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RELATIONS AMONG NEUTRAL CURRENT COUPLINGS TO TEST THE $SU(2)\otimes U(1)$ GAUGE GROUP STRUCTURE

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A B S T R A C T

We propose a test of the validity of $SU(2)\otimes U(1)$ as the gauge group of weak and electromagnetic interactions. We find that, among the seven empirical neutral current couplings, there are two restrictions which do not rely on a particular model. The relations are

$$2(G_V + G_A) + (\alpha + \beta) + 3(\gamma + \delta) = 0$$

and

$$2\varkappa - (\alpha + \beta) + 3(\gamma + \delta) = 0.$$

where α, β, γ, δ (G_V, G_A) are the couplings measured in neutrino-hadron (electron) scattering and $\varkappa$ is obtained from interference experiments. We also find several inequalities. Comparison with present data is given.

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According to commonly held present ideas, weak and electromagnetic interactions of the elementary fermions, leptons and quarks, are described by a gauge theory [1] based on some underlying non-Abelian group structure. The myriad of models constructed so far, following the gauge strategy, differ on two essential aspects : (i) the choice of the gauge group, (ii) the multiplet assignments for the fermions. However, a common feature of nearly all such models is that they maintain a $SU(2)\otimes U(1)$ substructure, with just four gauge bosons $W^{\pm,o}$ and B (to generate the physical bosons $W^{\pm}$, Z and γ), at least for low and medium energies. After agreeing on this, the models [a "few" typical examples are found in Refs. 1)-5)] differ by craving four, five... quarks and/or leptons and by the right-handed assignments of all fermions to multiplets. More recently, some experimental indications [6] of disagreement with the predictions of typical models have stimulated a new extension of the gauge strategy to larger groups [7].

Existing data for the various neutral current reactions already allow for the investigation of the structure of neutral currents without commitment to specific models. Analyses of the hadronic couplings have been performed for inclusive scattering [8),9)], single pion production [10] and elastic neutrino proton scattering [11]. Such a program is most meaningful at low and medium energies (below the threshold for new flavours), where charged weak and electromagnetic currents are understood. The question we have addressed ourselves is whether the requirement of the gauge group symmetry is able to provide restrictions on these empirical neutral current couplings, without relying on a particular model. In particular, one would like to know how to devise a test of the validity of $SU(2)\otimes U(1)$ as the gauge group of weak and electromagnetic interactions.

In this letter we wish to show that indeed these restrictions exist. Once neutral current couplings are extracted from experiment, there are two relations among them which are a consequence of the existence of an underlying $SU(2)\otimes U(1)$ gauge symmetry for low and medium energies. To reproduce the well established phenomenology of charged current weak interactions in this energy region, the left-handed fermion sector involving the familiar leptons and quarks will be fixed to its conventional assignment. We start showing, from an effective current $\times$ current Lagrangian, that there are seven independent neutral current couplings to be extracted from experiments at low and medium energies. The rôle of neutrino-electron, neutrino-hadron and interference sectors is given. Because of the fact that neutral current couplings only depend on the third component of weak isospin of the fermions, independently of the multiplet to which they belong, there are only five parameters within $SU(2)\otimes U(1)$ gauge theories. Therefore we look for, and find, two relations among the seven neutral current couplings.

CONVENTIONS AND MEASURABLES

We begin by giving a fleeting account of our conventions. For a low energy description, we write the neutral current effective Lagrangian in the form *)

$$\mathcal{L} = -\frac{G}{4\sqrt{2}}\,\varkappa\,[l_\lambda + h_\lambda]\,[l^\lambda + h^\lambda]^+$$

with

$$\begin{aligned} l_\lambda &= \bar{\nu}_e\,\gamma_\lambda(1-\gamma_5)\,\nu_e + \bar{e}\,\gamma_\lambda\,(v_e + a_e\gamma_5)\,e + \dots \\ h_\lambda &= \bar{u}\,\gamma_\lambda(v_u + a_u\gamma_5)\,u + \bar{d}\,\gamma_\lambda(v_d + a_d\gamma_5)\,d + \dots \end{aligned} \qquad (1)$$

We take G, the Fermi constant, to be positive. In the leptonic neutral current, the neutrino coupling is normalized to unity, whereby $\varkappa$ is the effective neutrino-neutrino coupling ; v_f (a_f) refers to the vector (axial) coupling of the fermion f, normalized to that of the neutrino. In a theory in which the interaction is mediated via an intermediate vector boson Z, $\varkappa$ will be proportional to M_Z^{-2}.

The empirical leptonic vector and axial vector couplings, supplied by neutrino-electron scattering experiments, are

$$G_V = \frac{\varkappa}{2}\,v_e \qquad , \qquad G_A = -\frac{\varkappa}{2}\,a_e \qquad (2)$$

The leptonic cross-sections are often [12] parametrized in the form

$$\sigma\,(\nu e \rightarrow \nu e) = C_{\nu e}\,\frac{E_\nu}{GeV}\,10^{-41}\,cm^2 \qquad (3)$$

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*) The μ-e universality and the two component neutrino theory are assumed throughout this work.

where E_ν is the neutrino energy. The reactor experiment [13], $\bar{\nu}_e e \to \bar{\nu}_e e$, gives $C_{\bar{\nu}_e e}$, while CERN experiments [14] give information on $C_{\nu_\mu e}$ and $C_{\bar{\nu}_\mu e}$.

In a model independent description of neutrino-hadron interactions one introduces four measurable couplings, usually denoted [8] by α, β, γ, and δ. These are related to the couplings in Eq. (1) through

$$\alpha + \gamma = \varkappa\, v_u \qquad -(\beta + \delta) = \varkappa\, a_u$$

$$-\alpha + \gamma = \varkappa\, v_d \qquad \beta - \delta = \varkappa\, a_d \tag{4}$$

They give the isovector vector (α) and axial (β) couplings and the isoscalar vector (γ) and axial (δ) couplings. Present information on these four couplings will be discussed below.

Counting the number of measurable quantities, we find that there are seven, viz., α, β, γ, δ (given by ν hadron experiments), G_V, G_A (determined in ν electron experiments) and $\varkappa$. The latter quantity, in practice, will be supplied by interference experiments without neutrinos, such as atomic experiments or $e^+e^- \to \mu^+\mu^-$, etc. For example, the quantity Q, measured in the first generation parity violation experiments in heavy atoms is given by [15]

$$Q = -2\,\frac{G_A}{\varkappa}\,\left[3\gamma A + \alpha(2Z - A)\right] \tag{5}$$

where A and Z are respectively the massic and atomic numbers. Any other observable associated with interference phenomena will equally contain $\varkappa$, apart from some of the other six empirical couplings.

In the realm of SU(2)×U(1) models, the elementary fermions are placed in multiplets with respect to the weak isospin. We may distinguish between two types of models [15] :

<u>Type 1</u> : a) There is no mixing of elementary fermions in the states ;
b) There is mixing in a state "i" and the orthogonal linear combination containing the same fermions has the same third component of weak isospin as the state "i" itself. All mixing angles drop out in the neutral current.

Type 2 : The third component of weak isospin of the orthogonal linear combination differs from that of "i". The neutral current will depend on specific mixing angles.

The couplings appearing in Eq. (1) are given by

$$v_f = 2\left(I_{3R}^f + I_{3L}^f\right) - 4 \sin^2\theta_W Q_f$$

$$a_f = 2\left(I_{3R}^f - I_{3L}^f\right) \tag{6}$$

Here f denotes the elementary fermion, Q_f is its charge ($Q_u = 2/3$, $Q_e = -1$, etc.) and I_{3R}^f (I_{3L}^f) is the true third component of the weak isospin for models of type 1. Otherwise I_{3R}^f and/or I_{3L}^f may be considered as a continuous variable. Finally θ_W is the Weinberg (unification) angle.

There is a further global parameter in these theories, viz.

$$\lambda = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \tag{7}$$

where M_W(M_Z) is the mass of the charged (neutral) intermediate boson. The specification of λ in the theory needs a particular mechanism to generate masses for W and Z bosons. No specific mechanism is assumed here, and λ is therefore an additional parameter in the theory.

By fixing the left-handed fermion sector involving the familiar leptons and quarks to its conventional assignment in order to reproduce the well known low energy charged current phenomenology, we have $I_{3L}^\nu = -I_{3L}^e = I_{3L}^u = -I_{3L}^d = \frac{1}{2}$. Thus there are <u>five unknowns</u> in SU(2)×U(1) gauge theories, viz., λ, $\sin^2\theta_W$, and I_{3R}^f ($f = u,d,e$). Since there are <u>seven independent measurables</u> we expect two relations among the measurable quantities.

THE RELATIONS

Consider first only the neutrino sector. Its theoretical description depends on a single parameter, namely λ. The "experimental" quantity associated with this sector is $\varkappa$. We find that in all $SU(2)\otimes U(1)$ gauge theories

$$\varkappa = \lambda \tag{8}$$

Next we introduce the electron, whereby there are three theoretical unknowns (λ, I_{3R}^{e} and $\sin^2\Theta_W$) and three measurable quantities (G_V, G_A and $\varkappa$). Thus the leptonic sector alone does not give any restriction among measurables, but we find an inequality to be satisfied by them, viz.

$$G_V + G_A = \lambda\left(-1+2\sin^2\Theta_W\right) \Rightarrow -\varkappa \leq G_V + G_A \leq \varkappa \tag{9}$$

It is interesting to note that due to positivity of λ the sign of $G_V + G_A$ is positive for $\sin^2\Theta_W > \frac{1}{2}$ and negative for $\sin^2\Theta_W < \frac{1}{2}$. Translated into cross-sections, Eq. (9) provides a lower limit on $\varkappa$

$$\varkappa^2 \geq 0.94\left[3\, C_{\nu_\mu e} - C_{\bar{\nu}_\mu e}\right] \geq 0 \tag{10}$$

where the C's were defined in (3). The quantities G_V and $\varkappa$ are also relevant for description for the weak effects [16] in $e^+e^-\to\mu^+\mu^-$. At sufficiently low energies where the Z propagator effect is not substantial *) we find

$$R^{\mu^+\mu^-} = \frac{\sigma(e^+e^-\to\mu^+\mu^-)}{\sigma(e^+e^-\to\mu^+\mu^-)_{QED}} \approx 1 - \frac{G}{\sqrt{2}\,\pi\alpha}\,\frac{G_V^2}{\varkappa}\,s \tag{11}$$

where s is the square of c.m.s. energy and α is the fine structure constant. From (9) and (10) we find

$$\frac{G_V^2}{\varkappa} \leq \frac{G_V^2 + G_A^2}{\varkappa} \leq \sqrt{0.94}\,\frac{C_{\nu_\mu e} + C_{\bar{\nu}_\mu e}}{\sqrt{3\, C_{\nu_\mu e} - C_{\bar{\nu}_\mu e}}} \tag{12}$$

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*) The magnitude of correction due to Z propagator may be estimated.

We now consider the neutrino-quark sector. Here there are four theoretical unknowns (λ, $\sin^2\Theta_W$, I^u_{3R} and I^d_{3R}) and four measurable quantities (α, β, γ and δ). Eliminating the right-handed couplings we find

$$2\lambda = \alpha + \beta - 3(\gamma + \delta) \tag{13}$$

$$\tan^2\theta_W = -3\,\frac{\gamma+\delta}{\alpha+\beta} \tag{14}$$

From positivity of the left-hand side of relations above, it follows

$$2\varkappa \geq \alpha + \beta \geq 0 \quad , \quad -\frac{2}{3}\varkappa \leq \gamma + \delta \leq 0 \tag{15}$$

These inequalities should serve in resolving [if SU(2)⊗U(1) gauge group is accepted] the isovector-isoscalar ambiguity discussed in the literature [8].

When the information from different sectors is put together we find that there are two relations among the measurable quantities. These read

$$\alpha + \beta + 3(\gamma + \delta) + 2(G_V + G_A) = 0 \tag{16}$$

and

$$\alpha + \beta - 3(\gamma + \delta) - 2\varkappa = 0 \tag{17}$$

Relation (16) combines the information extracted from neutrino-hadron and neutrino-electron experiments, whereas relation (17) needs information on neutrino-hadron and interference experiments. If any of these relations should be violated experimentally we would know that there is no hope for SU(2)⊗U(1) theories. We emphasize again that these relations should be checked at low and medium energies, where possible new degrees of freedom are not excited.

COMPARISON WITH DATA

Clearly relations (16) and (17) provide a very clean test of the underlying SU(2)×U(1) group structure. Present data, however, are not precise enough to allow us to check them. Therefore, we shall proceed by assuming their validity to obtain the allowed regions for couplings and group parameters, using the available data.

Hung and Sakurai [8] have outlined a method for determining (up to some ambiguities) the parameters α, β, γ and δ from deep inelastic neutrino-hadron data. Ecker has made the analysis of single pion production by neutrinos [10] and elastic neutrino-proton scattering [11]. From the deep inelastic data [17], and combining neutrino and antineutrino results on iso-scalar targets, we fix the sum of the squares of $\alpha+\beta$ and $\gamma+\delta$, viz.

$$(\alpha+\beta)^2 + (\gamma+\delta)^2 = \rho^2 , \quad \rho = 1.37 \pm 0.19 \tag{18}$$

In Fig. 1, we plot the result (18) in the plane $\gamma+\delta$ vs. $\alpha+\beta$. Due to the condition (15) only one quadrant in the plane is relevant for us.

Next we use the published leptonic data [13),14] to obtain our relevant combination $G_V + G_A$ given by

$$-0.68 \leq G_V + G_A \leq 0 \tag{19}$$

independent of the quadratic ambiguity. Thus using Eq. (9) we obtain the interesting result

$$\sin^2\theta_W \leq 1/2 \tag{20}$$

for all SU(2)×U(1) theories, on the basis of neutrino-electron data alone. Inserting (19) into Eq. (16) we find that the allowed domain in Fig. 1 is further restricted to lie between the lines $(\alpha+\beta) + 3(\gamma+\delta) = 0$ and $(\alpha+\beta) + 3(\gamma+\delta) = 1.36$. Requiring that the line (17) should go through the allowed domain in Fig. 1 gives restrictions on $\varkappa$

$$0.59 \leq \varkappa \leq 1.48 \tag{21}$$

REFERENCES

1) S. Weinberg - Phys.Rev.Letters 19, 1264 (1967) ; Revs.Modern Phys. 46 255 (1974) ;

A. Salam - Elementary Particle Physics, Ed. N. Svartholm (Almkvist and Wiksell, Stockholm, 1968).

2) J.C. Pati and A. Salam - Phys.Rev. D8, 1240 (1973) ;

H. Georgi and S.L. Glashow - Phys.Rev.Letters 32, 433 (1974) ;

H. Fritzsch and P. Minkowski - Ann.Phys.(N.Y.) 93, 222 (1975) ;

F. Gürsey, P.Ramond and P. Sikivie - Phys.Letters 60B, 177 (1976) ;

F. Gürsey and P. Sikivie - Phys.Rev.Letters 36, 775 (1976) ;

J.C. Pati and A. Salam - Preprint IC/76/76 (1976).

3) A. de Rujula, H. Georgi and S.L. Glashow - Phys.Rev.Letters 35, 69 (1975) ;

S. Pakvasa, W.A. Simmons and S.F. Tuan - Phys.Rev.Letters 35, 702 (1975) ;

H. Fritzsch, M. Gell-Mann and P. Minkowski - Phys.Letters 59B, 156 (1975).

4) A. de Rujula, H. Georgi, S.L. Glashow and H.R. Quinn - Revs.Modern Phys. 46, 391 (1974) ;

Y. Achiman, K. Koller and T. Walsh - Phys.Letters 59B, 961 (1975) ;

R.M. Barnett - Phys.Rev. D11, 326 (1975) ; D13, 671 (1976) ;

H. Fritzsch and P. Minkowski - Phys.Letters 63B, 99 (1976) ;

F.J. Yndurain - Phys.Letters 63B, 195 (1976).

5) S.L. Glashow and S. Weinberg - Preprint HUTP-76/A158 (1976).

6) P. Baird et al. - Nature 264, 528 (1976).

7) R.N. Mohapatra and D.P. Sidhu - BNL-22148 (1976) ;

A. de Rujula, H. Georgi and S.L. Glashow - HUTP-77/A002 (1977).

8) P.Q. Hung and J.J. Sakurai - Phys.Letters 63B, 295 (1976).

9) G. Rajasekaran and K.V.L. Sarma - Pramana 7, 194 (1976).

10) G. Ecker and R. Fischer - Nuclear Phys. B117, 209 (1976).

11) G. Ecker - Universität Wien Preprint (1977).

12) See, for example, B.W. Lee - Summary talk at the Neutrino Conference, ν'76, Aachen 1976, to appear in Proceedings.

13) F. Reines, H.S. Gurr and H.W. Sobel - Phys.Rev.Letters 37, 315 (1976).

14) The Gargamelle Collaboration, J. Blietschau et al. - Nuclear Phys. B114, 189 (1976).

15) J. Bernabéu and C. Jarlskog - CERN Preprint TH. 2206 (1976), to appear in Nuovo Cimento.

16) C. Jarlskog and F.J. Yndurain - Phys. Letters 63B, 215 (1976).

17) The Gargamelle Collaboration, J. Blietschau et al. - Nuclear Phys. B118, 218 (1977).

18) The Aachen-Padova Collaboration - to be published.

FIGURE CAPTIONS

Figure 1 The isoscalar coupling $(\gamma+\delta)$ vs. the isovector $(\alpha+\beta)$. The allowed region between full lines is obtained from deep inelastic neutrino data. Neutrino-electron data used in relation (16) give the region between broken lines. The allowed domain for the hadron couplings is indicated. Relation (17) (dot-dashed lines) restricts possible values of $\varkappa$.

Figure 2 Correlation on the gauge group parameters λ - $\sin^2\Theta_W$, as imposed by present data (full lines : deep inelastic neutrino data ; broken lines : neutrino-electron data) plus relations (16) and (17).

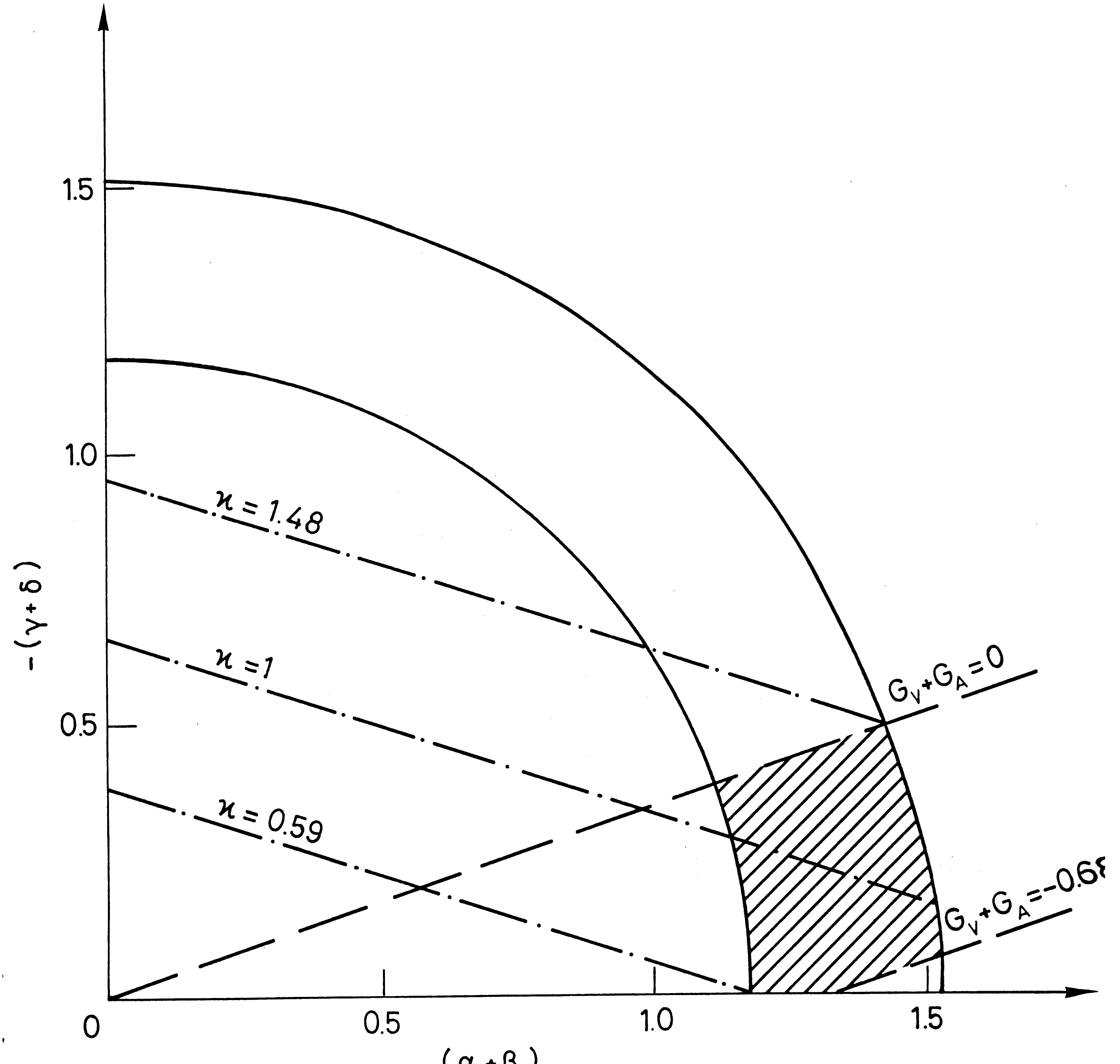
1.5
1.0
0.5
0
0.5
1.0
1.5
$-(\gamma+\delta)$
$(\alpha+\beta)$
$\varkappa = 1.48$
$\varkappa = 1$
$\varkappa = 0.59$
$G_V + G_A = 0$
$G_V + G_A = -0.68$

FIG.1

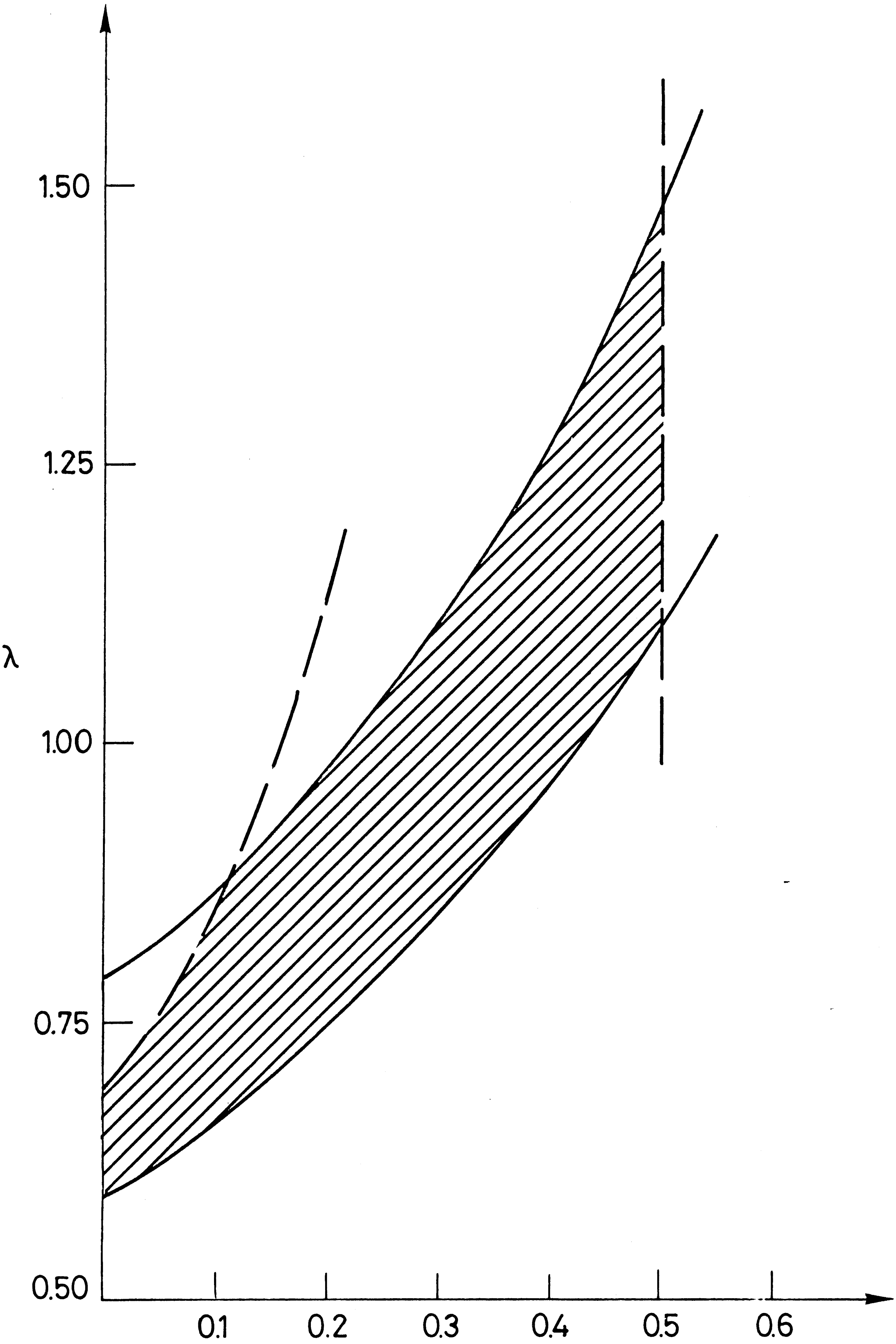

FIG. 2