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Flavour Violation in SUSY SU(5) GUT at Large tanβ

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Abstract

We study flavour violation in the minimal SUSY SU(5) GUT assuming all the third generation Yukawa couplings to be due to the renormalizable physics above GUT scale. At large $\tan\beta$, as suggested by Yukawa unification in SU(5), sizable flavour violation in the left (right) slepton (down squark) sector is induced due to renormalization effects of down type Yukawa couplings between GUT and Planck scales in addition to the flavour violation in the right slepton sector. The new flavour physics contribution to $K-\bar{K}$, $B-\bar{B}$ mixing is small but might be of phenomenological interest in the case of $b\to s\gamma$. The sign of the latter contribution is the same as the sign of the dominant chargino contribution, thus making the constraints on SUSY scale coming from $b\to s\gamma$ somewhat more restrictive. The most important feature of the considered scenario is the large rate of lepton flavour violation. Given the present experimental constraints, the $\mu\to e\gamma$ and $\mu-e$ conversion branching ratios are above the sensitivity of the planned experiments unless the SUSY scale is pushed above one TeV.

I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) [1,2], one of the best motivated extensions of the Standard Model (SM), has triggered intensive research both in theoretical as well as in experimental physics. Despite of non-trivial constraints on its parameter space coming from collider and low energy experiments the MSSM has so far successfully passed all the tests of precision physics.

In the most general case MSSM contains more than one hundred free parameters. Some of them may give rise to unobserved phenomena like proton decay, large electron dipole moments, large flavour violation, etc. To explain the absence of such phenomena additional assumptions are needed to explain the pattern of supersymmetry (SUSY) breaking parameters. An attempt towards that direction is to regard the MSSM as a low energy remnant of some grand unified theory (GUT) such as SU(5), which predicts unification of gauge couplings. Since the unification naturally occurs in the MSSM, one might try to apply a similar organizing principle to the soft SUSY breaking masses. Another prediction of GUTs is the unification of some or all Yukawa couplings of each generation. To achieve successful Yukawa unification [3–7] the ratio between the vacuum expectation values (vevs) of the two MSSM Higgs doublets, $\tan\beta$, is found to be in the range $\tan\beta \sim 30 - 50$ for tau-bottom unification and $\tan\beta \sim 50$ for tau-bottom-top unification [6]. However, large mixings in the lepton sector may somewhat change this picture [7].

Stringent tests of SUSY GUTs are offered by flavour violation experiments. The flavour violation in SUSY theories is not suppressed by the high scale where the SUSY breaking parameters are generated but rather by the mass scale of these terms themselves [8] which is believed to be of order TeV. In the scheme of the minimal flavour violation of Barbieri and Hall [9] the SUSY breaking parameters are generated above the GUT scale $M_{GUT} \sim 2\times 10^{16}$ GeV, at the reduced Planck scale $M_P \sim 2.4 \times 10^{18}$ GeV by gravitational interactions and are therefore universal at M_P. The renormalization group (RG) evolution below M_P induces non-universalities of the soft terms at the GUT scale where the SU(5) gauge group breaks down to the usual MSSM gauge symmetry group. Therefore the flavour mixings present in the Yukawa couplings at GUT scale cannot be rotated away and should be reflected at low energies in the squark and slepton mass matrices. The phenomenology of this scenario in SU(5) SUSY GUT has been studied with the assumption that the top quark Yukawa coupling is the only sizable one [10] which implies flavour violation in the right-handed slepton and left-handed down squark sectors. Rates of the lepton flavour violating (LFV) processes $\mu \to e\gamma$, $\mu - e$ conversion in nuclei and $\tau \to \mu\gamma$ are found to be large for some parts of the SUSY parameter space but due to the cancellations between gaugino and higgsino loops, they almost vanish for some other parts of the parameter space [11].

Another set of SUSY theories predicting large rates of flavour violation are the ones with right-handed massive neutrinos [7,12]. These models are motivated by the Super-Kamiokande results which imply maximal mixing between tau and muon neutrinos. The large mixing in the neutrino sector induces large flavour mixings in the left-handed slepton and right-handed down squark sectors in these models. The rates of flavour violating processes may in this case be much larger than in the minimal model, for example, the branching ratios of LFV processes can be close to or even exceed the present experimental bounds.

The aim of the present work is to revisit the minimal flavour violation scenario in the SUSY SU(5) grand unified theory. We extend the considerations of the previous papers [10,11] in several directions. Following the hints of possible Yukawa unification[1] we consider the case when all third generation Yukawa couplings are large and given by the renormalizable physics above the GUT scale. In this case, sizable flavour violation in the left-handed (right-handed) slepton (down squark) sector is induced due to the renormalization effects of down type Yukawa couplings between GUT and Planck scales, in addition to the flavour violation in the right-handed slepton sector. Thus the pattern of flavour violation in SUSY SU(5) GUT at large $\tan\beta$ resembles the flavour violation pattern in models with massive neutrinos, and deserves phenomenological studies. We do not make an attempt to identify and to study only that part of the soft terms parameter space in which tau-bottom Yukawa unification is achieved starting from the low energy parameters, but instead assume that there may be large corrections depending on the details of the GUT theory. We allow $\tan\beta$ to be a free, although large, parameter so that the effects of τ and b Yukawa couplings become non-negligible. The details of our calculations are given below. In our numerical analyses we consider the case where Yukawa unification is possible as well as the case where it is not.

In addition, we calculate the new flavour physics contributions to $K-\bar{K}$ and $B-\bar{B}$ mixings as well as to $b\to s\gamma$, $\mu\to e\gamma$, $\mu-e$ conversion in nuclei and $\tau\to\mu\gamma$ branching ratios. We also comment on the possibility of large GUT phases in this context. There is an increasing amount of constraints on the MSSM mass spectrum coming from direct collider searches as well as from indirect low energy measurements. We take these into account when calculating allowed ranges for the physical observables. In particular, the constraints coming from $b\to s\gamma$ turn out to allow only the LFV processes to be in the phenomenologically interesting ranges.

The main motivation for the present work is to study how sensitively the planned next generation LFV experiments will probe the SUSY model considered here. In the near future the branching ratio of the decay $\mu\to e\gamma$ will be probed with the sensitivity below 10^{-14} [15] and $\mu-e$ conversion in nuclei below 10^{-16} [16] (in more distant future μ factories may further reduce these numbers by orders of magnitude [17]). We shall show that if these experiments will indeed reach the planned sensitivity then, in the scenario considered in this work, LFV must be observed unless the SUSY scale $\sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}$ is above TeV scale. This conclusion remains valid even in the case where the only source of LFV is the right-handed slepton mass matrix as in Refs. [10] since the deep cancellation in the branching ratios observed in Ref. [11] occurs at slepton masses which imply multi-TeV SUSY scale. In this context it is interesting to note that if we require approximate τ-b Yukawa unification at GUT scale then the sparticle masses are required to be too high for direct production at future collider experiments. In this case, the observation of LFV may turn out to be the only signal of supersymmetry.

The paper is organized as follows. In the second section we present the crucial parts

[1]Values of $\tan\beta$ between 0.7 and 1.8 are excluded by the direct LEP2 searches for the lightest MSSM Higgs boson [13]. At the end of LEP2 run, values of $\tan\beta$ below 2.6 will be probed [14]. This constraint is not very restrictive but hints into the same direction as the Yukawa unification.

of model we are using throughtout this work, especially the relevant mixing matrices are introduced. In section III renormalization from the reduced Planck scale to the electroweak scale is discussed. In section IV we present our numerical results. Finally in section V we summarize our conclusions.

II. SOURCES OF FLAVOUR VIOLATION

In this section we present some details of the minimal flavour violation scenario considered in our work. We assume that the supersymmetric SU(5) grand unified theory is valid at mass scales between M_P and M_{GUT}. In the SU(5) model there are three generations of matter multiplets ψ_i and ϕ_i, $i = 1, 2, 3$, which form the $\mathbf{10}$ and $\mathbf{5}^*$ dimensional representations of SU(5), respectively, and $\mathbf{5}$ and $\mathbf{5}^*$ dimensional representations of Higgs multiplets, $\hat{H}_2$ and $\hat{H}_1$, respectively. The tenplets ψ_i contain the quark doublets, the charged lepton singlets and the up-type quark singlets, while the down-type quark singlets and the lepton doublets are included in the fiveplets ϕ_i. The Higgs fiveplet $\hat{H}_1$ contains the MSSM Higgs multiplet H_1 and a coloured Higgs multiplet H_{C1}, and $\hat{H}_2$ contains the second MSSM Higgs multiplet H_2 and another coloured Higgs multiplet H_{C2}. An adjoint representation Higgs multiplet Σ causing the breaking of SU(5) should also belong to the Higgs sector of the model. We neglect the Yukawa coupling and the soft SUSY-breaking parameters associated with it. Inclusion of these terms will *increase* the non-universalities at GUT scale and thus *increase* the amount of flavour violation in the model. Thus the terms in superpotential W relevant for our consideration are given by

$$W = \frac{1}{4} f_{u_{ij}} \psi_i^{AB} \psi_j^{CD} \hat{H}_2^E \epsilon_{ABCDE} + \sqrt{2} f_{d_{ij}} \psi_i^{AB} \phi_{jA} \hat{H}_{1B}, \tag{1}$$

where $A, B, ... = 1, ..., 5$ are the SU(5) indices. The relevant soft SUSY breaking terms associated with the SU(5) multiplets are

$$\begin{aligned} -\mathcal{L}_{\text{SUSY breaking}} = & (m_{10}^2)_{ij} \tilde{\psi}_i^\dagger \tilde{\psi}_j + (m_5^2)_{ij} \tilde{\phi}_i^\dagger \tilde{\phi}_j + m_{h_1}^2 h_1^\dagger h_1 + m_{h_2}^2 h_2^\dagger h_2 \\ & + \left\{ \frac{1}{4} A_{u_{ij}} \tilde{\psi}_i \tilde{\psi}_j h_2 + \sqrt{2} A_{d_{ij}} \tilde{\psi}_i \tilde{\phi}_j h_1 + h.c. \right\}, \end{aligned} \tag{2}$$

where $\tilde{\psi}_i$ and $\tilde{\phi}_i$ are the scalar components of the ψ_i and ϕ_i chiral multiplets, respectively, and h_1 and h_2 are the Higgs multiplets.

In the minimal SUGRA scenario the soft SUSY breaking parameters are generated by gravitational interactions and are universal at M_P:

$$\begin{gathered} (m_{10}^2)_{ij} = (m_5^2)_{ij} = \delta_{ij} m_0^2, \\ m_{h_1}^2 = m_{h_2}^2 = m_0^2, \\ A_{u_{ij}} = (A'_u \cdot f_u)_{ij},\ A_{d_{ij}} = (A'_d \cdot f_d)_{ij}, \\ A'_{u_{ij}} = A'_{d_{ij}} = \delta_{ij} A_0. \end{gathered} \tag{3}$$

Below M_P till M_{GUT} the parameters in Eq.(1) and Eq.(2) evolve with energy according to the renormalization group equations (RGE) of SUSY SU(5). These can be found for example in Ref. [18] and we do not present them here. Because the third generation Yukawa couplings

$f_{u_{33}}$ and $f_{d_{33}}$ are much larger than the Yukawa couplings of the first two generations, large hierarchies in the parameters of Eq.(3) are induced at M_{GUT}. Therefore the soft mass terms m_{10}^2, m_5^2 and $A'_{u,d}$ remain diagonal in the generation space but the third generation masses are smaller than the masses of the first two generations, which remain degenerate to a good approximation.

To understand the origin of flavour violation in SUSY SU(5) GUT we have to discuss the evolution of the Yukawa coupling matrices f_d and f_u with energy. At M_P we can choose the basis in which f_u is diagonal, $f_u^P = (f_u^P)_{ii}\delta_{ij}$. In that case the Yukawa matrix f_d can be diagonalized with a bi-unitary transformation,

$$V^{P\dagger}\Theta^P f_d^P U^P, \tag{4}$$

where V^P corresponds to the Cabibbo-Kobayashi-Maskawa (CKM) matrix at Planck scale and $\Theta^P = \text{ diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$, where $\phi_{1,2,3}$ are the GUT phases (satisfying $\phi_1+\phi_2+\phi_3=0$) present in addition to the phase in V^P. However, the mixing matrix U^P can be rotated away by a redefinition of the fiveplet fields ϕ. No trace of that rotation will remain in the soft SUSY breaking terms because these are universal and proportional to unit matrix at M_P. Below M_P the Yukawa matrices f_d and f_u run according to

$$\begin{aligned}
16\pi^2\frac{\mathrm{d}}{\mathrm{d}t}f_{u_{ij}} &= \left[-\frac{96}{5}g_5^2 + 3\mathrm{Tr}(f_u^\dagger f_u)\right] f_{u_{ij}} + 6(f_u f_u^\dagger f_u)_{ij} + 2(f_d f_d^\dagger f_u)_{ij} + 2(f_u f_d^* f_d^\top)_{ij}, \\
16\pi^2\frac{\mathrm{d}}{\mathrm{d}t}f_{d_{ij}} &= \left[-\frac{84}{5}g_5^2 + 4\mathrm{Tr}(f_d^\dagger f_d)\right] f_{d_{ij}} + 6(f_d f_d^\dagger f_d)_{ij} + 3(f_u f_u^\dagger f_d)_{ij}.
\end{aligned} \tag{5}$$

Because the third generation Yukawa couplings are large, of order unity, the running of the off-diagonal elements of the Yukawa matrices is significant. Notice that despite of the chosen basis, non-zero off-diagonal elements are generated also in f_u. To predict the running of the off-diagonal elements requires a precise knowledge of all the elements of the Yukawa matrices. This, however, goes beyond the assumptions made in our work: we assumed that only the third generation Yukawa couplings can be reliably estimated via the RGEs from the experimental data. Therefore, in our numerical estimates we assume that the off-diagonal elements of the rotation matrix U, which rotates the fiveplet fields ϕ are small, of order or smaller than the corresponding CKM matrix elements at GUT scale.

At GUT scale the Yukawa coupling matrices f_d and f_u get renormalized to f_d^G and f_u^G, respectively, and can be diagonalized again with bi-unitary transformations. However, now any rotation of the superfields ψ and ϕ is reflected in the soft SUSY breaking parameters because these are not universal at GUT scale any more. These rotations give rise to large flavour changing effects, as will be discussed in the following.

At M_{GUT} the SU(5) gauge group breaks spontaneously into the usual MSSM gauge group $\mathrm{SU}(3)_C \times\ \mathrm{SU}(2)_L \times\ \mathrm{U}(1)_Y$. The MSSM superpotential W valid below GUT scale is

$$W = Q_i^o(f_{u_{ij}})U_i^{co}H_2 - Q_i^o(f_{d_{ij}})D_j^{co}H_1 - E_i^{co}(f_{e_{ij}})L_j^o H_1 - \mu H_1 H_2 \tag{6}$$

and the soft SUSY breaking terms are given by

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} &= \tilde{L}_i^{o\dagger}(m_{\tilde{L}}^2)_{ij}\tilde{L}_j^o + \tilde{E}_i^{co*}(m_{\tilde{E}}^2)_{ij}\tilde{E}_j^{co} + \tilde{Q}_i^{o\dagger}(m_{\tilde{Q}}^2)_{ij}\tilde{Q}_j^o + \tilde{U}_i^{co*}(m_{\tilde{U}}^2)_{ij}\tilde{U}_j^{co} + \tilde{D}_i^{co*}(m_{\tilde{D}}^2)_{ij}\tilde{D}_j^{co} \\
&\quad + m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + \Big(\tilde{Q}_i^o(A_{u_{ij}})\tilde{U}_j^{co}H_2 - \tilde{Q}_i^o(A_{d_{ij}})\tilde{D}_j^{co}H_1 - \tilde{E}_i^{co}(A_{e_{ij}})\tilde{L}_j^o H_1 \\
&\quad + B_H H_1 H_2 + \frac{1}{2}M_1\tilde{B}\tilde{B} + \frac{1}{2}M_2\tilde{W}^a\tilde{W}^a + \frac{1}{2}M_3\tilde{g}^a\tilde{g}^a + h.c.\Big)\,.
\end{aligned} \tag{7}$$

Here all the fields are explicitly written in the flavour eigenstate basis denoted by the superscript zero. The boundary condition for the soft terms are specified via

$$\begin{aligned}(m^2_{\tilde{Q}})_{ij} = (m^2_{\tilde{U}})_{ij} = (m^2_{\tilde{E}})_{ij} = (m^2_{10})_{ij}\,,\\ (m^2_{\tilde{D}})_{ij} = (m^2_{\tilde{L}})_{ij} = (m^2_{5})_{ij}\,,\\ m^2_{H_1} = m^2_{h_1}\,,\quad m^2_{H_2} = m^2_{h_2}\,,\\ M_1 = M_2 = M_3 = M_0.\end{aligned} \tag{8}$$

Since there is no splitting among the gaugino masses above the GUT scale, it is most convenient to stipulate M_0 at the GUT scale. For the Yukawa couplings and trilinear terms this model predicts

$$f_e^G = f_d^G\,,\quad A_e^G = A_d^G \tag{9}$$

at GUT scale.

From the GUT scale down to the electroweak scale the Yukawa couplings and the soft parameters evolve with energy via the MSSM RGEs [19]. Once the electroweak symmetry is broken, rotations of the superfields

$$\begin{aligned}D^o = V_d D\,,\quad E^o = V_e E\,,\quad U^o = V_u U\,,\\ D^{co} = U_d^* D^c\,,\ E^{co} = U_e^* E^c\,,\ U^{co} = U_u^* U^c\,,\end{aligned} \tag{10}$$

bring quarks and leptons into their mass eigenstates with diagonal Yukawa couplings f'_d, f'_e and f'_u

$$f'_{d_i} = (V_d^T f_d U_d^*)_{ii}\,,\quad f'_{e_i} = (U_e^\dagger f_e V_e)_{ii}\,,\quad f'_{u_i} = (V_u^T f_u U_u^*)_{ii}\,. \tag{11}$$

At the GUT scale the down quark and lepton masses are predicted to be equal. Therefore the diagonalizing matrices are related at M_{GUT} as

$$V_d^G = U_e^{*G}\,,\quad U_d^G = V_e^{*G}. \tag{12}$$

This implies that at low energies the left-handed quark rotation matrix, the CKM matrix in the basis in which up quark Yukawa matrix is diagonal, can be related to the right-handed lepton rotation matrix and the right-handed down quark rotation matrix can be related to the left-handed lepton rotations via the RGEs.

At the same time the rotation in Eq.(10) changes the basis of the superpartners of quarks and leptons. For example, the mass eigenstates of the charged sleptons and sneutrinos can be expressed as

$$\tilde{E} = \Gamma_E \begin{pmatrix} V_e^\dagger \tilde{E}^o \\ U_e^\dagger \tilde{E}^{co*} \end{pmatrix}\,,\quad \tilde{\nu} = \Gamma_\nu V_e^\dagger \tilde{\nu}^o\,, \tag{13}$$

where Γ_E and Γ_ν are 6×6 and 3×3 rotation matrices, respectively. The slepton mass matrices are given by

$$\begin{aligned} m^2_{\tilde{E}} &= \Gamma_E \begin{pmatrix} V_e^\dagger m^2_{\tilde{L}} V_e + m_e^2 - m_Z^2\cos 2\beta(\frac{1}{2} - \sin^2\theta_W) & -\mu m_e \tan\beta + m_e U_e^\dagger A_e'^\dagger U_e \\ -\mu^* m_e\tan\beta + U_e^\dagger A'_e U_e m_e & U_e^\dagger m^2_{\tilde{E}} U_e + m_e^2 - m_Z^2 \cos 2\beta \sin^2\theta_W \end{pmatrix} \Gamma_E^\dagger \\ m^2_{\tilde{\nu}} &= \Gamma_\nu \left(V_e^\dagger m^2_{\tilde{L}} V_e + \frac{1}{2} m_Z^2 \cos 2\beta \right) \Gamma_\nu^\dagger\,, \end{aligned} \tag{14}$$

and analogously for squarks.

Notice that this definition of the diagonalizing matrices $\Gamma_{E,D,U}$ differs from the one originally given in Ref. [20] where the matrices Γ relate the flavour eigenstates directly to the mass eigenstates. The advantage of the present notation in calculating the flavour violating observables in our scenario is the following. It allows us to perform the MSSM RGE running of the soft terms (which are diagonal in the flavour space) and the CKM matrix elements (which induce the flavour mixings) separately without constructing the squark mass matrices at high scales and without running each element of the 6×6 sparticle mass matrices. The slepton and squark mass matrices are then constructed at low energies in terms of the low energy values of the soft terms, quark masses and the mixing matrices.

We use the standard notation for the neutralino and chargino mass matrices. The neutralino mass matrix $M_{\tilde{N}}$ in the basis $(\tilde{B}, \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0)$ can be diagonalized as

$$M_{\tilde{N}}^{diag} = N M_{\tilde{N}} N^\dagger \tag{15}$$

where N_{ij} is a 4×4 unitary matrix. Similarly, the two diagonalizing matrices $O_{L,R}$ in the chargino sector can be found from

$$M_{\tilde{\chi}}^{diag} = O_L \begin{pmatrix} M_2 & \sqrt{2} \sin\beta m_W \\ \sqrt{2} \cos\beta m_W & \mu \end{pmatrix} O_R^\dagger \tag{16}$$

For the further details see, *e.g.* , Ref. [2].

III. RENORMALIZATION PROCEDURE

Before calculating the rates of flavour violating observables, let us describe our procedure of calculating the input SUSY parameters via the RGE evaluation. We start the RGE running at M_Z where we introduce as the low energy input the values of the gauge coupling constants, the tau and bottom quark Yukawa couplings corresponding to the tau and bottom quark masses $m_\tau(M_Z) = 1.784$ GeV and $m_b(M_Z) = 3.0$ GeV [21], respectively. We parametrize the CKM matrix in the standard way [22] with $\theta_{12} = 0.22$, $\theta_{23} = 0.04$ and $\theta_{13} = 0.003$. We evolve these quantities from M_Z to m_t using two loop SM RGE-s for five flavours. At m_t we include top quark Yukawa coupling corresponding to the pole mass $m_t = 174$ GeV and run the SM RGE-s for six flavours up to the scale Q where superparticles are introduced. At the scale Q we convert the SM couplings to the MSSM ones fixing the value of $\tan\beta$ and include the SUSY loop corrections to the bottom quark and tau lepton Yukawa couplings according to Ref. [4,23]. Then the gauge and Yukawa couplings are evaluated at GUT scale with the help of two loop MSSM RGE-s. For the running of the CKM matrix elements we use the one loop RGE-s from Ref. [24].

The crucial issue in the present context is the correct tau-bottom Yukawa unification as predicted by SU(5). The Yukawa unification has been studied extensively in literature [3,4,6] and it turns out that unification is possible only for $\mu < 0$ and $30 \lesssim \tan\beta \lesssim 50$. This can be understood as follows. The correction to the bottom Yukawa coupling [4],

$$f_b \sim \frac{m_b}{1+\delta_b} \frac{1}{v_1} \tag{17}$$

which is numerically the most important one, is proportional to $\tan\beta$ and its sign depends on $sign(\mu)$. Tau-bottom Yukawa unification can be obtained if the top quark mass is approximately at a fixed point at $\tan\beta \approx 1.6$. On the other hand, small values of $\tan\beta$ are, anyway, excluded by the LEP 2 search for the lightest Higgs boson [13,14]. To achieve the tau-bottom Yukawa unification therefore a sizable negative δ_b is needed, which requires large values of $\tan\beta$ and fixes the sign of the μ parameter to be $sign(\mu) = -1$. On the other hand, it has been argued [5] that there might be important threshold corrections due to the coloured triplet Higgses at GUT scale modifying sizably the bottom quark Yukawa coupling already at M_{GUT}. In any case, the coupling which receives large corrections is the bottom Yukawa coupling, tau Yukawa coupling is less sensitive to these corrections and thus the prediction of tau Yukawa coupling at GUT scale is more robust. Therefore, in our numerical calculations we use actually the matching condition

$$f_d^G[\,\mathrm{SU}(5)] = f_\tau^G[\,\mathrm{MSSM}] \tag{18}$$

at M_{GUT} to give a numerical value to the SU(5) Yukawa coupling $f_{d_{33}}$, and allow the value of the MSSM coupling f_b^G to be numerically different. To achieve semi-realistic Yukawa coupling evolution we fix the magnitude of δ_b by hand for each considered value of $\tan\beta$ but keep its sign to be determined by the $sign(\mu)$. In such a case the Yukawa unification is achieved for $sign(\mu) = -1$ and not achieved for $sign(\mu) = +1$. In our numerical examples we consider both possibilities. For definiteness we consider two values of $\tan\beta$, $\tan\beta = 35$ (corresponds to $f_t^G \sim 2f_\tau^G$) and $\tan\beta = 48$ (corresponds to $f_t^G \sim f_\tau^G$).

At M_{GUT} we fix the leptonic mixing matrices V_e and U_e according to Eq.(12). Unlike the quark mixing matrices V_e and U_e do not run with energy due to the absence of neutrino Yukawa couplings in the MSSM. Choosing the basis in which the top Yukawa matrix is diagonal we have $V_d = V_{CKM}$ and therefore $|U_e|_{ij} = |V^G_{CKM}|_{ij}$. There is no experimental restriction on the mixing matrix of the right-handed quark fields. Therefore we assume that large couplings $f_{d_{33}}, f_{u_{33}}$ in Eq.(5) generate small non-zero (31) and (32) elements for U_d and V_e but the angle θ_{12} remains negligible in these matrices. Numerically we consider three cases: $V_e^{(31),(32)} = U_e^{(31),(32)}$, $V_e^{(31),(32)} = 0.1 \times U_e^{(31),(32)}$ and $V_e^{(31),(32)} = 0$.

Further, we run the SU(5) gauge and Yukawa couplings from M_{GUT} to M_P where we fix the values of m_0 and A_0. For the trilinear coupling we take always $A_0 = 0$ in our numerical calculations. Then we run all the parameters, including the SU(5) soft terms down to the GUT scale, apply the boundary conditions Eq.(8) and run all the MSSM parameters down to the scale Q which we take to be 200 GeV. At that scale we fix the SUSY parameters μ and B via the conditions of spontaneous symmetry breaking and calculate the masses and mixing matrices of the sparticles. The chargino and neutralino mass matrices are given by the standard expressions which can be found for example in Ref. [2]. In order not to run into contradiction with the mass bounds on the SUSY particles from direct searches we take $M_0 \gtrsim 150$ GeV and $m_0 \gtrsim 150$ GeV; the lightest sparticle masses corresponding to $m_0 = M_0 = 150$ GeV and $\tan\beta = 48$ are roughly $M_{\tilde\tau_1} = 73$ GeV, $M_{\tilde\chi_1^+} = 98$ GeV and $M_{\tilde\chi_1^0} = 58$ GeV and are just on the limit of the present LEP bounds.

IV. RATES OF FLAVOUR CHANGING PROCESSES

A. $K_L - K_S$ System

We start our numerical analyses by calculating the new SUSY flavour changing physics contribution to physical observables in the $K_L - K_S$ system. Recently it has been emphasized [25] that in models with massive neutrinos the SUSY flavour changing contribution to ϵ_K may be large, especially if new GUT phases are present which may maximize the effect. Because at large $\tan\beta$ the pattern of flavour violation in the SUSY SU(5) is the same as in the models with right-handed neutrinos we study this issue carefully.

The dominant new physics contribution to $\Delta S = 2$ processes comes from box diagrams with gluinos and down squarks running in the loop [20]. Because in our scenario the first two generation sfermions are almost exactly degenerate the appropriate tool to use is the mass insertion approximation [26]. The current state of the art on this subject is summarized in Ref. [27] in which the low energy $\Delta S = 2$ effective Hamiltonian is calculated including NLO QCD corrections and the relevant hadronic matrix elements are evaluated using the lattice results for the B_K-parameters. We use the expressions for the $\Delta S = 2$ Wilson coefficients, NLO QCD corrections and hadronic matrix elements as well as numerical input exactly as in Ref. [27] and therefore we do not copy them here. However, we need to specify the model dependent input.

In the mass insertion approximation the flavour violation is characterized with the dimensionless parameters $(\delta_{LL})_{ij}$, $(\delta_{RR})_{ij}$, $(\delta_{LR})_{ij}$ and $(\delta_{RL})_{ij}$ defined as

$$\begin{pmatrix} \delta_{LL} & \delta_{LR} \\ \delta_{RL} & \delta_{RR} \end{pmatrix} = \frac{1}{\tilde{m}^2} \begin{pmatrix} (m^2_{\tilde{D}})_{LL} & (m^2_{\tilde{D}})_{LR} \\ (m^2_{\tilde{D}})_{RL} & (m^2_{\tilde{D}})_{RR} \end{pmatrix} \tag{19}$$

where $(m^2_{\tilde{D}})_{MN}$, $M, N = L, R$ are the corresponding 3×3 components of the 6×6 down squark mass matrix and $\tilde{m}^2$ is an average squark mass appropriate for the problem under consideration. We calculate the down squark mass matrix as described in detail in previous sections. The mixings among the left down squark fields are generated by the CKM matrix V_{CKM}. The mixings among the right down squarks are assumed to be given by the matrix $U_{d_{ij}} = (V^G_{CKM})_{ij}$, $i, j = 2, 3$ with vanishing angle θ_{12}. Thus the off-diagonal elements of U_d are smaller than the elements of V_{CKM}. Because only the (12) components of δ_{MN} enter to the $\Delta S = 2$ process we identify $\tilde{m}^2 = (m^2_{\tilde{Q}})_{11}$.

The $K_L - K_S$ mass difference and the CP-violating parameter ϵ_K are given by:

$$\begin{aligned} \Delta M_K &= 2\, \mathrm{Re}\langle K^0 | H^{\Delta S=2}_{eff} | \bar{K}^0 \rangle \,, \\ \epsilon_K &= \frac{1}{\sqrt{2}\Delta M_K}\, \mathrm{Im}\langle K^0 | H^{\Delta S=2}_{eff} | \bar{K}^0 \rangle \,. \end{aligned} \tag{20}$$

It turns out that the new physics contribution to ΔM_K is very small in our model, typically at the permil level of the measured value. However, the new contribution to ϵ_K which is a small quantity in the SM can be sizable. In Fig. 1 we plot the gluino mediated contribution to ϵ_K as a function of the average squark mass $\tilde{m}$ fixing the gluino mass to be $M_{\tilde{g}} = 420$ GeV and assuming that the new GUT phases maximize the effect. For the squark masses of order 500 GeV and large $\tan\beta$ the contribution may exceed 25% of the measured value

FIGURES

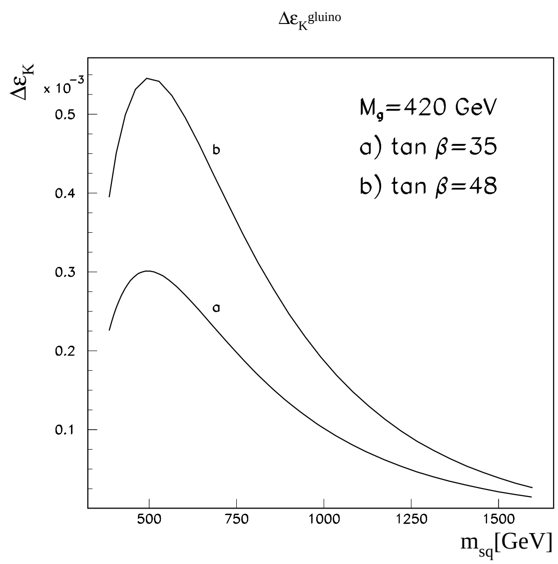

FIG. 1. *Gluino mediated contribution to ϵ_K as a function of the average down squark mass assuming that the possible new GUT phases maximize the effect.*

$\epsilon_K = 2.3 \cdot 10^{-3}$. However, because of the assumption of the GUT phases and because of large theoretical errors in the hadronic matrix elements no useful constraints can be derived from ϵ_K on the parameters of the model. The reason why the contribution to ϵ_K in SUSY SU(5) is smaller than in the models with massive neutrinos is the smallness of the $(\delta_{MN})_{12}$ elements. The magnitude of the largest mixing parameter $(\delta_{LL})_{12}$ is typically below 10^{-4} and for the chosen parameters $(\delta_{RR})_{12}$ is about factor of three smaller. The (LR) parameters are smaller by three orders of magnitude. We conclude that the small mixing angles in our approach suppress the contribution to the $K-\bar{K}$ mixing.

B. $B_d - \bar{B}_d$ System

Here we estimate the new physics contribution to the B_d meson system. The SM contribution to $\Delta M_{B_d} = 2|M_{12}|$ is given by [28]

$$M_{12}^{SM} = \frac{G_F^2}{12\pi^2}\eta_{QCD} B_{B_d} f_{B_d}^2 M_{B_d} M_W^2 (V_{td}V_{tb}^*)^2 S_0(z_t), \tag{21}$$

where

$$S_0(z_t) = \frac{4z_t - 11z_t^2 + z_t^3}{4(1-z_t)^2} - \frac{3z_t^3 \ln z_t}{2(1-z_t)^3}\,, \tag{22}$$

where $z_t = m_t^2/m_W^2$ and $\eta_{QCD} = 0.55\pm0.01$. With $B_{B_d} = 1.29\pm0.08\pm0.06$ and $f_{B_d} = 175\pm25$ MeV it successfully predicts the measured value $\Delta M_{B_d} = 0.470\pm0.019$ ps^{-1}. To estimate the magnitude of the dominant gluino induced contribution to $B-\bar{B}$ mixing in our scenario we use again the mass insertion approximation. Neglecting the small LR and RL contributions it reads [26]

$$M_{12}^{SUSY} = -\frac{\alpha_s^2}{216\tilde{m}^2}\frac{1}{3}B_{B_d}f_{B_d}^2 M_{B_d}\left\{((\delta_{LL})_{31}^2+(\delta_{RR})_{31}^2)(66\tilde{f}_6(x)+24xf_6(x))+\right. \tag{23}$$
$$\left.(\delta_{LL})_{31}(\delta_{RR})_{31}\left[\left(36-24\left(\frac{M_{B_d}}{m_b+m_d}\right)^2\right)\tilde{f}_6(x)+\left(72+384\left(\frac{M_{B_d}}{m_b+m_d}\right)^2\right)xf_6(x)\right]\right\},$$

$$f_6(x) = \frac{1}{6(x-1)^5}\left(x^3-9x^2-9x+17+6(1+3x)\ln x\right)\,, \tag{24}$$

$$\tilde{f}_6(x) = \frac{1}{3(x-1)^5}\left(-x^3-9x^2+9x+1+6x(1+x)\ln x\right)\,. \tag{25}$$

where $x = M_{\tilde{g}}^2/\tilde{m}^2$. In this case the common squark mass is taken to be $\tilde{m}^2 = (m_{\tilde{Q}}^2)_{33}$ and the mixing elements δ_{31} are calculated as in the previous subsection.

In the case where the sparticle masses are assumed to take their lower allowed limits, the SUSY contribution to ΔM_{B_d} turns out to be at most of order a few percent. Taking into account the large errors in the input parameter $f_{B_d}^2 B_{B_d}$ no useful constraints on our model can be derived drom the measurement of ΔM_{B_d}.

C. $b \to s\gamma$

The radiative decay $b \to s\gamma$ is known to get large contributions from SUSY particle loops and therefore this process implies strong constraints on the allowed SUSY parameter space [20,29,30]. Besides the SM W-boson-t-quark contribution there are also charged Higgs, chargino, neutralino and gluino contributions. At the electroweak scale all of them have been calculated in Ref. [20]. While in the SM the NLO QCD corrections are included to $b \to s\gamma$, in SUSY theories the NLO analyses has been performed only for specific scenarios [31]. The complication here lies in the new flavour structure of SUSY theories, as compared to the SM, which extends the operator basis beyond the SM one. Therefore the LO QCD corrections to the gluino contribution which gives the dominant new flavour physics contribution was calculated very recently in Ref. [32]. Before that, the gluino contribution in a generic SUSY flavour model was studied in [26] without including QCD corrections.

Here we shortly review the results of Ref. [32]. The effective Hamiltonian for $b \to s\gamma$ is expressed in two parts

$$\mathcal{H}_{eff} = \mathcal{H}_{eff}^{CKM} + \mathcal{H}_{eff}^{\tilde{g}}\,, \tag{26}$$

where $\mathcal{H}_{eff}^{CKM}$ is the effective Hamiltonian in which the structure of flavour violation is the same as in the SM and the gluino contribution $\mathcal{H}_{eff}^{\tilde{g}}$ exhibits the new flavour structure. The Wilson coefficients of the first term,

$$\mathcal{H}_{eff}^{CKM} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_i C_i(\mu)\mathcal{O}_i(\mu)\,, \tag{27}$$

contain the SM as well as the charged Higgs, chargino and neutralino contributions. The dominant operators in Eq.(27) are the dimension six magnetic operators ($\overline{m}_b(\mu)$ is the running mass)

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}\,, \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2}\, \overline{m}_b(\mu)\, (\bar{s}\sigma^{\mu\nu} T^a P_R b)\, G^a_{\mu\nu}\,, \tag{28}$$

and the operators $\mathcal{O}'_{7,8}$ obtained by $L \leftrightarrow R$. The gluino effective hamiltonian

$$\mathcal{H}^{\tilde{g}}_{eff} = \sum_i C_{i,\tilde{g}}(\mu) \mathcal{O}_{i,\tilde{g}}(\mu) + \sum_i \sum_q C^q_{i,\tilde{g}}(\mu) \mathcal{O}^q_{i,\tilde{g}}(\mu)\,. \tag{29}$$

contains in addition to the dimension six magnetic operators $\mathcal{O}_{7b,\tilde{g}}$, $\mathcal{O}'_{7b,\tilde{g}}$, $\mathcal{O}_{8b,\tilde{g}}$, $\mathcal{O}'_{8b,\tilde{g}}$ also operators of dimension five,

$$\mathcal{O}_{7\tilde{g},\tilde{g}} = e\, g_s^2(\mu)\, (\bar{s}\sigma^{\mu\nu} P_R b)\, F_{\mu\nu}\,, \qquad \mathcal{O}_{8\tilde{g},\tilde{g}} = g_s(\mu)\, g_s^2(\mu)\, (\bar{s}\sigma^{\mu\nu} T^a P_R b)\, G^a_{\mu\nu}\,, \tag{30}$$

in which the chirality-violating parameter is the gluino mass in the corresponding Wilson coefficients, as well as new four-quark operators $\mathcal{O}^q_{i,\tilde{g}}$, $q = u, d, c, s, b$, which are listed in Ref. [32]. These new operators change the structure of the LO QCD corrections in general SUSY flavour models.

The Wilson coefficients in Eq.(27) including the LO QCD corrections in the MSSM are well known. Their explicit expressions can be found in [30] and we do not rewrite them here. Instead, let us for a moment concentrate on the study of the effective hamiltonian Eq.(29) in our model.

At the electroweak scale the relevant Wilson coefficients are given by [32]

$$\begin{aligned}
C_{7b,\tilde{g}}(\mu_W) &= \; -\frac{e_d}{16\pi^2}\, C(R) \sum_{k=1}^{6} \frac{1}{m^2_{\tilde{d}_k}} \left(\Gamma^{kb}_{DL}\, \Gamma^{*\,ks}_{DL}\right) f_2(x_{gd_k})\,, \\
C_{7\tilde{g},\tilde{g}}(\mu_W) &= M_{\tilde{g}}\, \frac{e_d}{16\pi^2}\, C(R) \sum_{k=1}^{6} \frac{1}{m^2_{\tilde{d}_k}} \left(\Gamma^{kb}_{DR}\, \Gamma^{*\,ks}_{DL}\right) f_4(x_{gd_k})\,, \\
C_{8b,\tilde{g}}(\mu_W) &= \; -\frac{1}{16\pi^2} \sum_{k=1}^{6} \frac{1}{m^2_{\tilde{d}_k}} \left(\Gamma^{kb}_{DL}\, \Gamma^{*\,ks}_{DL}\right) \left[\left(C(R) - \tfrac{1}{2} C(G)\right) f_2(x_{gd_k}) - \tfrac{1}{2} C(G) f_1(x_{gd_k})\right]\,, \\
C_{8\tilde{g},\tilde{g}}(\mu_W) &= M_{\tilde{g}}\, \frac{1}{16\pi^2} \sum_{k=1}^{6} \frac{1}{m^2_{\tilde{d}_k}} \left(\Gamma^{kb}_{DR}\, \Gamma^{*\,ks}_{DL}\right) \left[\left(C(R) - \tfrac{1}{2} C(G)\right) f_4(x_{gd_k}) - \tfrac{1}{2} C(G) f_3(x_{gd_k})\right]\,,
\end{aligned} \tag{31}$$

Here the matrices Γ_{DL} and Γ_{DR} are the 6×3 submatrices of Γ_D,

$$\Gamma_D^{6\times 6} = \left(\, \Gamma_{DL}^{6\times 3} \;\; \Gamma_{DR}^{6\times 3}\,,\right) \tag{32}$$

and the ratios x_{gd_k} are defined as $x_{gd_k} \equiv M^2_{\tilde{g}}/m^2_{\tilde{d}_k}$. The Casimir factors $C(R)$ and $C(G)$ are $C(R) = 4/3$ and $C(G) = 3$ and the functions $f_i(x)$, $i = 1, ..., 4$, are given by

$$\begin{aligned}
f_1(x) &= \frac{1}{12\,(x-1)^4} \left(x^3 - 6x^2 + 3x + 2 + 6x \log x\right)\,, \\
f_2(x) &= \frac{1}{12\,(x-1)^4} \left(2x^3 + 3x^2 - 6x + 1 - 6x^2 \log x\right)\,, \\
f_3(x) &= \frac{1}{2\,(x-1)^3} \left(x^2 - 4x + 3 + 2 \log x\right)\,, \\
f_4(x) &= \frac{1}{2\,(x-1)^3} \left(x^2 - 1 - 2x \log x\right)\,.
\end{aligned} \tag{33}$$

The Wilson coefficients of the corresponding primed operators are obtained through the interchange $\Gamma^{ij}_{DR} \leftrightarrow \Gamma^{ij}_{DL}$. At the low scale μ_b, the LO renormalization group improved coefficients become

$$C_{7\tilde{g},\tilde{g}}(\mu_b) = \eta^{\frac{27}{23}}\, C_{7\tilde{g},\tilde{g}}(\mu_W) + \frac{8}{3}\left(\eta^{\frac{25}{23}} - \eta^{\frac{27}{23}}\right) C_{8\tilde{g},\tilde{g}}(\mu_W)\,,$$
$$C_{7b,\tilde{g}}(\mu_b) = \eta^{\frac{39}{23}}\, C_{7b,\tilde{g}}(\mu_W) + \frac{8}{3}\left(\eta^{\frac{37}{23}} - \eta^{\frac{39}{23}}\right) C_{8b,\tilde{g}}(\mu_W) + R_{7b,\tilde{g}}(\mu_b)\,, \tag{34}$$

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$. The remainder function $R_{7b,\tilde{g}}(\mu_b)$ turns out to be numerically very small [32] and we neglect it in our numerical computation. The low-scale Wilson coefficients for the corresponding primed operators are obtained by replacing in (34) all the unprimed coefficients with primed ones.

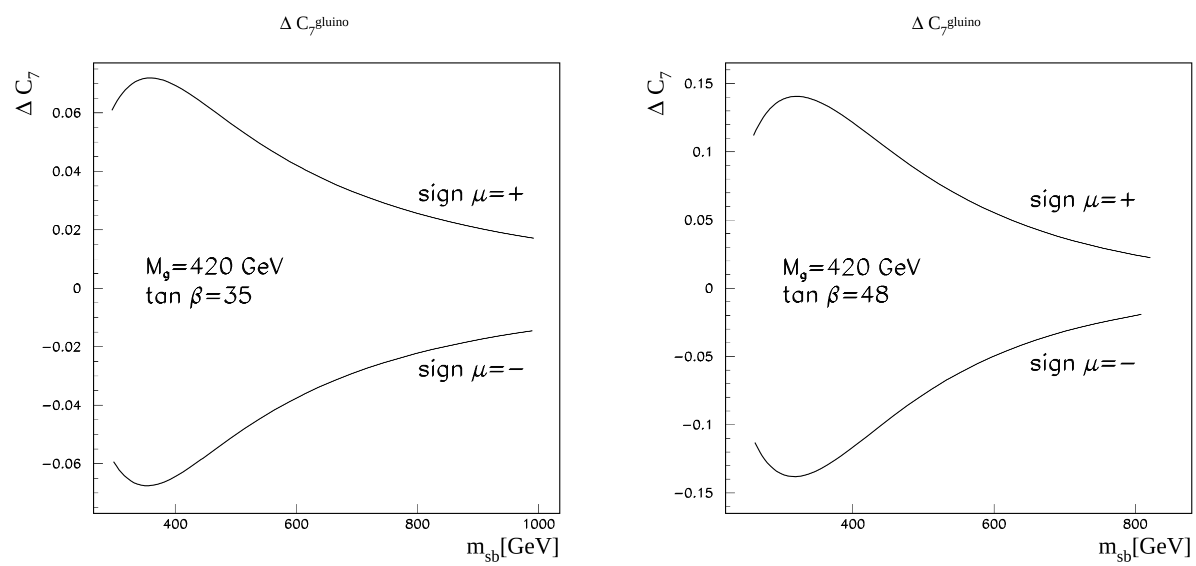

FIG. 2. *Gluino mediated contribution to the Wilson coefficient* C_7 *as a function of the sbottom mass for fixed gluino mass* $M_{\tilde{g}} = 420$ *GeV and* $\tan\beta = 35$, *48 as indicated in figures. The sign of the contribution depends on sign*(μ).

The decay width of $b \to s\gamma$ can be written

$$\Gamma(b \to s\gamma) = \frac{m_b^5\, G_F^2\, |V_{tb}V_{ts}^*|^2\, \alpha}{32\pi^4} \left|C_7^{eff}\right|^2\,, \tag{35}$$

where in our model

$$\left|C_7^{eff}\right|^2 = \left|C_7 + C_7^{\tilde{g}}\right|^2 + \left|C_7^{\prime\tilde{g}}\right|^2\,. \tag{36}$$

Here

$$C_7^{\tilde{g}} = -\frac{16\sqrt{2}\pi^3\alpha_s(\mu_b)}{G_F\, V_{tb}V_{ts}^*}\left[C_{7b,\tilde{g}}(\mu_b) + \frac{1}{m_b}C_{7\tilde{g},\tilde{g}}(\mu_b)\right]\,,$$
$$C_7^{\prime\tilde{g}} = -\frac{16\sqrt{2}\pi^3\alpha_s(\mu_b)}{G_F\, V_{tb}V_{ts}^*}\left[C'_{7b,\tilde{g}}(\mu_b) + \frac{1}{m_b}C'_{7\tilde{g},\tilde{g}}(\mu_b)\right]\,, \tag{37}$$

and C_7 stands for the total contribution from the effective Hamiltonian Eq.(27). In the SM its value is $C_7^{SM} = -0.3$ [28]. In the MSSM C_7 receives large contribution from charged Higgs loops which add constructively with C_7^{SM} and at large $\tan\beta$ also from chargino loops which add constructively (destructively) with C_7^{SM} for $sign(\mu) = -1$ ($sign(\mu) = +1$). Therefore for large $\tan\beta$ and small Higgs and sparticle masses the C_7 may be completely dominated by SUSY. Therefore the present experimental result [33]

$$2 \times 10^{-4} < \mathrm{BR}(\bar{B} \to X_s \gamma) < 4.5 \times 10^{-4}, \tag{38}$$

which favours the following allowed range:

$$0.25 < |C_7^{eff}| < 0.375\,, \tag{39}$$

implies strong constraints on the SUSY masses.

Let us now study the gluino contribution to C_7^{eff} numerically. In Fig. 2 we plot the values of $C_7^{\tilde{g}}$ as a function of the lightest bottom squark mass $m_{\tilde{b}_1}$ for a fixed value of the gluino mass $M_{\tilde{g}} = 420$ GeV and for two values of $\tan\beta$ and $sign(\mu)$, as indicated in the figure. Note that this is the smallest gluino mass consistent with chargino mass bounds in our model. The values of $C_7^{\tilde{g}}$ might be sizable for small sparticle masses but decreases rapidly if the masses are higher. The important behaviour to notice is that the sign of $C_7^{\tilde{g}}$ depends on the $sign(\mu)$ exactly the same way as the dominant contribution C_7. This implies that C_7 and $C_7^{\tilde{g}}$ add up constructively and no cancellation between them is possible. It has been argued in Ref. [32,34] that the constraints on the SUSY parameter space coming from C_7 which are rather restrictive can be relaxed by new flavour contributions. However, in our model this does not happen and the $b \to s\gamma$ bounds cannot be relaxed with help of $C_7^{\tilde{g}}$.

The gluino contribution $C_7^{\tilde{g}}$ is induced by the mixings in the left-handed down squark sector, thus by $V_d \equiv V_{CKM}$. However, the coefficient $C_7^{'\tilde{g}}$ is almost entirely induced by the mixings in the right-handed down squark sector. If these mixings are similar in size to the CKM mixings, as we argue here, then in our scenario with large $\tan\beta$, $C_7^{'\tilde{g}}$ is as large as $C_7^{\tilde{g}}$. While its absolute contribution to C_7^{eff} is subdominant (see Fig. 2) it still may have important phenomenological consequences. Namely, while the mixing induced time dependent CP asymmetry in $B \to M_s\gamma$, where M_s is some CP eigenstate, is predicted to be very small, a few percent, in the SM; our model, where it can be expressed as

$$\frac{\Gamma(t) - \bar{\Gamma}(t)}{\Gamma(t) + \bar{\Gamma}(t)} = A_t \sin \Delta M_{B_d} t\,, \qquad A_t = \frac{2\ \mathrm{Im}\left[e^{-i\theta_{B_d}}(C_7 + C_7^{\tilde{g}})C_7^{'\tilde{g}}\right]}{|C_7^{eff}|^2}\,, \tag{40}$$

with $\theta_{B_d} = arg(M_{12}^{B_d})$ being the phase in the $B - \bar{B}$ mixing amplitude, asymmetries A_t of more than 10% (which would be a clear and powerful signal of beyond the Sm physics) are allowed. Similar conclusions hold also in models with right-handed neutrinos [35].

D. Lepton Flavour Violation

So far we have shown that in SUSY SU(5) the new flavour physics contribution to flavour changing hadronic observables is subdominant. At the same time the SUSY contribution to $b \to s\gamma$ induced by the effective Hamiltonian Eq.(27) constrains severely the SUSY scale in our model. Now we turn to study the LFV processes which are completely dominated by the new physics. The amplitude for the process $l_j \to l_i \gamma^*$ where the photon is off-shell can be written as

$$M = e\epsilon^{\alpha *}\bar{l}_i \left[q^2\gamma_\alpha \left(A_1^L P_L + A_1^R P_R \right) + m_{l_j} i\sigma_{\alpha\beta} q^\beta \left(A_2^L P_L + A_2^R P_R \right) \right] l_j \,, \tag{41}$$

where q is the photon momentum and $A_{1,2}^{L,R}$ are the form factors giving rise to the process. Note that all the form factors contribute to $\mu - e$ conversion but only $A_2^{L,R}$ give rise to the lepton radiative decays. The form factors are induced by two types of loop diagrams with neutralinos and charged sleptons in the loop, and charginos and sneutrinos in the loop:

$$A_{1,2}^{L,R} = A_{1,2}^{(n)L,R} + A_{1,2}^{(c)L,R} \,, \tag{42}$$

where schematically written

$$\begin{aligned} A_{1,2}^{(n)L,R} &= A_{1,2}^{(n)L,R}\left(M_{\tilde{\chi}^0}, m_{\tilde{l}}, N, \Gamma_E \right) \,, \\ A_{1,2}^{(c)L,R} &= A_{1,2}^{(c)L,R}\left(M_{\tilde{\chi}^+}, m_{\tilde{\nu}}, O_{L,R}, \Gamma_\nu \right) \,, \end{aligned} \tag{43}$$

depend on the slepton masses and mixings as well as on the neutralino and chargino masses and mixings. We have adopted the formulas for the form factors from Ref. [12] and we do not present them here. The decay rate of $l_j \to l_i \gamma$ is then given by

$$\Gamma(l_j \to l_i\gamma) = \frac{e^2}{16\pi} m_{l_j}^5 \left(|A_2^L|^2 + |A_2^R|^2 \right) \,. \tag{44}$$

We start with studying the decay $\mu \to e\gamma$ in the case where the only source of LFV is the mixing in the right-handed slepton sector as given by Eq.(12), and the mixing matrix V_e is the unit matrix. This is the case studied in Ref. [11]. In Fig. 3 we plot the branching ratio of $\mu \to e\gamma$ on the plane of the lightest charged slepton mass $m_{\tilde{l}_1}$ and the lightest chargino mass $M_{\tilde{\chi}_1^+}$ for two values of $\tan\beta = 35$, 48 and $sign(\mu) = -1$ (to achieve Yukawa unification). We have taken into account the experimental constraints coming from $b \to s\gamma$ by requiring that the total value of C_7^{eff} is in the allowed range (39). As seen in Fig. 3 the constraint Eq.(39) puts strong lower bounds on the lightest sparticle masses. Our values of the branching ratio of $\mu \to e\gamma$ are much smaller than the quoted values in Ref. [11]. The reason is twofold. First, the $b \to s\gamma$ constraint pushes the sparticle masses to high values[2] and this has not been taken into account in Ref. [11]. Second, the authors of Ref. [11] fix the top Yukawa coupling at M_{GUT} to be $f_t^G = 1.4$ which for large $\tan\beta$ implies by far a too large top quark mass. For $m_t = 174$ GeV and $\tan\beta = 35$, the correct value is $f_t^G = 0.56$. For large part of the parameter space the destructive interference between the gaugino and higgsino contributions suppresses the $\mu \to e\gamma$ branching ratio to almost vanishing values.

---

[2]It has been noticed in Ref. [36] that for a particular corner of the parameter space where $M_0 \ll m_0 \sim A$ one can supress $b \to s\gamma$ and still have light gauginos. However, also LFV processes are suppressed for this parameter space.

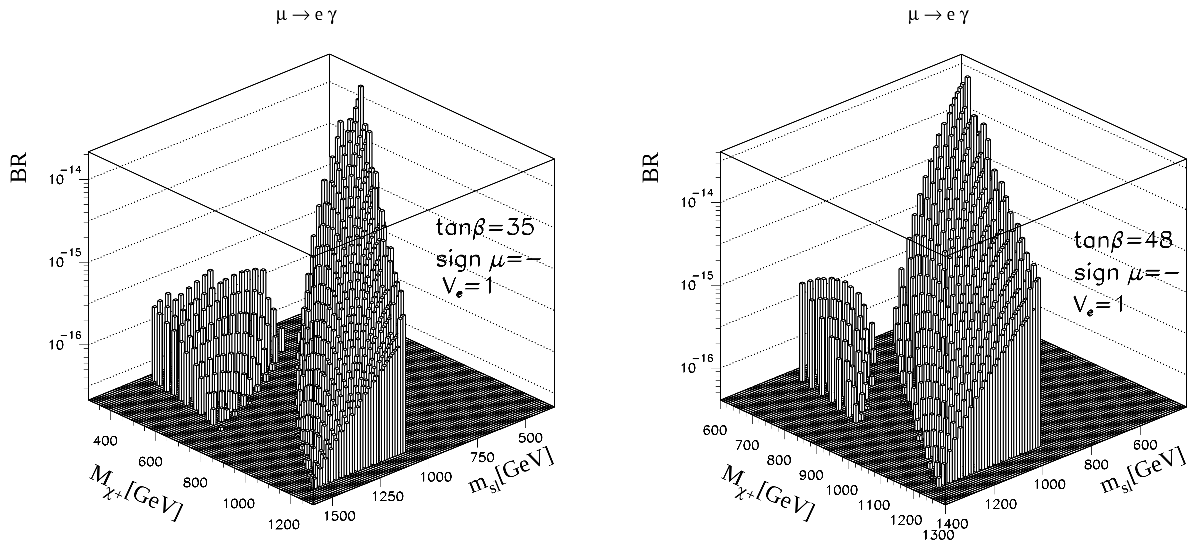

FIG. 3. *The branching ratio of $\mu \to e\gamma$ as a function of the lightest charged slepton and the lightest chargino mass assuming that the flavour mixing in the left-slepton sector is vanishing, $V_e = \mathbf{1}$. We have fixed sign(μ) $= -$ and values of tanβ are indicated in the figures. The destructive interference between the gaugino and higgsino contributions suppresses the branching ratio in large regions of the parameter space. The constraints coming from $b \to s\gamma$ are taken into account.*

The situation changes completely if we allow also flavour mixings in the left-slepton sector and allow $sign(\mu)$ to be also positive. Taking $V_e^{ij} = U_e^{ij} = (V_{CKM}^G)^{ij}$ as we predict in our model we plot the $\mu \to e\gamma$ branching ratio for $\tan\beta = 35$ in Fig. 4 and for $\tan\beta = 48$ in Fig. 5. The branching ratios are about two orders of magnitude higher than in Fig. 3 and no cancellation occurs for any sparticle masses. This implies that $\mu \to e\gamma$ is dominated by the sneutrino-chargino contribution. The LFV pattern here is exactly the same as in models with right-handed neutrinos. Notice that for $sign(\mu) = +1$ the sparticle masses are allowed to be much smaller than in the other case. This is because for $sign(\mu) = +1$ the chargino contribution to $b \to s\gamma$ interferes destructively with the SM and charged Higgs contributions. In particular, for $\tan\beta = 48$ and for very small chargino and slepton masses the chargino contribution can be so large that it cancels the SM and H^+ contributions and gives the allowed C_7^{eff} value with an opposite sign. This is seen in Fig. 5 for $sign(\mu) = +1$ in which a small parameter region around $M_{\tilde{\chi}_1^+} \approx 100$ GeV and $m_{\tilde{l}_1} \approx 300$ GeV is allowed. This region can be excluded by collider searches for a very light chargino or by improving the bound on the $\mu \to e\gamma$ branching ratio by a factor of few.

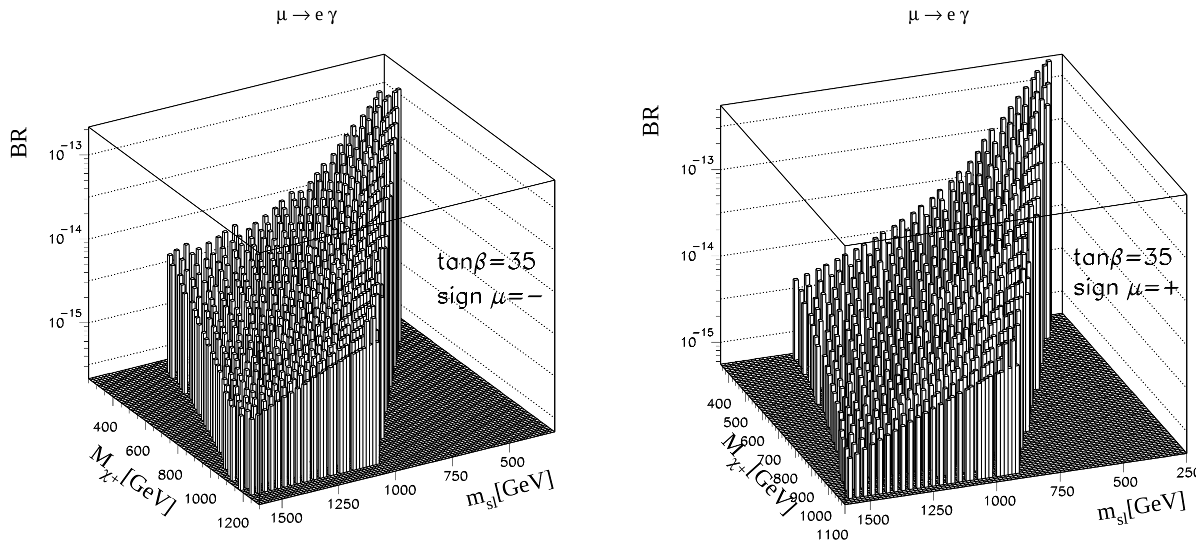

FIG. 4. *The branching ratio of $\mu \to e\gamma$ as a function of the lightest charged slepton and the lightest chargino mass for fixed $tan\beta = 35$. The sign of the μ parameter is indicated in the figures. Here and in the following $V_e^{ij} = U_e^{ij} = (V_{CKM}^G)^{ij}$. The constraints on SUSY mass spectrum coming from $b \to s\gamma$ are taken into account.*

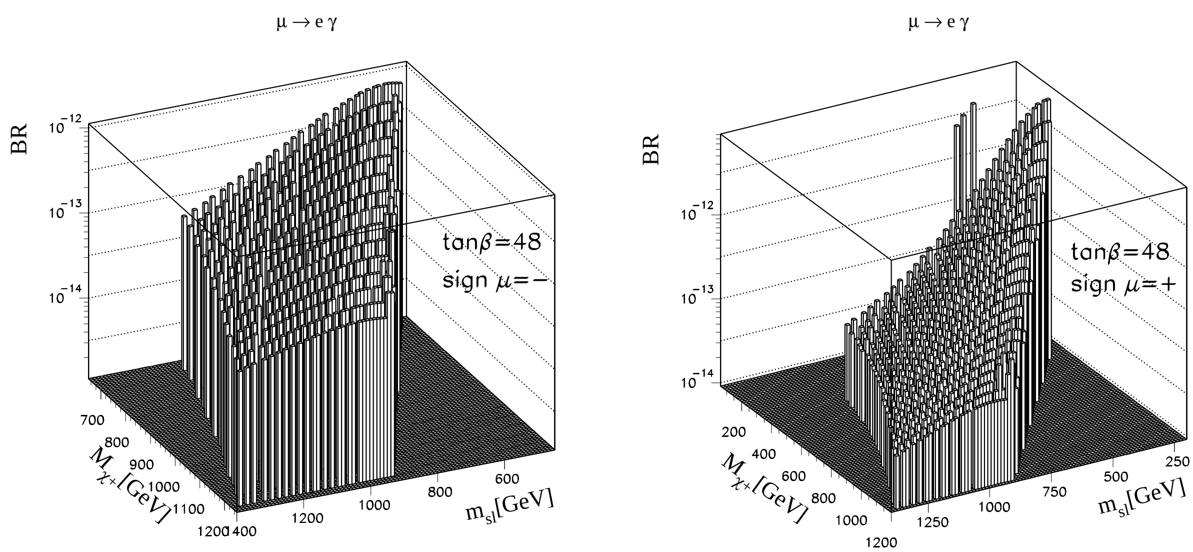

FIG. 5. *The same as in Fig. 4 but for $tan\beta = 48$.*

We have shown that the non-vanishing flavour mixings in the left-slepton sector are crucial to ensure the detectibility of $\mu \to e\gamma$ in the planned experiments. Since there is no experimental information on these mixings the central question to ask now is how small the off-diagonal elements of V_e can be and still allow successful determination of $\mu \to e\gamma$. To analyze this question we plot in Fig. 6 the $\mu \to e\gamma$ branching ratio against the lightest slepton mass $m_{\tilde{l}_1}$ for a fixed $M_2 = 460$ GeV which is roughly the minimal chargino mass for $sign(\mu) = -1$. The curves denoted by a correspond to $V_e = \mathbf{1}$, curves denoted by b to $V_e^{ij} = 0.1 \times U_e^{ij}$, $i \neq j$ and curves denoted by c to $V_e^{ij} = U_e^{ij}$. As can be seen, if the off diagonal elements of V_e are as small as 10% of the corresponding U_e elements then the deep cancellation is superseeded. Let us also mention that for the chosen chargino mass in Fig. 6 the SUSY scale, $M_{SUSY} = \sqrt{m_{\tilde{l}_1} m_{\tilde{l}_2}}$, is $M_{SUSY} \approx 1200$ GeV for the minimally allowed slepton mass. Thus the LFV processes are sensitive to SUSY scale above TeV.

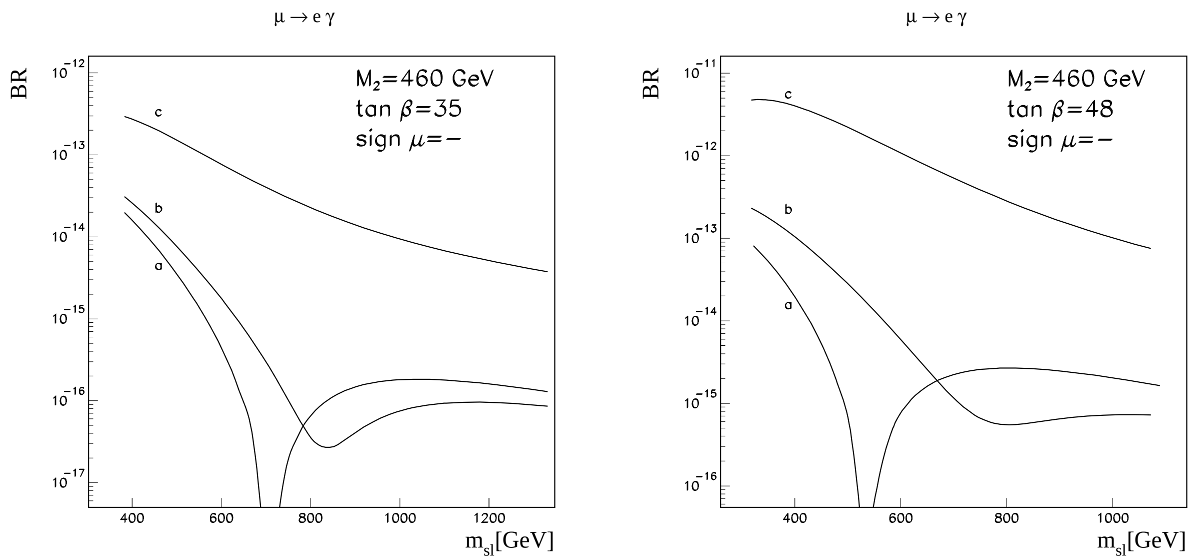

FIG. 6. *The branching ratio of $\mu \to e\gamma$ as a function of the lightest charged slepton mass for fixed wino mass $M_2 = 460$ GeV. We have fixed $sign(\mu) = -$ and values of $tan\beta$ are indicated in the figures. For the curves denoted by (a) $V_e^{ij} = 0$, $i \neq j$, thus they correspond to Fig. 3. For the curves (b) $V_e^{ij} = 0.1 \times U_e^{ij}$ and for the curves (c) $V_e^{ij} = U_e^{ij}$.*

Our calculations show that the rate of $\mu - e$ conversion in nuclei is about 6×10^{-3} times the branching ratio of $\mu \to e\gamma$. The qualitative behaviour of the $\mu - e$ conversion rate with the sparticle masses is the same as in the case of $\mu \to e\gamma$. Thus the results for $\mu - e$ conversion can be obtained by rescaling the figures for $\mu \to e\gamma$. Therefore we do not present any new plots for the $\mu - e$ conversion process. We conclude that the planned $\mu - e$ conversion experiments [16] are as sensitive to our models as the planned $\mu \to e\gamma$ experiments [15].

Finally let us discuss the decay $\tau \to \mu\gamma$. In Fig. 7 we plot the branching ratio of the decay $\tau \to \mu\gamma$ for the same choice of parameters as in Fig. 6. Even for $\tan\beta = 48$ the branching ratio is always below a few times 10^{-9} and unobservable in the planned experiments. Thus if $\tau \to \mu\gamma$ will be discovered at these experiments this implies some other LFV scenario than the one considered in this work.

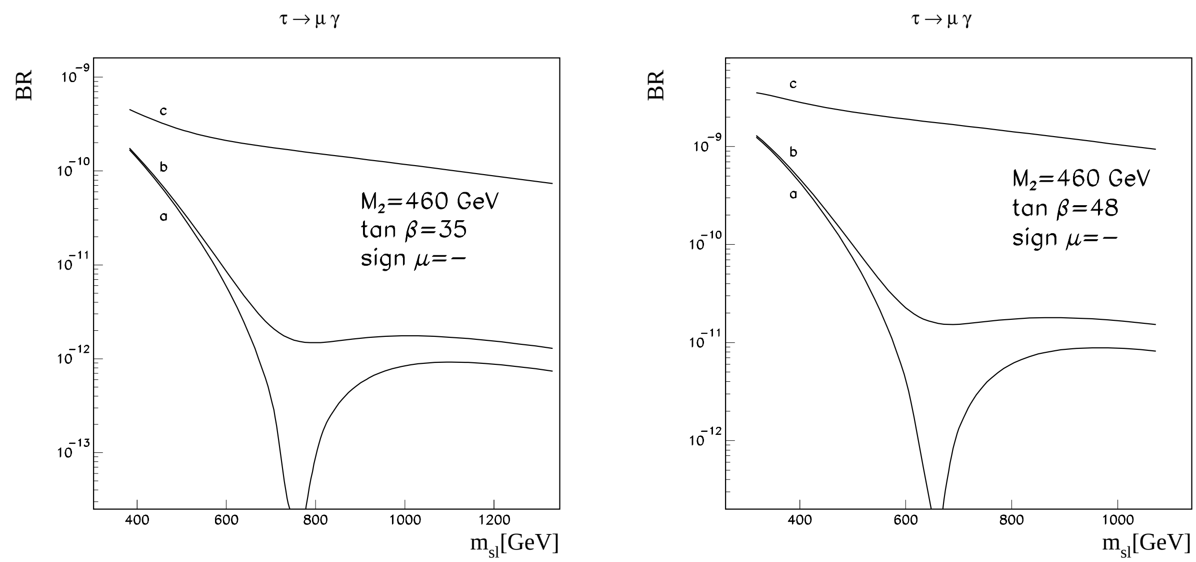

FIG. 7. *The same as in Fig. 6 but for the decay $\tau \to \mu\gamma$.*

V. CONCLUSIONS

Motivated by the sensitivity of the running or approved experiments to flavour violating processes we have studied flavour violation in the minimal SUSY SU(5) GUT at large $\tan\beta$. In this case the flavour mixing occurs both in the left and right slepton and squark mass matrices and are enhanced by the large value of $\tan\beta$. We have calculated the new physics contributions to $K-\bar{K}$ and $B-\bar{B}$ mixings and to the decays $b\to s\gamma$, $\mu\to e\gamma$, $\tau\to\mu\gamma$ and to $\mu-e$ conversion in nuclei. To predict reliably the rates of these processes we have correctly taken into account the measured values of the low energy parameters as well as the constraints on the SUSY particle masses.

We found that in our model the new physics contributions to ΔM_K and ΔM_B are negligible, but might reach a 10% level in ϵ_K if there exist new GUT phases. No useful constraints on the model parameters can be derived from these processes.

The decay $b\to s\gamma$ receives contributions from two sources of flavour violation: from the loops proportional to the CKM matrix elements and from the loops exhibiting new flavour violation in the squark mass matrices. The latter contribution interferes constructively with the dominant chargino contribution. At large $\tan\beta$ the experimental constraints on the $b\to s\gamma$ branching ratio imply stringent constraints on the SUSY particle masses, especially for $sign(\mu)=-1$ as required by Yukawa unification. In this case, the SUSY scale is constrained to be at least TeV. For such a high squark masses the new flavour physics contribution to $b\to s\gamma$ branching ratio is a few percent. Nevertheless, this may induce CP asymmetries considerably larger than in the SM.

There is a competition between the sensitivity of the future LFV experiments to the new flavour physics and the constraints on the SUSY scale coming from the $b\to s\gamma$ branching ratio. If the branching ratio of the decay $\mu\to e\gamma$ will be tested down to 10^{-14} and the SUSY scale is below 1 TeV then, the present scenario predicts that $\mu \to e\gamma$ should be discovered in these experiments. The branching ratio of the decay $\tau \to \mu\gamma$ is, however, predicted to be below a few times 10^{-9} and should not be seen at LHC in the minimal SUSY SU(5).

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