

# Oscillations, Neutrino Masses and Scales of New Physics

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## Abstract

We show that all the available experimental information involving neutrinos can be accounted for within the framework of already existing models where neutrinos have zero mass at tree level, but obtain a small Dirac mass by radiative corrections.

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The results of the Super Kamiokande collaboration convincingly proved that an initial beam of atmospheric muon neutrinos oscillates into other neutrino species. As a nontrivial mass sector for neutrinos is definitely outside of what the minimal standard model can predict, at tree level or through radiative corrections, these results provide a first clear evidence for the existence of new physics beyond the minimal standard model. In this note we address the question whether the two rather different mass differences which are needed to explain all neutrino anomalies simultaneously, the Super Kamiokande results, the solar neutrino deficit, and the LSND data, can be understood in a natural manner within mild and plausible extensions of the minimal model. We point out that this is indeed possible with implications which, though simple, have far reaching and testable consequences.

As is well known, in the minimal version of the standard model, the left-handed neutrino is the only member of each family which does not have a right-handed partner. The model makes a clear distinction between left and right in the classification of leptons and, therefore, in their interactions with the gauge particles. Indeed, with left-chiral fields in doublets, and right-chiral fields in singlets with respect to  $SU(2)_L$ , charged current interactions show maximal violation of parity and charge conjugation. Regarding neutrino mass terms there are then two choices: either right-chiral neutrinos are part of the particle spectrum of the model, with all  $SU(2)_L$  and  $U(1)_Y$  quantum numbers vanishing so that they do not interact with the gauge particles, and they appear in the mass matrix due to physics beyond the minimal model. In this case it is natural to expect neutrino masses to be of Dirac type and to find a close relationship between mass matrices and family mixing.

The other possibility is that right-chiral neutrino fields do not appear in the model at all. In this case only Majorana masses are possible. For example, as shown in [1], effects of quantum gravity could generate lepton number violating, non-renormalizable operators of the form (using the usual short-hand notation)

$$\lambda_L \frac{LLHH}{M_{\text{Planck}}} + \text{h.c.}$$

which in turn would be induced by a Majorana mass term of the kind  $\mu \nu_L^T C^{-1} \nu_L$ . Here  $M_{\text{Planck}} \approx 1.22 \cdot 10^{19}$  GeV denotes the Planck mass, while  $\lambda_L$  is an effective dimensionless coupling constant that one would expect to be of order one. Such a mass would lead to squared mass differences of the order of  $10^{-9}$  eV<sup>2</sup>, by far too small to explain any of the existing asymmetries, even with “anomalous” fine tuning of  $\lambda_L$ . Of course, one might ask whether the mass scale in the denominator could be chosen differently, for example, by taking it to be the grand unification scale  $M_{\text{GUT}} \approx 2 \cdot 10^{16}$  GeV. Unfortunately, this not only would yield masses which are still too low, but also would call by itself for new physics beyond the standard model. (What other reason would there be to introduce a new scale?) Thus, it appears that the observed neutrino anomalies cannot be accommodated within the minimal standard model in a natural manner, even with inclusion of effects from quantum gravity, and, therefore, the need arises to extend the model such that neutrino physics can be understood in a framework which already contains the other fermions, scalars and gauge bosons (the known ones and, hopefully, the ones to be discovered soon).

In this situation and with neutrino oscillations being positively established the challenge for theorists is to explain why the masses of neutrinos are small, as compared to the ones of their charged partners, and to find out whether or not there is violation of total lepton number

$$L := L_e + L_\mu + L_\tau . \quad (1)$$

As is well known Dirac masses are compatible with  $L$ -conservation while Majorana masses imply  $\Delta L = \pm 2$ .

Many extensions of the standard model assume neutrinos to be Majorana particles and understand the smallness of their mass in terms of the see-saw mechanism. The underlying idea is quite simple: In a basis of Majorana fields<sup>1</sup> the mass matrix has the general form

$$M = \begin{pmatrix} \mu_M^{(l)} & m_D \\ m_D & \mu_M^{(h)} \end{pmatrix} . \quad (2)$$

Here, when dealing with one family,  $\mu_M^{(l)}$  and  $\mu_M^{(h)}$  are Majorana mass terms which may be chosen real and, say, positive,  $m_D$  is a Dirac term and may still be relatively complex,  $m_D = \mu_D e^{i\phi}$ . In the case of three families these quantities are corresponding block matrices. It seems natural to assume  $\mu_D$  to be of the order of the charged partner's mass,  $\mu_D \approx m(l)$ , and, from the smallness of physical neutrino masses, to assume the lower Majorana term to fulfill  $\mu_M^{(l)} \ll \mu_D$ . In contrast to the former, the second Majorana term is expected to be large as compared to the charged partner's mass,  $\mu_M^{(h)} \gg \mu_D$ , and to represent the signal of a new mass scale. The eigenstates of  $M$ , eq. (2), are Majorana fields, the corresponding eigenvalues being approximately [3]

$$m_1 \approx \frac{\mu_D^2}{\mu_M^{(h)}} , \quad m_2 \approx \mu_M^{(h)} .$$

The first of these is small and proportional to the *square* of the charged partner's mass, while the second is large and proportional to the new mass scale<sup>2</sup>. Since the heavier neutrino state is absent in the standard model and has not been observed experimentally, it is plausible to assume that the new scale is associated with either a local [2] or global [4]  $B - L$  symmetry. As this mass must be in the range of a TeV or higher, a natural explanation for the smallness of neutrino masses is achieved. However, the proportionality to the squared charged lepton masses,  $m(\nu_i) \propto m^2(l_i)$ , leads to difficulties in trying to fit all neutrino anomalies simultaneously. Typically, these models contain two very different mass differences. Therefore, if one wishes to explain the atmospheric and LSND neutrino data with the same mass difference (as should be done if the three anomalies are to be

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<sup>1</sup>We remind the reader that in studying spinor representations of the Lorentz group, for fermions which do not carry any additively conserved quantum number, one naturally finds Majorana spinors  $\Psi_M^{(i)}$  rather than Dirac spinors. Only when two Majorana fields happen to have the same mass can one form Dirac fields such as  $\Psi_D = \Psi_M^{(1)} + i\Psi_M^{(2)}$ . From this point of view one might say that Majorana fields are more fundamental than Dirac fields.

<sup>2</sup>We note in passing that if the two Majorana terms were equal and if  $\phi$  were an odd multiple of  $\pi/2$  the eigenvalues of (2) would be equal. In this case a Dirac state is formed.

understood within three flavours, i.e. with two mass differences only), the remaining mass difference is too small to account for the vacuum oscillation solution of the solar neutrino data while being too large for the MSW effect to be operative.

Let us then turn to the possibility that neutrinos are Dirac particles, not Majorana, and that the leptonic mass matrices conserve total lepton number<sup>3</sup>. Indeed, an attractive possibility seems to us that neutrinos initially are massless in a tree level Lagrangian, and that their small (Dirac) masses are due entirely to quantum corrections. Such ideas were discussed first in [6] and were fully developed later, when they were studied in the context of extensions of the minimal model, most notably left-right symmetric models [7] where this possibility finds a rather economical and elegant realization. It is the purpose of this work to embed the empirical solutions of the neutrino anomalies found in [8] in these models and to analyze the predictions this gives rise to. We outline somewhat schematically the rich set of phenomenological implications which follow from this and which are interesting in their own right.

Going backwards in time, one realizes that, though neutrinos were considered to be Dirac particles in the original version of the left-right symmetric models [9], the smallness of their mass was not understood. Later on, a version of these models was developed [10] wherein neutrino masses vanish at tree level but arise at one-loop or two-loop level, thus rendering them naturally much smaller than the corresponding charged fermion masses. An interesting feature of these models that makes them particularly predictive, is that neutrino masses scale *linearly* with the charged fermion masses. As they are inherently different from the class of left right symmetric models that lead to Majorana masses we give here a brief sketch but refer the interested reader to the original literature for a more detailed discussion.

The electroweak sector of the models we consider is based on left-right symmetry with gauge group  $SU(2)_L \times SU(2)_R \times U(1)$ . The assignment of the known quarks and leptons of every generation is as follows,

$$\begin{aligned}
 Q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L & : \quad \left(2, 1, \frac{1}{3}\right) \quad , \quad Q_R \equiv \begin{pmatrix} u \\ d \end{pmatrix}_R & : \quad \left(1, 2, \frac{1}{3}\right) \\
 \Psi_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L & : \quad (2, 1, -1) \quad , \quad \Psi_R \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_R & : \quad (1, 2, -1)
 \end{aligned}
 \tag{3}$$

At this point we depart from the minimal left-right symmetric model, the difference being that our models also assume heavy singlet quarks and leptons with vector-like couplings and make use of the see-saw mechanism for quarks and charged leptons instead of neutrinos. The idea of partial see-saw mechanism for down type quarks and charged leptons was first discussed in [7]. Later it was proposed [11] that the entire fermion spectrum be generated via the see-saw mechanism, by postulating the following heavy

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<sup>3</sup>Possible variations of the see-saw mechanism were proposed so as to realize this possibility also in a natural manner [5] by postulating the existence of a new scale. This time, however, the scale is not connected with any obvious symmetry and therefore, such models are less appealing than the original see-saw scheme.

fermions in the left-right symmetric model, in addition to the known fermions: two singlet quarks denoted P and N, and a singlet lepton, denoted E,

$$P : \left(1, 1, \frac{4}{3}\right) , \quad N : \left(1, 1, -\frac{2}{3}\right) , \quad E : (1, 1, -2) , \quad (4)$$

An advantage of these models is their relatively simple Higgs structure.

For the sake of simplicity we focus on three models which differ by their Higgs content. In the first of them, say (A), only one pair of Higgs doublets and a parity odd real singlet scalar (needed to generate the left-right symmetry) are needed,

$$\chi_L : (2, 1, 1) , \quad \chi_R : (1, 2, 1) , \quad \sigma : (1, 1, 0) , \quad (5)$$

In this model, neutrino masses vanish at tree level and arise only at the two-loop level. The relevant two-loop graph is shown in Fig 1(a), it leads to small and finite Dirac masses given by

$$m(\nu_i) = \left(\frac{\alpha}{4\pi \sin^2 \theta_w}\right)^2 \left(\frac{m_t m_b}{M_{W_L}^2}\right) \left(\frac{m_E^2}{M_{W_R}^2}\right) m(l_i) . \quad (6)$$

Here  $m(l_i)$  is the mass of the charged lepton of the  $i$ -th generation,  $m_E$  is the mass of the singlet lepton.

In the second scenario, say (B), the following set of Higgs bosons is chosen in order to break the gauge symmetry down to  $U(1)_{em}$  and to give the fermions their masses

$$\phi_2 : (2, 1, 0)$$

$$\chi_L : (2, 1, 1) , \quad \chi_R : (1, 2, 1) \quad (7)$$

$$n_a : (1, 1, 0) , \quad a = 1, 2 .$$

The one-loop graph which contributes to Dirac neutrino masses in this case, is shown in Fig 1(b) and can be estimated to yield

$$m(\nu_i) = \left(g \frac{m(l_i)}{2M_{W_L}}\right) \left(g \frac{m_t}{2M_{W_L}}\right) \frac{m_b}{16\pi^2} \left(\frac{v_R}{\sigma}\right)^3 \quad (8)$$

where  $\sigma$  is the vacuum expectation value (vev) of the singlets  $n_a$ , and where  $v_R$  is the usual vev of the neutral member of the right-handed doublet. Notice that the ratio of scales in this case is somehow the inverse of the previous case: In eq. (6) the right-handed scale appears in the denominator while the vector-like scale  $m_E$  is in the numerator.

As a third model, say (C), let us introduce the following Higgs multiplets, in addition to those of model (B),

$$\phi_l : (2, 2, 0) \quad (9)$$

$$n_3 : (1, 1, 0)$$

In this model finite neutrino masses arise from the graph of Fig 1(c). Since in this case  $W_L - W_R$  mixing is finite (in the previous cases the  $W_L - W_R$  mixing vanishes at tree level and is induced at one-loop level only), we find

$$m(\nu_i) = \left( \frac{\alpha}{4\pi \sin^2 \theta_w} \right)^2 \left( \frac{m_t m_b}{M_{W_R}^2} \right) m(l_i). \quad (10)$$

We now turn to the phenomenological neutrino spectrum that is required to fit all anomalies simultaneously, i.e, the deficits observed in the solar neutrino flux and in the atmospheric neutrino data, as well as the appearance signal of electron anti-neutrinos in the initial  $\bar{\nu}_\mu$  beam of the LSND experiment. Obviously, in a mixing scheme involving three flavours one has only two mass differences. In the models just described these are given by the mass difference of the corresponding charged leptons divided by the right-hand scale (model (C)), or multiplied by a ratio of scales (models (A) and (B)). Therefore, once one of these mass differences is fixed, the other is predicted. Furthermore, in fixing any one of the two mass differences we obtain a prediction for the scale of new physics, the two quantities being directly related by eq. (6), eq. (8), or eq. (10). The new scale can be probed through the predictions it implies, either in the lepton sector (magnetic moment of the neutrinos, radiative decays, etc.) or in the quark sector (e.g. sizeable CP violating rate asymmetries), or else its existence can be disproved, for example, by an erroneous  $K_L - K_S$  mass difference.

Following our previous work [8] we assume that at low energies there are just three neutrino flavours and, hence, only two differences of squared masses. In order to account for both, the atmospheric neutrino anomaly at low energy as well as the observation of LSND we fix the larger of these to be

$$\Delta M^2 = m_3^2 - m_2^2 = .3 \text{ eV}^2. \quad (11)$$

In the case of model (C), eq. (10), this means that

$$m^2(\nu_\tau) - m^2(\nu_\mu) = \left( \frac{\alpha}{4\pi \sin^2 \theta_w} \right)^4 \left( \frac{m_t m_b}{M_{W_R}^2} \right)^2 (m^2(\tau) - m^2(\mu)) \approx .3 \text{ eV}^2. \quad (12)$$

Inserting the known quantities in this expression we deduce the value

$$M_{W_R} \approx 4 \text{ TeV} \quad (13)$$

for the right-hand scale. When we calculate the second mass difference from this estimate we find

$$\Delta m^2 = m_2^2 - m_1^2 \approx m^2(\nu_\mu) - m^2(\nu_e) \approx 10^{-3} \text{ eV}^2, \quad (14)$$

i.e. exactly the value that we needed to explain the solar neutrino deficit [8]. Notice that  $\Delta m^2$  is an output of our model and that no additional input was used to fix its value. Such a mass difference is also called to play an important role in the atmospheric neutrino anomaly for upward going events [12].

The value (13) for the right hand scale which is implied by our model is high enough to evade the stringent bounds from the  $K_L - K_S$  mass difference. On the other hand, it leads

to strong predictions which should be observable in ongoing and future experiments. For example, at the forthcoming B-factories the CP violating rate asymmetries should show clear deviations from the values expected within the minimal standard model.

Regarding the other two scenarios, (A) and (B), only the ratio of the different scales involved can be predicted and we cannot obtain as definite predictions as those we have obtained before, in model (C). However, we still can vary these ratios within reasonable limits and can explore the range of admissible masses.

Let us begin with model (A). The experimental lower bound on the mass of the heavy charged lepton from direct searches,  $M_E > 42.8$  GeV, automatically places a lower bound on the right-handed gauge boson mass

$$M_{W_R} > 2.2 \text{ TeV} . \quad (15)$$

This bound ensures that the new contributions to flavour changing neutral current processes such as  $K_L \rightarrow \mu \bar{\mu}$  and the  $K_L - K_S$  mass difference are well within present experimental limits. Even more, the range of masses of the heavy charged lepton required in order to have a non-decoupled right-handed gauge boson is within what is being tested now and what will be tested in the near future in accelerator experiments. As can be seen from Fig 2, only masses in the range  $m_E = 43 - 420$  GeV can give phenomenologically interesting right-handed gauge bosons, of masses lighter than 20 TeV. Therefore if model (A) has some truth in it this will soon be seen either by an observation of this relatively light charged lepton  $E$  (and the associated heavy vector-like quarks), or by indirect effects of the right-handed gauge bosons.

In model (B), the quantity  $\sigma$  is again associated with the mass of the heavy charged lepton, which is  $M = f\sigma$  with  $f$  a coupling constant presumably of order one. As the smallness of the masses of the down type quarks is explained in this model by a see-saw type mechanism

$$m_{qd} \propto \frac{v_L v_R}{\sigma} , \quad (16)$$

$v_R$  is expected to be much smaller than  $\sigma$ . This leads to a pattern of masses which is just the inverse of the previous one, i.e. now the heavy lepton masses are much larger than the mass of the right-handed gauge boson. For the same range of phenomenologically acceptable masses of the right-handed gauge bosons we have used before,  $1.6 \text{ TeV} \leq M_{W_R} \leq 20 \text{ TeV}$ , the mass of the heavy charged lepton is now in an interval spanning from 195 TeV to 2440 TeV, cf. Fig. 3. Thus, we have a model where the vector-like quarks are almost completely decoupled. Therefore, unlike the previous case, this model resembles the original left-right model in the sense that the low energy phenomenology associated with the enlargement of the fermion and Higgs sectors is reduced to radiatively induced Dirac masses of neutrinos only.

In summary we show that it is possible to explain all neutrino observations within the framework of previously developed models where neutrinos have zero masses at tree level, but obtain small Dirac masses by radiative corrections. Phenomenological patterns of neutrino masses consistent with the observed anomalies were proposed previously, their sole purpose being to explain the anomalies, but, to the best of our knowledge, they were not part of any complete model. We do not know of any model accounting for

all available experimental information involving neutrinos where the mass differences are somehow justified by an underlying principle. As far as we know, our models provide the first attempt to do so. The choice to describe the light neutrinos as Dirac particles, as opposed to Majorana particles, is largely dictated by lepton number conservation, which seems to be conserved in nature. By the same token, this choice puts all fermions on equal footing as far as their additive quantum numbers and their masses are concerned.

To be specific we made use of three simple extensions, known since long, of the left-right symmetric model in which neutrinos are Dirac particles, to account for all experimental information revealed by the observed neutrino oscillations. The masses of our neutrinos arise either at the one-loop or the two-loop level, thus explaining why they are small as compared to the masses of their charged partners. In some variations, the scale for new physics that has to be introduced, (i.e. the  $W_R$  and the vector-like fermion mass) could be light enough to be probed in the next generation of experiments.

We emphasize again the importance of avoiding any ad-hoc assumptions. Roughly speaking, instead of introducing model assumptions designed to deal with the neutrino anomalies we proceed in the opposite way. By fixing one mass difference, within a specific model and not meant to explain any neutrino deficit or appearance, the remaining difference as well as the new scales are obtained as definite predictions. This observation illustrates that, despite the danger of a phenomenological disaster (such as an unacceptably low  $W_R$  mass, or flavour-changing neutral currents that are too large), there are models in which the problems raised by neutrinos can be solved in a consistent way and without ad-hoc assumptions. At the same time such models have very interesting phenomenological implications to which we hope to return in the near future.

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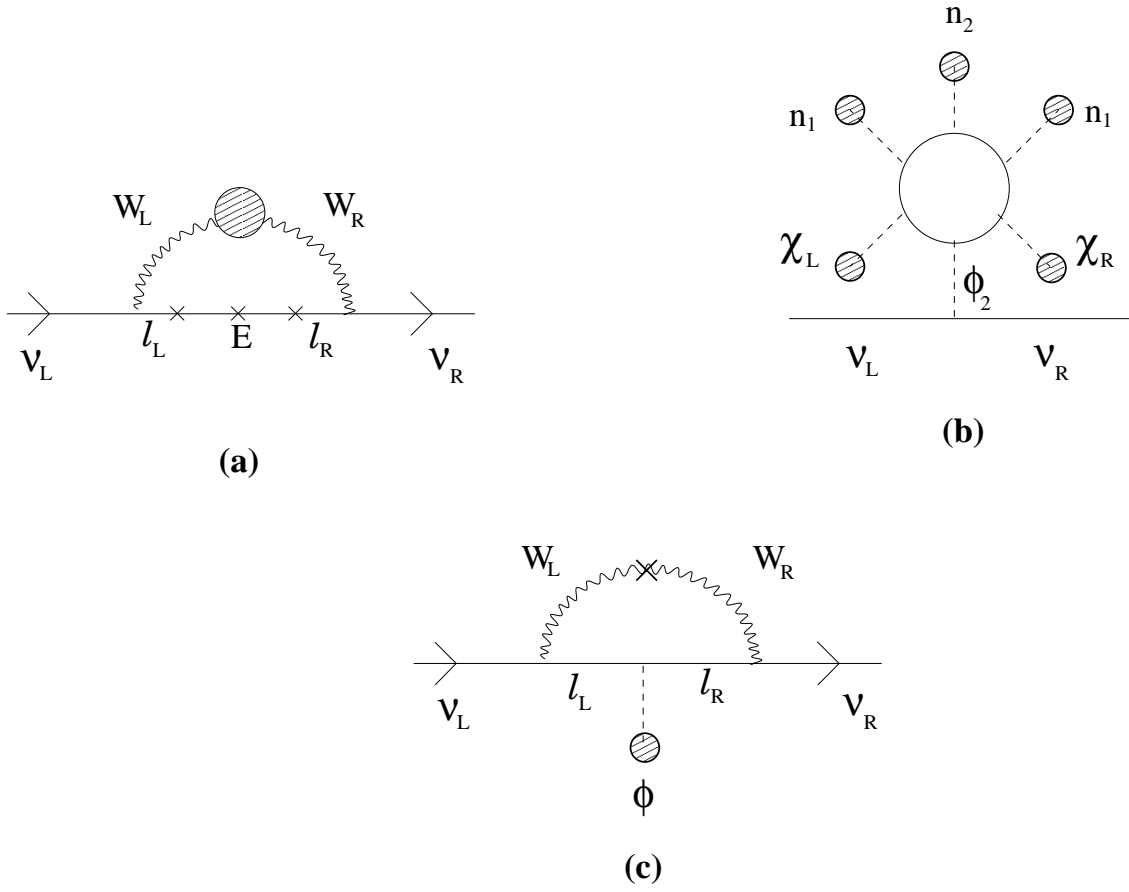


Figure 1: Radiatively generated Dirac masses for the different models

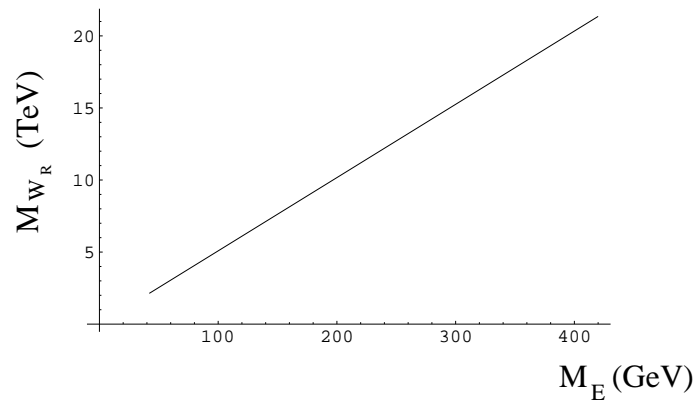


Figure 2: Right handed gauge boson mass as a function of the heavy charged lepton mass

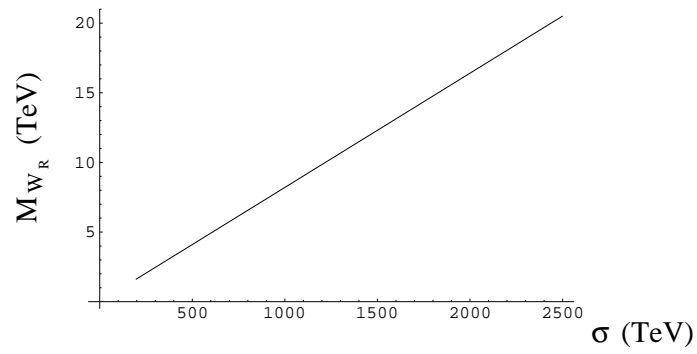


Figure 3: Right handed gauge boson mass as a function of the singlet's vev