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CP VIOLATION AT THE Z° PEAK

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ABSTRACT

The measurement of a non-vanishing asymmetry $\alpha \equiv \{\Gamma(\bar{s}b)-\Gamma(s\bar{b})\}/\{\Gamma(\bar{s}b)+\Gamma(s\bar{b})\}$ would signal CP violation in Z° decays. We study here this effect within the standard model: it appears at the one-loop level with a key ingredient provided by the possible on-shellness of the intermediate quarks. In the three-generation case, the α value comes out small due to the effective degeneracy of u and c quarks at these high energies. In the four-generation case, results are encouraging for LEP: one could have a branching ratio of this flavour-changing decay to the flavour-conserving one of $O(10^{-6})$ and reach α values near unity.

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In this letter we would like to discuss the prospects for CP-violating effects at the e^+e^- colliders on top of the Z° peak. The observable proposed is the asymmetry in the flavour-changing decay of the Z° to $b\bar{s}$ and $s\bar{b}$

$$\alpha = \frac{\Gamma(Z^\circ \to s\bar{b}) - \Gamma(Z^\circ \to b\bar{s})}{\Gamma(Z^\circ \to s\bar{b}) + \Gamma(Z^\circ \to b\bar{s})} \tag{1}$$

as well as its trivial generalization to other $q\bar{q}$ pairs such as $d\bar{s}$, $b\bar{d}$. For reasons which will become clear below, the prospects are better for down quarks in the final state, at least in the standard theory. The consideration of up-quark final pairs such as $\bar{t}c$, $t\bar{c}$ could be appropriate in other models.

It is of crucial importance to look for CP-violating effects in physical situations other than the K°-$\bar{K}^\circ$ system. In this context, experimental studies[1)] in the B-$\bar{B}$ or other heavy flavour systems are interesting, although the theoretical expectation is rather gloomy (in the standard model). Those proposals keep the same profile as the K-$\bar{K}$ ones insofar as they go either through the analysis of the mass matrix[2)] or through exclusive/semi-inclusive decays[3)] to hadronic final states.

However, from now on, it is imperative to find out new CP violation guides appropriate to the colliders and supercolliders. In high energy collisions, the natural asymptotic states are jets, not hadronic states. The question is to find out the optimal observables at every energy range. Furthermore, one should try to isolate observables in which the ingredients are intrinsic to the assumed basic theory responsible for CP violation.

Let us start this programme in the framework of the standard electroweak theory with three generations. There the CP-violating effects stem[4)] from flavour-mixing of quarks non-degenerate in mass. One should be aware that the high statistics of future colliders does not necessarily imply good prospects for CP searches. In fact, with growing energy, although many channels open, the effective quark masses vanish and the CP-violating effects are correspondingly small. The situation could change drastically in theories where CP violation appears at high scales.

At the supercollider energies, the appropriate and reasonable observables should no longer explicitly depend on the different flavours. However, in the first generation of accelerators, it is perhaps still possible to identify the flavours at the jet level. Let us consider LEP I to start with. This Z°-factory will provide several millions of Z° per year. It is worth having a look at its flavour-changing decays and seeing whether there is already any CP effect at all in the standard theory.

The answer is affirmative. The interference of amplitudes with different weak phases as well as different absorptive parts provides a contribution at the one-loop level (Fig. 1) for these flavour-changing decays. The absorptive parts are available because the energy involved is $M_{Z^\circ} \gtrsim (2\ m_q)$, where m_q is the mass of the quark in the loop. These intermediate quarks may now be on mass-shell, a situation to be contrasted with typical low-energy analyses. As a consequence, different absorptive contributions appear in our case due to unitarity. Note that such a CP-violating effect does not need an interference between topologically or chirally different diagrams; while the weak phase changes sign in going to the conjugate channel, the absorptive parts do not[5].

Let us consider the flavour-changing decays of the Z°. To our knowledge, the corresponding rates have been estimated[6] in view of the future colliders, but no attention has been paid to the CP non-conserving effects in that context.

In the 't Hooft-Feynman gauge, the diagrams responsible for the flavour-changing vertex $Z^\circ d_i d_j$ are shown in Fig. 1. In a self-explanatory notation, the invariant T matrix element is given by

$$T = \frac{g}{\cos\theta_W}\, \bar{u}_i(p_1,\lambda_1)\, \Gamma^\mu\, v_j(p_2,\lambda_2)\, \epsilon_\mu(q,\lambda) \tag{2}$$

where, in the limit of zero <u>external</u> quark masses, the induced current at the one-loop level is of the V-A type

$$\Gamma^\mu = \frac{g^2}{2(4\pi)^2} \sum_k \xi_k\, I(r_k, s)\, \gamma^\mu L \tag{3}$$

In Eq. (3), g is the SU(2) gauge coupling, $\xi_k \equiv U^*_{ki} U_{kj}$ where U_{ki} is the flavour matrix element in generation space and k labels the quark running in the loop, $s \equiv (M_k/M_W)^2$, $r_k \equiv (m_k/M_W)^2$ and L is the left-handed projector $L \equiv (1-\gamma_5)/2$. The limit of the zero quark mass for external legs is well defined in each diagram by itself, but for the self-energy ones in which one has to sum (1c) + (1d) and (2c) + (2d) (see Fig. 1).

Our calculation of the flavour-mixing form factor $I(r_k,s)$ induced by the $Z°$ boson is an exact one, with no approximations in the internal quark masses with respect to the W mass. It contains interesting features, both from the point of view of higher-order electroweak interactions such as finite renormalization and gauge-dependent cancellations, and for the possibilities of generating different absorptive parts of the amplitude from different quark masses of the internal legs. A detailed study of all these matters will be presented elsewhere for real and virtual $Z°$, where explicit formulae for the form factor $I(r_k,s)$ are given for arbitrary values of r_k and s. We present here the points relevant to our discussion for $Z°$ decays.

The diagrams (2) are at least of order r_k, whereas this is not so a priori for the diagrams (1). However, due to the unitarity of the flavour-mixing matrix, the pieces independent of r_k are irrelevant in the effective vertex Γ^μ (GIM cancellation). The final result for the flavour-changing form factor is finite. In the 't Hooft-Feynman gauge the cancellations operate in the following form. Diagrams (1e) and (1f) are finite. In diagrams (1a), (1b) and (1c)+(1d), the pole term in the dimensionally-regularized amplitude is quark-mass-independent and so it is GIM-cancelled. The divergences in diagrams (2) cancel among themselves for a given quark intermediate state (fixed k).

We have checked from our expressions that the limit $s \to 0$ reproduces the results already known in the literature[7]. In this low-energy limit, the amplitudes do not provide any absorptive parts. In fact, by keeping only the terms which survive in the GIM-mechanism, one gets for the summed contribution

$$I(r, s=0) = -\frac{3r}{1-r} - \frac{r \ln r}{(1-r)^2} + \frac{1}{2}\frac{r^2}{1-r} - \frac{3}{2}\frac{r^2 \ln r}{(1-r)^2} \qquad (4)$$

For our purposes, we have to analyze the form factor for $s = (M_Z/M_W)^2$, and its behaviour changes drastically for on-shell $Z°$ bosons. Diagrams (1a) and (2a) have then an absorptive part for quark masses such that $r < s/4$, with a value which is r-dependent. Diagrams (1b), (1e), (1f) and (2b) are real as long as $s < 4$, as corresponds to the actual physical situation. Diagrams (1c)+(1d) and (2c)+(2d) are real.

The result for the decay widths reads

$$\Gamma(Z^0 \to d_i \bar{d}_j) = \frac{g^2}{\cos^2\theta_W} M_{Z^0} \frac{1}{8\pi} \left[\frac{g^2}{2^5 \pi^2}\right]^2 \cdot \left| \sum_k \xi_k I(r_k, s) \right|^2$$

$$\Gamma(Z^0 \to d_i \bar{d}_i) = \frac{g^2}{\cos^2\theta_W} M_{Z^0} \frac{1}{8\pi} 2 (a^2+v^2) \tag{5}$$

where

$$a^2 + v^2 = \frac{1}{16} \left\{ 1 + \left(1 - \frac{4}{3}\sin^2\theta_W\right)^2 \right\}$$

Defining functions F_2 and F_3 by $F_2 \equiv I_2 - I_1$, $F_3 = I_3 - I_1$, and using the unitarity of the KM matrix in three generations $\xi_1+\xi_2+\xi_3 = 0$, the rate between flavour-changing and flavour-conserving decays can be written as:

$$R = \frac{\Gamma(Z^0 \to d_i \bar{d}_j)}{\Gamma(Z^0 \to d_i \bar{d}_i)} = \frac{G_F^2 \; M_W^4}{\pi^4 \left\{1 + \left(1 - \frac{4}{3}\sin^2\theta_W\right)^2\right\}} \left| \xi_2 F_2 + \xi_3 F_3 \right|^2 \tag{6}$$

which implies

$$R \simeq 4 \cdot 10^{-5} \left| \xi_2 F_2 + \xi_3 F_3 \right|^2 \tag{7}$$

Taking, for instance, $m_u \simeq 0$, $m_c \simeq 1.5$ GeV, $m_t \simeq 45$ GeV, the function F gets values $F_2 = (-6+i7)\cdot 10^{-4}$, $F_3 = -0.26 + i\,0.73$. The corresponding results for R are shown in the Table. Only the channel $s\bar{b}$ offers some prospects of having a branching ratio close to the experimental possibilities of future e^+e^- colliders.

With the same parametrization, the asymmetry

$$\alpha \equiv \frac{\Gamma(Z^0 \to d_i \bar{d}_j) - \Gamma(Z^0 \to d_j \bar{d}_i)}{\Gamma(Z^0 \to d_i \bar{d}_j) + \Gamma(Z^0 \to d_j \bar{d}_i)}$$

is given by

$$\alpha = \frac{-4\,\mathrm{Im}(\xi_2 \xi_3^*)\,\mathrm{Im}(F_2 F_3^*)}{|\xi_2 F_2 + \xi_3 F_3|^2 + |\xi_2^* F_2 + \xi_3^* F_3|^2} \simeq -2\,\frac{\mathrm{Im}(\xi_2 \xi_3^*)\,\mathrm{Im}(F_2 F_3^*)}{|\xi_2 F_2 + \xi_3 F_3|^2} \tag{8}$$

where the expression to the right is obtained by considering ξ_2 and ξ_3 approximately real in the denominator. Finally,

$$\alpha \simeq 10^{-12} \sin\delta / R \tag{9}$$

for present accepted values of the mixing Kobayashi-Maskawa angles.

From the Table and Eq. (9) it is observed that the rate for a given flavour-changing channel and the CP-violating effect are correlated: when the rate grows, the asymmetry diminishes by the same amount. This is a feature of the Kobayashi-Maskawa model with three generations, because in it the number of invariant parameters associated with CP violation is a unique universal one. Furthermore, the small value of F_2 (compare with F_3!) is understood as a consequence of the near-degeneracy, at these energies, of the u and c quarks. Remark that the CP-violating effect is always proportional to the combination of form factors Im $(F_2F_3^*)$. Writing

$$\mathrm{Im}\;(F_2 F_3^*) = \mathrm{Im}\left(I_1 I_2^* + I_2 I_3^* + I_3 I_1^*\right) \tag{10}$$

one sees that the result is antisymmetric under the exchange of any pair of intermediate quarks. As long as the different I_k are the same functions of m_K $[I_k = I(m_k)]$, Eq. (10) gives zero in the limit when any two of the three quarks are degenerate. This is the reason why the measurable asymmetry comes out small in the standard model: at the Z° energies, m_c and m_u behave as if almost degenerate. Furthermore, in the limit when $m_k \ll M_W$, we can extract the explicit dependence of the observable by performing an expansion in r

$$I\,(r,s) \simeq I(0,s) + I'(0,s)\cdot r + \frac{1}{2} I''(0,s)\cdot r^2 + \ldots\ldots \tag{11}$$

where the coefficients are universal (independent of flavour) functions of s. Then

$$\mathrm{Im}\,(F_2 F_3^*) \simeq \frac{1}{2}\,\mathrm{Im}\left\{ I'(0,s)\, I''(0,s)^* \right\}\Big|_{r=0} (r_1 - r_2)\,(r_2 - r_3)\,(r_3 - r_1) \tag{12}$$

In view of the above remarks, it is relevant to consider what the situation would be in the standard electroweak theory, but with four generations of quarks. The new CP-violating quantities allow us to replace in that case the (r_1-r_2) factor by (r_1-r_4), for example. One then gets the CP-violating observable proportional to

$$\sum_{j>i=2}^{4} \mathrm{Im}\,(\xi_i\, \xi_j^*)\ \ \mathrm{Im}\,(F_i\, F_j^*) \tag{13}$$

and, neglecting F_2 for the above reasons,

$$\alpha \simeq -2\,\frac{\mathrm{Im}\,(\xi_3\, \xi_4^*)\ \ \mathrm{Im}\,(F_3\, F_4^*)}{|\xi_3 F_3 + \xi_4 F_4|^2} \tag{14}$$

where $\mathrm{Im}\,(\xi_3\xi_4^*)$ is not a unique universal quantity: it now depends on the channel considered.

In order to illustrate the situation, let us still consider $s\bar{b}$ as the final state. For $m_t' \simeq M_Z$, $F_4 = -0.50 + i\,0.75$, so that $\mathrm{Im}\,(F_3F_4^*) \simeq -0.2$, which is a typical value increasing somewhat with the mass of t', t' being the 2/3-charged quark of the fourth generation. We have analyzed the four-generation mixing matrix using the parametrization of Ref. 8). Two of the new angles are constrained, whereas the other one, s_u, remains free. One gets the following behaviour

$$R\,(Z^\circ \to s\,\bar{b}) \simeq 4\cdot 10^{-5}\, s_u^2\, c_u^2\, \lambda^2\, |F|^2 \tag{15}$$

where $F \sim F_3$, F_4 and λ is the parameter which, in the Wolfenstein parametrization[9)] of the KM matrix, takes the value 0.22. This expression is to be compared with the corresponding one for three generations where

$$R\,(Z^\circ \rightarrow s\bar{b}) \simeq 4 \cdot 10^{-5}\, \lambda^4\, |F_3|^2 \qquad (16)$$

In this channel $s\bar{b}$ one gets an "enhancement factor" when going to four generations, which is $(s_u c_u/\lambda)^2$. In the most favourable case, the flavour-changing rate to $s\bar{b}$ would increase by an order of magnitude. A similar situation occurs in the $d\bar{b}$ channel; however, the channel $d\bar{s}$ now has a factor $(c_u/\lambda)^4$, which could mean an enhancement of three orders of magnitude.

It is the CP symmetry itself that increases remarkably in the four-generation case. In fact, one sees from Eq. (14) that in this case α could reach values near unity because $|\xi_3| \sim |\xi_4|$ and $|F_3| \sim |F_4|$ and one has appropriate relative phases between ξ_3 and ξ_4 as well as Im $(F_3 F_4^*) \simeq -0.2$.

As a consequence, in the limit where both the u and c quark masses are effectively zero ($F_2 \sim 0$), the CP-violating effect does not disappear and α could reach values near unity. The relevant product $\alpha \cdot R \simeq 10^{-6}$, for four generations in the standard model, is probably a result not far from the capabilities of the next generation of e^+e^- colliders.

Let us remark that we have focused our analysis of flavour-changing Z° decays on final states made out of down quark pairs. The reason, as explained above, is that in order to obtain a sizeable asymmetry in the standard model with (three) four generations, one needs at least two quarks with masses comparable to $M_{W,Z}$. This does not need to be the case in other models.

Let us conclude by pointing out that any other CP-violating model which is independent of the quark mass difference $m_c^2 - m_u^2$ should also give predictions not outside the realm of the first generation of colliders. This could be the case, for instance[10], for left-right symmetric models, Higgs models of CP-violation and supersymmetric models with phases in the gaugino and squark mass matrices. The corresponding detailed analysis should be done in order to determine if and when the effect is phenomenologically interesting.

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	R		$\|\alpha\|$	
	$R^{(3)}$	$R^{(4)}$	$\|\alpha^{(3)}\|$	$\|\alpha^{(4)}\|$
$Z° \rightarrow d\bar{s}$	10^{-11}	10^{-8}	$10^{-1} s_\delta$	s_ϕ
$Z° \rightarrow d\bar{b}$	10^{-9}	10^{-8}	$10^{-3} s_\delta$	s_ϕ
$Z° \rightarrow s\bar{b}$	10^{-7}	10^{-6}	$10^{-5} s_\delta$	s_ϕ

Table

Estimates of the branching ratio R of the flavour-changing decays $Z° \rightarrow d_i \bar{d}_j$ to the flavour-conserving decay in the standard theory with three and with four generations, as well as the corresponding CP asymmetries α. s_δ is the sinus of the KM phase; s_ϕ is the sinus of a combination of the phases for four generations.

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FIGURE CAPTION

Diagrams responsible for the flavour-changing decay $Z^\circ \to d_i\bar{d}_j$ in the 't Hooft-Feynman gauge. External particles and momenta are indicated in diagram 1a.

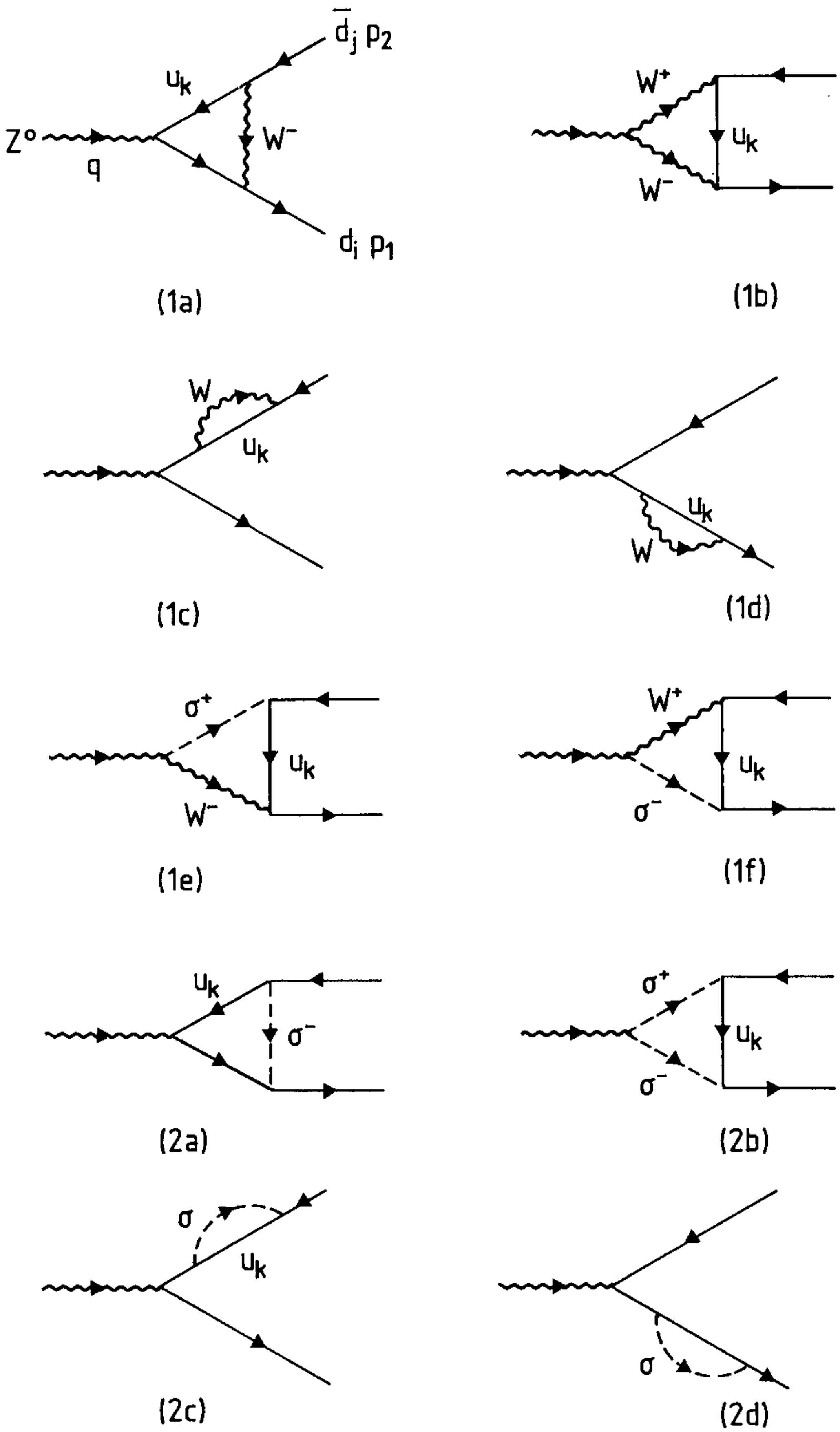
$\bar{d}_j$ p_2
u_k
Z^o
q
W^-
d_i p_1
(1a)
W^+
u_k
W^-
(1b)
W
u_k
(1c)
u_k
W
(1d)
σ^+
u_k
W^-
(1e)
W^+
u_k
σ^-
(1f)
u_k
σ^-
(2a)
σ^+
u_k
σ^-
(2b)
σ
u_k
(2c)
σ
(2d)

Fig. 1