

# Non-decoupling and lepton number violation in left-right models

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## Abstract

We argue that large non-decoupling effects of heavy neutrinos can appear naturally in manifestly left-right symmetric models due to the minimization conditions of the scalar potential and the structure of vev's imposed by phenomenology. We derive constraints on off-diagonal light-heavy and heavy-heavy neutrino mixings from the searches for lepton violating decays  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow ee^-e^+$  and  $\mu - e$  conversion in nuclei. The most stringent limits come from the latter process because its amplitude shows a quadratic non-decoupling dependence on the heavy neutrino mass. Due to the suppression of right-handed currents by large  $W_R$  mass the present experiments are not sensitive to the intergenerational mixings between heavy neutrinos if  $M_{W_R} > 200$  TeV.

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# 1 Introduction

Although the current experimental data are consistent with the standard model of weak and electromagnetic interactions (SM) with an impressive accuracy there are several hints that the SM is not the ultimate theory of Nature. Indeed, the anomalies measured in the solar [1] and atmospheric [2] neutrino fluxes seems to require that neutrinos have a tiny mass, manifested in these phenomena through flavour oscillations. The observations of COBE satellite [3] indicate the existence of a hot neutrino component of dark matter. These phenomena cannot be explained in the framework of the SM and, therefore, motivate us to search for physics beyond the electroweak scale.

The new physics at higher energies should incorporate new heavy states which, in general, mix with the light ones. These mixings can occur, for example, in the neutral fermion sector between light and heavy neutrinos or in the gauge boson sector between the SM gauge bosons and the gauge bosons of the new interactions modifying slightly the SM couplings of charged and neutral currents. Such effects have been searched for using collider as well as low energy precision data. Since no deviations from the SM predictions have been found one has been able to constrain the mixing angles and masses of the new particles considerably [4, 5, 6]. In the following discussion we shall concentrate on the properties of heavy neutrinos i.e. neutral, weakly interacting fermions with masses larger than  $M_Z$ .

Motivated by the effective field theory approach it is natural to expect that heavy particles decouple from low energy life, i.e. the effects of heavy particles in the virtual intermediate states would be suppressed by inverse powers of the heavy scale. Indeed, since the low energy theory of any extended gauge model (where the new heavy states are singlets of the SM gauge group) should be the SM then due to its renormalizability the heavy scale decouples [7]. The above result was first derived by Appelquist and Carazzone [8]. However, large effects of the heavy states much below their production threshold are possible. They occur due to the loop corrections which grow with the mass of heavy particle to some power. These non-decoupling effects have been studied previously in many works [9] in which loops involving heavy neutrinos have also been considered. If the smallness of ordinary neutrino mass is explained by the see-saw mechanism [10] then the non-decoupling effects are cancelled by the small see-saw mixing angles. However, if the see-saw mechanism is not active then these effects can be significant.

Non-decoupling effects are, indeed, phenomenologically very interesting because they allow to explore the physics at high scales through low energy processes. To enhance the effects we shall consider possibilities other than the see-saw mechanism to keep the masses of the known neutrinos below the laboratory limits and at the same time allow mixing angles between heavy and light neutrino states to be large. Such alternative scenarios, where vanishingly small neutrino masses are ensured by some symmetry arguments have been considered in many works [11, 12, 13]. Some of them, motivated by the generalized  $E_6$  models, suggest a very specific form of neutrino mass matrix involving a large number of new heavy doublet and singlet neutrinos [12]. The others demand relations for neutrino mass matrix [13] which cannot be reasonably justified in the SM enlarged by adding right-handed singlet neutrinos. Since the only energy scale involved in the SM is the electroweak breaking scale the latter case, thus, introduces a fine tuning for Yukawa couplings in the neutrino sector.

However, this is not the case in models with extended gauge sector. One of the most interesting extensions of the SM is based on the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  [14]. At the Lagrangian level this model is left-right symmetric but, in order to explain the observed parity violation of the weak interaction, the vacuum is not invariant under the left-right symmetry. The left-right model with triplet representations of Higgs fields can accommodate the see-saw mechanism for neutrino masses [15]. As we are going to argue, due to the built in hierarchy of vacuum expectation values (vevs) of the Higgs fields and relations between the vevs obtained in minimization of the Higgs potential, large non-decoupling effects of heavy neutrinos can occur in the model with manifest left-right symmetry.

In this work we are going to constrain the off-diagonal elements of light-heavy and heavy-heavy neutrino mixing matrices from the lepton flavour violating processes  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow ee^-e^+$  and  $\mu - e$  conversion in nuclei in the manifestly left-right symmetric model. These processes involve virtual heavy, predominantly right-handed, neutrinos and gauge bosons as well as their mixing angles with the light states. The amplitudes of the two latter processes grow with heavy neutrino mass to the second power showing a genuine non-decoupling dependence. This non-decoupling behaviour is comparable to the top mass dependence of the  $\rho$  parameter and of the  $Z \rightarrow b\bar{b}$  vertex. We show that the perturbative unitarity bound on heavy neutrino mass in the left-right model is not as restrictive as it is in the  $E_6$  based extensions of the SM [16] and, therefore, experiments can bound the flavour changing mixings more strictly than in the latter case.

The outline of the paper is the following. In Section 2 we present basics of the left-right symmetric model and discuss the mechanism for natural occurrence of large mixings of heavy neutrinos. In Section 3 we consider the dominant decays of heavy neutrinos and derive the perturbative unitarity condition for neutrino masses. In Section 4 we calculate the branching ratios of the processes  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow ee^-e^+$  and  $\mu - e$  conversion in nuclei. We discuss the non-decoupling effects and derive limits on the off-diagonal light-heavy and heavy-heavy neutrino mixings. Our conclusions are given in Section 5.

## 2 Light-heavy mixings in left-right symmetric models

We begin with presenting the minimal  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  model with a left-right discrete symmetry. In the left-right symmetric models each generation of quarks and leptons is assigned to the multiplets

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} , \quad \psi = \begin{pmatrix} \nu \\ l \end{pmatrix} , \quad (1)$$

with the quantum numbers  $(T_L, T_R, B - L)$

$$\begin{aligned} Q_L &: \left( \frac{1}{2}, 0, \frac{1}{3} \right) , & \psi_L &: \left( \frac{1}{2}, 0, -1 \right) , \\ Q_R &: \left( 0, \frac{1}{2}, \frac{1}{3} \right) , & \psi_R &: \left( 0, \frac{1}{2}, -1 \right) . \end{aligned} \quad (2)$$

Concerning the Higgs sector, in order to give masses to fermions, all the left-right models should contain a bidoublet

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} = \left( \frac{1}{2}, \frac{1}{2}, 0 \right). \quad (3)$$

Since the bidoublet does not break the right-handed symmetry the Higgs sector has to be enlarged. This procedure is not unique but interesting models are obtained by adding the scalar triplets

$$\Delta_{L,R} = \begin{pmatrix} \frac{\Delta_{L,R}^+}{\sqrt{2}} & \Delta_{L,R}^{++} \\ \Delta_{L,R}^0 & -\frac{\Delta_{L,R}^+}{\sqrt{2}} \end{pmatrix} \quad (4)$$

with the quantum numbers  $\Delta_L : (1, 0, 2)$  and  $\Delta_R : (0, 1, 2)$ , respectively. In addition, we require the full Lagrangian of the model to be manifestly left-right symmetric i.e. invariant under the discrete symmetry

$$\psi_L \longleftrightarrow \psi_R, \quad \Delta_L \longleftrightarrow \Delta_R, \quad \phi \longleftrightarrow \phi^\dagger. \quad (5)$$

This symmetry plays a role in minimizing the Higgs potential and, therefore, is crucial for obtaining the non-decoupling effects in the model.

In general, our symmetry breaking would be triggered by the vevs

$$\langle \phi \rangle = \begin{pmatrix} \frac{k_1}{\sqrt{2}} & 0 \\ 0 & \frac{k_2}{\sqrt{2}} \end{pmatrix}, \quad \langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ \frac{v_{L,R}}{\sqrt{2}} & 0 \end{pmatrix}. \quad (6)$$

The vev  $v_R$  of the right triplet breaks the  $SU(2)_R \times U(1)_{B-L}$  symmetry to  $U(1)_Y$  and gives masses to new right-handed particles. Since the right-handed currents are not observed,  $v_R$  should be sufficiently large [6, 17]. Further, the vevs  $k_1$  and  $k_2$  of the bidoublet break the SM symmetry and, therefore, are of the order of electroweak scale. The vev  $v_L$  of the left triplet, which contributes to the  $\rho$  parameter, is quite tightly bounded by experiments [18] and should be below a few GeV-s. Thus, the following hierarchy should be satisfied:  $|v_R| \gg |k_1|, |k_2| \gg |v_L|$ . In principle, due to the underlying symmetry two of the vevs can be chosen to be real but two of them can be complex leading to the spontaneous  $CP$  violation [19]. To simplify the discussion we assume in the following that all the vevs are real.

The most general Yukawa Lagrangian for leptons invariant under the gauge group is given by

$$\begin{aligned} -\mathcal{L}_Y = & f_{ij} \bar{\psi}_L^i \phi \psi_R^j + g_{ij} \bar{\psi}_L^i \tilde{\phi} \psi_R^j + \text{h.c.} \\ & + i(h_M)_{ij} \left( \psi_L^{iT} C \tau_2 \Delta_L \psi_L^j + \psi_R^{iT} C \tau_2 \Delta_R \psi_R^j \right) + \text{h.c.}, \end{aligned} \quad (7)$$

where  $f$ ,  $g$  and  $h_M$  are matrices of Yukawa couplings. The left-right symmetry (5) requires  $f$  and  $g$  to be Hermitian. The Majorana couplings  $h_M$  can be taken to be real and positive due to our ability to rotate  $\psi_L$  and  $\psi_R$  by a common phase without affecting  $f$  and  $g$ .

According to the Lagrangian (7), neutrino masses derive both from the  $f$  and  $g$  terms, which lead to Dirac mass terms, and from the  $h_M$  term, which leads to large Majorana

mass terms. Defining, as usual,  $\psi^c \equiv C(\bar{\psi})^T$ , the mass Lagrangian following from Eq.(7) can be written in the form

$$-\mathcal{L}_{mass} = \frac{1}{2}(\bar{\nu}_L^c M \nu_R + \bar{\nu}_R M^* \nu_L^c), \quad (8)$$

where  $\nu_L^c = (\nu_L, \nu_R^c)^T$  and  $\nu_R = (\nu_L^c, \nu_R)^T$  are six dimensional vectors of neutrino fields. The neutrino mass matrix  $M$  is complex-symmetric and can be written in the block form

$$M = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}, \quad (9)$$

where the entries are  $3 \times 3$  matrices given by

$$M_L = \sqrt{2}h_M v_L, \quad M_D = h_D k_+, \quad M_R = \sqrt{2}h_M v_R. \quad (10)$$

Here we have defined  $h_D = (fk_1 + gk_2)/(\sqrt{2}k_+)$ , where  $k_+^2 = k_1^2 + k_2^2$ . The masses of charged leptons are given by  $M_l = (gk_1 + fk_2)/\sqrt{2}$  and, therefore, without fine tuning of  $f$  and  $g$  one has  $M_D \simeq M_l$ . Moreover, on the basis of avoiding possible fine tunings it is natural to assume that all the Yukawa couplings  $h_{M,D}$  are of similar magnitude for a certain lepton family. In this case the mass matrix (9) has a strong hierarchy between different blocks which is set by the hierarchy of vevs.

Since the neutrino mass matrix is symmetric it can be diagonalized by the complex orthogonal transformation

$$U^T M U = M^d, \quad (11)$$

where  $M^d$  is the diagonal neutrino mass matrix. If we denote

$$U = \begin{pmatrix} U_L^* \\ U_R \end{pmatrix}, \quad (12)$$

then in the basis where the charged lepton mass matrix is diagonal (we can choose this basis without loss of generality) the physical neutrino mixing matrices which appear in the left- and right-handed charged currents are simply given by  $U_L$  and  $U_R$ , respectively. Indeed,  $U_L, U_R$  relate the left-handed and right-handed neutrino flavour eigenstates  $\nu_{L,R}$  with the mass eigenstates  $\nu_m$  according to

$$\nu_{L,R} = U_{L,R} \nu_m. \quad (13)$$

In order to find relations between the different parameters of the theory we have to solve the first derivative equations of the scalar field potential  $V$

$$\frac{\partial V}{\partial v_{L,R}} = 0, \quad \frac{\partial V}{\partial k_{1,2}} = 0. \quad (14)$$

The full potential, invariant under the transformation (5), can be found in Ref.[18, 19]. When considering the minimization of the potential we get a see-saw type relation among the vevs

$$v_L v_R \simeq k_1 k_2. \quad (15)$$

Here we have excluded a possibility of fine tuning and assumed that all the parameters of the potential are of the same order.

Having this in mind we can now start to study mass matrices of type (9). For a simple one family case we find

$$\det M \sim 2v_L v_R - k_+^2 = 2v_L v_R - (k_1^2 + k_2^2). \quad (16)$$

If  $k_1 \neq k_2$ , then Eq.(15) implies that  $M$  is not singular and we have the usual see-saw mechanism. It is convenient to employ the self-conjugated spinors

$$\nu \equiv \frac{\nu_L + \nu_L^c}{\sqrt{2}}, \quad N \equiv \frac{\nu_R + \nu_R^c}{\sqrt{2}}, \quad (17)$$

which are also the approximate mass eigenstates with masses

$$\begin{aligned} m_\nu &\simeq \sqrt{2} \left( h_M v_L - \frac{h_D^2 k_+^2}{2h_M v_R} \right), \\ m_N &\simeq \sqrt{2} h_M v_R, \end{aligned} \quad (18)$$

respectively. The mixing between them depends on the ratio of the masses as  $\sin^2 \theta \sim m_\nu/m_N$  being vanishingly small for the present experimental values of  $m_\nu$  and  $m_N$  [20].

On the other hand, if  $k_1 = k_2$ , then it follows from Eq.(15) that  $M$  is singular. Indeed, this is the most natural case since  $k_1$  and  $k_2$  are vevs of the same bidoublet. The discrete left-right symmetry (5) together with the imposition of the discrete symmetry  $\phi_1 \longleftrightarrow \phi_2$  [21] ensures that the scalar fields  $\phi_1$  and  $\phi_2$  are indistinguishable at the Lagrangian level. As they both acquire vevs at the same stage of the symmetry breaking it is natural to expect that also the vacuum respects this symmetry. In this case the mixing angle  $\sin \theta \sim k/v_R$  is no longer related to the ratio of the light and heavy neutrino masses and would be allowed to be as large as  $O(10^{-1})$ . The masses then, are given by

$$m_\nu \simeq 0, \quad m_N \simeq \sqrt{2} h_M v_R, \quad (19)$$

which explains the smallness of the mass of the known neutrino. According to this one generation example the singularity of the mass matrix, indeed, may arise naturally in our model. However, it should be noted that it is still an imposed relation which can be justified by some higher symmetry.

This given example can be generalized to the three generation model. The sub-matrices  $M_{L,D,R}$  of the mass matrix (9) are proportional to the corresponding vevs and, together with Eq.(15), there is a natural structure for a singular mass matrix with three times degenerate zero eigenvalue. The remaining three neutrinos have masses of the order of  $v_R$ . The neutrino mixing matrixes  $U_L$  and  $U_R$  consist of two  $3 \times 3$  blocks. In  $U_L$  one of them presents light-light and another light-heavy mixings while  $U_R$  contains light-heavy and heavy-heavy mixings. According to our model, the light-light and heavy-heavy mixing angles are naturally maximal while the light-heavy mixing angles are of the order of  $k/v_R$ . This implies that the light-heavy mixings can also be substantial. The intergenerational mixings among light neutrinos can be constrained, for example, in neutrino oscillation experiments [1, 2]. In the present paper we are going to constrain the off-diagonal light-heavy and heavy-heavy mixings from flavour changing processes.

### 3 Perturbative unitarity bound on massive neutrinos

As mentioned in the Introduction, due to the non-decoupling effects in our model, the branching ratios of the flavour changing processes we are considering are increasing functions of neutrino masses. Therefore, in order to bound neutrino mixing angles from the existing measurements we must first study the viable range of masses the heavy neutrinos can take in our model. In general, if neutrino is extremely massive its decay width becomes so broad that one cannot identify it as a particle. In this way, assuming that

$$\Gamma \lesssim \frac{m_N}{2}, \quad (20)$$

where  $\Gamma$  is the total width of heavy neutrino, one can obtain a perturbative unitarity bound on the heavy neutrino mass. It turned out that in the SM with right-handed singlet neutrinos this bound is very restrictive demanding the heavy neutrino mass to be below  $\mathcal{O}(1)$  TeV. This is an expected result since the unique energy scale in this model is the electroweak breaking scale and, therefore, neutrinos cannot be considerably heavier. Consequently, the experimental bounds on off-diagonal heavy-light neutrino mixings in models of this type are affected by this result [16].

In left-right models there are new right-handed interactions and a new energy scale associated with the breaking of the right-handed symmetry. Due to these new ingredients the perturbative unitarity bound on neutrino mass is not related any more to the scale of left-handed interactions but to the right-handed one. Therefore, in the left-right models one can expect to obtain more stringent bounds on the light-heavy neutrino mixings than in the models with right-handed singlets.

The dominant decay modes of the heavy neutrino are

$$N \rightarrow e + W_R, \quad (21)$$

$$N \rightarrow e + W_L, \quad (22)$$

$$N \rightarrow \nu + Z_L. \quad (23)$$

We assume that possible decays to other heavy neutrinos and  $Z_R$  or Higgs bosons are suppressed by phase space factors due to the large masses of the final state particles. If  $N$  is much heavier than  $W_R$  then the first decay occurs without any suppression while the latter two are suppressed either by the gauge boson mixing angle (decay (22)) or by light-heavy neutrino mixing angle (decay (23)). Since we are interested in obtaining an upper bound on the neutrino mass then we assume that the condition  $m_N > M_{W_R}$  holds and we expect the decay (21) to be dominant. The decay rates of (21) and (22) can be written as

$$\Gamma_{W_{L,R}} = \frac{g^2}{64\pi M_{W_{L,R}}^2} \beta_{L,R}^2 m_N^3 \left(1 + \frac{M_{W_{L,R}}^2}{2m_N^2}\right) \left(1 - \frac{M_{W_{L,R}}^2}{m_N^2}\right)^2, \quad (24)$$

where  $\beta_R = 1$  and  $\beta_L = \xi$  is the gauge bosons mixing angle. Similarly we obtain for the decay (23)

$$\Gamma_{Z_L} = \frac{g^2}{64\pi M_{W_L}^2} \theta^2 m_N^3 \left(1 + \frac{M_{Z_L}^2}{2m_N^2}\right) \left(1 - \frac{M_{Z_L}^2}{m_N^2}\right)^2, \quad (25)$$

where  $\theta$  is the mixing angle between  $N$  and  $\nu$ .

Let us first analyse the decay (22). In the limit  $M_{W_L}^2 \ll m_N^2$  substituting the numerical values to Eq.(24) we get  $m_N < 30 M_{W_L}/\xi$ . In the left-right model, assuming  $k_1 = k_2$ , the mixing angle between left and right gauge bosons is given by  $\xi = M_{W_L}^2/M_{W_R}^2$  and we obtain

$$m_N < 30 \frac{M_{W_R}}{M_{W_L}} M_{W_R}. \quad (26)$$

On the other hand, from the decay (21), which does not depend on any mixing angle, we get in the limit  $M_{W_R}^2 \ll m_N^2$

$$m_N < 30 M_{W_R}, \quad (27)$$

a much stronger requirement than the previous one. Indeed, since  $\xi$  depends on the gauge boson masses to the second power one expects to have weaker bound from the decay involving  $W_L$  than from that involving  $W_R$ . The right-hand side of Eq.(26) is enhanced by a factor of  $M_{W_R}/M_{W_L}$  if compared with Eq.(27) which means that, in the case of very heavy neutrino, the decay mode (21) dominates over (22).

However, in models with non-decoupling of heavy neutrinos the neutrino mixing angle  $\theta$  is given by  $\theta = k/v_R = M_{W_L}/M_{W_R}$ , the ratio of the masses to the first power. Taking into account the expression for neutrino mixing angle we obtain from Eq.(25) exactly the bound (27) also for the decay mode (23). We know from precision measurements that the mixing of light neutrinos with heavy neutral fermions are limited to  $\theta_e^2 < 0.005$ ,  $\theta_\mu^2 < 0.002$  and  $\theta_\tau^2 < 0.01$  [5], where the subscripts denote corresponding generations. These limits are of the same order of magnitude as the limit predicted by our model taking into account the present lower bound on right-handed gauge boson mass  $M_{W_R} \gtrsim 1.4$  TeV [17]. Consequently, if non-decoupling is the true scenario then for very heavy neutrinos the decay rates of the processes (21) and (23) are almost equal in magnitude. For lighter neutrinos (but still heavier than  $W_R$ ) the former decay can be suppressed by a phase space factor which implies that the heavy neutrino decays (23) dominate. However, if non-decoupling does not occur then the decay mode (21) dominates. In any case, the perturbative unitarity bound on heavy neutrino mass is with a good precision given by Eq.(27). With this result we proceed to the analysis of lepton flavour violating processes.

## 4 Constrains on neutrino mixings from flavour changing processes

Indirect limits on intergenerational light-heavy neutrino mixing angles can be derived from the precision measurement limits on the diagonal mixing angles by using the Schwartz inequality. The resulting bounds are all in the range of  $(U_L U_L^\dagger)_{ab} < (0.003 - 0.004)$  [16], where  $a, b$  stand for any generation index. They are more stringent than any direct bound from tree level processes [22]. Nevertheless, loop diagrams involving heavy neutrinos give rise to unobserved rare processes such as  $l_i \rightarrow \gamma l_j$ ,  $l_i \rightarrow l_j \bar{l}_j l_k$ , etc. where  $i$  is either a tau or a muon while  $j$  and  $k$  are electron or muon in the former case and only electron in the latter case. Taking into account the indirect limits the rates for all the processes involving



violation of the tau lepton number turn out to be below the experimental sensitivity even for extreme values of the heavy neutrino masses [23]. However, due to the extraordinary sensitivity of the experiments looking for flavour changing processes involving the first two families the obtained constraints are significantly stronger than the indirect limits. Because of this we are going to consider the first two families only.

Let us first consider the decay  $\mu \rightarrow e\gamma$  induced in left-right models by the one-loop Feynman diagrams depicted in Fig.1. The additional contribution arising from the diagrams involving lepton flavour violating triplet Higgs bosons  $\Delta^{++}$  have been studied in Ref.[24]. The triplet Higgs bosons must be heavy because their masses are set by  $v_R$ . Since there is no non-decoupling effect working in these diagrams (lepton flavour violation is induced in the vertices not in the internal lines) their contribution to the  $\mu$  decay is suppressed by  $v_R^{-4}$  and can be neglected in our approach. The dominant diagrams involve both left- and right-handed charged current interactions and, consequently, both left- and right-handed neutrino mixing matrices  $U_{L,R}$ . The branching ratio for the process takes the form

$$B(\mu \rightarrow e\gamma) = \frac{3\alpha}{8\pi} (|g_L|^2 + |g_R|^2), \quad (28)$$

where

$$g_{L,R} = \eta_{L,R} \sum_i (U_{L,R}^{ei} U_{L,R}^{i\mu\dagger}) F(x_{L,R}^i), \quad (29)$$

where

$$F(x_{L,R}^i) = \left[ \frac{x_{L,R}^i (1 - 5x_{L,R}^i - 2(x_{L,R}^i)^2)}{2(x_{L,R}^i - 1)^3} + \frac{3(x_{L,R}^i)^2}{(x_{L,R}^i - 1)^4} \ln x_{L,R}^i \right]. \quad (30)$$

In these expressions  $\eta_L = 1$ ,  $\eta_R = M_{W_L}^2/M_{W_R}^2$ ,  $x_{L,R}^i = (m^i/M_{W_{L,R}})^2$  and summation goes over all neutrino masses  $m^i$ .

The function  $F(x)$  varies from 0 to 1 as  $x$  goes from 0 to  $\infty$ . Therefore, only the heavy neutrinos contribute to the  $\mu$  decay and we can constrain the light-heavy and heavy-heavy mixings only. Functions  $g_L$  and  $g_R$  represent contributions from the left-handed and right-handed interactions, respectively. In the case of  $M_{W_R} = 1.4$  TeV and neutrino masses as high as argued in Section 3 one has  $F(x_L) \approx 1$  and  $F(x_R) \approx 0.98$  which implies that there is no significant suppression due to the function  $F$  in the right-handed sector. However, the suppression appears due to the parameter  $\eta$  which is small in the case of right-handed currents. On the other hand, since our model allows  $|U_R|^{e\mu}$  to be of order one, we still can bound the heavy-heavy mixings even for quite large  $W_R$  masses. For fixed neutrino mixing angles, the branching ratio for the process approaches a constant value when neutrino masses are sufficiently large. If we let  $v_R$  to grow then the light-heavy mixings approach zero and the amplitude of the process vanishes. Due to the mixing between left- and right- handed gauge bosons one would also expect to get terms proportional to neutrino mass but these terms vanish in the case of real photon.

The present experimental bound on the branching ratio for the process is  $B(\mu \rightarrow e\gamma) < 4.9 \cdot 10^{-11}$  [25]. This result is not sensitive to the left- and right-handed contributions separately. Therefore, in order to constrain mixings in the left and right sector we

assume that only one of them is dominant at the time. Limits obtained in this way are optimistic ones, if both terms contribute the bounds would be more stringent. Therefore, the following limits should be taken as suggested ones just to get a feeling of what one can expect in this class of models.

We assume for simplicity that the heavy neutrino masses are similar in magnitude. Then, in the first approximation, the values of  $F(x)$  can be taken to be equal for all neutrino generations and we can easily constrain the quantity  $|U_{L,R}|^{e\mu} = \left(\sum_i U_{L,R}^{ei} U_{L,R}^{i\mu\dagger}\right)^{\frac{1}{2}}$ . Here the summation goes over the heavy neutrinos only which means that in the case of  $U_L$  we can bound light-heavy mixings and in the case of  $U_R$  heavy-heavy mixings. As expected, the process is not sensitive to light-light mixings. In Fig.2 we plot the constrains on  $|U_L|^{e\mu}$  and in Fig.3 the constrains on  $|U_R|^{e\mu}$  from the searches of the decay  $\mu \rightarrow e\gamma$  (curves denoted by  $a$ ) as functions of the heavy neutrino mass. In Fig.3 we take  $M_{W_R} = 1.4$  TeV. As can be seen, the curves become almost constant for neutrino masses allowed by the perturbative unitarity. The limiting value for  $|U_L|^{e\mu}$ , which does not depend on  $M_{W_R}$  but on the sensitivity of the experiment only, converges at  $|U_L|^{e\mu} = 0.016$  which agrees with similar bounds obtained in other models [16]. The bound on  $|U_R|^{e\mu}$ , however, depends almost linearly on the right gauge boson mass. In order to get limits on  $|U_R|^{e\mu}$  for other values of  $M_{W_R}$  one has to scale the curve  $a$  in Fig.3 by an appropriate factor. With the present value of  $M_{W_R}$  the neutrino mass should exceed 0.6 TeV in order to bound  $|U_R|^{e\mu}$  below unity. If the mass of  $W_R$  exceeds 5 TeV then the sensitivity of the experiment is not sufficient to constrain  $|U_R|^{e\mu}$  any more even for the maximally allowed neutrino masses.

In the future more stringent constaraints on  $|U_{L,R}|^{e\mu}$  can be obtained from  $\mu \rightarrow e\gamma$  searches. According to the proposals [26] new experiments will not only rise the sensitivity but also use almost 100% polarized muons. This will allow one to get direct information about the right- and left-handed interaction contributions to the decay and, thus, constrain the model further. At present the curves denoted by  $a$  in Figs.2,3 imply, indeed, severe constraints on our model.

However, one can do better even with the present experimental data. The extraordinary sensitivity of the experiments looking for  $\mu - e$  conversion in nuclei together with the fact that the branching ratio of the conversion grows with the heavy neutrino mass will provide us with constraints that are stronger than those from  $\mu \rightarrow e\gamma$ .

The  $\mu - e$  conversion in nuclei is induced by the flavour changing  $Z\bar{e}\mu$  current which can be parametrized as

$$J_{Z\bar{e}\mu}^\mu = \frac{g^3}{(4\pi)^2 \cos\theta_W} \bar{e}\gamma^\mu (f_L P_L + f_R P_R)\mu, \quad (31)$$

where  $g$  is the weak coupling constant and  $P_{R,L} = (1 \pm \gamma_5)/2$ . The corrections from photon exchange to  $\mu - e$  conversion are small in the case of heavy neutrinos [27] since the  $\gamma\bar{e}\mu$  amplitude does not grow with the heavy neutrino mass and we ignore them. We also neglect the contributions from the new right-handed  $Z_R$  exchange and diagrams involving  $\Delta^{++}$  since they are suppressed by the  $Z_R$  and  $\Delta^{++}$  masses, respectively. In this approximation the process receives contributions from the loop diagrams in Fig.4. We have calculated the parameters  $f_{L,R}$  of the effective vertex (31) providing the results

$$f_{L,R}(x_{L,R}^i) = \frac{\rho_{L,R}}{2} \left[ \sum_i (U_{L,R}^{ei} U_{L,R}^{i\mu\dagger}) \left( \frac{(x_{L,R}^i)^2 - 6x_{L,R}^i}{2(x_{L,R}^i - 1)} + \frac{3(x_{L,R}^i)^2 + 2x_{L,R}^i}{2(x_{L,R}^i - 1)^2} \ln x_{L,R}^i \right) \right]$$

$$\begin{aligned}
& -\frac{3x_{L,R}^i}{4(x_{L,R}^i - 1)} + \frac{(x_{L,R}^i)^3 - 2(x_{L,R}^i)^2 + 4x_{L,R}^i}{4(x_{L,R}^i - 1)^2} \ln x_{L,R}^i \Big) \\
& + \sum_{ij} (U_{L,R}^{ei} (U_{L,R}^\dagger U_{L,R})_{ij}^* U_{L,R}^{j\mu\dagger}) \left( \frac{x_{L,R}^i x_{L,R}^j}{2(x_{L,R}^i - x_{L,R}^j)} \ln \frac{x_{L,R}^j}{x_{L,R}^i} \right) \\
& + \sum_{ij} (U_{L,R}^{ei} (U_{L,R}^\dagger U_{L,R})_{ij} U_{L,R}^{j\mu\dagger}) \sqrt{x_{L,R}^i x_{L,R}^j} \left( \frac{1}{4} \right. \\
& - \frac{(x_{L,R}^i)^2 - 4x_{L,R}^i}{4(x_{L,R}^i - 1)(x_{L,R}^i - x_{L,R}^j)} \ln x_{L,R}^i \\
& \left. - \frac{(x_{L,R}^j)^2 - 4x_{L,R}^j}{4(x_{L,R}^j - 1)(x_{L,R}^j - x_{L,R}^i)} \ln x_{L,R}^j \right) \Big] , \tag{32}
\end{aligned}$$

where  $\rho_L = 1$ ,  $\rho_R = \eta/(1 - \eta)$  and summation goes over all neutrino species. We see from these expressions that the leading terms go as  $x^i \ln x^i$ . Therefore the branching ratio of  $\mu - e$  conversion in nuclei grows as the fourth power of the heavy neutrino mass and the contribution from the light neutrinos is negligible. The right-handed current contribution is suppressed by the factor of  $\rho_R$  and also by the smaller values of  $x_R$  if compared with  $x_L$ . Eq.(32) has terms both proportional to the second and fourth power of the mixing matrices which are multiplied by  $x^i$  in the leading order. In the light of our analysis of the process  $\mu \rightarrow e\gamma$ , which constrains the mixings well below unity, we conclude that only the terms containing  $|U|^2$  are relevant in our analysis. For the left-handed terms this is obvious also from the theoretical point of view since the light-heavy mixings cannot exceed  $k/v_R$ . We emphasise that, unlike in the  $\gamma\bar{e}\mu$  interaction case, if  $v_R \rightarrow \infty$  then the current (31) does not vanish and approaches a constant value proportional to the electroweak breaking scale.

The best limit on the flavour changing current  $J_{Z\bar{e}\mu}^\mu$  arises from the search for  $\mu - e$  conversion in nuclei. With the general couplings defined in Eq.(31) the total nuclear muon capture rate for nuclei with atomic number  $A \lesssim 100$  is [27]

$$B \approx \frac{G_F^2 \alpha^3}{\pi^2} m_\mu^3 p_e E_e \frac{Z_{eff}^4}{Z} |F(q)|^2 \frac{2}{\Gamma_{capture}} (f_L^2 + f_R^2) Q_W^2 , \tag{33}$$

where  $p_e$  and  $E_e$  are the electron momentum and energy,  $E_e \approx p_e \approx m_\mu$  for the present process,  $F(q)$  is the nuclear form factor, measured for example in electron scattering processes [28] and  $Z_{eff}$  has been determined in Ref.[29]. The coherent nuclear charge  $Q_W = (2Z + N)v_u + (Z + 2N)v_d$  associated with the vector current of nucleon is a function of the quark couplings  $v_{u,d}$  to the  $Z$  boson and the nucleon charge and atomic number  $A = Z + N$ . For  ${}_{22}^{48}Ti$  we use the experimental data  $\Gamma_{capture} = (2.590 \pm 0.012) \cdot 10^6 \text{ s}^{-1}$  [30],  $F(q^2 \sim m_\mu^2) = 0.54$  and  $Z_{eff} = 17.6$ . With the present experimental bound  $B(\mu - e) = 4 \cdot 10^{-12}$  [30] the resulting limit on the flavour changing couplings  $f_{L,R}^2$  is

$$(f_L^2 + f_R^2) < 2.6 \cdot 10^{-13} . \tag{34}$$

From this constraint we get bounds on the flavour changing neutrino mixings. Again, we assume that either the left- or right-handed currents dominate when bounding the

light-heavy or heavy-heavy mixings. Since only the heavy states contribution is significant we take for simplicity the masses of heavy neutrinos to be almost degenerate i.e.  $|m_i^2 - m_j^2|/(m_i^2 + m_j^2) \ll 1$ . In this case, the bounds on  $|U_L|^{e\mu}$  and  $|U_R|^{e\mu}$  obtained from  $\mu - e$  conversion in nuclei are plotted in Fig.2 and Fig.3, respectively, and denoted by  $b$ . As previously, we take  $M_{W_R} = 1.4$  TeV. Indeed, these bounds on  $|U_L|^{e\mu}$  are always more stringent than the ones from  $\mu \rightarrow e\gamma$  and become very restrictive (of the order of  $10^{-5}$ ) if the neutrino masses approach the maximum value allowed by perturbative unitarity. If we rise the mass of  $W_R$  then also  $m_i$  can be bigger and the bound on  $|U_L|^{e\mu}$  approaches zero for  $m_i \rightarrow \infty$ . Let us note that the indirect limit  $|U_L|^{e\mu} < 0.063$  is worse than those in Fig.2.

As can be seen in Fig.3, even with the present lower bound on  $M_{W_R}$  we have a lower bound of 1.1 TeV on the heavy neutrino mass in order to bound  $|U_R|^{e\mu}$  below unity. For  $m_N < 1.3$  TeV the best limit on  $|U_R|^{e\mu}$  comes from the decay  $\mu \rightarrow e\gamma$ . For heavier neutrino the  $\mu - e$  conversion limit becomes more restrictive approaching a few times  $10^{-3}$  in the case of maximally allowed neutrino masses. The right-handed current contribution to the branching ratio of  $\mu - e$  conversion is suppressed by the factor  $\rho_R$  but, due to the almost linear dependence on  $m_N$ , this suppression is partly compensated by the large values of neutrino mass. Therefore, quite large values of  $M_{W_R}$  for which we still can constrain  $|U_R|^{e\mu}$  for some neutrino masses are allowed. In this case we cannot bound the heavy-heavy neutrino mixings if  $M_{W_R}$  exceeds 200 TeV.

For completeness we have also analysed the process  $\mu \rightarrow ee^+e^-$ . In the left-right model it arises from tree level diagram of  $\Delta^{++}$  exchange and loop diagrams which involve heavy neutrinos and gauge bosons. The contribution from the tree level graph has been analysed in Ref.[31]. Assuming that  $\Delta^{++}$  is very heavy and the dominant contribution comes from the loop diagrams we have calculated the branching ratio of the process. In addition to the  $J_{Z\bar{e}\mu}^\mu$  current sub-diagrams, there are also box diagrams which give rise to the quadratic dependence with the neutrino mass. The bounds on off-diagonal neutrino mixings derived from the experimental limit  $B(\mu \rightarrow ee^+e^-) < 1.0 \cdot 10^{-12}$  [32] are complementary to the bounds from  $\mu \rightarrow e\gamma$  and  $\mu - e$  conversion in nuclei but the limits are about 2-3 times less stringent than those from the  $\mu - e$  conversion. Therefore, we do not present our lengthy expressions here. We note that this result agrees with similar ones in Refs.[16, 27].

## 5 Conclusions

We have shown that the mixings between light and heavy neutrinos in  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  models with the discrete left-right symmetry (5) can be naturally significant. Minimizing the full scalar potential of the theory and avoiding fine tunings of the parameters of the potential, we obtain the relation  $v_L v_R \simeq k_1 k_2$  for the vevs. This leads naturally to a singular neutrino mass matrix allowing to explain the smallness of the ordinary neutrino masses and keep the mixings with heavy states large at the same time. We emphasise that, unlike in the SM extended with heavy right-handed neutrinos where one has to fine tune the neutrino Yukawa couplings, the singularity of the mass matrix in the left-right models may come even if all the Yukawa couplings are equal. We have derived constraints on the intergenerational neutrino mixings from the searches for the rare flavour changing processes. These processes are sensitive not only to the mixings of

light and heavy neutrinos but, due to the right-handed charged currents present in the model, also to the off-diagonal mixings between heavy neutrinos.

For the allowed values of neutrino masses the best constraints on light-heavy mixings occur from  $\mu - e$  conversion in nuclei. If the masses of heavy neutrinos are below  $\sim 1.3$  TeV and  $M_{W_R} = 1.4$  TeV then the best constraints on heavy-heavy mixings come from the decay  $\mu \rightarrow e\gamma$ . For heavier neutrinos, up to the perturbative unitarity bound, the best limits arise from the data of  $\mu - e$  conversion in nuclei. The effective  $Ze\mu$  coupling, which induces the conversion process, shows a strong non-decoupling behaviour being quadratically dependent of the heavy neutrino mass while the amplitude of the process  $\mu \rightarrow e\gamma$  is almost constant for the neutrino masses above the electroweak scale. Therefore, the heavier the neutrinos, the more stringent are the constraints from  $\mu - e$  conversion. In our model, the perturbative unitarity requirement relates the upper bound on the mass of heavy neutrino to the mass of new right-handed gauge boson as given in Eq.(27). Consequently, rising the right-handed scale the constraints on light-heavy mixings can be more stringent than e.g. in the SM with singlet neutrinos where the heavy neutrino masses are limited to be below 1 TeV.

Since the right-handed charged currents are suppressed by the large mass of  $W_R$  also the constraints on off-diagonal heavy-heavy mixings depend on  $M_{W_R}$ . The present experiments on  $\mu \rightarrow e\gamma$  can constrain the heavy-heavy mixings only if  $M_{W_R} < 5$  TeV. In the case of  $\mu - e$  conversion, however, this suppression is much weaker and the corresponding bound is 200 TeV.

The bounds obtained from the analyses of decay  $\mu \rightarrow ee^+e^-$  are complementary to the previous ones but about two times less stringent than those from the  $\mu - e$  conversion in nuclei. This result agrees with the results in Refs.[16, 27].

The searches for flavour changing processes imply, indeed, very strong constraints on our model. The planned experiments looking for  $\mu - e$  conversion and  $\mu \rightarrow e\gamma$  with polarized muons are specially suitable for finding signals arising from the type of models we have considered.

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## Figure captions

**Fig.1.** Feynman diagrams which involve heavy neutrinos and contribute to the decay  $\mu \rightarrow e\gamma$ .

**Fig.2.** Constraints on the off-diagonal mixing  $|U|_{e\mu}$  between light and heavy neutrinos as functions of the heavy neutrino mass derived from  $\mu \rightarrow e\gamma$  and  $\mu - e$  conversion in nuclei and denoted by  $a$  and  $b$ , respectively.

**Fig.3.** Constraints on the intergenerational mixing  $|U|_{e\mu}$  among heavy neutrinos as functions of the heavy neutrino mass derived from  $\mu \rightarrow e\gamma$  and  $\mu - e$  conversion in nuclei and denoted by  $a$  and  $b$ , respectively. The mass of the right-handed gauge boson is taken to be 1.4 TeV.

**Fig.4.** Feynman diagrams which involve heavy neutrinos and contribute to the  $\mu - e$  conversion in nuclei.



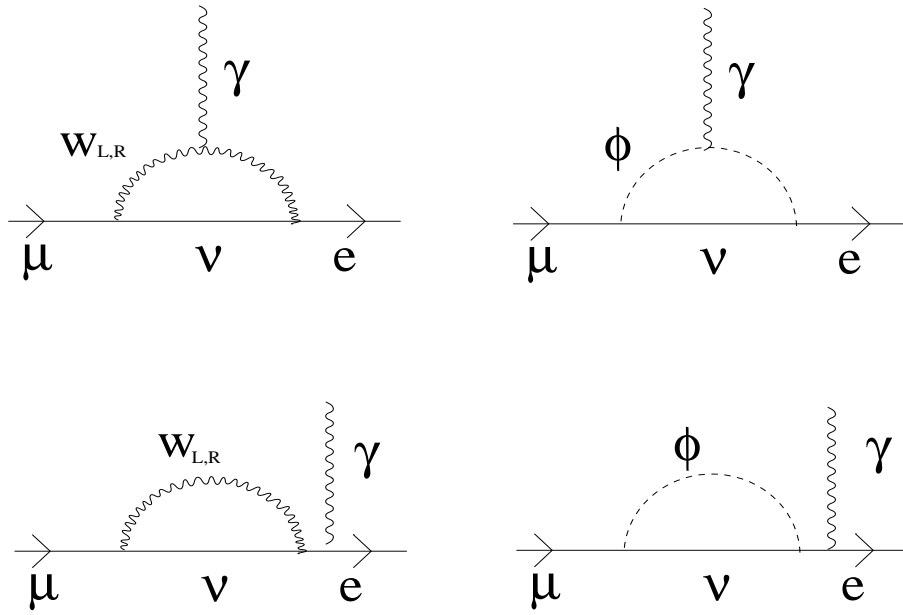


Figure 1:

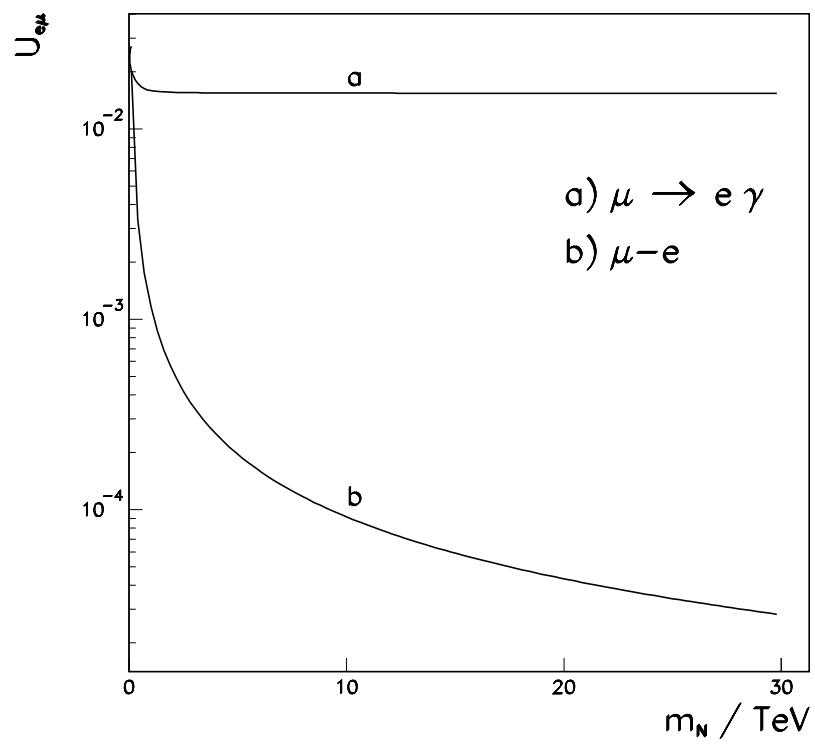


Figure 2:

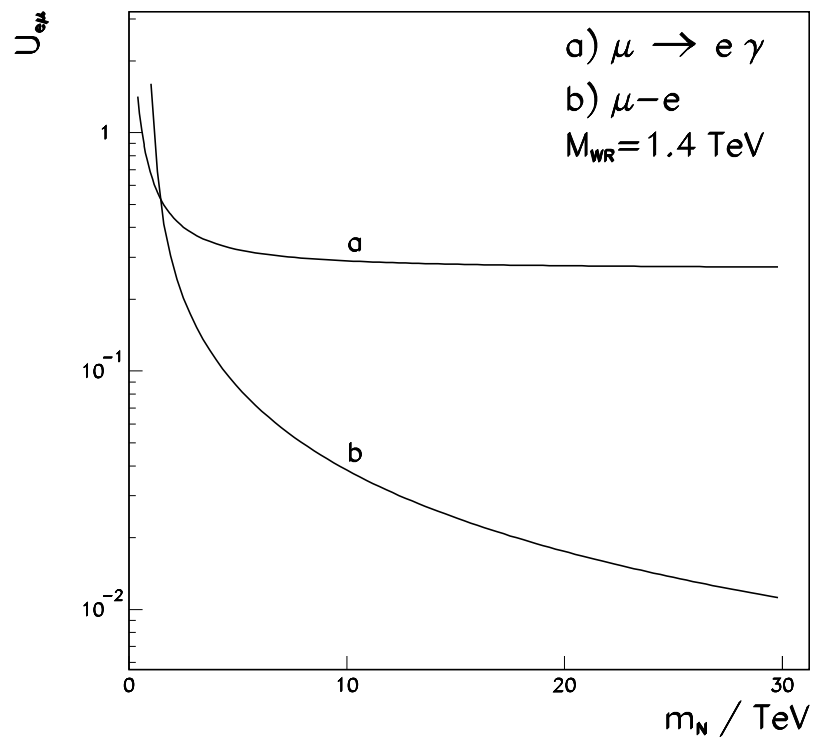


Figure 3:

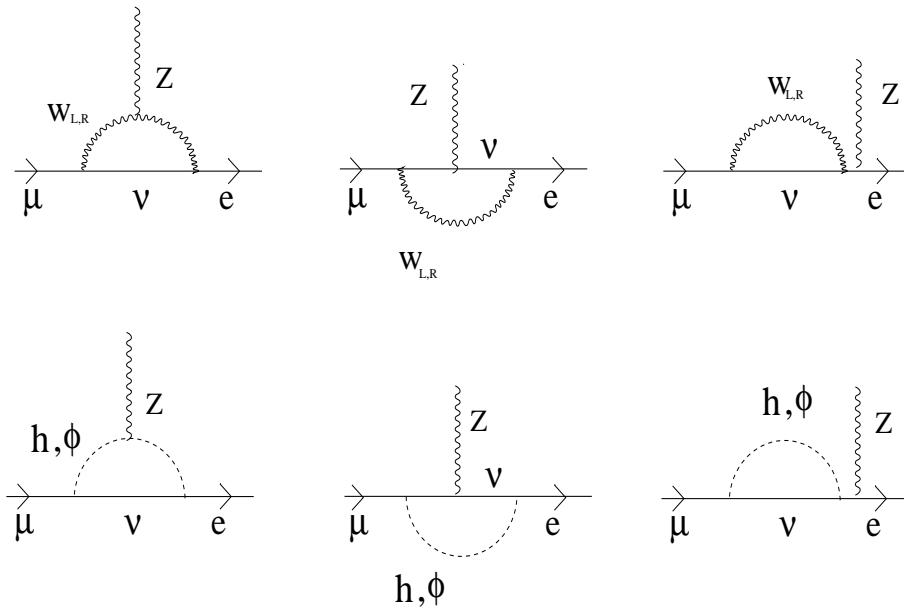


Figure 4: