IMPLICATIONS OF PARITY VIOLATION IN ATOMS FOR GAUGE THEORIES

J. Bernabéu and C. Jarlskog[+)]

CERN -- Geneva

A B S T R A C T

The knowledge to be gained on neutral currents from parity violating observables in heavy atoms is studied. After isolating the relevant couplings, the major part of the analysis is done within the framework of $SU(2) \otimes U(1)$ unified gauge theories of weak and electromagnetic interactions. The leptonic and hadronic sectors of these models are studied separately, using the available information from neutrino physics, to impose restrictions on the mass of the neutral intermediate boson and the unification angle. The observable in atoms, which provides a link between the two sectors, is found to be powerful in discriminating among models.

[+)] Permanent address after August 1, 1976 : Department of Physics, University of Bergen, Norway.

1. INTRODUCTION

Three years have elapsed since the experimental discovery[1] of the weak neutral currents in neutrino and antineutrino induced reactions. Theoretically, the central issue involved is whether these currents are due to unification of weak and electromagnetic interactions and, if so, which is the correct unified theory.

During the past years, the quantitative experimental information has been acquired slowly and it has been limited to a few types of processes[2]. Recently, however, the experimental picture is changing rapidly[3] and knowledge is being gained on a variety of neutral current processes. This has led several authors[4-7] to take a fresh look at the consequences of neutrino-hadron interactions for various fashionable theoretical models based on the gauge strategy. Independent of the gauge theory prejudice, the question of isolating the space-time and isospin properties of the hadronic neutral currents, using inclusive neutrino data, has been considered by Quang Hung and Sakurai[8]. Other methods, such as the study of nuclear transitions[9] and diffractive neutrino production of vector mesons[10], have also been proposed.

The gross features of neutrino neutral current data[3] seem to agree with the predictions of the popular Weinberg-Salam (abbreviated W-S) model[11] with four leptons and as many quark flavours[12]. However, there are indications that the true theory is more involved than the W-S model, viz.

i) The μ-e events[13] from SLAC, believed to be the attribute of a heavy lepton. Such a lepton may be accommodated in the W-S model if it is of the sequential type (i.e. it has its own neutrino and lepton number) and provided more quarks are introduced.

ii) The reported[14] antineutrino charged current anomaly observed in counter experiments at Fermilab. Although different observables, such as the y distribution and the ratio of antineutrino-to-neutrino cross-sections, seem to prove that the anomaly is there, the situation is not completely settled[15]. This issue should get resolved in the near future.

These and other phenomenological reasons, together with theoretical arguments, have led to numerous proposals of alternative models. Present data on neutrino and antineutrino interactions do not (quite) distinguish[4-7] among these models, apart from the fact that the pure vector model[16] [or pure axial vector[17]] for hadrons is ruled out[18].

The search for parity violating effects in atomic physics has recently obtained a rapid development. It opens a new and completely independent source of information on weak neutral currents. These effects are a signal of a new interaction between charged leptons and hadrons. Such an interaction is expected if the neutrino-hadron scattering events and the neutrino-electron scattering events are believed to be mediated via the exchange of the same heavy neutral boson Z. As several experiments[19] are approaching the level which is expected in certain gauge models, it is useful to study the restrictions that the atomic results will impose on the theoretical models. In this paper we wish to point out[20] that the atomic physics experiments are going to play a substantial rôle both for the determination of hadronic coupling constants and in distinguishing among different gauge theory models. As an illustration, theories with similar predictions for neutrino neutral current scattering can give completely different results for parity violation in atoms (sometimes even with opposite signs).

This paper is organized as follows: In section 2 we isolate the quantity (denoted by Q) measured in the first generation atomic experiments. This quantity is made-up of three factors, namely the electron axial vector coupling a_e, the hadronic weak neutral charge Q^h and a factor κ, related to the mass of the neutral boson mediating the interaction. We emphasize the importance of Q^h in resolving the ambiguities encountered in the analysis of neutrino inclusive interactions. In section 3 we study $SU(2) \times U(1)$ gauge models which have the conventional (Cabibbo) structure for the left-handed electron and up as well as down quarks. We classify the hadronic sector of $SU(2) \times U(1)$ gauge models, according to the weak isospin assignment for the right-handed up and down quarks (in section 4) and give the weak neutral charge Q^h for the different models. The major conclusion of that section is that models with right-handed current for the up quark give Q^h with opposite sign as compared with the conventional (W-S) model. We conclude section 4 by giving the restrictions imposed on the unification angle and the parameter κ, from inclusive neutrino neutral current data, for two popular models. These restrictions would further limit the predicted range of Q in these models if a_e were known. In order to learn about a_e, in section 5 we study the leptonic sector of gauge models. Here we give bounds on a_e and κ using also the purely leptonic neutrino neutral current data. Finally, our main conclusions are summarized in section 6.

2. THE EFFECTIVE WEAK NEUTRAL CHARGE

Parity violation experiments in atoms measure the γ-Z interference for the interaction between an electron (or muon) and hadrons. Particular observables, both in electronic [21] and muonic [22] atoms, and experimental methods [23] for measuring them have been discussed extensively in the literature. Here we shall restrict ourselves to the theoretical aspects of parity violation in atoms.

We start by writing the neutral current interaction between the electron (muon) and hadrons in the form[*)]

$$ \mathcal{L} = J_\lambda^Z \, Z^\lambda $$
$$ J_\lambda^Z = N \, [\ell_\lambda + h_\lambda] \tag{1} $$

where ℓ_λ (h_λ) is the leptonic (hadronic) neutral current and Z denotes the neutral intermediate vector boson (generalization to several Z-fields being trivial). In Eq. (1) a normalization factor N has been introduced for convenience [see after Eq. (9)]. We define

$$ \ell_\lambda = \bar{e} \, \gamma_\lambda \left(v_e + a_e \, \gamma_5 \right) e \; + \; \text{muonic term} \tag{2} $$

$$ h_\lambda = \sum_{q=u,d,\dots} \bar{q} \, \gamma_\lambda \left(v_q + a_q \, \gamma_5 \right) q $$

Here the v's (a's) denote the vector (axial vector) couplings and u(d) are the up (down) quarks. We employ the quark model language since it facilitates comparison with theoretical models. Furthermore, we have restricted the analysis to vector and axial vector interaction since these are the only ones which interfere with the electromagnetic current.

In the language of effective Lagrangian of four-fermion type, Eq. (1) gives the following interaction between leptons and hadrons

$$ \mathcal{L}^{eff} = - \frac{N^2}{M_Z^2} \, \ell_\lambda \, h^\lambda \equiv - \frac{G \, \kappa}{2 \sqrt{2}} \, \ell_\lambda \, h^\lambda \tag{3} $$

where G is the Fermi coupling constant and

$$ \kappa = \frac{2 \sqrt{2} \, N^2}{G \, M_Z^2} \tag{4} $$

---

*) Our notations and conventions are as in Ref. 24).

The parity violating perturbating Hamiltonian contains two pieces, one proportional to $a_e v_q$ and the other proportional to $v_e a_q$. As the matrix element of the time component of the hadronic vector current (for low enough momentum transfers) adds up coherently for composite objects, the atomic physics experiments in heavy elements are mainly sensitive to the first piece. The perturbating Hamiltonian will induce an admixture of states with opposite parity in the atomic states. It is given by

$$H_{PV} = \frac{G}{2\sqrt{2}} \, Q \, e^+ \gamma_5 \, e \, \rho(\vec{r}) \qquad (5)$$

where $\rho(\vec{r})$ is the nuclear density (normalized to one). One sees that the main theoretical ingredient Q may be written as[*)]

$$Q = \kappa \, a_e \, Q^h \qquad (6)$$

$$Q^h = v_u \, N_u + v_d \, N_d$$

where N_u (N_d) is the number of up (down) quarks inside the atomic nucleus. For an element $^A Z$ one has

$$Q^h = \frac{3}{2} (v_u + v_d) A + \frac{1}{2} (v_u - v_d)(2Z - A) \qquad (7)$$

with the two terms showing the isoscalar and isovector vector couplings, respectively. In medium-heavy atoms Q is more sensitive to the isoscalar coupling; as an illustration the ratio (A-2Z)/A is 21% for ^{209}Bi and 17% for ^{133}Cs. When discussing complete experiments in inclusive neutrino reactions, the authors of Ref. 8) notice two ambiguities for the hadronic couplings: i) vector-axial vector ambiguity; ii) isoscalar-isovector ambiguity. Both would be simply resolved with the determination of Q given in Eq. (5) for different elements $^A Z$.

The present knowledge about the couplings appearing in Eq. (7) is very limited. The purely leptonic reactions have been observed, but data is too scarce to allow a separation of the axial coupling a_e. In section 5 we will analyze the present situation. From data on single pion production it is known[25] that the hadronic neutral current is not purely isoscalar. Finally, from inclusive reactions[18] one knows that both vector and axial vector components must be there

---

*) Our normalizations will be chosen such that in the W-S model $a_e = \kappa = 1$, giving $Q = Q^h$, where "h" refers to hadronic.

for the hadronic current. Therefore, if $a_e \neq 0$ (see section 5), the parity violating effects in atomic physics are expected to be there. One can conclude that they will provide information on weak interaction theory if the atomic physics part of the problem is well under control. It seems that even in complex systems, such as Bi or Cs, one has access to a theoretical estimate[26,27] for the relevant matrix element induced by Eq. (5) which is accurate to better than 20%. In the case of muonic atoms, the problem is hydrogen-like to a good approximation and there will be no important uncertainties originating from atomic wave functions. We end this section by concluding that the atomic physics experiments can be used to learn about the isoscalar-vector and isovector-vector couplings appearing in the effective neutral charge Q^h, given by Eq. (7).

3. SU(2) $\otimes$ U(1) GAUGE MODELS

In this section we wish to study the quantity Q, which will be supplied by the atomic experiments, in several gauge theory models which make reasonably well-defined predictions for weak neutral current processes. If the gauge group is SU(2) $\otimes$ U(1), the coupling constants g and g' [for the SU(2) and U(1) groups, respectively] are constrained to satisfy

$$ g' \cos \theta_w = g \sin \theta_w \qquad (8) $$

where θ_w is the unification angle, i.e. the mixing angle between the neutral gauge fields to reproduce the physical neutral fields A_μ and Z_μ. Once the assignment for the left-handed and right-handed fermion multiplets (ψ_L and ψ_R) fixed (this assignment defines the particular model), each model depends on just two parameters: θ_w and the mass of the intermediate Z boson. The neutral current J_μ^Z [see Eq. (1)] in any SU(2) $\otimes$ U(1) model is given by

$$ J_\mu^Z = \frac{e}{\sin \theta_w \cos \theta_w} \left\{ \sum_R \overline{\psi}_R I_{3R} \gamma_\mu \psi_R + \sum_L \overline{\psi}_L I_{3L} \gamma_\mu \psi_L - \sin^2 \theta_w J_\mu^{em} \right\} \quad (9) $$

where the first (second) sum runs over the right-handed (left-handed) multiplets. In Eq. (9), $I_{3R}(I_{3L})$ denotes the third component of the weak SU(2) in the corresponding right-(left-) handed multiplet.

In general a state i with the third component of isospin I_{3R}^i or I_{3L}^i (appearing in ψ_R or ψ_L) may contain a mixture of two or more states of elementary fermions[*]. We may distinguish between two types of models:

Type 1 : The orthogonal linear combination to i containing the same fermions has the same third component of weak isospin as the state i itself. In this case all the mixing angles among elementary fermions drop out in J_μ^Z when the sums in Eq. (9) are performed.

Type 2 : The third component of weak isospin of the orthogonal linear combination differs from that of i. Then J_μ^Z will depend on specific mixing angles.

Below we shall first consider Type 1. Generalization to Type 2 is then trivial (the factors I_{3R} and I_{3L} need to be weighted appropriately).

It is convenient to choose the normalization factor N such that the vector and axial couplings [see Eq. (2)], obtained from Eq. (9), are [see the footnote concerning Eq. (6)]

$$v_i = 2(I_{3R}^i + I_{3L}^i) - 4 \sin^2\theta_W Q_i$$

$$a_i = 2(I_{3R}^i - I_{3L}^i) \tag{10}$$

where i stands for e (electron) or q (quark) and Q_i denotes the charge ($Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$). For the choice (10) the normalization factor N [see Eq. (3)] is given by

$$N = \frac{e}{4\sin\theta_W \cos\theta_W} \tag{11}$$

Combining this relation with the equality

$$\frac{G}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W M_W^2}$$

(M_W being the mass of $W^\pm$) we find the parameter κ in Eqs. (3) and (4) is given by

$$\kappa = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \tag{12}$$

---

[*] This is, for example, the case for the left-handed hadronic sector, due to the Cabibbo structure.

For the simplest realization of spontaneous symmetry breaking in the W-S model $M_W^2 = M_Z^2 \cos^2 \theta_W$ and thus $\kappa = 1$. However, most models require a more complicated symmetry breaking mechanism and, in general, κ will not equal unity. Therefore, we shall keep κ (or equivalently the Z mass) arbitrary in our analysis. It is important to note that κ enters linearly in the atomic physics observables but quadratically in neutrino neutral current cross-sections.

For our considerations [see Eq. (7)] only v_u, v_d and a_e are relevant. In the W-S model for leptons[11] $(I_{3L}^e = -\tfrac{1}{2}, I_{3R}^e = 0)$

$$v_e = -1 + 4 \sin^2 \theta_W \quad , \qquad a_e = 1 \tag{13}$$

With the conventional extension to hadrons[12] one obtains

$$v_u = 1 - \frac{8}{3} \sin^2 \theta_W \quad , \qquad a_u = -1$$

$$v_d = -1 + \frac{4}{3} \sin^2 \theta_W \quad , \qquad a_d = 1 \tag{14}$$

In the W-S model and for $\sin^2 \theta_W = \tfrac{3}{8}$ *) v_u vanishes and parity violation is due to the down quarks. With the choice $\kappa = 1$, we find that the values of Q [Eq. (6)] for ^{209}Bi and ^{133}Cs are

$$Q_0 (Bi) = -167.5 \quad , \qquad Q_0 (Cs) = -105.5 \tag{15}$$

In the following we shall take this value as our "normalization point". Atomic experiments are then crucial in distinguishing between the W-S model and those models which predict widely different values for Q than that given in Eq. (15). With the prospect of precision experiments in atoms[28], in future, models with slightly different Q's should also be distinguishable.

Now we examine the predictions of several classes of popular models for the quantity Q in order to see how these models may be distinguished by atomic experiments.

---

*) This value is obtained in grand unified schemes, which unify weak, electromagnetic and strong interactions. Furthermore, it is compatible with all present neutrino neutral current data[3].

We shall restrict ourselves to those models which have the W-S model assignment for the <u>left-handed</u> leptons and up as well as down quarks (i.e. I_{3L}^{u} = $= -I_{3L}^{d} = -I_{3L}^{e} = \frac{1}{2}$). The difference in models lies in the way the right-handed multiplets are introduced. Moreover, we consider only models where right-handed quarks are members of weak isospin doublets or singlets; however, the right-handed electron (or muon) need not be in a singlet or doublet. For all such models [see Eq. (10)]

$$a_e = 1 + 2 \, I_{3R}^{e}$$

$$v_u = 1 - \frac{8}{3} \sin^2 \theta_W + 2 \, I_{3R}^{u} \qquad (16)$$

$$v_d = -1 + \frac{4}{3} \sin^2 \theta_W + 2 \, I_{3R}^{d}$$

if the model is of Type 1 [see after Eq. (9)]. If the model is of Type 2, the I_{3R}'s get multiplied by appropriate mixing angles as we shall comment on below for each specific case.

In the next two sections we study the two relevant sectors, hadronic and electronic, respectively, including the information available from neutrino physics.

4. HADRONIC SECTOR OF MODELS

We discuss the quantity Q^h in different models. Depending on the isospin of the right-handed up and down quarks we have four possibilities (a)-(d), viz.

$$(a) \quad I_{3R}^{u} = 0 \quad , \quad I_{3R}^{d} = 0 \qquad (17)$$

$$(b) \quad I_{3R}^{u} = 1/2 \quad , \quad I_{3R}^{d} = 0 \qquad (18)$$

$$(c) \quad I_{3R}^{u} = 0 \quad , \quad I_{3R}^{d} = -1/2 \qquad (19)$$

$$(d) \quad I_{3R}^{u} = 1/2 \quad , \quad I_{3R}^{d} = -1/2 \qquad (20)$$

For all these cases we have

$$-3A - Z \leq Q^h \leq 3Z \qquad (21)$$

independent of θ_W and mixings in the fermion multiplets.

Case (a) is the conventional extension to hadrons of the W-S model. For this case

$$Q^h = - A + 2 Z (1 - 2 \sin^2 \theta_W) \qquad (22)$$

i.e. Q^h is negative for all nuclei (with possible exception for hydrogen and helium-three).

Case (b) is phenomenologically appealing because, with the introduction of a doublet[29] such as $\binom{u}{b}_R$, where b is a new quark, one may explain[30] the anomalous y distributions in charged current neutrino interactions. This explanation, although the favourite of experimentalists, is not unique. Other suggestions, such as massive colour gluon excitations[31] have also been given in the literature. Such a doublet is present in a class of models[32] based on the exceptional Lie group E_7 as the fundamental symmetry of weak, electromagnetic and strong interactions. In these models

$$Q^h = (3 - 4 \sin^2 \theta_W) Z \qquad (23)$$

Here Q^h is positive (unless $\sin^2 \theta_W > 3/4$), thus enabling distinction between (a) and (b). This is fortunate, since the predictions of these models for inclusive neutral current reactions in neutrino physics are rather similar[4] to those of the W-S model. We note also that the sign of Q^h is independent of the mechanism which gives mass to the Z boson (i.e. independent of κ).

In Fig. 1, we show Q^h/Q_0 [see relation (15)] in models (a) and (b) for Bi and Cs. The predictions of these models are strikingly different. Now if the up quark in the right-handed doublet is mixed with a new quark, say t, and the orthogonal linear combination is a singlet we must replace I_{3R}^u by $\frac{1}{2} \cos^2 \alpha$, where α is the mixing angle between the u and t quarks. Then the results (a) and (b) could be connected continuously by varying $\cos \alpha$ from zero to one. In terms of this mixing, the results of Fig. 1 are connected by the expression

$$Q^h/Q_0 = 0.26 - 1.74 \cos^2 \alpha + 1.98 \sin^2 \theta_W \qquad (24)$$

for Bi.

Cases (c) and (d) : For case (c) we have

$$Q^h = - 3 A + Z (3 - 4 \sin^2 \theta_W) \qquad (25)$$

which is always negative and typically much larger in magnitude than for case (a). For case (d)

$$Q^h = -2A + 4Z(1 - \sin^2\theta_W) \qquad (26)$$

Q^h/Q_0 for cases (c) and (d), for Bi and Cs, are also shown in Fig. 1. Note that Q^h for the case (d) differs only by $2Z-A$ from that for case (a). This is due to the fact that the isoscalar coupling only depends on the sum $I^u_{3R} + I^d_{3R}$. The hadronic sectors of the two models are substantially different, however the above sum is the same for both. In case (d) the axial couplings a_u and a_d vanish [see Eq. (10)] and thus the theory is purely vector for u and d quarks. Such theories are ruled out by experiments[18]. Again by introducing mixings of the type discussed for case (b), one may dispose of the pure vector feature of the theory.

From Fig. 1 we see that the $\sin^2\theta_W$ dependence of Q^h is very similar for Bi and Cs. In both experiments, one is exploring a very similar coupling, as it was already understood after Eq. (7).

In view of the interesting result that the atomic experiments discriminate between models (a) and (b), by the virtue of the sign of the effect, we analyze now the available restriction on the value of κ. We use the results of inclusive neutrino-hadron (and antineutrino-hadron) reactions to obtain this information.

The theoretical expressions for inclusive neutrino reactions have been discussed repeatedly in the literature. Here we shall employ the results obtained by Albright et al.[4]. Furthermore we use[33]

$$R_\nu = .28 \pm .04 \quad , \quad R_{\bar\nu} = .38 \pm .06 \qquad (27)$$

for the ratio of neutral current to charge current reactions. These results, which are of the CERN Gargamelle Collaboration, obtained at neutrino energies below 10 GeV, are compatible with other determinations at higher energies. The advantage of low-energy data is, of course, that there is no problem of interpretation of the quantity R (e.g. due to new phenomena in charged currents).

In Fig. 2, we show the domain of κ^2, as a function of $\sin^2\theta$, which is allowed by Eq. (27) (continuous and broken lines refer to neutrino and antineutrino values, respectively) for the model (a). With no prejudice on the Z mass, the range $0 \leq \sin^2\theta \leq 0.42$ is allowed, whereas κ must be near 1. For $\kappa = 1$ one obtains

$$\sin^2\theta = .34 \pm .06 \qquad \text{(W-S model)} \qquad (28)$$

Qualitatively, large values of $\sin^2 \theta$ are possible only if the experimental result for $R_{\bar{\nu}}$ is increased and/or R_ν decreases substantially.

In Fig. 3, the same analysis is presented for the model (b). Again, independent of κ we obtain the limits $0 \leq \sin^2 \theta \leq 0.6$, and for $\kappa = 1$ we have

$$\sin^2 \theta = .42 \pm .02 \tag{29}$$

The qualitative features of the curves in Fig. 3 are similar to those of model (a), Fig. 2, if $\sin^2 \theta$ is not too large.

With the information contained in the results shown in Figs. 2 and 3 we may return to the observable quantity in atoms Q and show the present restriction on the product κQ^h. In Fig. 4 we show the prediction for $\kappa(Q^h/Q_0)$ in models (a) and (b) for the case of ^{209}Bi. This provides an illustration of the power of the atomic experiments in discriminating between two models which give similar results for neutrino neutral current processes. A possible connection between (a) and (b), through a mixing of Type 2 (see section 3) has already been discussed above with the result given in Eq. (24). Note, however, that such mixings affect the neutrino and antineutrino charged current phenomenology[7].

5. LEPTONIC SECTOR OF THE MODELS

The quantities discussed in the last section are the ones measured in the atomic physics experiments if $a_e = 1$. From Eq. (16) we know that this value corresponds to the singlet assignment for the right-handed electron, irrespectively of whether it is mixed with other heavy leptons or not. In this section we shall also consider models in which the right-handed electron is not assigned to a singlet. We shall assume that atomic experiments imply parity violation in atoms, i.e. $a_e \neq 0$. Then we may conclude

$$I_{3R}^e \neq -1/2 \tag{30}$$

Then from relation (16) it follows that the parity violating charge is crucially dependent on I_{3R}^e. For example

$$a_e \geq 2 \qquad \text{for} \qquad I_{3R}^e \geq 1/2 \tag{31}$$

$$a_e \leq -2 \qquad \text{"} \qquad I_{3R}^e \leq -3/2 \tag{32}$$

$$a_e = -1 \qquad \text{"} \qquad I_{3R}^e = -1 \tag{33}$$

Thus the sign of the effect is opposite for $I_{3R}^e \geq 0$ as compared to $I_{3R}^e \leq -1$, and the magnitude of parity violation is expected to be large for large $|I_{3R}^e|$. Note that the case in Eq. (33) is especially interesting since it gives the same magnitude but opposite sign[*] as compared with the W-S model, provided the hadronic sectors for up and down quarks are maintained.

Fortunately, one may obtain further information on I_{3R}^e from purely leptonic processes

$$\nu_\mu e \rightarrow \nu_\mu e \qquad \text{and} \qquad \bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$$

Especially the antineutrino, being itself right-handed, generally responds strongly to the right-handed coupling of the electron, viz.

$$\sigma(\nu_\mu e \rightarrow \nu_\mu e) = c \, \kappa^2 \left[(2\sin^2\theta - 1)^2 + \frac{1}{3}(2\sin^2\theta + 2 I_{3R}^e)^2 \right] \quad (34)$$

$$\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) = c \, \kappa^2 \left[\frac{1}{3}(2\sin^2\theta - 1)^2 + (2\sin^2\theta + 2 I_{3R}^e)^2 \right] \quad (35)$$

with the same proportionality constant C in Eqs. (34) and (35); κ is defined in Eqs. (3), (4) and (12). The present experimental data from the CERN Gargamelle Collaboration read[33]

$$\sigma(\nu_\mu e \rightarrow \nu_\mu e)/c \leq .65$$

and

$$.02 \leq \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)/c \leq .70$$

(36)

These values substituted into Eqs. (34) and (35) provide bounds for the allowed region in the $\kappa^2 - \sin^2\theta$ plane. In Figs. 5 and 6 we show these regions for (W-S model) $I_{3R}^e = 0$ and a "triplet model" with $I_{3R}^e = -1$, respectively. The two figures are obtained one from the other by mirroring above the line $\sin^2\theta = \frac{1}{2}$. Furthermore, the antineutrino data are very sensitive to the model. For example, for the extreme values of $\sin^2\theta$ (near zero or unity) the antineutrino cross-section differs by a factor of $\underline{13}$ in the two models. For $\sin^2\theta = \frac{3}{8}$ the corresponding factor is 2.7.

---

[*] This peculiarity has been emphasized by H. Fritzsch (see Ref. 34).

With the present results for purely leptonic neutral current processes[33] any
value of $\sin^2 \theta_W$ is allowed if κ^2 is not restricted. The main information, inde-
pendent of $\sin^2 \theta$, is that $\kappa^2 < 2$, thus giving a lower limit to the Z mass. For
$\kappa^2 = 1$ we reproduce the known result[33]

$$. 10 \leq \sin^2 \theta \leq .42 \qquad (37)$$

for W-S type model (i.e. $I^e_{3R} = 0$).

For the future goal of determining the multiplet assignment of the right-
handed electron from only purely leptonic processes, the ratio r of the ν_μ-e and
$\bar{\nu}_\mu$-e cross-sections, which is independent of κ, is presented in Fig. 7 for these
two models with opposite sign of a_e. Up to now, only the preliminary result[35]
from the Aachen-Padova collaboration is available, with a value r = 0.44 ± 0.2.
If this value were confirmed, Fig. 7 would impose new restrictions on $\sin^2 \theta_W$ in
these models.

Now we shall combine the results of this section with the ones of section 4,
to learn about the compatibility of the hadronic and leptonic sectors of differ-
ent models. The values of κ and $\sin^2 \theta_W$, associated with the vector bosons, must
be the same. This analysis will be restricted to the popular models (a) and (b)
for the hadronic sector. From the comparison of these results, we conclude that
the "singlet" assignment (Fig. 5) for the right-handed electron is compatible
with the model (a) for hadrons (without any additional restrictions than the ones
shown in Fig. 2). It is also compatible with the model (b) for hadrons, but the
allowed region of $\kappa - \sin^2 \theta_W$ (Fig. 3) is restricted to the maximum values $\kappa \approx 1$,
$\sin^2 \theta_W \approx 0.42$. However, the "triplet" assignment (Fig. 6) for the right-handed
electron is neither compatible with the model (a) nor with the model (b), unless
the actual value of the $\bar{\nu}_\mu$-e cross-section is more than twice the upper limit
published by the Gargamelle collaboration.

In view of these results, one is led to the following question: What are
the effective values of a_e which are allowed by the above comparison? We have
explored similar restrictions to the ones shown in Figs. 5 and 6 varying a_e "con-
tinuously" from $a_e = +1$ to negative values. The answer we obtain is $a_e \gtrsim -0.5$,
as a requirement imposed by (36) plus the allowed domains shown in Figs. 2 and/or
3. This result means that the right-handed electron cannot have $I^e_{3R} \leq -1$, unless
it is mixed with a heavy one (in the sense of a mixing of Type 2, as discussed in
section 3) with a probability amplitude cos $\alpha \leq 87\%$.

6. <u>DISCUSSION AND CONCLUDING REMARKS</u>

In this paper we have studied the information contained in the first generation parity violation experiments in heavy atoms within the context of vector and axial-vector neutral current interactions in order to generate the weak electromagnetic interference. Atomic experiments will provide information on the quantity Q, Eq. (6), which consists of three factors namely the axial vector (neutral current) coupling a_e, a "strength" parameter κ and the hadronic neutral weak charge Q^h. Assuming that neutral currents are mediated via exchange of a single vector boson Z, the coupling factors appearing in Q also enter in neutral current neutrino interactions. The "strength" parameter κ is then proportional to the inverse of the Z mass squared.

If $Q \neq 0$, measuring it in different elements, supplies the ratio of the iso-vector-vector and isoscalar-vector couplings and thus should serve to resolve the ambiguities encountered[8] in the model independent analysis of neutrino inclusive data.

We have further specialized our analysis to SU(2) × U(1) gauge models of weak and electromagnetic interactions, which have the conventional structure for the left-handed electron and up as well as down quarks. In view of agreeing with the charged current phenomenology such a restriction seems quite reasonable. The various models differ by the way in which the right-handed fermions (specially the electron, up and down quarks) are introduced. They are all described by five parameters, three "isospin" parameters (I_{3R}^u, I_{3R}^d and I_{3R}^e) which could be weighted by mixing angles (if the model is of Type 2 as discussed in section 3) as well as $\sin^2 \theta_W$ and κ. We classified the hadronic sector of these models in section 4 and the leptonic sector in section 5.

The predictions for Q^h depend only on the hadronic sector of the models. These were given in section 4 where we found that the sign and magnitude of this quantity is very model dependent and should be useful in testing models. For example, models with right-handed current for the up quark give opposite sign for Q^h as compared to the W-S model (see Fig. 1).

In any SU(2) × U(1) model, the measured inclusive neutrino and antineutrino cross-sections supply limitations on $\sin^2 \theta_W$ and κ. In section 4 we demonstrated these limitations (Figs. 2 and 3) for two popular models, viz. the W-S model and a model with right-handed current for the up quark. Such a study gives allowed domains for κQ^h as a function of $\sin^2 \theta_W$ as shown in Fig. 4. For these two interesting models there is no overlap in their predictions. The only remaining

"unknown" in Q, namely a_e, is studied in purely leptonic neutrino neutral current reactions. Similar for the hadronic sector, also the leptonic data supply us with allowed regions in $\kappa - \sin^2 \theta_W$ plane (see Figs. 5 and 6) as we have demonstrated for two popular models. It is also possible to get restrictions on a_e without assuming any specific model. Such a study gives $a_e \geq -\frac{1}{2}$. This already serves to exclude a large number of possibilities, e.g. a model in which the right-handed electron is in a triplet (with $I_{3R}^e = -1$) is ruled out.

With the restriction $-0.5 \leq a_e \leq 1$, the results of Fig. 4 show that, for typical SU(2) × U(1) gauge models [case (a) and (b), see Eqs. (17)-(18)], the expected value of Q cannot be in magnitude much larger than Q_o, as given in Eq. (15). Models without mixing, or with mixings of Type 1, in the leptonic sector require (from our analysis) to have $a_e = +1$. In that case, the knowledge of the sign and magnitude of the experimental parity violating effect in atomic physics would allow to distinguish models with similar predictions for neutrino-hadron physics.

REFERENCES

1) CERN - Gargamelle collaboration, F.J. Hasert et al., Phys. Letters 46B, 121 (1973); ibid. 46B, 138 (1973).

2) See e.g. the talks by Bertrand-Coremans and by H. Faissner in Neutrino '75, IUPAP Conference, Balatonfüred, Proceedings Vol. 1, pp. 6, 116, 156 (Eds A. Frenkel and G. Marx 1975).

3) See the contributions presented at the neutral current session of the Neutrino Conference, Aachen (1976).

4) C.H. Albright, C. Quigg, R.E. Schrock and J. Smith, Fermilab-Pub-76/40 - THY (1976).

5) V. Barger and D.V. Nanopoulos, COO-546 (1976).

6) R.M. Barnett, Harvard University preprint (1976).

7) K.P. Das and N.G. Deshpande, preprint OITS-62 (1976).

8) Pham Quang Hung and J.J. Sakurai, CERN preprint TH.2171 (1976).

9) R.A. Eramzhyan et al., JINR preprint (1974);
T.W. Donnelly et al. Phys. Letters 49B, 8 (1974).

10) M.K. Gaillard, S. Jackson and D. Nanopoulos, Nucl. Phys. B102, 326 (1976).

11) S. Weinberg, Phys. Rev. Letters 19, 1264 (1967), Rev. Mod. Phys. 46, 255 (1974);
A. Salam, Elementary Particle Physics, Ed. N. Svarthalm (Almkvist and Wiksell, Stockholm, 1968).

12) S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).

13) M.L. Perl et al., Phys. Rev. Letters 35, 195 (1975);
M.L. Perl, talk presented at the Neutrino Conference, Aachen (1976)

14) A. Benvenuti et al., Phys. Rev. Letters 37, 189 (1976).

15) M. Derrick, F.A. Nezrick, talks presented at the Neutrino Conference, Aachen (1976).

16) A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. Letters 35, 69 (1975);
S. Pakvasa, W.A. Simmons and S.F. Tuan, Hawaii preprint UH-511-196-75 (1975);
H. Fritzsch, M. Gell-Mann and P. Minkowski, Phys. Letters 59B, 156 (1975).

17) H. Fritzsch and P. Minkowski, CERN preprint TH-2165 (1976) submitted to Physics Letters.

18) V. Brisson, talk at Rencontre de Moriond, Flaine (1976);
A. Bodock,   "   "   "   "   "   "   "   ;
A. Benvenuti et al., HPWF collaboration, report 76/4; see also ref. (3)

19) D.C. Soreide et al., Phys. Rev. Letters 36, 352 (1976) and references therein; P.G.H. Sandars, private communication; M.A. Bouchiat and L. Pottier, Phys. Letters 62B, 327 (1976).

20) Preliminary results of this work have been presented in
J. Bernabéu, talk given at the VII Seminar on Theoretical Physics, GIFT, L'Escala (Gerona, Spain), June 7-12 (1976);
C. Jarlskog, talk given at the Neutrino Conference, Aachen, Germany, June 8-12 (1976).

21) See e.g. C. Bouchiat in Neutrino 1975 Proceedings, Editors A. Frenkel
   and G. Marx (Budapest, 1975), p.241;
   M.A. Bouchiat and C.C. Bouchiat, Phys. Letters 48B, 111 (1974) and
   references therein.

22) J. Bernabéu, T.E.O. Ericson and C. Jarslkog, Phys. Letters 50B,
   467 (1974);
   G. Feinberg and M.Y. Chen, Phys. Rev. D10, 190 (1974);
   L.M. Simons, Helv. Phys. Acta 48, 141 (1975).

23) M.A. Bouchiat and L. Pottier, J. Phys. Lett. 36, L-189 (1975);
   D.C. Soreide et al., Phys. Rev. Letters 36, 352 (1976) and references
   therein;
   P.G.H. Sandars in Atomic Physics 4, Ed. C. Zu Putlitz, E.W. Weber
   and A. Winnecher, (Plenum, New York, 1975)
   L.M. Simons, Proc. Symposium on Interaction Studies in Nuclei,
   Mainz (1975).

24) J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics (McGraw-
   Hill, Inc., New York, 1964).

25) G. Bertrand - Coremans et al., Phys. Letters 61B, 207 (1976).

26) M. Brimicombe, C.E. Loving and P.G.H. Sandars, preprint;
   E.M. Henley and L. Wilets, RLO-1388-713 preprint (1976).

27) M.A. Bouchiat and C. Bouchiat, J. Physique 36, 493 (1975);
   C.E. Loving and P.G.H. Sandars, J. Phys. B. Atom. Mol. Phys.
   8L, 336 (1975).

28) V. Telegdi, private communication;
   P.G.H. Sandars, loc. cit. Ref. 23.

29) A. De Rújula, H. Georgi, S.L. Glashow and H.R. Quinn, Revs. Modern
   Phys. 46, 391 (1974);
   Y.A. Achiman, K. Koller and T.F. Walsh, Phys. Letters 59B, 261 (1975);
   R.M. Barnett, Phys. Rev. Letters 34, 41 (1975); Phys. Rev. Letters
   D11,3246 (1975).

30) R.M. Barnett, loc. cit. Ref. 29 .

31) B.P. Sidhu, R.N. Mohapatra and J.C. Pati, BNL-21581 preprint (1976),
   presented at Neutrino Conference in Aachen, Germany, June 8-12
   (1976).

32) F. Gürsey and P. Sikivie, Phys. Rev. Letters 36, 775 (1976);
   P. Ramond, CALT-68-540 preprint.

33) W. van Domick, talk given at the Neutrino Conference, Aachen,
   Germany, June 8-12 (1976).

34) H. Fritzsch, talk given at the Neutrino Conference, Aachen, Germany,
   June 8-12 (1976).

35) H. Faissner, Seminar given at CERN, July 1976.

Figure captions

Fig. 1   :   The hadronic weak neutral charge Q^h normalized to the "canonical" value Q_o given in Eq. (15), as function of $\sin^2 \theta_W$. The indications (a), (b), (c), (d) refer to the four standard possibilities (17), (18), (19), (20) for the right-handed third components of weak isospin of up and down quarks. Full lines correspond to the case of ^{209}Bi, broken lines to ^{133}Cs.

Fig. 2   :   Restrictions between κ^2, given in Eq. (12), and $\sin^2 \theta_W$ for the model (a) of the hadronic sector. Full lines determine the strip allowed from inclusive neutrino neutral current scattering, broken lines from antineutrino scattering results. The limits on $\sin^2 \theta_W$ imposed by the choice $\kappa = 1$ are explicitly indicated.

Fig. 3   :   Same as Fig. 2, but for the model (b) of the hadronic sector.

Fig. 4   :   The allowed domains for κQ^h, the quantity measured in atomic physics, in the models (a) and (b), for ^{209}Bi. The restrictions from Figs. 2 and 3 have been incorporated.

Fig. 5   :   Restrictions between κ^2 and $\sin^2 \theta_W$ for the conventional model (e_R^- is a singlet) of the leptonic sector. Full lines give the allowed region from neutrino-electron scattering, broken lines from antineutrino-electron results.

Fig. 6   :   Same as Fig. 5, but for a model in which e_R^- has $I_{3R}^e = -1$.

Fig. 7   :   The ratio r of the neutrino-electron to antineutrino-electron cross-sections, as a function of $\sin^2 \theta_W$. r_s (full line) corresponds to the model with $I_{3R}^e = 0$, r_t (broken line) to the model with $I_{3R}^e = -1$.
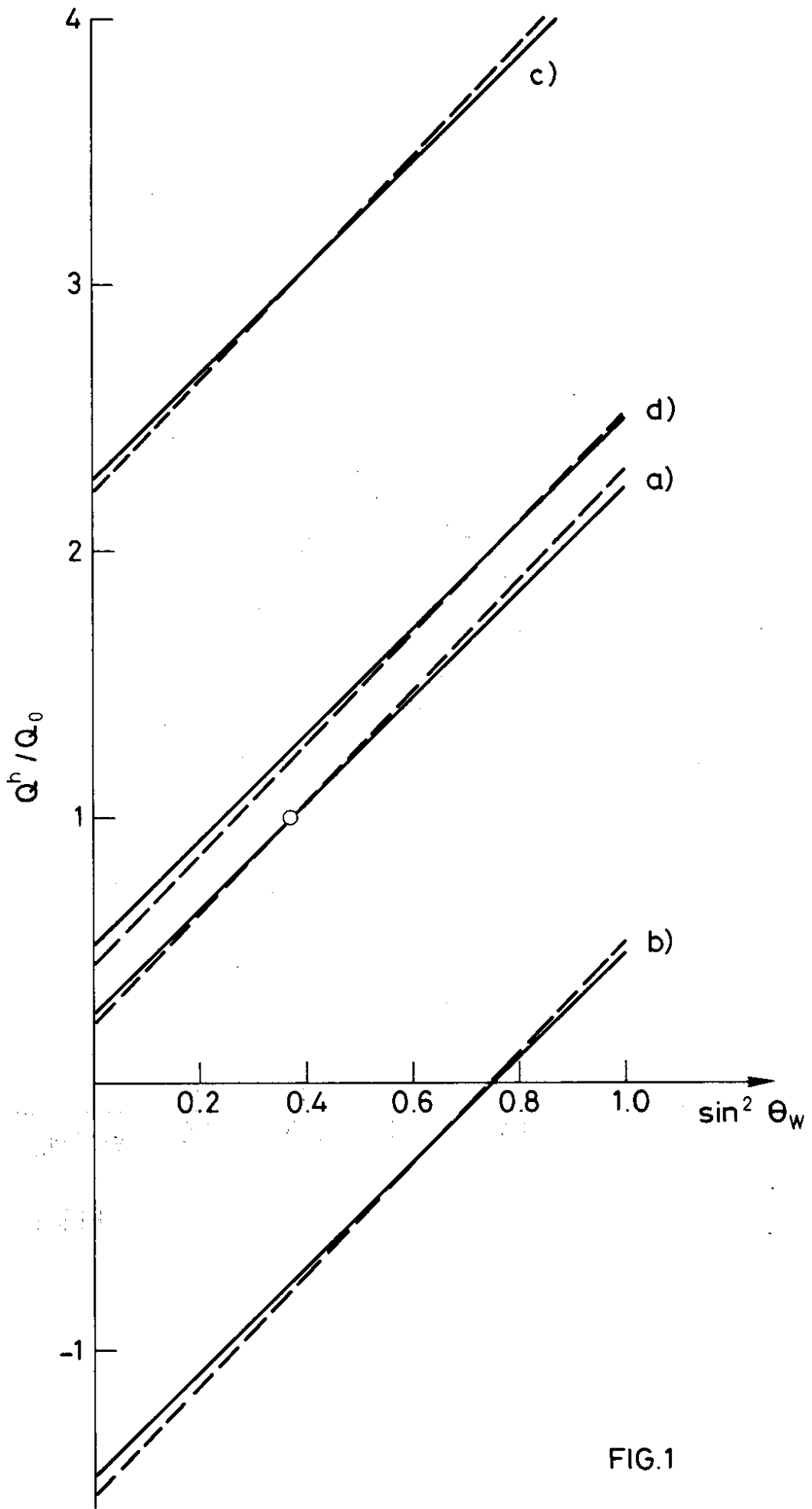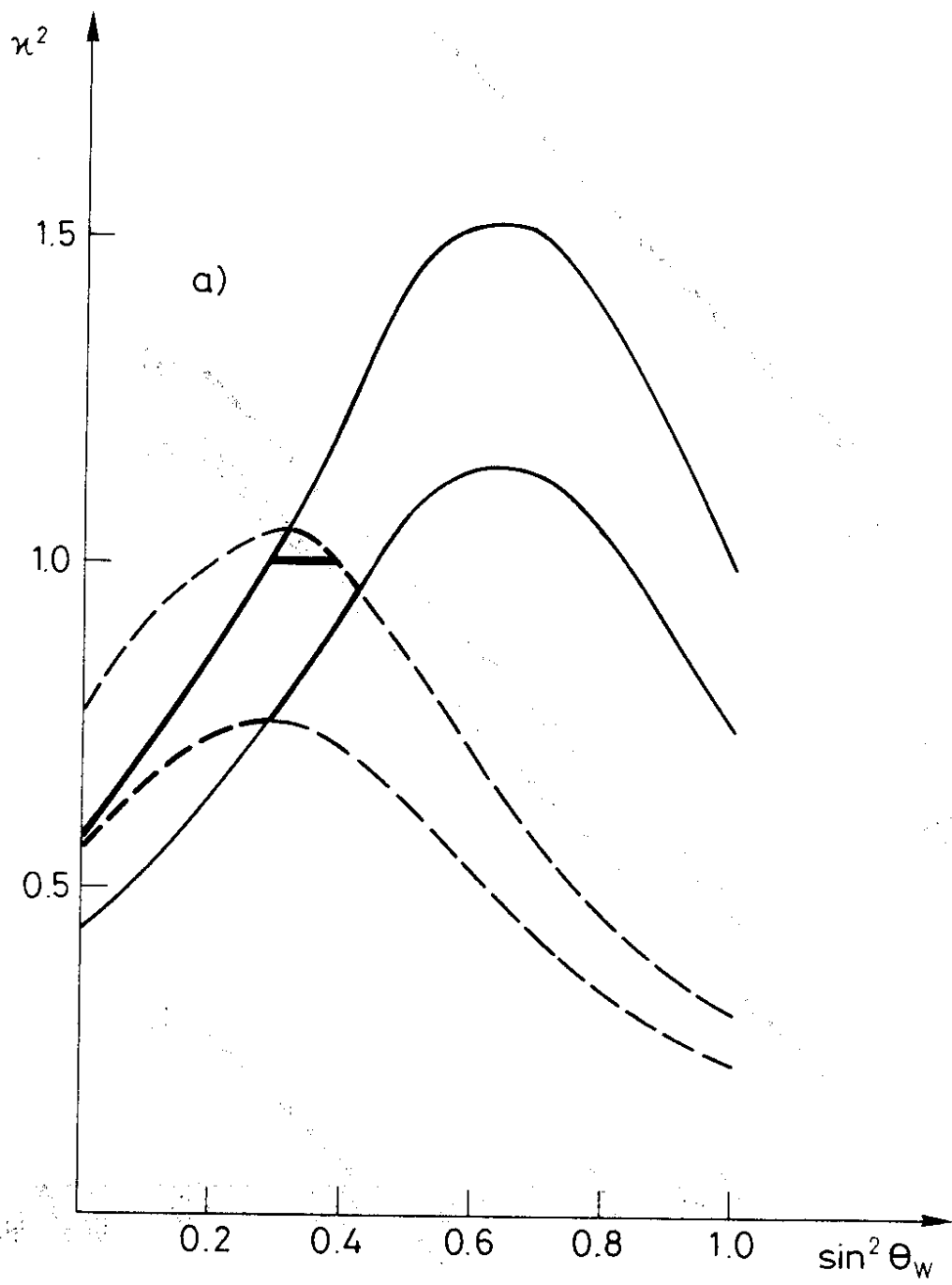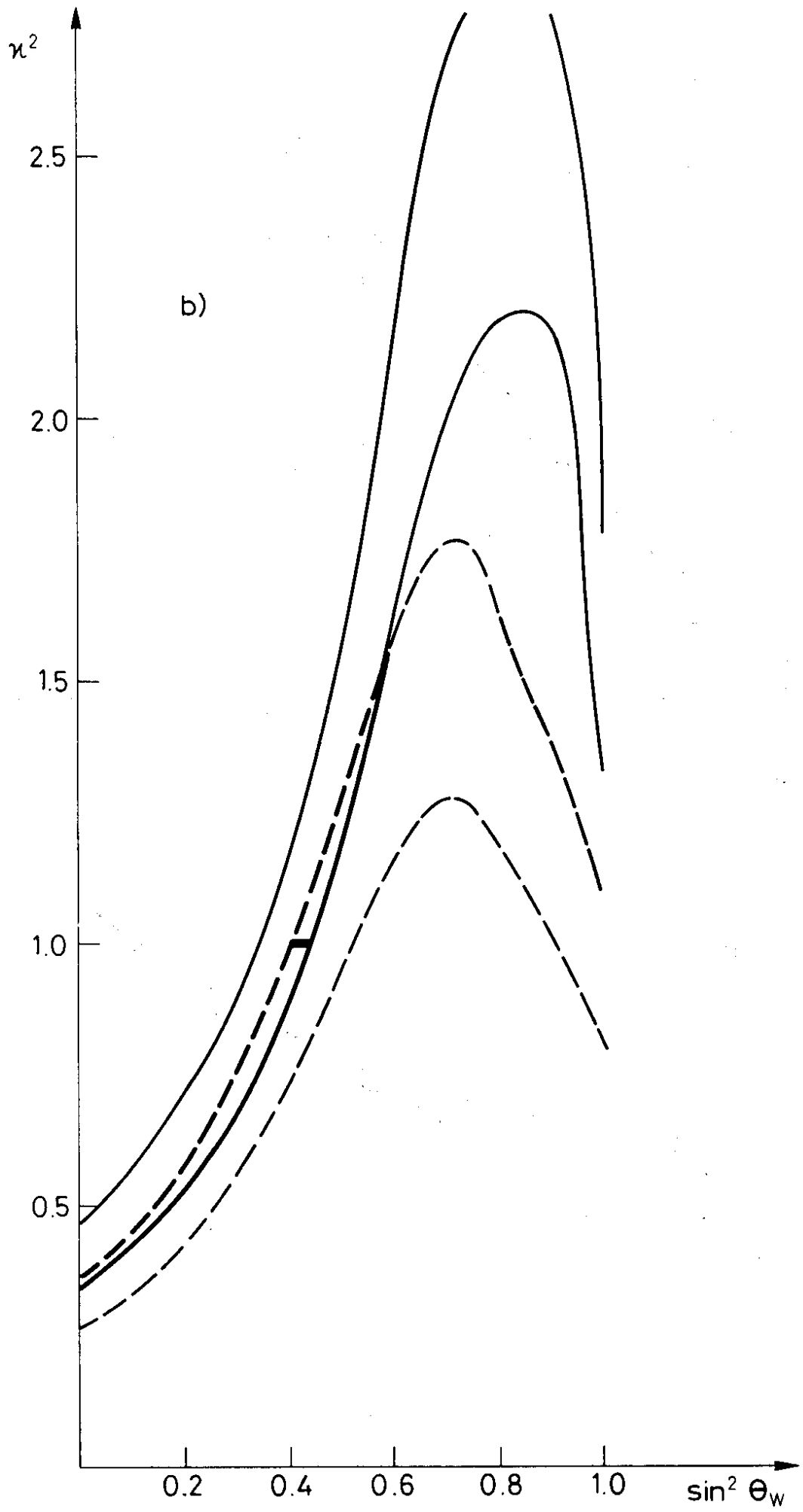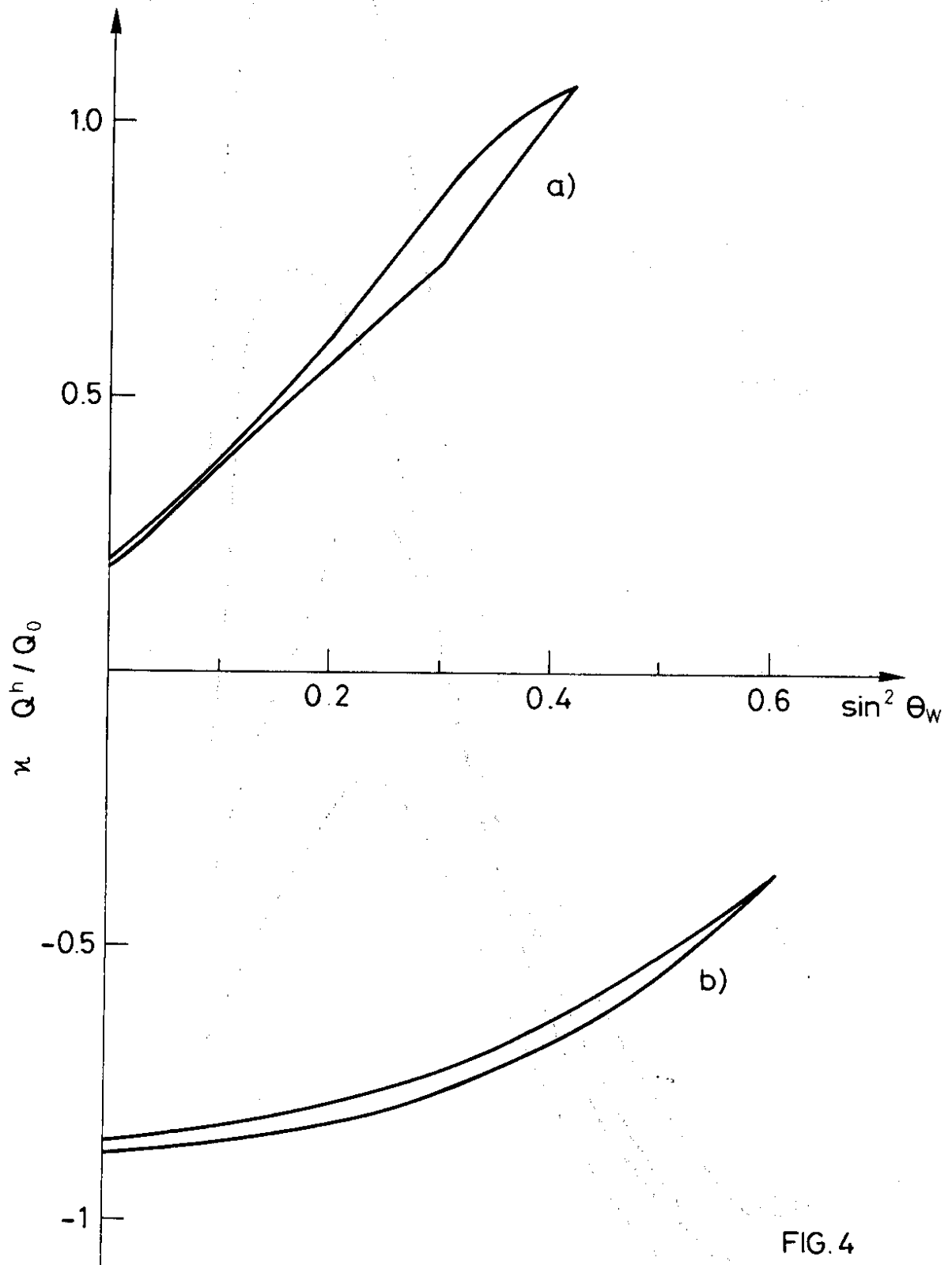
FIG.1

FIG.2

FIG.3

FIG. 4

x^2

2.5

2.0

1.5

1.0

0.5

e_R^- singlet

0.2   0.4   0.6   0.8   1.0 $\sin^2 \Theta_W$

FIG. 5

FIG. 6

FIG. 7

28