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Neutrino Mixing and Masses from Long Baseline and Atmospheric Oscillation Experiments

Gabriela Barenboim* and Florian Scheck†

Institut fűr Physik - Theoretische Elementarteilchenphysik
Johannes Gutenberg-Universitǎt, D-55099 Mainz, Germany

Abstract

We argue that regardless of the outcome of future Long Baseline experiments, additional information will be needed to unambiguously decide among the different scenarios of neutrino mixing.

We use, for this purpose, a simple test of underground data: an asymmetry between downward and upward going events. Such an asymmetry, in which matter effects can be crucial, tests electron and muon neutrino data separately and can be compared with the theoretical prediction without relying on any simulation program.

Keywords: Neutrino oscillations; athmospheric neutrinos, long baseline experiments, matter effects on neutrino beams

*gabriela@thep.physik.uni-mainz.de

†Scheck@dipmza.physik.uni-mainz.de

If the experimental evidence for neutrino oscillations and for nonvanishing neutrino masses consolidates further, the primary goals of leptonic electroweak physics will be threefold: definite identification of the flavour channels into which given initial states oscillate, determination of the actual number and nature of neutral leptonic states which participate in the mixing, and measurement of the (squared) mass differences of the states involved in the mixing. Obviously, the analysis of any single experiment, or subset of experiments, in terms of only *two* families can yield no more than a parametrization of that experiment, or group of data, and no special physical significance can be attached to the numerical values of the parameters. In particular, the differences of squared masses extracted in this way may lead to erroneous conclusions when applied to the analysis of other experiments. Although there is growing evidence, by now more or less accepted by the community, that a scenario of three flavours (e, μ, τ) all of which mix strongly, is compatible with all data showing neutrino anomalies, a mixing pattern involving a fourth, sterile, neutrino ν_S cannot yet be excluded.

In the endeavour to answer these questions much hope is placed in the planned or forthcoming long baseline (LB) oscillation experiments [1]. Although these experiments will be very important, because of their much increased sensitivity as compared to short baseline oscillations, they are not sufficient to fix the parameters of the mixed neutrino sector. As we show in this note, LB experiments need complementary information for an unambiguous determination of oscillation channels and mass differences and for discriminating between the setup with three flavours and an alternative which includes a sterile neutrino state. We point out that the up-down asymmetry (zenith-antipode asymmetry) of atmospheric neutrino beams might prove to be the complementary information that is needed, provided the matter effects on the neutrinos coming from the antipode are properly taken into account. We calculate these effects by numerical integration of the neutrino's evolution equation and, depending on the assumed scenario and on the neutrino energy, find them to be important. As the mass distribution in the earth's interior is well known, and as asymmetries are independent, to a large extent, of systematic errors, a clean analysis should be possible.

The interpretation of the modulation of neutrino events with zenith angle reported by Super Kamiokande in terms of $\nu_\mu - \nu_\tau$ oscillations, obviously, would receive support by observing appearance of ν_τ in LB experiments. This could be established either directly, through observation of τ-leptons, or indirectly, via an enhancement of the neutral to charged current ratio, provided no large ν_e appearance is found. The dependence on zenith angle of the atmospheric neutrinos is confirmed by recent data from Super Kamiokande. The analysis is performed separately on sub-GeV ($p < 1.3$ GeV) and multi-GeV ($p > 1.3$ GeV) data samples. Electron-like (ν_e scattering) candidates and muon-like (ν_μ scattering) candidates are presented separately as a function of the zenith angle (see Fig. 4 of [2]). Looking at the electron-flavour data, one notes that there appears to be a small excess in the lowest $\cos\theta_z$ bin. This fact allows for the Super Kamiokande data to accommodate some $\nu_\mu \longrightarrow \nu_e$ at low Δm^2 which should be confirmed or disproved with further and more precise data by Super Kamiokande itself, or by the forthcoming LB neutrino experiments.

With three generations of neutrinos, one expects a more complicated oscillation phenomenology that includes transitions between all three pairs of flavour channels. In particular, an experiment such as Super Kamiokande which measures ν_μ disappearance, might in fact be seeing a combination of both $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_e$. In turn, if one analyzes the data in terms of a *two*-flavour scenario, say $\nu_\mu \to \nu_\tau$ to name the most popular, one will extract a mixing angle and a unique, effective $\Delta\mathcal{M}^2$ which is some convolution of the two physical differences $\Delta M^2 = m_3^2 - m_2^2$ and $\Delta m^2 = m_2^2 - m_1^2$ but by itself has no real physical meaning.

Regarding LB experiments, and keeping in mind the lesson we have learned from LSND [3], we already know that even if there is a large $\nu_\mu - \nu_\tau$ appearance signal it is not granted that the energy distribution of the appearance signal alone would allow to extract Δm^2 or, at least, to reduce significantly the available parameter space. To witness, one should keep in mind that even though LSND reports an excess of 50 events (plus a small background) they are not able to distinguish the low and high Δm^2 cases from their $\nu_\mu - \nu_e$ data.

It follows that a positive signal at any of the LB experiments cannot be taken as support for atmospheric ν_μ oscillating into ν_τ unless the corresponding ΔM^2 can be inferred from the *same* signal. The reason for this is clear: The same signal (for both appearance and disappearance) can be hiding a maximal $\nu_\mu - \nu_S$ oscillation with a small $\nu_\mu - \nu_\tau$ contamination (with $\sin^2(2\theta) \approx 10^{-3}$) or, alternatively, a three neutrino mixing scheme with a low $\nu_\mu - \nu_e$ mixing rate. It is also important to notice that sterile neutrinos can only be invoked in cases where a deficit is observed, as opposed to a signal. So, sterile neutrinos may provide the explanation for the deficits in atmospheric or solar neutrinos, but LSND must indeed be $\nu_\mu - \nu_e$. At this point it may worth stressing that although the LSND signal is somehow under suspicion because it lies very close to the region already excluded by KARMEN [4], one has to keep in mind that in their running to date, KARMEN sees no event indicative for $\nu_\mu - \nu_e$ from the expected 3 background + 1 signal events (taking LSND at face value). From this result the authors conclude that not seeing any event allows them to exclude the LSND signal at 90 % C.L. . However, had they observed the expected background events, this sensitivity would not have been sufficient to exclude the LSND result to this confidence level.

Thus, independently of the outcome of LB neutrino experiments we are urged to look for complementary information if an unambiguous interpretation of their future results is to be possible. The complementary experiment we need must be one that allows us to distinguish in a clear and unambiguous manner three possibilities in interpreting the anomaly in atmospheric neutrino fluxes: (i) the two-flavour interpretation in terms of the ν_μ and ν_τ channels, (ii) the hypothesis of the ν_μ channel oscillating into an otherwise sterile neutrino ν_S, and (iii) the scenario of three strongly mixing flavours with two differences of squared masses $\Delta M^2 = m_3^2 - m_2^2$ and $\Delta m^2 = m_2^2 - m_1^2$. In the absence of such a cross check, none of these hypotheses can be excluded or taken for granted.

We argue, following a proposal by Flanagan, Learned and Pakvasa [5], that a quantity as simple as a directional asymmetry of atmospheric neutrino fluxes might prove to be just what we need for the purpose of distinguishing the alternatives described above. We feel encouraged to explore this possibility by the recent sizeable upgrading of Super Kamiokande which already yields better statistics than the one achieved in the previous, entire Kamiokande project.

The up-down asymmetry in e- or μ-flavour events, $A^{(e)}$ or $A^{(\mu)}$, respectively, is defined as

$$A^{(f)} = \frac{D^{(f)} - U^{(f)}}{D^{(f)} + U^{(f)}}\ , \quad f = \mu, e\,, \tag{1}$$

where D is the number of downward going events, produced in the atmosphere in the zenith, for either electron or muon neutrinos, while U is the number of upward going events, stemming from the antipode and having passed through the center of the earth. For the case of μ-flavour, D and U are

$$D^{(\mu)}(U^{(\mu)}) = N^0_\mu \left[P^{D(U)}_{\mu\mu} + \frac{1}{r} P^{D(U)}_{\mu e} \right] \tag{2}$$

and similarly for electron flavour. Here $N^0_{\mu(e)}$ is the initial flux of muon (electron) neutrinos and $r = N^0_\mu / N^0_e$ is the expected ratio of fluxes without oscillations. This ratio varies somewhat with energy and zenith angle [6] but typical values are $\sim 1/3$ for Multi-GeV and $\sim 1/1.6$ for sub-GeV fluxes.

The important point to note is that matter effects on neutrino oscillations due to the charged current interaction with electrons in the earth can play a considerable role. As was shown long ago by Mikheyev and Smirnov [7], under certain conditions the presence of matter can lead to a resonant amplification of the neutrino transitions, even if these transitions are strongly suppressed in vacuum.

Matter effects on the oscillation of neutrino beams which cross the earth were studied previously in the two generation case in [8], and in the three generation case in [9]. Here we study these effects for oscillations involving the three flavour neutrinos, ν_e, ν_μ and ν_τ, by solving the neutrino evolution equation. As the matter distribution of the earth neither is homogeneous nor can be modeled by means of a simple analytic function, we cannot rely on any simplifying assumptions and, therefore, we obtain solutions of the evolution equation by numerical integration. Note that with minor modifications our results can be extended to oscillations of anti-neutrinos, $\overline{\nu_e}$, $\overline{\nu_\mu}$ and $\overline{\nu_\tau}$.

In the presence of matter the neutrino wave function $\Psi^{(\nu)} = \{ \mid \nu_e \rangle, \mid \nu_\mu \rangle, \mid \nu_\tau \rangle \}^T$ obeys the evolution equation

$$i \frac{d}{dl} \Psi^{(\nu)} = \frac{1}{2E} M(l) \Psi^{(\nu)} \tag{3}$$

with

$$M(l) = U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a(l) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

Here m_i are the neutrino mass eigenvalues, and $U_{\alpha i}$ is the 3x3 mixing matrix in vacuum. The mixing matrix relates the weak interaction states α and the mass eigenstates i in the leptonic sector, viz.

$$\mid \nu_\alpha \rangle = \sum_{j=1}^{3} U_{\alpha i} \mid n_i(p, m_i) \rangle \tag{5}$$

with $\mid n_i(p,m_i)\rangle$ denoting the state vector of the mass eigenstate with momentum p and mass m_i in vacuum. It will prove to be convenient to use the following parametrization of the mixing matrix U in vacuum,

$$U = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{-i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix} \tag{6}$$

The background density of electrons, $N_e(l)$ induces a mass-like interaction term for the electron neutrino

$$a(l) = 2\sqrt{2}\, G_F\, E\, N_e(l) \tag{7}$$

with E the neutrino energy.

For a concrete application to the case at stake, these equations must be solved numerically. Note, however, that one can write down a formal solution making use of the fact that the mass matrix in matter, $M(l)$, can be diagonalized at each position l by means of a mixing matrix $U_M(l)$. The formal solution reads

$$\mathcal{A}\left(n_i\left(l=0\right) \to n_j\left(l=L\right)\right) = P\ \exp\left[-i\int_0^L dl \left(\frac{1}{2E}\widetilde{M}(l) - iU_M^\dagger(l)\frac{d}{dl}U_M(l)\right)\right] \tag{8}$$

where $\widetilde{M}$ is a diagonal matrix containing the eigenvalues of $M(l)$ while P denotes path ordering. Here $\mathcal{A}\left(n_i\left(l=0\right) \to n_j\left(l=L\right)\right)$ is the transition amplitude for the ith mass eigenstate created at $l=0$ to turn into the jth mass eigenstate detected at $l=L$. The probability of one flavour eigenstate to propagate to another is then given by

$$\mathcal{P}\left(\nu_\alpha\left(l=0\right) \to \nu_\beta\left(l=L\right)\right) = |\langle \nu_\beta \mid U_M(L)\mathcal{A}\left(n_i\left(l=0\right) \to n_j\left(l=L\right)\right) U_M^\dagger(0) \mid \nu_\alpha\rangle|^2 \tag{9}$$

In the case of oscillations involving anti-neutrinos the amplitude $\mathcal{A}$ of finding the anti-neutrino β at $l=L$ is obtained from an evolution equation similar to Eq.(3). The corresponding probabilities are obtained from Eq.(9) by making the formal change $U \to U^*$ and $a(l) \to -a(l)$. Note, however, that when there is a MSW resonance in the particle sector due to level crossing, there is no resonance in the antiparticle sector.

In our analysis of the effects of earth on the neutrino beam from the antipode, we have used the density distribution as given by what is called the preliminary reference earth model [10], $\rho_E(r)$, r denoting the distance from the center of the earth. According to this model, $\rho_E(r)$ increases from an initial value of 1.02 gr/cm^3 in the earth surface to its maximum of 13.1 gr/cm^3 in the center of the earth. There are basically nine regions in which $\rho_E(r)$ varies continuously. The discontinuities of $\rho_E(r)$ at the borders of these regions are described by step functions. The most pronounced of these jumps (all of which are rather small) takes place at the border between the mantle and the core where $\rho_E(r)$ changes by 4.3 gr/cm^3. This happens at a distance $r=$ 3480 km from the center of the earth.

In our calculation we assume that the ratio of the electron density to the nucleon density is everywhere the same and is equal to 1/2, $N_E = N_N/2$. N_N is the nucleon density and is given by $N_N = \rho_E N_A$ with N_A Avogadro's number.

The result of this tedious calculation can be summarized as follows: For upgoing neutrinos originating from the antipode, matter effects affect substantially neutrino transitions if

$$10^3 \ \text{GeV/eV}^2 \leq \quad \frac{E}{\Delta m^2} \ , \ \frac{E}{\Delta M^2} \quad \leq 10^5 \ \text{GeV/eV}^2 \tag{10}$$

or equivalently

$$10^{-14} \ \text{eV} \leq \quad \frac{\Delta m^2}{E} \ , \ \frac{\Delta M^2}{E} \quad \leq 10^{-12} \ \text{eV} \ . \tag{11}$$

(As $\Delta m^2 \ll \Delta M^2$ we have taken $m_3^2 - m_1^2 \simeq \Delta M^2$). The magnitude of the interval (10) or (11) is determined by the range of values of the electron number density that we find in earth when crossing it along a diameter. Remember that matter effects are important only if the interaction energy squared (7)

$$a(l) \approx 7.7 \cdot 10^{-5} \text{eV}^2 \left(\frac{\rho}{\text{gr/cm}^3}\right) \left(\frac{E}{\text{GeV}}\right) \tag{12}$$

is comparable to or larger than either Δm^2 or ΔM^2, cf. Eqs. (13) and (14) below. The enhancement typically shows up in the dependence of the probability of a given transition on the neutrino energy as an irregular sequence of two or three well pronounced local maxima with different heights.

For $E/\Delta m^2 (M^2) \gg 10^5 \text{GeV/eV}^2$ or, equivalently, $\Delta m^2 (M^2)/E \ll 10^{-14} \text{eV}$, the resonant densities $|m_i^2 - m_j^2| \cos(2\theta)/(2\sqrt{2} G_F E)$ are much smaller than the electron density in the earth which, as mentioned before, varies from 1 to 13 gr/cm^3. In this case the charged-current interaction with electrons in the earth dominates and oscillations are suppressed. For $E/\Delta m^2 (M^2) \ll 10^3 \text{GeV/eV}^2$ or, equivalently, $\Delta m^2 (M^2)/E \gg 10^{-12} \text{eV}$, the resonant densities are much larger than the electron density and neutrinos oscillate like in vacuum.

When a beam of electron neutrinos crosses the earth there can potentially be two resonances, in the $\nu_e \to \nu_\mu$ and the $\nu_e \to \nu_\tau$ channels. In the case of muon neutrinos there can only be one resonance, the one in the $\nu_\mu \to \nu_e$ channel. In the parametrization (6) the resonance densities are given by

$$N^{(res)}_{\nu_e \to \nu_\mu} = \frac{\Delta m^2 \cos 2\theta_1}{2\sqrt{2} E G_F} = N^{(res)}_{\nu_\mu \to \nu_e} \ , \tag{13}$$

$$N^{(res)}_{\nu_e \to \nu_\tau} = \frac{\Delta M^2 \cos 2\theta_3}{2\sqrt{2} E G_F} \ . \tag{14}$$

These resonances would show up wonderfully if the vacuum mixing angles, $\sin\theta_1 \ll 1$ and $\sin\theta_3 \ll 1$ were small, [11], provided they are sufficiently separated [12], i.e. provided

$$\Delta M^2 - \Delta m^2 \gg \left(\Delta M^2 \sin\theta_3 + \Delta m^2 \sin\theta_2\right) \ .$$

It can be shown that the resonances take place at densities at which the differences of local mass eigenvalues $M_2(l) - M_1(l)$ and $M_3(l) - M_1(l)$ are minimal. One advantage of the parametrization (6) we have chosen for the mixing matrix is that $M_1(l)$, $M_2(l)$ and

$M_3(l)$ do not depend on the CP violating phase δ and therefore the very existence of these resonances as well as the electron number densities at which they occur are independent of δ. Indeed, the eigenvalues of $M(l)$ are given by

$$M_1^2(l) = m_1^2 + \frac{A}{3} - \frac{1}{3}\sqrt{A^2 - 3B}\left(\cos\omega + \sqrt{3}\sin\omega\right) \tag{15}$$

$$M_2^2(l) = m_1^2 + \frac{A}{3} - \frac{1}{3}\sqrt{A^2 - 3B}\left(\cos\omega - \sqrt{3}\sin\omega\right) \tag{16}$$

$$M_3^2(l) = m_1^2 + \frac{A}{3} + \frac{2}{3}\sqrt{A^2 - 3B}\cos\omega \tag{17}$$

where

$$A = 2\Delta m^2 + \Delta M^2 + a(l) \tag{18}$$

$$B = \left(\Delta M^2 + \Delta m^2\right)\ \Delta m^2 + a(l)\left[\left(\Delta M^2 + \Delta m^2\right)\ \cos^2\theta_3 + \Delta m^2\left(\cos^2\theta_3\ \cos^2\theta_1 + \sin^2\theta_3\right)\right] \tag{19}$$

$$C = a(l)\ \left(\Delta M^2 + \Delta m^2\right)\ \Delta m^2\ \cos^2\theta_3\ \cos^2\theta_1 \tag{20}$$

$$\omega\ = \frac{1}{3}\arccos\frac{2A^3 - 9AB + 27C}{2\left(A^2 - 3B\right)^{\frac{3}{2}}}\ . \tag{21}$$

Note, however, that the transition probabilities do depend in a nontrivial way on δ.

We now turn to the analysis of the up-down asymmetry for both electron and muon neutrinos. We calculate this asymmetry for the three scenarios listed in the introduction, i.e. the two-flavour interpretation, oscillation into a sterile ν_S channel, and strong mixing of three flavours with two rather different mass differences, all of which would give similar and hardly distinguishable signals in LB experiments.

The two-flavour mixing is the simplest possible case, although it is bound to explain the atmospheric neutrino anomaly only, while discretely keeping silent about the solar neutrino deficit and the LSND result. Here, the electron neutrino flux is unaffected and therefore, the corresponding electron asymmetry $A^{(e)}$ is essentially zero. In order to calculate the muon asymmetry, we have to take the limit $\sin\theta_1 \to 0$ and $\sin\theta_3 \to 0$ in the three neutrino case outlined before. Although $A^{(\mu)}$ is nonzero, matter effects are negligible in this case.

In the scenario of three strongly mixing flavours [13] the larger of the two differences $\Delta M^2 \equiv m_3^2 - m_2^2 = .3$ eV^2 is found to account for both, the atmospheric anomaly at low energy as well as the observations of LSND. The second mass difference $\Delta m^2 \equiv m_2^2 - m_1^2 \simeq 10^{-3} - 10^{-4}$ eV^2 explains the solar neutrino deficit but it is also called to play an important role in the atmospheric neutrino anomaly for upward going events. In this case, as of the two would-be resonances the one corresponding to $\nu_e \to \nu_\mu$ is reached in the core of the earth, *matter effects are dominant*.

Finally, another way to accommodate all observations is to introduce at least one additional neutrino state ν_S which mixes with the active neutrinos but otherwise is sterile.

In Ref [14] the four possible types of neutrino spectra were considered. It was shown there that only two out of all possibilities are compatible with the existing bounds. In these two schemes, the four neutrino masses are grouped in two pairs of closeby masses, the two groups being separated by a gap of the order of 1 eV. Thus, it is assumed that $\Delta m^2_{41} = 1$ eV^2 (we again use the abbreviation $\Delta m^2_{ij} = m^2_i - m^2_j$). In one scheme, say (A), the choices are $\Delta m^2_{21} \simeq 10^{-3}$ eV^2 (in view of explaining the atmospheric neutrino anomaly) and $\Delta m^2_{43} \simeq 10^{-10}$ or 10^{-5} eV^2 (needed for the suppression of the solar electron neutrinos). Here the numbers refer to the vacuum oscillation solution and the MSW solution, respectively. In the other scheme, say (B), the roles of Δm^2_{21} and Δm^2_{43} are reversed. A detailed analysis of these models as well as the bounds following from them can be found in [15].

What is important to notice is that the effective Hamiltonian of the interaction of neutrinos with matter in the case of models containing active and sterile neutrinos, has an additional neutral-current term, apart from the usual charged-current term, which is proportional to the background density of neutrons, and therefore has half the charged-current term strength.

Cutting this long story short, in both models (A) and (B), oscillations between the electron neutrino and the sterile neutrino are not affected by matter effects. In the case of model (A) the $\nu_e \to \nu_\mu$ oscillations are in the appropriate range for resonant effects to occur while crossing the earth and, therefore, electron neutrino beams will oscillate and get depleted substantially. The resulting electron asymmetry $A^{(e)}$ would be strikingly different from the three-flavour case: its property of having the opposite sign in the two scenarios would make it a unique and easy criterion to distinguish these alternatives. Furthermore, in these models with a sterile neutrino $A^{(e)}$ and $A^{(\mu)}$ will differ only slightly from one another; they are basically similar and always have the same sign.

Figure 1 shows our results for the electron neutrino asymmetry $A^{(e)}$, Fig. 2 gives the corresponding results for the muon neutrino asymmetry $A^{(\mu)}$, as a function of the neutrino energy and for the three models described above. It is clear that given sufficient statistics the three scenarios can be clearly distinguished by both, the energy dependence and the relative signs of the asymmetries $A^{(\mu)}$ and $A^{(e)}$. In particular, it is noteworthy that in the case of the electron asymmetry, for which the two-flavour oscillation model predicts a vanishing result, both observation or non-observation of such an effect would be relevant. Finally, although we have restricted our analysis to these three models, it will be easy to calculate the expected asymmetries also for other scenarios or for a different choice of parameters.

In conclusion, whatever the outcome of future Long Baseline neutrino experiments will be, complementary information will be crucial for an unambiguous interpretation of their results. A complemementary experiment is needed in order to distinguish in a clean manner between the $\nu_\mu \to \nu_\tau$, $\nu_\mu \to \nu_S$ and the three-flavour alternatives in interpreting the atmospheric neutrino anomaly. For this purpose, we proposed to use the up-down asymmetries of electron and muon neutrinos that are detected in underground detectors and that we calculated by numerical integration of the evolution equation in earth. These asymmetries have the virtue that they do not rely on any simulations and that they require only identification of charged particles and measurement of their energies and directions. Contrary to the zenith angle dependence, the asymmetries test electron and muon neutrino data separately and do not require knowledge of the fluxes. Given sufficient statistics, the asymmetries $A^{(e)}$ and $A^{(\mu)}$, when added to the results of forthcoming long baseline experiments, will help to distinguish the different scenarios that are being considered and will allow for an unambiguous determination of the mixing matrix and the differences of squared neutrino masses.

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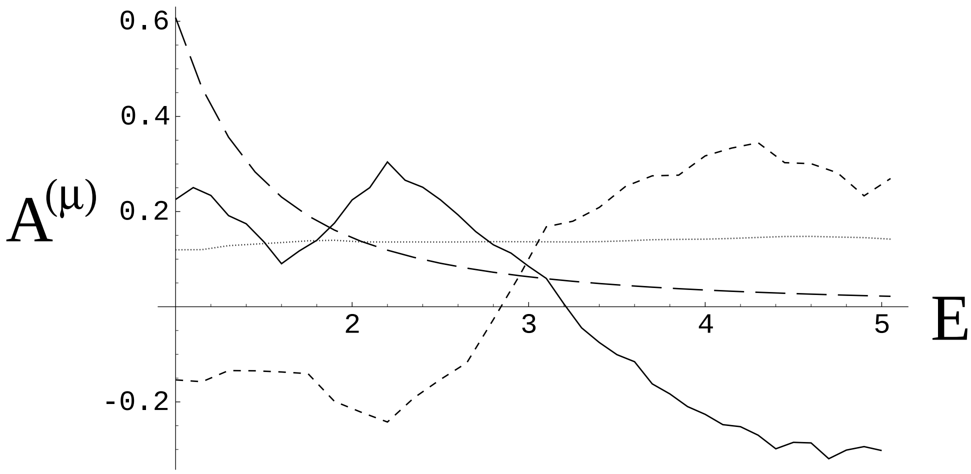

Figure 1: Muon neutrino asymmetry, $A^{(\mu)}$, versus energy (GeV) for the two flavour mixing scheme (long-dashed line), the three strongly mixed flavours scenario (solid line) and the three active plus one sterile neutrino model (A) (dotted line) and (B) (short-dashed line)

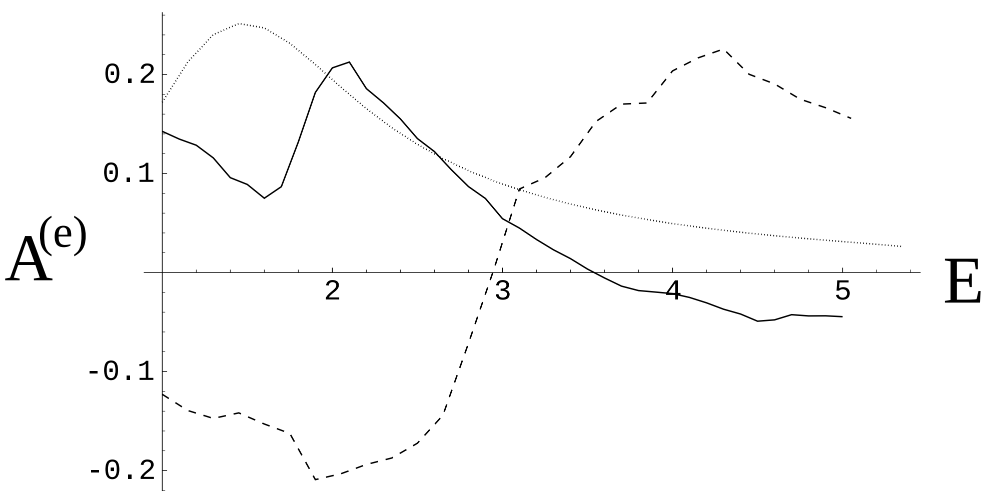

Figure 2: Electron neutrino asymmetry, $A^{(e)}$, versus energy (GeV) for the same models as in Fig. 1.