Ref.TH.2304-CERN

THE AXIAL CURRENT IN EXTENDED SYSTEMS

T.E.O. Ericson and J. Bernabéu *)

CERN - Geneva

A B S T R A C T

We establish a connection between the nuclear axial current and pionic properties in the medium. As a strict consequence of PCAC alone, the time component has its matrix elements determined by s wave pion absorption to leading order in the momentum transfer. The properties of the system are governed by scale parameters associated both with its size and with the local interaction structure. Based on this observation we suggest a locality condition for the part of the axial current which has no pion pole. It follows that the full axial current can be explicitly constructed from the local s and p wave pion source functions. A model illustration is presented.

---

*) On leave of absence from Departamento de Fisica Téorica, Universidad de Barcelona, Barcelona, Spain.

Ref.TH.2304-CERN
5 April 1977

Our purpose in the present article is to provide an explicit construction of the axial current in an extended object (such as a nucleus) and to show that it is naturally linked to the local s and p wave interaction of pions in the system. The problem is therefore to establish a possible connection of the axial current with pionic properties in the nuclear medium. It is no surprise for the reader to say that one of the corner-stones in our considerations is the PCAC relation [1].

For the nucleon the axial current is indeed connected to pions in this way for quasi-elastic processes, provided the momentum transfer is so small that multi-meson excitations are not explored. In this limit the "pionic" form factor of the nucleon is nearly a constant. The system can then be regarded as effectively pointlike, provided the long-range pion pole contribution is first separated out (the pseudoscalar form factor). The axial form factor follows then from the pion source by the Goldberger-Treiman relation [2].

Extended systems like nuclei differ fundamentally from the nucleon in that the leading excitations are not the multi-meson states. Instead there is characteristically a vast number of states below pion threshold. The nuclear size is the leading scale in this problem ; it is not related (at least in any direct manner) to the multi-meson scale. These two scales are distinctly different. If we now choose to consider non-mesic substructures in the nucleus (nucleons, "quasideuterons", etc.) these will characteristically have a mesonic scale rather than a long-range scale like the size of system. This suggests that there is a substantial region of momentum transfers for which the constituents can be considered pointlike in the previous sense. In this region it may therefore be possible to give an explicit program linking the axial current to the pion interaction. Since the local scale parameter must enter into this consideration, we must be prepared to use the local properties of the pion source in the medium in an essential way.

The conventional approach to the study of the axial current in nuclei has implicitly recognized this point. It describes a nucleus as consisting of a set of nucleons in interaction and takes the impulse approximation as a starting point. When the kinematics is close to the quasi-free process on nucleons, this approximation has proved to be quite good (β decay, μ capture). The corrections to this picture involve the interacting meson field and are introduced as "exchange currents" [3]. This amounts to add a two-nucleon current (predominantly from pion exchange) to the single nucleon current of the impulse approximation. In a recent paper, J. Delorme et al. [4] have shown that, for static nucleons, the axial current can be related to the pionic field in the nucleus, in such a way that all modifications to the impulse approximation are identical for the axial current and for the pion source. From this important result it follows that the axial current in this case is a pionic phenomenon. It has in particular the consequence that the renormalization of the axial coupling constant in the nuclear medium is seen as a result of the change in the pion coupling. A close parallel between the behaviour of the electrostatic displacement vector $\vec{D}$ and the axial vector $\vec{A}$ has been established [5], in the sense that both acquire an induced "polarization" coming from the induced "dipoles" in the medium.

While the picture of Delorme et al. [4] can to some extent be generalized also to the nearly static case, it leaves the question of the axial current at large energy transfers open (large on the nuclear scale). In this case one expects the impulse approximation to fail severely as it is known to do in π capture [6]. It is then necessary to take a different viewpoint of the problem. As outlined before, the emphasis will be put on the local distribution of the pionic source in the medium and an explicit reference to how and by what these pionic properties are generated will be avoided.

In the following we will proceed in two steps. We first show that, as a consequence of PCAC, the time component A_o of the axial current has its matrix elements completely determined by s wave pion absorption on the system to leading order in the momentum transfer $\vec{q}$. This rigorous result from PCAC determines a piece of the current which contains both pion-pole and pole-free contributions. Its pole-free part would not exist if the current were built from individual nucleon currents in the limit of very massive nucleons. For large values of the energy transfer q_o (on the nuclear scale), the s wave pion absorption becomes very important, however, and to speak in terms of the impulse approximation is then meaningless.

When the energy transfer q_o is large, it is not possible to derive the spatial part of the axial current from PCAC alone, even for a momentum transfer $\vec{q} = 0$. However, since for small systems the pole-free part of this current is closely related to the p wave pion amplitude, we will conjecture "axial locality". By this we mean that the pole-free axial spatial current $\tilde{\vec{A}}$ is in essence given by the <u>local</u> p wave interaction in the medium. This local connection is established in terms of the full pionic source at the same point, and it will be discussed in detail below. This conjecture provides a natural basis for a unified discussion of the axial current in terms of local pion properties in the system. As a consequence, it becomes essential to understand the physics of off-shell pions in the nuclear medium in order to quantitatively understand the axial current.

The PCAC (partially conserved axial current) requirement for the axial current $A_\mu(x)$ in terms of the pion field operator $\Phi(x)$ is

$$\partial^\mu A_\mu(x) = f_\pi m_\pi^2 \Phi(x) \tag{1}$$

where f_π is the charged pion decay constant $f_\pi = 0.94\ m_\pi$. It is illuminating for the further discussion to rewrite Eq. (1) identically in such a way that the "pion pole contribution" to $A_\mu(x)$ becomes explicit. If the pole free part of the axial current is denoted by $\tilde{A}_\mu(x)$ we have

$$A_\mu(x) \equiv A_\mu^{pole}(x) + \tilde{A}_\mu(x) = -f_\pi \partial_\mu \Phi(x) + \tilde{A}_\mu(x) \tag{2}$$

Equation (1) then imposes the following condition for $\tilde{A}_\mu(x)$

$$\partial^\mu \tilde{A}_\mu(x) = f_\pi j(x) \tag{3}$$

where $j(x) = (\partial^\mu \partial_\mu + m_\pi^2)\Phi(x)$ defines the pion source. It is of great importance for the following to note that the source $j(x)$ is not only the local emission (or absorption) vertex for pions, but that it also contains the induced source terms generated by pion re-scattering in the medium. The pole contribution A_μ^{pole} has the following form in momentum space :

$$A_\mu^{pole}(q_0,\vec{q}) = -i f_\pi q_\mu \frac{M(q_0,\vec{q})}{m_\pi^2 - q^2} \tag{4}$$

where $M(q_0,\vec{q})$ is the off-shell transition amplitude for pions. In particular, $M(q_0,\vec{q})$ contains all re-scattering effects mentioned above. The presence of the "free" pion pole in Eq. (4) does not mean that the transition amplitude is the "free" on-shell amplitude, which would be a grossly incorrect interpretation. In fact, the result (4) is not the pion-pole contribution as determined by a dispersive approach.

The local pole-free current associated with the energy transfer q_0 satisfies the relation

$$q_0 \tilde{A}_0 (q_0, \vec{x}) + i \vec{\nabla} . \tilde{\vec{A}} (q_0, \vec{x}) = - i f_\pi j(q_0, \vec{x}) \tag{5}$$

Clearly Eq. (5) is a restriction on the longitudinal current only.

The volume integral of Eq. (5) gives the time component of the axial current at $\vec{q} = 0$:

$$\tilde{A}_0 (q_0, \vec{q} = 0) = - i \frac{f_\pi}{q_0} M(q_0, \vec{q} = 0) \tag{6}$$

The sum of the pole term (4) and the term (6) determines completely the time component of the axial current as a pion amplitude

$$A_0 (q_0, \vec{q} = 0) = - i f_\pi \frac{m_\pi^2}{q_0} \frac{M(q_0, \vec{q} = 0)}{m_\pi^2 - q_0^2} \tag{7}$$

One notes that only s wave pions contribute in Eq. (7) because of the spherical symmetry for $\vec{q} = 0$. It corresponds in the nuclear case to s wave pion absorption at the off-shell value $q_0 \neq m_\pi$. It is remarkable that this contribution is very small (a velocity term) for a single nucleon. For appreciable values of q_0, it is well known that M becomes very important, so that A_0 is fundamentally outside of any impulse approximation approach. To our knowledge this part of the current has not previously been discussed in the literature. The reason is probably that it has been considered as a very small ingredient.

Equation (7) is therefore a rigorous consequence of PCAC for small $\vec{q}$. In order to gain further insight into the relation of the axial current to the pion source we construct the source function $j(q_0,\vec{x})$ as s and p waves taken locally

$$j(q_0, \vec{x}) = s(q_0, \vec{x}) - \vec{\nabla} \cdot \vec{S}(q_0, \vec{x}) \tag{8}$$

We emphasize that the transition densities $s(\vec{x})$ and $\vec{S}(\vec{x})$ contain the full effects of pion rescattering in the medium [*].

If the system were pointlike we can introduce the effective parametrization

$$s(q_0, \vec{x}) = s(q_0)\,\delta(\vec{x}) \quad , \quad \vec{S}(q_0, \vec{x}) = \vec{S}(q_0)\,\delta(\vec{x}) \tag{9}$$

It follows immediately from Eqs. (5) and (8) that a pointlike axial current $\tilde{A}_\mu(q_0,\vec{x}) \propto \delta(\vec{x})$ is given by the s and p wave parameters

$$\tilde{A}_0(q_0, \vec{x}) = -i\,\frac{f_\pi}{q_0}\, s(q_0)\,\delta(\vec{x}) \tag{10a}$$

$$\tilde{\vec{A}}(q_0, \vec{x}) = f_\pi\, \vec{S}(q_0)\,\delta(\vec{x}) \tag{10b}$$

For a single nucleon current, Eqs. (10) are automatically satisfied if one imposes (and they are equivalent to) the Goldberger-Treiman relation [2]

$$g_A = f_\pi \frac{g_r}{\sqrt{2}\,M} \tag{11}$$

This is the content of Eqs. (10) for a pointlike system. The axial current is the equivalent of a pion s and p wave for the time and space components, respectively.

For an extended system such a simple relation with the global pion amplitudes cannot be expected to be true. In this case an s wave or p wave with respect to a local point in the system may even correspond

---

*) The terms in Eq. (8) correspond to an expansion with respect to the short-range scale (higher derivative terms are for this reason neglected). The $\vec{x}$ dependence of these source functions reflects the long-range scale in the medium.

to a very high partial wave with respect to its centre of mass. If one instead takes the attitude that Eqs. (10) express that the (pole free part of the) axial current is given by the local matter density, it becomes reasonable to insist that this should be so also in an extended system. The equivalent relations of Eqs. (10) in this case are particularly easy to visualize, if we think of the extended system as consisting of small, discrete constituents. From Eqs. (8) and (5) we then propose the following explicit form for $\tilde{A}_\mu(q_o,\vec{x})$:

$$\tilde{A}_o(q_o,\vec{x}) = -i\,\frac{f_\pi}{q_o}\,s(q_o,\vec{x}) \tag{12a}$$

$$\tilde{\vec{A}}(q_o,\vec{x}) = f_\pi\,\vec{S}(q_o,\vec{x}) \tag{12b}$$

Equations (12) express a hypothesis of "axial locality" : the pole free part of the axial current is given by the <u>local</u> s and p wave parts of the pion source.

We emphasize that the conjecture (12) is consistent with PCAC, but it does not follow from it for an extended system. In addition it should be clearly realized that the proposed range of validity of Eqs. (12) is on the nuclear scale of energies and distances : the short range behaviour in the source cannot be explored [see after Eq. (8)]. The implication of the locality principle is that it provides a well-defined model independent frame for calculation of the nuclear axial current by insisting that it is identical to the local pion current. All that is needed is therefore the correct pion sources $s(\vec{x})$ and $\vec{S}(\vec{x})$ whether these are derived from a particular theoretical model or are extracted from the physical π nuclear interaction.

In certain cases the consequences of "axial locality" can be obtained using weaker assumptions. If the PCAC relation (5) is multiplied by $\vec{x}$ and integrated by parts, it follows from the pion source (8) that

$$\int d^3x\;\tilde{\vec{A}}(q_o,\vec{x}) = \int d^3x\;\vec{x}\left[f_\pi\,j(q_o,\vec{x}) - i q_o\,\tilde{A}_o(q_o,\vec{x})\right]$$

$$= f_\pi\int d^3x\;\vec{S}(q_o,\vec{x}) + \int d^3x\;\vec{x}\left[f_\pi\,s(q_o,\vec{x}) - i q_o\tilde{A}_o(q_o,\vec{x})\right] \tag{13}$$

Since physically for $\vec{q}=0$, the very existence of $\tilde{A}_0$ is linked to s wave pion absorption [see Eq. (6)], it is natural to assume this link to hold also locally, with Eq. (12a) valid for any $\vec{x}$. With this partial locality assumption, the last term in Eq. (13) cancels exactly with the result

$$\int d^3x \ \vec{\tilde{A}}(q_0,\vec{x}) = f_\pi \int d^3x \ \vec{S}(q_0,\vec{x}) \tag{14}$$

Therefore, the use of Eq. (12a) for locality for the time component automatically implies that the space part of the axial current is determined from the <u>local</u> p wave π interaction in the medium to leading order in $\vec{q}$.

A case for which Eq. (14) is valid as has recently been discussed in the literature by Delorme et al. [4]. They consider a system of static nucleons with meson exchange terms in the limit $q_0=0$. Then both $q_0 A_0 \to 0$ and the s wave pion interaction $s(q_0,\vec{x}) \to 0$, so that the equivalent of Eq. (14) results as a strict consequence of PCAC. Their analysis of the structure of the nuclear axial current shows that it is natural to identify the local $\vec{\tilde{A}}(\vec{x})$ with the p wave pion source term $\vec{S}(\vec{x})$ for this special case $q_0=0$.

From the axial locality (12) we have obtained an explicit construction of the axial current. The PCAC relation (5) is, however, a restriction on the longitudinal component of the axial current only. We are in particular free to add transverse terms of the type $\vec{A}_t(\vec{x}) = \vec{\nabla} \wedge \vec{B}(\vec{x})$ to the axial current. They cannot be connected with pionic phenomena. For vanishing momentum transfer $\vec{q}=0$, such terms do not contribute. The axial locality (12b) is in particular equivalent to the assumption that there are <u>no</u> such transverse components in the axial current.

The considerations in this note were triggered by our wish to discuss μ capture at very large energy transfer $q_0 \simeq m_\mu$ [7]. In this case, the s wave pion source is important, contrary to the static limit. In addition the pion source for this energy $w_\pi = m_\mu$ is almost the one for physical pions at threshold, at least for light nuclei.

As an illustration of the content of Eqs. (12) we give the explicit expression for the axial current in a model. Consider a direct absorption (or emission) vertex for a pion in the nuclear medium. We describe this by a local s and p wave transition (pseudo)potential

$$j_{direct}(\vec x) = \sigma(\vec x) - \vec\nabla\,\vec\Sigma(\vec x)$$

In addition the pion has a source connected to its rescattering in the nuclear medium. We describe this by a non-local optical potential of the usual type dominated by local s and p wave pion scattering

$$2\, m_\pi V = c(\vec x) + \vec\nabla\cdot\alpha(\vec x)\,\vec\nabla$$

Although of little importance to our argument, we note that repeated absorption and emission of pions also generates scattering in the medium. Such effects are included into the phenomenological optical potential above to the extent consistent with no double counting [8]. With these interactions the pion field in the medium, associated with total energy q_0, is denoted by $\Phi(q_0,\vec x)$ and the full pion source functions are given by

$$s(q_0,\vec x) = \sigma(\vec x) - c(\vec x)\,\Phi(q_0,\vec x)$$

$$\vec S(q_0,\vec x) = \vec\Sigma(\vec x) + \alpha(\vec x)\,\vec\nabla\Phi(q_0,\vec x) \qquad (15)$$

From Eqs. (2) and (12) the total axial current is constructed as

$$A_0(q_0,\vec x) = -i\,\frac{f_\pi}{q_0}\left[q_0^2\,\Phi(q_0,\vec x) + \sigma(\vec x) - c(\vec x)\,\Phi(q_0,\vec x)\right]$$

$$\qquad (16)$$

$$\vec A(q_0,\vec x) = f_\pi\left[-\vec\nabla\Phi(q_0,\vec x) + \vec\Sigma(\vec x) + \alpha(\vec x)\,\vec\nabla\Phi(q_0,\vec x)\right]$$

When written in this form, the axial current shows an interesting analogy with the displacement vector $\vec D(\vec x)$ in an inhomogeneous diaelectric. This analogy has been pointed out [5] for the static case $q_0 = 0$. In a medium with a permanent dipole density $\vec d(\vec x)$, the three terms in Eq. (16) would correspond to

$$\vec D(\vec x) = \vec E(\vec x) + \vec d(\vec x) + \vec P(\vec x)$$

The contributions originate from the external field $\vec{E} \rightleftarrows -\vec{\nabla}\Phi$, the permanent sources in the medium $\vec{d} \rightleftarrows \vec{\Sigma}$ and the polarization vector $\vec{P} \rightleftarrows +\alpha\,\vec{\nabla}\Phi$ due to the induced dipoles. We see that the same kind of structure remains for the non-static case $q_0 \neq 0$. The time component of the axial current has a completely analogous structure, but it is now associated with s waves and not p waves (dipoles). However, this component has no electromagnetic correspondence.

ACKNOWLEDGEMENTS

We have benefited from a number of interesting discussions on these topics with Professors M. Ericson and C. Jarlskog.

REFERENCES

1) Y. Nambu - Phys.Rev.Letters 4, 380 (1960) ;
M. Gell-Mann and M. Lévy - Nuovo Cimento 16, 605 (1960).

2) M.L. Goldberger and S.B. Treiman - Phys.Rev. 111, 354 (1958).

3) M. Chemtob and M. Rho - Nuclear Phys. A163, 1 (1971).

4) J. Delorme, M. Ericson, A. Figureau and C. Thévenet - Ann.Phys. 102, 273 (1976).

5) M. Ericson - Lectures given at the International School "Interface between Particle Physics and Nuclei", Erice (September 1976).

6) J. Le Tourneux - Nuclear Phys. 81, 665 (1966).

7) J. Bernabéu, T.E.O. Ericson and C. Jarlskog - CERN Preprint TH. 2301 (1977).

8) T. Mitzutani and D.S. Koltun - University of Rochester Preprint UR-602, COO-2171-69 (1976) ;
A.S. Rinat - Weizmann Institute Preprint WIS-77/9-Ph (1977).