Tau neutrinos from muon storage rings

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Abstract

Charged tau leptons emerging in a long baseline experiment with a muon storage ring and a far-away detector will positively establish neutrino oscillations. We study the conversion of ν_μ ($\overline{\nu}_\mu$) and of $\overline{\nu}_e$ (ν_e) to ν_τ or $\overline{\nu}_\tau$ for neutrinos from a 20 GeV muon storage ring, within the strong mixing scheme and on the basis of the squared mass differences which are compatible with all reported neutrino anomalies, including the LSND data. In contrast to other solutions which ignore the Los Alamos anomaly, we find charged tau production rates which should be measurable in a realistic set up. As a consequence, determining the complete mass spectrum of neutrinos as well as all three mixing angles seems within reach. Matter effects are discussed thoroughly but are found to be small in this situation.

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1 Introduction

Neutrino beams from a muon storage ring provide an ideal tool for the next round of experiments, aiming at establishing quantitatively oscillations and, possibly, CP-violation in the leptonic sector [1, 2]. The main reason for this is that both composition and spectrum of such beams are perfectly known and, in addition, the beam energy may be tuned. For instance, the neutrino beams from a storage ring such as the ones described in [1, 2] would contain an equal number of muon neutrinos and electron antineutrinos, or, depending on the sign of the parent muon, an equal number of muon antineutrinos and electron neutrinos, without any contamination from other neutrino species.

The physics potential of a storage ring and a far-away detector, with regard to oscillations, was studied, e.g., in [3, 4], and, with regard to CP-violation, in [5, 6]. In this paper we address the question of appearance of tau neutrinos in the processes

$$\nu_\mu \longrightarrow \nu_\tau \qquad \text{and} \qquad \nu_e \longrightarrow \nu_\tau\,,$$

due to oscillations. We calculate the number of produced τ^+ and τ^-, taking into account matter effects on the neutrino beam on its way from the source to the detector, within the strong mixing scheme of three flavours only that we had proposed earlier [7]. The latter assumption is receiving growing support by the fact that new data from KAMIOKANDE seem to disfavour the existence of a fourth, sterile, neutrino, while still showing the characteristic modulation of events as a function of zenith angle. As a consequence of the strong mixing which helps to populate strongly the ν_τ and $\overline{\nu}_\tau$ final channels from both the $\overline{\nu}_e$ (ν_e) and the ν_μ ($\overline{\nu}_\mu$) initial states, we find $\tau^\pm$ production rates one to two orders of magnitude larger than within a model ignoring the LSND data.

In the next section we summarize some relevant formulae and collect the parameters extracted earlier from the analysis of all neutrino anomalies. We then describe and discuss matter effects in the Earth's crust and calculate the event rates for the $\tau^\pm$ appearance channels. The article ends with our results as well as some conclusions.

2 Formulae and parameters

In a scenario with three families of leptons, the mixing between three neutrino species is described by a conventional Cabibbo-Kobayashi-Maskawa matrix U relating flavour states to mass eigenstates. If neutrinos are Dirac particles, U has the form

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}e^{i\delta} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{1}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. In the case they are of Majorana type, extra phases appear but their effects on neutrino observables are of order m_ν/E_ν [8] and are generally negligible.

The oscillation probabilities in vacuum take the form [9]

$$P(\nu_i \to \nu_j) = P_{CP-even}(\nu_i \to \nu_j) + P_{CP-odd}(\nu_i \to \nu_j) \tag{2}$$

$$P(\bar{\nu}_i \to \bar{\nu}_j) = P_{CP-even}(\nu_i \to \nu_j) - P_{CP-odd}(\nu_i \to \nu_j), \tag{3}$$

where

$$P_{CP-even}(\nu_i \to \nu_j) = \delta_{ij} - 4\mathrm{Re}J_{12}^{ji}\sin^2\Delta_{12} - 4\mathrm{Re}J_{23}^{ji}\sin^2\Delta_{23} - 4\mathrm{Re}J_{31}^{ji}\sin^2\Delta_{31}, \tag{4}$$

$$P_{CP-odd}(\nu_i \to \nu_j) = -8\sigma_{ij}J\sin\Delta_{12}\sin\Delta_{23}\sin\Delta_{31},$$

with J the Jarlskog invariant and

$$J_{kh}^{ij} \equiv U_{ik}U_{kj}^\dagger U_{jh}U_{hi}^\dagger, \quad \Delta_{ij} \equiv \Delta m_{ij}^2 L/4E, \quad \sigma_{ij} \equiv \sum_k \varepsilon_{ijk} \tag{5}$$

In order to account for all reported neutrino anomalies one needs only two squared mass differences. The LSND result can be understood in terms the mass difference

$$\Delta M^2 := m_3^2 - m_2^2 \approx 0.3 \text{ eV}^2 \tag{6}$$

The second mass difference is tuned so as explain the observed deficit of electron neutrinos coming from the sun,

$$10^{-4} \text{ eV}^2 \leq \Delta m^2 \leq 10^{-3} \tag{7}$$

with $m_2^2 - m_1^2 \equiv \Delta m^2$, while the atmospheric neutrino data depend on both differences and are found in agreement with the prediction. Given the squared mass differences, the mixing angles are easily found to be [7] (solution I)

$$\theta_{12} \approx 35.5^0, \quad \theta_{23} \approx 27.3^0, \quad \theta_{13} \approx 13.1^0. \tag{8}$$

This solution is favoured by the existing oscillation data and, thus, implies simultaneous and strong mixing of all three flavours.

3 Matter effects

Of all neutrino species, only electron neutrinos can scatter elastically in the forward direction off electrons in matter, via charged current interactions. When the electron neutrinos oscillate into either muon or tau neutrinos, this introduces an additional term in the diagonal element of the neutrino flavour evolution matrix corresponding to $\nu_e \rightarrow \nu_e$ [10]. It is useful to define an effective mass term which stems from elastic scattering due to charged weak currents,

$$a = 2\sqrt{2} G_F n_e E = 7.7 \cdot 10^{-5} \mathrm{eV}^2 \left(\frac{\rho}{\mathrm{gr/cm}^3} \right) \left(\frac{E_\nu}{\mathrm{GeV}} \right) \tag{9}$$

where n_e is the electron number density in matter of density ρ and E is the neutrino energy.

As is well-known matter effects become important [11] only when a is comparable to, or larger than, the quantity $\Delta_{m_{ij}^2} = m_i^2 - m_j^2$ for some mass difference and neutrino energy. Given our neutrino mass spectrum and taking into account that for the Earth's crust $\rho \simeq 3\mathrm{gr/cm}^3$, we are far from being in a range where matter effects would be dominant and could not be neglected. This would be the case, for example, in the case of $\nu_\mu \rightarrow \nu_e$ where large CP asymmetries are expected but will be masked by matter effects. In the case that we discuss here, matter effects are not expected to play a significant role but we will include them anyway. (In the case of electron neutrinos oscillating into tau neutrinos, and with our parameters, they represent at most a 2 % effect.)

In order to exhibit the essential mechanisms we assume for a moment the two lightest neutrinos to be degenerate in mass. Indeed, this limiting case is close to the realistic situation where $\Delta M^2 \gg \Delta m^2$, cf. eqs. (6), (7). In this case, the transition probabilities in vacuum for the "terrestrial" experiments depend only on three variables, i.e., θ_{23}, θ_{13} and ΔM^2, as follows,

$$
\begin{aligned}
P\left(\nu_e \rightarrow \nu_\tau\right) &\approx 4 U_{13}^2 U_{33}^2 \sin^2\left(\frac{\Delta M^2 L}{4E} \right) \\
&= \cos^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2\left(\frac{\Delta M^2 L}{4E} \right), \tag{10} \\
P\left(\nu_\mu \rightarrow \nu_\tau\right) &\approx 4 U_{23}^2 U_{33}^2 \sin^2\left(\frac{\Delta M^2 L}{4E} \right) \\
&= \cos^4(\theta_{13}) \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta M^2 L}{4E} \right). \tag{11}
\end{aligned}
$$

Note that in this limit of setting Δm^2 equal to zero the probabilities are independent of the angle θ_{12}.

When matter effects are included the above formulae are still valid provided one makes the replacements,

$$\Delta M^2 \longrightarrow \Delta M^2 + \frac{\left(3s_{13}^2 - 1\right)a}{2} \tag{12}$$

$$s_{13}^2 \longrightarrow s_{13}^2 \left[1 - \frac{2a\left(s_{13}^2 - 1\right)}{\Delta M^2}\right], \tag{13}$$

with s_{23} unchanged. Here we have assumed that $\Delta M^2 \gg a, \Delta m^2$, a hierarchy which is respected by the mass spectrum we chose.

From these formulae it is clear that the probability for a muon neutrino to oscillate into a tau neutrino in matter will not be different from its vacuum value. The reason for this is clear: the interpretation of the modulation of neutrino events with zenith angle reported by SuperKamiokande in terms of a muon neutrino oscillating into a tau neutrino either requires a nearly maximal θ_{23}, in two-neutrino mixing schemes, or still a sizeable one, in three-neutrino mixing schemes. (As noted above, the data strongly disfavour schemes with three active and one sterile neutrinos.) Furthermore, reactor experiments set a strict upper bound on θ_{13}, giving $\sin^2 \theta_{13} \leq .005$. Thus, the factor $\cos^4(\theta_{13})$ in eq. (11), to a good approximation, is equal to 1. Likewise the factor $\sin^2(2\theta_{23})$ in the same equation is also close to 1. Therefore, $\nu_\mu \to \nu_\tau$ oscillations are favoured and the influence of matter effects on them is negligible.

Introducing the terms containing Δm^2 does not modify this simple picture because they are suppressed by the huge gap between the two squared mass differences. It is important to notice that although we have assumed degeneracy between the two lightest neutrinos, this degeneracy is broken by matter effects which introduce an "effective" mass difference of the order $a(s_{13}^2 - 1)/2$.

In the case of electron neutrinos oscillating into tau neutrinos, however, the story is different. In this case, the factor $\sin^2(2\theta_{13})$ in eq. (10) is a suppressing one and could compensate, at least partially, the gap in the squared mass differences. With our parameters, the contributions proportional to Δm^2 do not really compete with the one proportional to ΔM^2 but they account for a sizeable correction. In our results presented below we use the full expression for the transition probabilities.

At this point it is important to stress, that unlike our scheme, in schemes where only two anomalies are taken into account, disregarding the Los Alamos result, the typical mass differences are $\Delta M^2 \approx 10^{-3}$ eV^2 and $\Delta m^2 \approx 10^{-10}$ eV^2, or 10^{-6} eV^2 to 10^{-5} eV^2, depending on whether in the vacuum solution the small-angle MSW solution or the large-angle MSW solution to the solar neutrino problem is chosen. Under these assumptions, ΔM^2 is just in the appropriate range to exhibit sizeable matter effects so that these cannot be neglected.

The conclusion so far is clear: If MiniBooNe confirms the LSND evidence for oscillations, then ΔM^2 is too large to cause a significant modification of the oscillation

probabilities due to matter effects. As we will see below, the prospects of discovering both $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$ oscillations then are promising indeed. However, if KamLAND obtains a positive result in disappearance of electron neutrinos, corroborating the large-angle MSW solution for solar oscillations, or if KARMEN definitely and conclusively excludes the LSND result, then ΔM^2 will be in a range to produce a significant modification of the oscillation probability due to matter. In this case, the rates for tau appearance being lower by more than an order of magnitude, an experiment would be more difficult and would probably require a more intense neutrino source than that assumed here.

4   Event rates

The calculation of event rates for tau lepton (anti tau lepton) production from electron neutrinos (anti muon neutrinos) as a result of oscillations is straightforward. The total number of events in the two channels is given by

$$n_{\tau^-} = N_{\mu^+} \; N_{kT} \; \frac{10^9 \; N_A \; E_\mu^3}{m_\mu^2 \; \pi \; L^2} \; \int_{E_{\min}}^{E_\mu} \; g_{\nu_e} \; \epsilon \; \sigma_{\nu_\tau}^{\mathrm{CC}} \; P\left(\nu_e \to \nu_\tau\right) \; dE \tag{14}$$

$$n_{\tau^+} = N_{\mu^+} \; N_{kT} \; \frac{10^9 \; N_A \; E_\mu^3}{m_\mu^2 \; \pi \; L^2} \; \int_{E_{\min}}^{E_\mu} \; g_{\bar{\nu}_\mu} \; \epsilon \; \sigma_{\bar{\nu}_\tau}^{\mathrm{CC}} \; P\left(\bar{\nu}_\mu \to \bar{\nu}_\tau\right) \; dE \tag{15}$$

where N_{μ^+} is the number of positive muon decays, σ_{ν_τ} is the charged current cross section per nucleon , $P(\nu_e \to \nu_\tau)$ and $P\left(\bar{\nu}_\mu \to \bar{\nu}_\tau\right)$ are the oscillation probabilities for neutrinos traveling inside the Earth taking into account matter effects. N_{kT} is the size of the detector in kilotons, $10^9 \; N_A$ is the number of nucleons in a kiloton, E_μ is the energy of the muons in the ring and $E_{\min} = 5$ GeV is a lower cut on the neutrino energies that we assume, for the sake of an example. For our numerical calculation, the energy spectrum (normalized to 1) of the neutrinos is taken to be

$$g_{\nu_e} = 12x^2(1-x) \tag{16}$$
$$g_{\bar{\nu}_\mu} = 2x^2(3-2x) \tag{17}$$

where $x = E_\nu/E_\mu$ is the fractional neutrino energy. It is straightforward to obtain the corresponding expression in the case of either electron anti neutrinos or muon neutrinos.

Regarding the cross section, our calculation is based on the renormalization group improved parton model, focusing on the inclusive process $\nu_\tau(\bar{\nu}_\tau) + N \longrightarrow \tau^-(\tau^+) +$ anything. Note that in this case, unlike the charged current cross section for the muon, the terms proportional to the charged lepton mass are not negligible. The differential cross section is [12],

$$\frac{d\sigma}{dx\,dy} = \frac{G_F^2 M E_\nu}{\pi} \left(\frac{M_W^2}{M_W^2 + Q^2}\right)^2 \left\{ F_2 \left(1 - y - \frac{Mxy}{2E}\right) + F_1 xy^2 \pm xF_3 \left(y - \frac{y^2}{2}\right) \right.$$

$$+ \frac{m_\tau^2}{ME} \left(-F_2 \left(\frac{M}{4E} + \frac{1}{2x} \right) + \frac{F_1 y}{2} \mp \frac{F_3 y}{4} \right) \bigg\}$$ (18)

where we use the Bjorken scaling variables $x = Q^2/2M\nu$ and $y = \nu/E_\nu$. Here $-Q^2$ is the invariant momentum transfer between the incident neutrino and outgoing tau, $\nu = E_\nu - E_\tau$ is the energy loss in the laboratory frame while M and M_W are the nucleon and W-gauge boson masses respectively.

The Callan-Gross relation $2xF_1 = F_2$ simplifies the above equation further. It is then dependent on only two form factors, F_2 and F_3, which are given in terms of the parton distributions by

$$F_2 = \sum_i x \left(q_i + \bar{q}_i \right)$$ (19)

$$F_3 = \sum_i \left(q_i - \bar{q}_i \right)$$ (20)

For our calculations we have used the MRST99 [13] parton distribution. Although there does not seem to be a general consensus about the best way of combining the quasi-elastic resonance and the deep inelastic scattering cross sections at fixed energy to form the total cross section, a number of different approaches were proposed in the literature or were used in practice [14]. We adopt the simplest of them, and include only the deep inelastic part. In any case, the error one makes in the prediction due to uncertainties in the neutrino spectrum and by assuming a constant matter distribution in the Earth's crust is larger than the uncertainties in the cross section itself.

5   Results and conclusions

We consider a 20 GeV muon storage ring at CERN with a neutrino beam from one of its straight sections that points to the Gran Sasso underground laboratory, at a distance of 732 km. This is about the same distance as the one between FNAL and the Soudan mine.

In order to mimic a nearly realistic experimental situation we set a lower cutoff in energy at 5 GeV and we assume a detection efficiency of 30%. Also, for two cases, production of τ^+ from the process $\overline{\nu}_\mu \to \overline{\nu}_\tau$ and production of τ^- from the process $\nu_e \to \nu_\tau$, we show our results in bins of 3 GeV so as to get a feeling for the number of events to be expected with our parameters.

Fig. 1 shows the predicted spectrum for tau lepton appearance coming from $\nu_e \to \nu_\tau$, while Fig. 2 shows the predicted spectrum of anti taus coming from $\overline{\nu}_\mu \to \overline{\nu}_\tau$. For the sake of comparison in Figs. 3 and 4 we plot the same observables for the choice of parameters reported in Ref [3] which disregards the LSND data (note the different

scales in the ordinates!). Results similar to these were also reported in [15]. Fig. 5, finally, shows our result for τ^- and τ^+ production, grouped in bins of 3 GeV each.

Provided the difference ΔM^2 is in the range required by the LSND data, it is clear from Figs. 1, 2, and 5 that an experiment using a 20 GeV muon storage ring and a baseline of about 730 km between source and detector (corresponding to the distance CERN-Gran Sasso, or FNAL to the Soudan mine) would have a fair chance to see the appearance of charged tau leptons. The average oscillation probability could be measured with a statistical precision better than 3% [16] and ΔM^2 could be determined with a precision of about 3%. Note that since matter effects are small in this case, there is no additional uncertainty on this mass difference arising from an incomplete knowledge of the oscillation mode.

All in all, with the choice of parameters that we obtained by a simultaneous explanation of all neutrino anomalies, a 10 kT detector some 732 km downstream would probe $\cos^2(\theta_{23})\sin^2(2\theta_{13})$ to very low values. When combined with measurements of $\nu_e \to \nu_\mu$ oscillations (see for example [17]) and with further results from atmospheric neutrinos, a precise determination of the complete mass spectrum of neutrinos and of all mixing angles seems possible.

Acknowledgements

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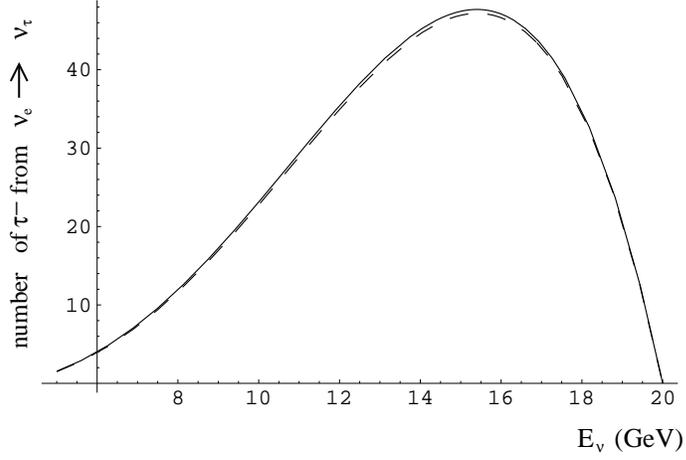
Figure 1: Number of tau leptons detected in one year's time in a 732 km baseline and assuming 30% detecting efficiency. The solid line correspond to setting the CP violating phase $\delta = 0$ while the dashed line correspond to $\delta = \pi/2$
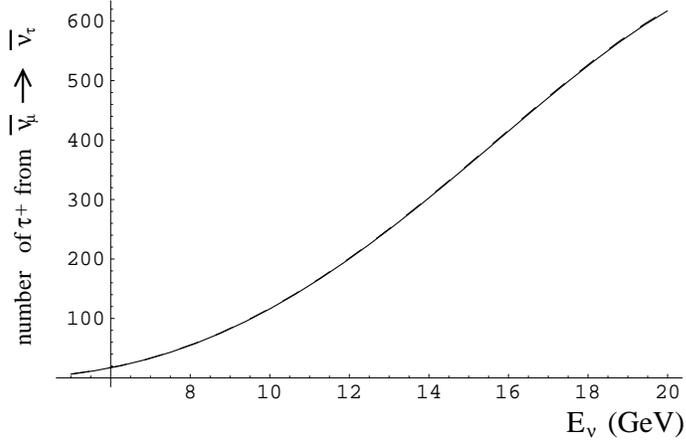
Figure 2: Number of antitau leptons detected in one year's time in a 732 km baseline and assuming 30% detecting efficiency. The solid and the dashed curves (defined as in Fig.1) are indistinguishable.
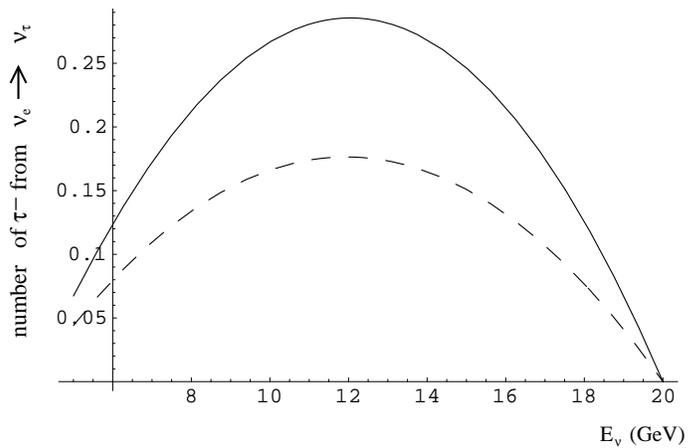
Figure 3: Number of tau leptons detected in one year's time in a 732 km baseline and assuming 30% detecting efficiency for the parameters of ref. [3]. Dashed and solid lines as before.
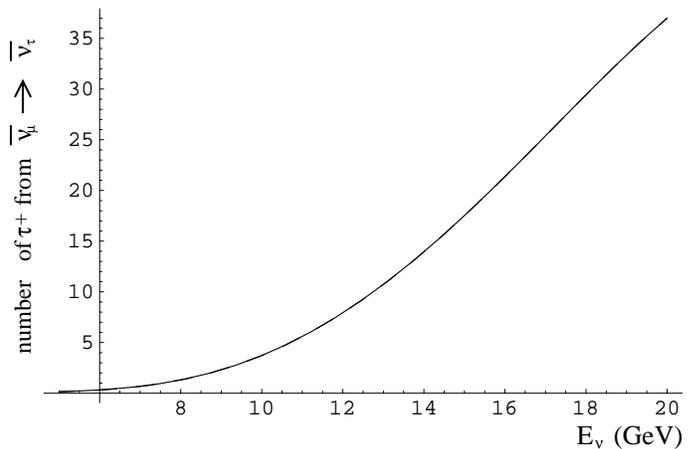
Figure 4: Number of tau leptons detected in one year's time in a 732 km baseline and assuming 30% detecting efficiency for the parameters of ref. [3]. The solid and the dashed curves are indistinguishable.
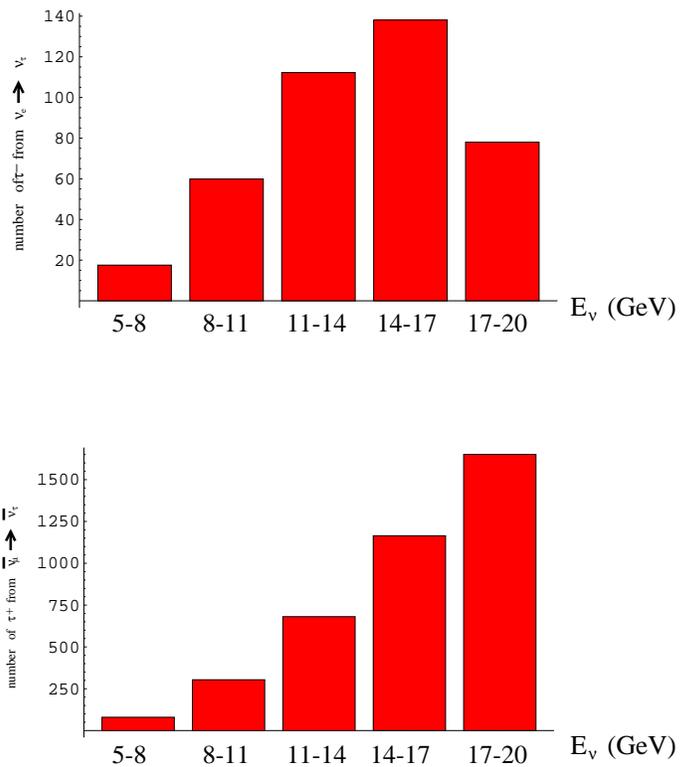
Figure 5: Number of tau (above) and antitau (below) leptons detected in one year's time in a 732 km baseline and assuming 30% detecting efficiency grouped in bins of 3 GeV.