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LONG-RANGE PARITY VIOLATING INTERACTION IN MUONIC ATOMS

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ABSTRACT

Long-range parity violating forces are induced in muonic atoms by virtual γ-Z^o conversion between the muon and the nucleus. They are of order $G_F\alpha$ with range $(2m_e)^{-1}$. The relevant diagrams in unified electroweak interactions are calculated and the effects of the corresponding potential on parity admixtures in muonic levels are studied. It is proved that they are negligible for n=3 orbits, but they have overwhelmed the conventional short-range contribution for n=5.

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1. INTRODUCTION

After the experimental discovery[1)] of weak neutral current interactions in physics, and following the lines of the suggested parity violating effects in ordinary atoms[2)], experiments to look for these effects have been discussed for muonic atoms[3)]. Theoretically, this possibility allowed a hydrogen-like analysis for any Z muonic system, thus avoiding the uncertainties of the many body atomic problem. The higher $q^2 \sim (m_\mu Z\alpha)^2$ of this case was also an argument in favor of these proposals. Due to the short range character of the direct neutral current interaction, one preferred to stay in the lowest angular momentum orbits. As only the circular orbits $\ell=n-1$ are strongly populated during the muonic cascade in the atom, this situation centered the discussion around the 2S-2P admixtures and M1-E1 interferences in radiative transitions to the 1S level.

The difficulty to observe the M1 transitions has obliged to move the discussion to higher orbits and looking for parity violating asymmetries in E2-E1 interferences has been suggested[4)]. In these cases, the short-range interaction only proceeds through the finite size of the nucleus. Apart from arguments coming from level degeneracies, it is thus advantageous to go to heavy systems. In this context, after a careful study of experimental conditions and expected theoretical effects, Missimer and Simons[5)] have proposed to look for the E2-E1 interference induced by the $3p_{3/2}$ admixture into the $3d_{3/2}$ level of muonic $^{175}Lu_{71}$.

Once the considered orbits do not overlap strongly with the nucleus, one question naturally arises: Are there any long-range parity violating interactions which would affect these higher orbits? Even if the strength of this possible contribution corresponds to a radiative order α correction, the long range character with better overlap could compensate the situation in this problem. Of course, the concept of long-range interaction depends on the system to be considered. Apart from any true long-range interaction, i.e., an inverse r power behaviour, also the case for which the range is larger than the size of the muonic atom would be relevant for us. This is the situation when the interaction is mediated by neutrinos or electrons exchanged in the t channel. It is easy to prove that, to leading order α, a long range parity violating interaction is originated only from the virtual γ-Z^o conversion diagram between the muon and the nucleus.

In this paper we study the existence of this parity violating long-range interaction for muonic systems; the induced effective potential is established and its effects on $n>2$ orbits in muonic atoms are explored. The paper is organized as follows. In Section 2 we discuss the relevant diagrams and calculate the invariant amplitude. The q^2 behaviour is analyzed and its branch point singularities are pointed out. From the absorptive part in the t channel, we build the effective potential in Section 3 and its region of validity is discussed. The muonic atom is clearly within the range of the Compton wave-length of the electron. The effects of the long-range parity violating potential are calculated in Section 4 and the results are compared with the conventional short-range effects in muonic atoms. Some conclusions are drawn in Section 5.

2. LONG-RANGE INTERACTION

In this Section we discuss the diagrams which provide a long-range interaction between muons and quarks. In order to single out the radiative corrections of interest, we work in the framework of the SU(2)×U(1) theory of weak and electromagnetic interactions[6]. One can imagine loop and vertex diagrams in which two neutrinos are exchanged in the t channel, thus giving a candidate for a true long-range parity violating interaction. Radiative corrections similar to these have been discussed by several authors[7] in another context. It is easy to prove that the non-analytic terms, such as $q^2 \ln(-q^2)$, originated by these diagrams are of order G_F^2 and thus too small to be considered here. On the other hand, the size of a muonic atom is smaller than the Compton wave-length of the electron, so that any diagram in which two electrons are exchanged in the t channel is a candidate for a long-range interaction between the muon and the quark in the muonic system. The Compton wave-length of the muon being of the order of the nuclear size, any diagram with muons as an intermediate state will give an effective short-range interaction. With these criteria, the only diagrams which contribute, to order $G_F\alpha$, to a long-range parity violating interaction between muons and quarks are the ones for virtual γ-Z^o conversion through an electron loop, and they are depicted in Fig. 1. with the relevant momenta.

The vertices of the diagrams are extracted from the Lagrangian

$$L(x) = e\, Q_f\, \bar{f}(x)\, \gamma^\lambda f(x)\, A_\lambda(x) + g\, \bar{f}(x)\, \gamma^\lambda (v_f + a_f \gamma_5) f(x)\, Z_\lambda(x) + \dots \tag{1}$$

where $f(x)$ denotes any fermion field, Q_f its charge and $v_f(a_f)$ its vector (axial vector) neutral current coupling. In the standard theory[6], with the simplest Higgs structure, one has

$$g = \frac{e}{4\sin\theta\cos\theta} \quad , \quad a \equiv a_e = a_\mu = -1 \quad , \quad a_u = -a_d = 1$$

$$v \equiv v_e = v_\mu = -1 + 4\sin^2\theta \, , \quad v_u = 1 - \frac{8}{3}\sin^2\theta \, , \quad v_d = -1 + \frac{4}{3}\sin^2\theta \qquad (2)$$

with θ the weak mixing angle. Present findings[8)] from various experimental sources favour the standard theory with $\sin^2\theta \simeq 0.23$.

The invariant T amplitude corresponding to the sum of diagrams a) and b) can be written in the form

$$T = i\,\frac{g^2 e^2}{M_Z^2}\,\frac{1}{q^2}\Big\{ Q_\mu Q_e\, \bar{u}_\mu \gamma^\lambda u_\mu\, \bar{u}_q \gamma^\rho (v_q + a_q\gamma_5) u_q\, I_{\lambda\rho} + Q_q Q_e\, \bar{u}_\mu \gamma^\lambda (v + a\gamma_5) u_\mu\, \bar{u}_q \gamma^\rho u_q\, \tilde{I}_{\lambda\rho} \Big\} \qquad (3)$$

with the corresponding integrals over the electron loop

$$I_{\lambda\rho} = \frac{1}{(2\pi)^4}\int d^4k\, \frac{Tr\{(\not{k}+\not{q}+m_e)\gamma_\lambda(\not{k}+m_e)\gamma_\rho(v+a\gamma_5)\}}{[(k+q)^2-m_e^2]\,[k^2-m_e^2]}$$

$$\tilde{I}_{\lambda\rho} = \frac{1}{(2\pi)^4}\int d^4k\, \frac{Tr\{(\not{k}+\not{q}+m_e)\gamma_\lambda(v+a\gamma_5)(\not{k}+m_e)\gamma_\rho\}}{[(k+q)^2-m_e^2]\,[k^2-m_e^2]}$$

(4)

Clearly, the γ_5 terms in (4) are not going to give any contribution to the integrals since we have only one "external" momentum q at our disposal. This means that the γ-Z^0 conversion only proceeds through the vector coupling of the neutral current and, therefore, the two tensors $I_{\lambda\rho}$ and $\tilde{I}_{\lambda\rho}$ are equal.

A straightforward evaluation of the integral, keeping the terms contributing to the discontinuity through the q^2 cut above $4m_e^2$, gives *) the following result

$$I_{\lambda\rho} = -i\, g_{\lambda\rho}\, \frac{v\, q^2}{2\pi^2} \int_0^1 dx\; x(1-x)\, \ln\left[1 - x(1-x)\frac{q^2}{m_e^2}\right] \tag{5}$$

where we dropped induced terms $q_\lambda q_\rho$ which do not contribute to the contraction in Eq.(3). Collecting all these expressions, we obtain the invariant amplitude

$$T = \frac{G_F\,\alpha}{3\pi\sqrt{2}}\, v\, Q_e \Big\{ Q_\mu\, \bar{u}_\mu \gamma^\lambda u_\mu\, \bar{u}_q \gamma_\lambda (v_q + a_q \gamma_5) u_q + Q_q\, \bar{u}_\mu \gamma^\lambda (v + a\gamma_5) u_\mu\, \bar{u}_q \gamma_\lambda u_q \Big\} \cdot$$

$$\left\{ -\frac{5}{6} - \frac{2 m_e^2}{q^2} + \frac{m_e^2}{q^2}\left(1 + \frac{q^2}{2m_e^2}\right)\sqrt{1 - \frac{4m_e^2}{q^2}}\, \ln \frac{\sqrt{1-\frac{4m_e^2}{q^2}} + 1}{\sqrt{1-\frac{4m_e^2}{q^2}} - 1} \right\} \tag{6}$$

As anticipated, this amplitude presents an absorptive part only for $q^2 > 4m_e^2$ which value can be obtained from the last term of Eq.(6). The q^2 behaviour in the physical region $(q^2<0)$ is not apparent in Eq.(6) because of some cancellations. In fact, for $(-q^2) \ll 4m_e^2$, the behaviour of the bracket is $O(-q^2)$. In the intermediate region of interest for muonic atoms, with $(-q^2) > 4m_e^2$, one obtains the typical non-analytic behaviour $\ln(-q^2)/m_e^2$.

Some characteristic features of Eq.(6) are worth discussing. The algebraic structure of the amplitude is similar to the one of the direct neutral current contraction of muons and quarks. In particular, if one is interested in the parity violating piece and the coherent nuclear effect is exploited,

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*) The diagrams are ultra-violet divergent and must be renormalized. This is, however, of no relevance for us since we are interested in the terms with non-analytic behaviour.

the dominant amplitude to be considered is the one coming from the axial vertex in the muon

$$Q_q \; a \; \bar{u}_\mu \gamma^\lambda \gamma_5 u_\mu \; \bar{u}_q \gamma_\lambda u_q \; F(q^2) \tag{7}$$

where $F(q^2)$ is the last bracket in Eq.(6). It is similar to the usual short-range interaction, but with a completely different q^2 behaviour which becomes long-ranged for muonic atoms. This is considered now.

3. THE POTENTIAL

Consider the invariant amplitude obtained in the previous Section. Up to constants given by possible subtractions, this amplitude satisfies the dispersion relation

$$F(q^2) = \frac{1}{\pi} \int_{4m_e^2}^{\infty} dt' \, \frac{\mathrm{Abs}\, F(t')}{t' - q^2} \tag{8}$$

from which we are going to obtain an effective potential through the Fourier transform of Eq.(8). The absorptive part of $F(q^2)$ is immediately calculated from the bracket in Eq.(6) and it gives

$$\mathrm{Abs}\, F(t) = -\pi \frac{m_e^2}{t} \left(1 + \frac{t}{2m_e^2}\right) \sqrt{1 - \frac{4m_e^2}{t}} \quad ; \quad t > 4m_e^2 \tag{9}$$

The parity violating potential, which this absorptive amplitude originates[9], is then written in the following form

$$V_{PV} = \frac{G_F \alpha Z}{12 \pi^2 \sqrt{2}} \, v a \left\{ -V(r) \frac{\vec{\sigma} \cdot \vec{p}}{2m_\mu} - \frac{\vec{\sigma} \cdot \vec{p}\,'}{2m_\mu} V(r) \right\} \tag{10}$$

where $\vec{p}(\vec{p}\,')$ is the momentum operator acting on its right (left) and the function $V(r)$ is given by

$$V(r) = \frac{2 m_e^2}{r} \int_1^\infty dx \left(2 + \frac{1}{x^2}\right) \sqrt{x^2-1}\; e^{-2 m_e x r} \tag{11}$$

In Fig. 2 we plot the dimensionless quantity $G(r) \equiv r^3 V(r)$. The behaviour of this function is easily understood from the following considerations. The asymptotic dependence of $V(r)$ is

$$V(r) \underset{r\to\infty}{\sim} \frac{3\sqrt{\pi}}{2} \frac{m_e^{1/2}}{r^{5/2}} e^{-2 m_e r} \tag{12}$$

and it is shown in Fig. 2 as a broken line. The asymptotic potential (12) is, however, not precise enough for our problem, at least for medium-heavy muonic atoms. The size of the atom being smaller than $(2me)^{-1}$, the atomic wave-functions overlap with the potential in a region which corresponds more to the "small" r behaviour of (11). Of course, this is still far from the nuclear size, where short-range interactions come into the game. With the definition chosen for $V(r)$, one has $G(r=0)=1$.

The parity violating potential (10) obtained here compares with the conventional one in the following way. The strength of the short-range potential is

$$\frac{G_F}{2\sqrt{2}}\, a \left[(2 v_u + v_d) Z + (2 v_d + v_u) N \right]$$

and $V(r)$ is replaced by the nuclear density $\rho(r)$, normalized to 1.

The region of validity of our long-range potential, in its complete form (11), is the one which avoids short-range effects, i.e. one must have $r > R_{\text{nucleus}}$ and $r > (2m_\mu)^{-1}$. Up to distances of thousands of fm, this is the dominant parity violating force existing in Nature.

4. MUONIC ATOMS

Here we apply the potential (10) to calculate the parity admixture induced in a muonic atom level $(n\ell)_j$. Apart from the radial integrations, the matrix element

$$\langle n' \ell' j m | -V(r) \frac{\vec{\sigma}.\vec{p}}{2 m_\mu} - \frac{\vec{\sigma}.\vec{p}'}{2 m_\mu} V(r) | n \ell j m \rangle \tag{13}$$

is the same for the short and long-range contributions. As Eq.(10) is scalar, under rotations of the muon co-ordinates, this connects only states of the same jm and (13) becomes m independent. Given the state $(n\ell)$, admixtures are induced to levels $(n'\ell')$ with $\ell'=\ell\pm1$. We obtain

$$\langle n', \ell-1, j\, m | - \left\{ V(r), \frac{\vec{\sigma}.\vec{p}}{2 m_\mu} \right\}_+ | n, \ell, j\, m \rangle = i \frac{\sqrt{6\ell}}{2 m_\mu} W(\ell 1 j \tfrac{1}{2} | \ell-1 \tfrac{1}{2})$$

$$\int_0^\infty dr\; r^2\, V(r) \left\{ R_{n',\ell-1} \frac{d R_{n,\ell}}{dr} - R_{n,\ell} \frac{d R_{n',\ell-1}}{dr} + \frac{2\ell}{r} R_{n',\ell-1} R_{n,\ell} \right\} \tag{14}$$

where $R_{n,\ell}(r)$ are the corresponding radial wave functions. In practice, the muon cascade in the atom follows the circular orbits $\ell=n-1$, so that (14) is the piece of interest. Furthermore, because of possible strong degeneracies, the main admixtures are expected within the same orbit $n'=n$. Our program now is to establish a comparison between the expected parity mixings in $(n,\ell=n-1)$ states induced by the long-range interaction [Eqs.(10) and (11)] and the conventional short-range one. For the latter, we need a model for the nuclear density and it is simply assumed

$$\rho(r) = \frac{3}{4\pi R^3}\, \theta(R-r) \tag{15}$$

This allows a close analytic expression for the matrix element of the short-range interaction

$$M_S(Z,n) \equiv \langle n\,(n-2)_{n-3/2} |\, V_{PV}^{S} \,| n\,(n-1)_{n-3/2} \rangle$$

$$= -i \frac{G_F}{\pi\sqrt{2}}\, a \left[(2v_u + v_d)Z + (2v_d + v_u)N\right] \frac{3 \cdot 2^{2n-5} (\alpha Z)^{2n} m_\mu^{2n-1}}{n^{2n+1} (2n-3)! \sqrt{2n-1}}\, R^{2n-4}\, e^{-2m_\mu Z\alpha R/n} \tag{16}$$

which shows explicitly the strong decrease of the effect for higher n orbits. The recently suggested transition from the n=3 orbit in muonic $^{175}Lu_{71}$ gives a value for the matrix element (16) ≃ $-i5\times10^{-3}$ eV, very similar to the one obtained in Ref. 5).

For the long-range interaction matrix element we obtain

$$M_L(Z,n) \equiv \langle n(n-2)_{n-3/2} |\, V_{PV}^{L} \,| n\,(n-1)_{n-3/2} \rangle = -i\, v a\, \frac{G_F (\alpha Z)^3}{\pi^2 \sqrt{2}} \frac{\sqrt{2n-1}}{6 n^3}\, m_e^2 m_\mu$$

$$\int_1^\infty dx \left(2 + \frac{1}{x^2}\right)\sqrt{x^2-1}\, \frac{1}{\left(1 + \frac{m_e n}{m_\mu \alpha Z} x\right)^{2n-2}} \left\{ 1 - \frac{2n-2}{2n-1} \frac{1}{1 + \frac{m_e n}{m_\mu \alpha Z} x} \right\} \tag{17}$$

A numerical computation of this integral allows us to obtain the results plotted in Fig. 3, for n=3,4,5 orbits, as function of Z. An explicit analytic result is only possible with some approximation for V(r) given in Eq.(11). We find that, within an error less than 10%, it is possible to use the behaviour $V(r) \simeq r^{-3}$ for Z≳4 in n=3 orbits, Z≳11 in n=4 and Z≳20 in n=5. In this situation, and imposing n>2, one obtains

$$M_L(Z,n) = -i \frac{G_F}{2\pi^2\sqrt{2}}\, v a\, \frac{(\alpha Z)^5\, m_\mu^3}{n^5 (n-2)(2n-3)\sqrt{2n-1}} \tag{18}$$

which shows a much smoother behaviour with n than the one of (16). It is apparent in the comparison that for $n \gtrsim 3$, the result (18) can be competitive and it cannot be neglected *a priori*. There is a fact which tends to suppress the contribution (18), in general. In the standard theory and $\sin^2\theta_W = \frac{1}{4}$, the vector neutral current coupling to electrons vanishes so that, with the present analyses, this interaction is universally decreased by an order of magnitude *). Independently of this accidental cancellation, one is interested in knowing what are the regions of the atomic number Z and the orbit n in muonic atoms, for which it is safe to keep only the conventional short-range interaction in the theoretical analysis. In Fig. 3 we plot the ratio of Eq.(16) to Eq.(17) in the following form

$$R(Z,n) = \frac{\dfrac{N\, M_S(Z,n)}{a\left[(2v_u+v_d)Z+(2v_d+v_u)N\right]}}{\dfrac{M_L(Z,n)}{v\, a}} \tag{19}$$

We conclude from this comparison that the short-range interaction clearly dominates the parity violating effects in n=3 orbits for all Z but very light nuclei, in n=4 for $Z \gtrsim 50$ elements and it has been overwhelmed by the long-range interaction in n=5 orbits. For the proposed n=3 level in $^{175}Lu_{71}$, the short-range effect is larger by more than two orders of magnitude.

A different attitude would be to look for favourable conditions under which the long-range parity violating interaction could manifest itself. This would imply to go to n=4, $Z \simeq 20$ elements or n=5, $Z \simeq 50$, where precision experiments on QED vacuum polarization studies have been made and are still being pursued[10]. In fact, the criteria to go to these levels for vacuum polarization are on the same footing as the ones discussed here for the long-range parity violating interaction. Any hope to detect asymmetries at the expected level must depend on dramatic level degeneracies in these regions of orbits and atoms, and there is a limit to that from the widths of the levels[11].

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*) This γ-Z^o conversion suppression, through the leptonic loop, in the standard theory and $\sin^2\theta_W \simeq \frac{1}{4}$ is the same in the absorptive part that prevents the γ-Z^o interference in e^+e^- machines.

5. CONCLUSION

In view of the necessity, for parity violation studies in muonic atoms, to move from the lower orbits in order to look for favourable transitions, we have addressed in this work the question of whether long-range parity violating interactions would exist and could affect the analysis. In unified theories of weak and electromagnetic interactions, a defined recipe to consider the relevant radiative corrections exists. We conclude that, to the dominant $G_F\alpha$ order, only the virtual γ-Z^o conversion diagram, through an electron loop, is able to provide a long-range interaction on the scale which is appropriate for muonic atoms.

The effective potential thus obtained is given in Eqs.(10) and (11) and plotted in Fig. 2. The relevant r region is the one around 60 fm, where a smooth departure from an r^{-3} behaviour is seen. This should be compared to a radial dependence of the conventional short-range interaction which is sharply cut down for r values of few fm.

A comparative analysis of the parity admixtures induced by the short-range and long-range interactions indicates that it is safe to neglect the radiative contributions as long as one stays in $n=3$ orbits or in the case $n=4$ for $Z\gtrsim 50$. Higher orbits are, however, controlled in their parity violating properties by the long-range interaction discussed here.

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FIGURE CAPTIONS

Fig. 1.- Virtual γ-Z^o conversion diagrams, giving origin to a $G_F\alpha$ order long-range interaction for muonic atoms.

Fig. 2.- The effective long-range potential $G(r) \equiv r^3 V(r)$ is plotted versus r. The dotted line corresponds to its asymptotic behaviour.

Fig. 3.- The ratio $R(Z,n)$ of short-range to long-range induced parity admixtures is plotted for $n=3,4,5$ versus Z. The dotted lines give the relevant long-range matrix element divided by iva.

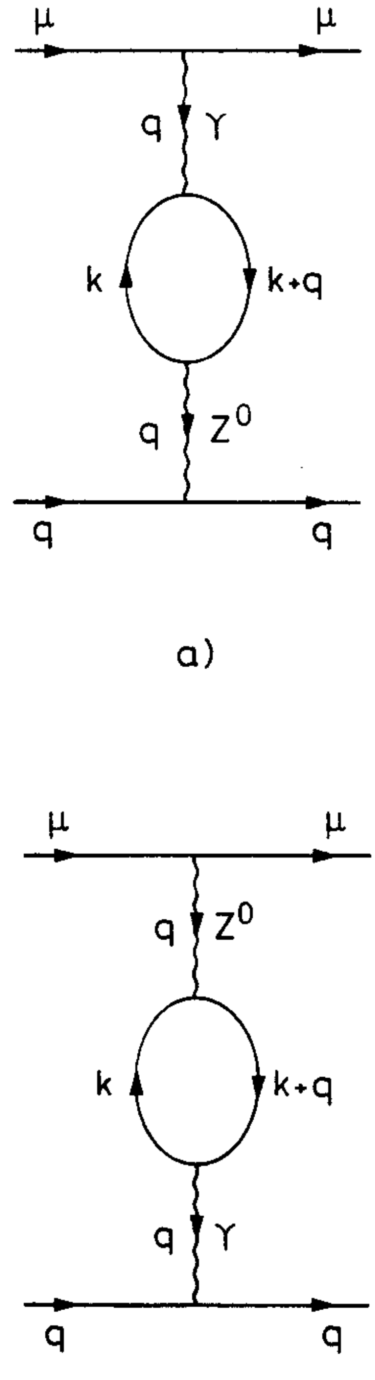


FIG.1

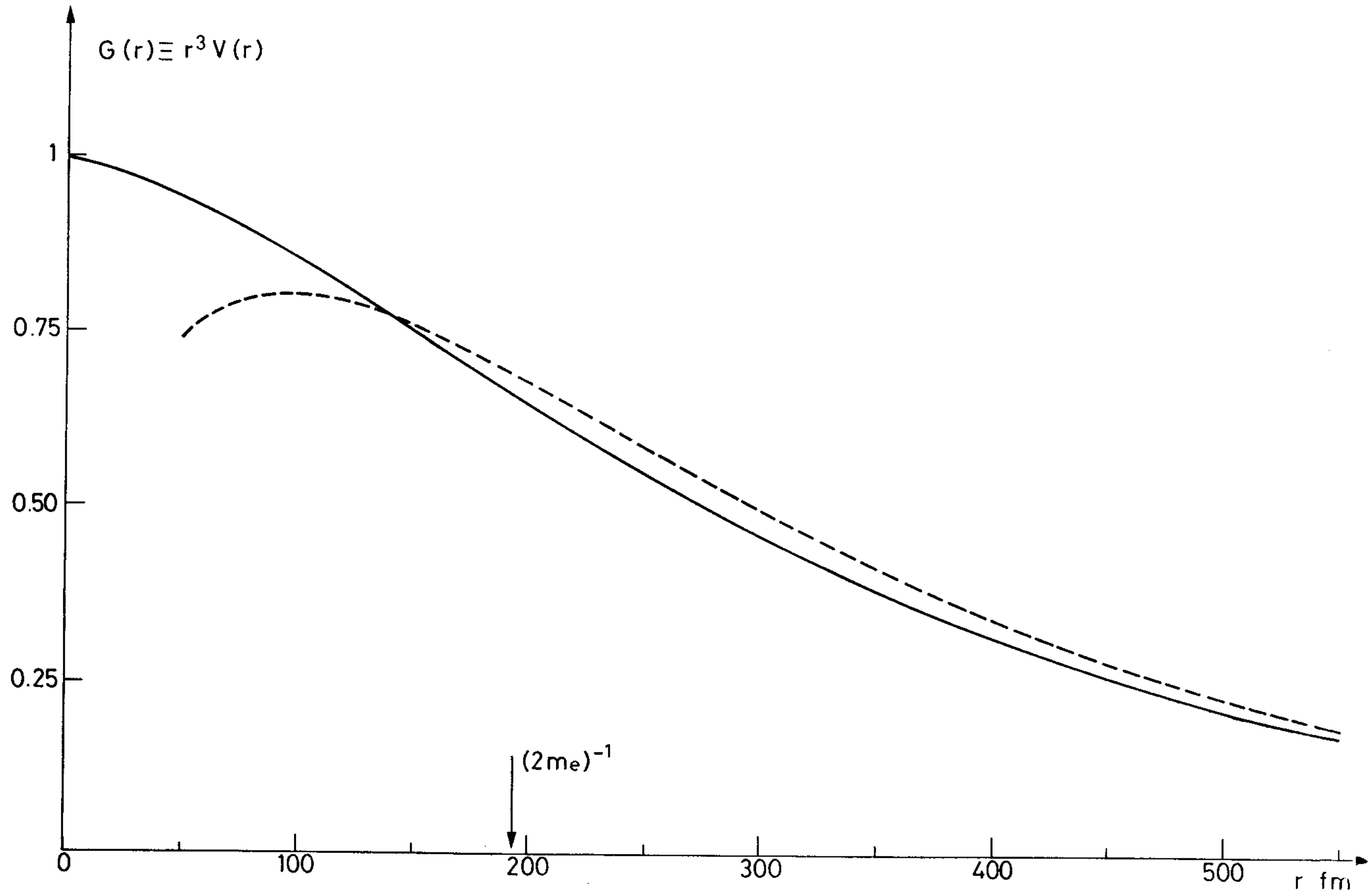
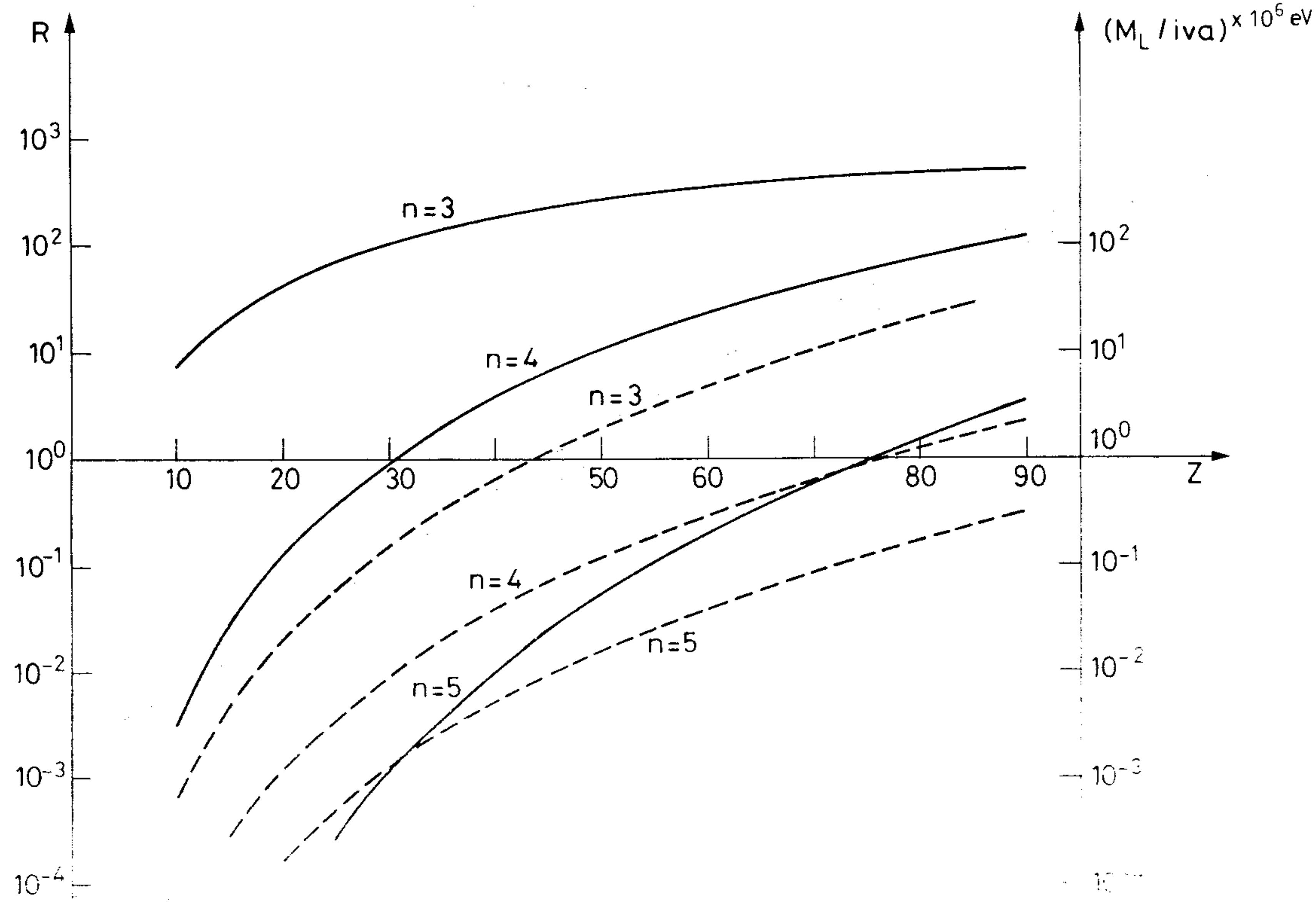


FIG. 2


R
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$(M_L / iva) \times 10^6$ eV
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10
20
30
50
60
80
90
Z
n=3
n=4
n=3
n=4
n=5
n=5