar independently of the set of parameters chosen (see Tab. III).

At this point, we apply the sum rule of Eq. (12) to the present case. We first need to extrapolate the amplitudes to the complex plane and to look for the complex pole $\sqrt{s_0}$ in
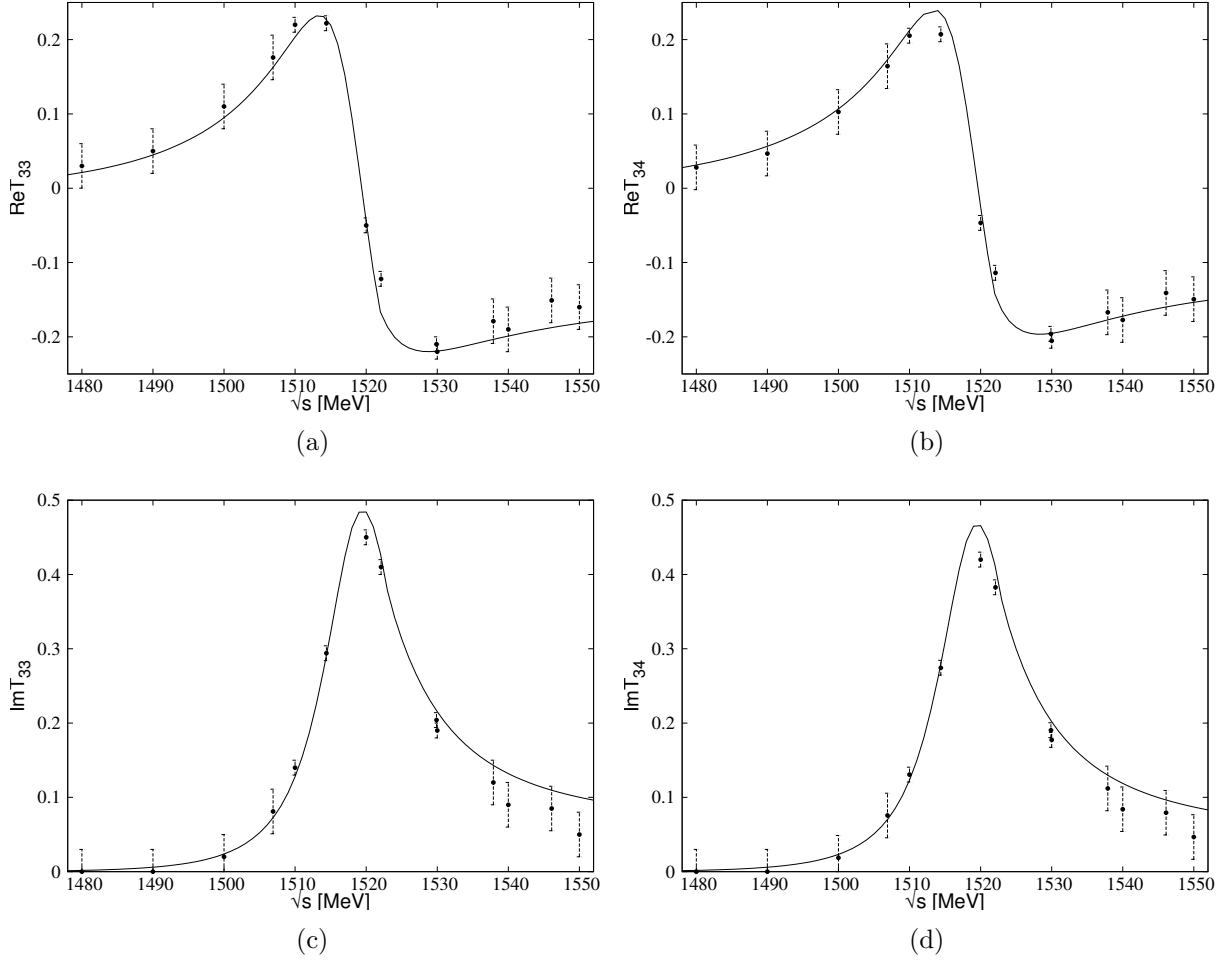
FIG. 1: Fit N. 1) to the experimental amplitudes for the transitions $\bar{K}N \to \bar{K}N$ in Figs. a) and c), and $\bar{K}N \to \pi\Sigma$ in Figs. b) and d).

the second Riemann sheet. This is done by changing $G_i^{(s)}$ and $G_i^{(d)}$ to $G_i^{II(s)}$ and $G_i^{II(d)}$ in Eqs. (16) and (19), in the channels which are open. The functions $G_i^{II(s)}$ and $G_i^{II(d)}$ are the analytic continuations of the loop functions in the second Riemann sheet and are defined as

$$G_i^{II(s)}(\sqrt{s}) = G_i^{I(s)} + \frac{i}{2\pi} \frac{M_i}{\sqrt{s}} q_i \; ,$$

$$G_i^{II(d)}(\sqrt{s}) = G_i^{I(d)} + \frac{i}{2\pi} \frac{M_i}{\sqrt{s}} \frac{q_i^5}{q_i^4(m_{\lambda^*})} \; , \qquad Im(q_i) > 0 \; , \tag{23}$$

where $G_i^{I(s)}$ and $G_i^{I(d)}$ are the loop functions in the first Riemann sheet given by Eqs. (16) and (19).

The values of the poles that we get from the five sets are listed in the first column of Tab. II.

Now we can evaluate the couplings of the resonance to the different channels as the residues at the pole of the amplitudes,

$$g_i^2 = \lim_{\sqrt{s} \to \sqrt{s_0}} (\sqrt{s} - \sqrt{s_0}) T_{ii}^{II} \; , \tag{24}$$

| Set | $\sqrt{s_0}$ [MeV] | $g_{\pi\Sigma^*}$ | $g_{K\Xi^*}$ | $g_{\bar{K}N}$ | $g_{\pi\Sigma}$ |
|---|---|---|---|---|---|
| 1) | $1518.7 - i6.4$ | $0.70 - i0.01$ | $-0.40 + i0.05$ | $0.54 - i0.06$ | $0.43 - i0.05$ |
| 2) | $1519.1 - i6.7$ | $0.78 - i0.07$ | $-0.35 + i0.07$ | $-0.56 + i0.05$ | $-0.45 + i0.03$ |
| 3) | $1518.3 - i6.5$ | $0.73 + i0.01$ | $-0.31 + i0.03$ | $0.53 - i0.06$ | $0.44 - i0.06$ |
| 4) | $1519.9 - i6.5$ | $0.74 + i0.00$ | $-0.34 + i0.04$ | $0.53 - i0.07$ | $0.44 - i0.04$ |
| 5) | $1518.5 - i6.4$ | $0.63 + i0.02$ | $-0.35 + i0.03$ | $-0.53 + i0.07$ | $-0.43 + i0.05$ |

TABLE II: Pole positions and values of the couplings of the $\Lambda(1520)$ to the four different channels of the model.

| Set | $X_{\pi\Sigma^*}$ | $X_{K\Xi^*}$ | $X_{\bar{K}N}$ | $X_{\pi\Sigma}$ | $1 - Z$ |
|---|---|---|---|---|---|
| 1) | 0.084 | 0.002 | 0.494 | 0.214 | 0.79 |
| 2) | 0.089 | 0.001 | 0.526 | 0.225 | 0.84 |
| 3) | 0.093 | 0.001 | 0.541 | 0.239 | 0.99 |
| 4) | 0.093 | 0.001 | 0.518 | 0.237 | 0.87 |
| 5) | 0.072 | 0.002 | 0.531 | 0.237 | 0.84 |

TABLE III: Values of the weights $X_i$ of the different channels in the wave function of the $\Lambda(1520)$ and the total $1 - Z = \sum_i X_i$

and apply the sum rule to evaluate the contribution of a single channel to the $\Lambda(1520)$:

$$X_i = -Re\left[g_i^2\left[\frac{dG_i^{II}(s)}{d\sqrt{s}}\right]_{\sqrt{s}=\sqrt{s_0}}\right] . \tag{25}$$

The couplings that we find using Eq. (24) are shown in Tab. II. Note that there is an ambiguity in the sign of $g_{\bar{K}N}$ and $g_{\pi\Sigma}$ among the different sets but the product $g_{\bar{K}N}g_{\pi\Sigma}$ has the same sign. This is because we fit the transition $\bar{K}N \to \pi\Sigma$ which determines the relative sign but not the overall one referred to the $\pi\Sigma^*$ channel.

From these values we can obtain the relevance of the different channels in the wave function of the $\Lambda(1520)$ resonance, using Eq. (25). The values of the different weights are shown in Tab. III.

We can now estimate the composite character of the $\Lambda(1520)$ resonance since, according to Eq. (12)

$$\sum_i X_i = 1 - Z , \tag{26}$$

where $Z$ is a measure of the presence in the state of something different from the meson-baryon components considered, (genuine components). We obtain for $1 - Z$ the values shown in the last column of Tab. III. Taking the average of the last column we have $1 - Z = 0.87 \pm 0.10$, which indicate an appreciable weight of meson-baryon character in the resonance with less than 15% weight for other genuine components. It is worth noting that numerically, the value of $Z$ corresponds [33] to

$$Z = -\sum_{ij}\left[g_i G_i^{II}(\sqrt{s})\frac{\partial V_{ij}(\sqrt{s})}{\partial\sqrt{s}}G_j^{II}(\sqrt{s})g_j\right]_{\sqrt{s}=\sqrt{s_0}} . \tag{27}$$

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Therefore, the diversion of $\sum_i X_i$ from unity is due to the smooth energy dependence of the s-wave elements of the potential (see Eq. (18)). In ref. [25] it was shown that in cases where there is an explicit CDD pole in the potential that accounts for a genuine state, or when one channel has been eliminated introducing an equivalent effective potential, which has a specific energy dependence, Eq. (27) indeed accounts for the probability, or weight, of the missing channels. Yet, it is not clear that the small magnitude obtained from the smooth energy dependence of the Weinberg Tomozawa interaction can be attributed to missing channels. We prefer to think that this amount can be considered as an uncertainty in the method to determine $Z$. In the present case we also see that this amount is of the same order of magnitude as the statistical uncertainties. Anyway, the fact that we get a good fit without needing to include a CDD pole is an indication of the low weight of genuine components in the building up of the $\Lambda(1520)$ resonance.

On the other hand, we can see in Tab. II that the coupling of the resonance to the $\pi\Sigma^*$ channel is the largest one. Yet, in terms of weight (probability of the state if it was a bound state) it represents only about 10%. This small probability can be deceiving, because the relevance of each channel is usually tied to the values of the wave function at the origin, more than to the probability. This is why in each particular process one has to find out the relevance of each channel. For instance, in the radiative decay $\Lambda(1520) \to \gamma\Lambda$, $\gamma\Sigma^0$ it was found that the $\pi\Sigma$, $\pi\Sigma^*$ channels did not contribute to the $\gamma\Lambda$ decay, but in the case of $\gamma\Sigma^0$ decay channel the $\pi\Sigma^*$ and $\pi\Sigma$ ($d$-waves) component gave the largest contribution to the decay width [36].

We should stress that one must be careful asserting the relevance of the channels from the weight obtained. Indeed, for the open channels, $\pi\Sigma$, $\bar{K}N$, the value of $X$ corresponds to the integral of the wave function squared, which goes as $e^{-iqr}/r$ for large $r$. While the integral of the modulus squared of the wave function diverges, this is not the case for the wave function squared where the oscillations of the $e^{-2iqr}$ factor lead to large cancellations at large $r$. Yet, it is clear that for the open channels one is getting contributions to $X$ from larger values of $r$ than in the bound channels, $\pi\Sigma^*$, $K\Xi^*$. Yet, the wave function at large values of $r$ will not have relevance in most processes involving short distances. In this sense, the couplings in the normalization that we have, or the relative values of $X$ in the $s-$waves, or $d-$waves, channels are the magnitudes that more fairly indicate the relevance of the different channels, but ultimately it is the specific dynamics of a given process that will determine the relevance of the channels, as seen in [36].

## V. SUMMARY AND CONCLUSIONS

We have applied the compositeness condition for resonances in higher partial waves to the case of the $\Lambda(1520)$ resonance. The aim was to quantify the weight of the meson-baryon components (s-waves $\pi\Sigma^*$, $K\Xi^*$ and d-waves $\bar{K}N$, $\pi\Sigma$) into the $\Lambda(1520)$ wave function. The meson-baryon scattering amplitudes are obtained implementing the techniques of the chiral unitary approach where some unknown parameters (five d-wave coefficients and two cutoffs) are fitted to $\bar{K}N$ and $\pi\Sigma$ experimental scattering data.

The momentum dependence coming from the d-wave channels are incorporated into the loop function leaving a smooth energy dependent potential for which the techniques developed in ref. [23] can be applied. From the knowledge of the loop functions and the couplings of the scattering amplitudes to the different channels, obtained from the residues of the amplitudes at the pole positions, the addends in the sum rule of Eq. (7), and the total sum

rule itself, can be evaluated which are a measure of the weight of the different channels into the $\Lambda(1520)$ wave function.

While the largest coupling obtained is to $\pi\Sigma^*$ (see Tab. II), the largest weight $X_i$ is to $\bar{K}N$ (see Tab. III). This is not contradictory since they represent different concepts. The coupling (actually the product of the coupling times the loop function, $g_iG_i$) accounts for the wave function at the origin [23] for $s-$waves while, as already explained, $X_i = -g_i\frac{\partial G_i}{\partial E}$ is a measure of the probability to find that channel.

We also explained that the large weight obtained for the open channels was a consequence of the contribution to the integral of the wave function squared from larger values of $r$ than for the bound channels, and not a measure of the contribution of the channel in different processes, most of which are sensible to short distances. The values of the couplings and the specific dynamics of those processes are what finally determine the relevance of each of the channels.

## ACKNOWLEDGMENTS

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