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CP PHASES IN THE CHARGED CURRENT AND HIGGS SECTORS FOR MAJORANA NEUTRINOS

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ABSTRACT

The diagonalization of the leptonic mass matrices is performed in the framework of the triplet model to generate Majorana mass terms for neutrinos. This allows the understanding of the role played by the CP-violating phases in the Higgs sector and their relation with those of the charged-current Lagrangian. It is shown that all the leptonic mixings, including those of the Higgs couplings, can be given in terms of a Kobayashi-Maskawa matrix and the relative Majorana phases of the neutrino fields. The characteristic Majorana phases, always appearing together with the neutrino mass, are present in $|\Delta L| = 2$ pieces and they show up in processes with a) neutrino-antineutrino propagation, and/or b) at least two different neutrinos as asymptotic states, and/or c) a vertex with a doubly-charged scalar. The phenomenological implications for processes with these characteristics are given.

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1. - INTRODUCTION

If neutrinos are massive, an appealing situation would be that they are of Majorana type[1]. In fact, this kind of neutrinos emerge naturally from grand unified schemes like SO(10) [2]. From the point of view of the fermionic content of the theory, Majorana masses are the most economical to generate, because no right-handed ν_R neutrino is needed. A simple mechanism to do so is provided by the Gelmini-Roncadelli model[3),4] which enlarges the Higgs sector of the standard minimal $SU(2)_W \otimes U(1)_Y$ theory by adding a Higgs triplet to the usual doublet.

It is known[5)-7] that Majorana-type mass terms lead, after diagonalization, to the appearance of CP-violating phases in the leptonic charged current, in addition to the usual Kobayashi-Maskawa ones[8]. These additional phases are characteristic of Majorana neutrinos and they can be related to the mass insertion present in the neutrino-antineutrino propagation[9].

In this paper we tackle the question of the origin of the CP phases for Majorana particles, related to the particular mechanism used to generate neutrino masses. Following the accepted wisdom, the fermionic mass terms originate, after spontaneous symmetry breaking, from the Yukawa couplings with the scalar fields. Therefore there are two possible sources of phases:

a) the Yukawa couplings may not be relatively real, so that explicit phase dependences appear in the Lagrangian, and

b) in theories with more than one neutral scalar, the different scalar vacuum expectation values can have relative phases leading to spontaneous CP-violation[10)-12].

In any case, the resulting phase effects will show not only in the charged current piece of the Lagrangian, the one studied in previous works, but also in the charged Higgs couplings to leptonic fields.

In order to work out the entire implications of the leptonic mass term diagonalization, we choose a definite theoretical scheme to generate the neutrino masses. We will work in the framework of the Gelmini-Roncadelli (GR) model, selected because it leads to Majorana neutrino masses in a very simple and natural way. The simplicity of this model, where no spontaneous CP-violation can occur (due to the lepton number symmetry of the Lagrangian), offers a good scenario for clarifying these ideas.

We shall show explicitly the appearance of phases in the scalar sector, and also that, contrary to some suggestions, these are exactly the same phases that have been discussed previously for the charged gauge piece. This fact is motivated by the common phase-sources of both sectors, with $\Delta L = 0$ and $|\Delta L| = 2$ pieces.

In order to fix the notation, we present briefly the GR model in Section 2. The diagonalization of the leptonic mass matrices is made in Section 3, where the lepton-boson interaction Lagrangians are given in terms of the mass eigenstates. The observable Majorana phases are discussed in Section 4, and applications to particular processes are studied in Section 5. Finally, the main conclusions are summarized in Section 6.

2. - THE TRIPLET MODEL

If we do not enlarge the leptonic content of the standard theory, there are only two possible scalar multiplets: the usual doublet $\varphi \sim (\frac{1}{2},\frac{1}{2},0)$ and a scalar triplet $\vec{\phi} \sim (1,1,2)$. [The numbers between brackets refer to the $SU(2)_W$, $U(1)_Y$ and $U(1)_L$ transformation properties.] The Yukawa Lagrangian is then given by:

$$\mathcal{L}_{F,\phi} = f_{ab}\,\bar{\ell}'_{aL}\varphi\,\ell'_{bR} + \tilde{f}_{ab}\,\epsilon_{ij}\,\bar{\ell}'^{c}_{ajL}(\chi\,\ell'_{bL})_i \tag{1}$$

where a,b are flavour indices, i,j are $SU(2)_W$ ones and ℓ'_{aL} is the usual left-handed leptonic doublet $(\nu'_a, \ell'_a)_L$. The super-index c means charge conjugation. We have put primes in the fields to indicate that they are not mass eigenstates. The scalars with definite charge are

$$\varphi = \begin{pmatrix} \varphi^{(+)} \\ \varphi^{(0)} \end{pmatrix} \quad , \quad \chi \equiv \frac{1}{\sqrt{2}}\,\vec{\tau}\cdot\vec{\phi} = \begin{bmatrix} \phi^{(+)}/\sqrt{2} & \phi^{(++)} \\ \phi^{(0)} & -\phi^{(+)}/\sqrt{2} \end{bmatrix} \tag{2}$$

The most general Higgs potential invariant under $SU(2)_W \otimes U(1)_Y \otimes U(1)_L$ can be written in the following form[4]:

$$V(\varphi,\phi) = \lambda_1\left(\varphi^+\varphi - \frac{1}{2}u^2\right)^2 + \lambda_2\left(Tr\,(\chi^+\chi) - \frac{1}{2}v^2\right)^2$$
$$+ \lambda_3\left(\varphi^+\varphi - \frac{1}{2}u^2\right)\left(Tr\,(\chi^+\chi) - \frac{1}{2}v^2\right) \tag{3}$$
$$+ \lambda_4\left(\varphi^+\varphi\, Tr\,(\chi^+\chi) - \varphi^+\chi\,\chi^+\varphi\right) + \lambda_5\left((Tr(\chi^+\chi))^2 - Tr(\chi^+\chi\chi^+\chi)\right)$$

which for λ_1, λ_2, λ_4 and λ_5 positive and $|\lambda_3| < 2.\ \sqrt{\lambda_1\lambda_2}$ gets its minimum when

$$\langle\varphi\rangle = \begin{Bmatrix} 0 \\ u/\sqrt{2} \end{Bmatrix} \quad ; \quad \langle\chi\rangle = \begin{bmatrix} 0 & 0 \\ v/\sqrt{2} & 0 \end{bmatrix} \tag{4}$$

With these vacuum expectation values, we parametrize the neutral Higgs in the form

$$\varphi^{(0)} = \frac{1}{\sqrt{2}}\left(u + \rho_D + i\,\eta_D\right) \quad ; \quad \phi^{(0)} = \frac{1}{\sqrt{2}}\left(v + \rho_T + i\,\eta_T\right) \tag{5}$$

Both vacuum expectation values can be chosen real by means of two suitable independent $SU(2)_W$ and $U(1)_L$ global transformations[3], so, as we have indicated before, this model cannot have spontaneous CP-violation. If $v \neq 0$, the global $U(1)_L$ symmetry is spontaneously broken because the triplet field carries non-zero leptonic number.

The spontaneous symmetry-breaking mechanism generates the following $W^{\pm}$ and $Z°$ masses

$$M_W^2 = \frac{e^2}{4\sin^2\theta_W}\left(u^2 + 2v^2\right) \quad ; \quad M_Z^2 = \frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}\left(u^2 + 4v^2\right) \tag{6}$$

If $v \neq 0$, the standard relation $M_W = M_Z \cdot \cos\theta_W$ is violated to the order $(v/u)^2$. So the experimental success of the $\Delta I = \frac{1}{2}$ predictions implies that $v \ll u \sim 250$ GeV.

Once the gauge bosons have acquired a mass through the Higgs mechanism, seven physical scalars remain in the theory. One of them is a massless Goldstone boson associated with the global $U(1)_L$ breakdown, the majoron,

$$\theta = -\sin\beta\ \eta_D + \cos\beta\ \eta_T \tag{7}$$

The other six fields describe massive scalars

$$\begin{aligned}
\omega^{(\pm)} &= -\sin\alpha\ \varphi^{(\pm)} + \cos\alpha\ \phi^{(\pm)} \\
&\phi^{(\pm\pm)} \\
\rho_H &= \cos\gamma\ \rho_D + \sin\gamma\ \rho_T \\
\rho_L &= -\sin\gamma\ \rho_D + \cos\gamma\ \rho_T
\end{aligned} \tag{8}$$

with masses

$$\begin{aligned}
m_\omega^2 &= \frac{1}{4}\lambda_4 (u^2 + 2v^2) \\
m_\phi^2 &= \frac{1}{2}(\lambda_4 u^2 + 2\lambda_5 v^2) \simeq 2 m_\omega^2 \\
m_{\rho_H}^2 &\simeq 2\lambda_1 u^2 \\
m_{\rho_L}^2 &\simeq (4\lambda_1\lambda_2 - \lambda_3^2)\, v^2/2\lambda_1
\end{aligned} \tag{9}$$

The parameters α, β and γ which determine the physical bosons are given by

$$\cos\alpha = \frac{u}{\sqrt{u^2+2v^2}} \quad ; \quad \cos\beta = \frac{u}{\sqrt{u^2+4v^2}}$$

$$\cos\gamma \simeq \frac{2\lambda_1 u}{\sqrt{4\lambda_1^2 u^2 + \lambda_3^2 v^2}} \tag{10}$$

It is interesting to remark that this pattern of masses is not substantially modified when one tries to embed the model in an SU(5) GUT[13].

3. - DIAGONALIZATION OF THE LEPTONIC MASS MATRICES

In terms of charge eigenstates, the Yukawa Lagrangian is given by

$$\mathcal{L}_{F,\phi} = f_{ab}\left(\bar{\nu}'_{aL}\varphi^{(+)} + \bar{\ell}'_{aL}\varphi^{(0)}\right)\ell'_{bR}$$
$$+ \tilde{f}_{ab}\left[\bar{\ell}'^{c}_{aL}\left(\frac{1}{\sqrt{2}}\phi^{(+)}\nu'_{bL} + \phi^{(++)}\ell'_{bL}\right) - \bar{\nu}'^{c}_{aL}\left(\phi^{(0)}\nu'_{bL} - \frac{1}{\sqrt{2}}\phi^{(+)}\ell'_{bL}\right)\right] + h.c. \tag{11}$$

From this expression one immediately realizes that each vacuum expectation value gives masses separately to only one kind of leptons: the doublet to the charged ones and the triplet to the neutrino species. The mass terms which arise are

$$\mathcal{L}_{mass} = -\left(\bar{\ell}'_L M^{(1)}\ell'_R + \bar{\ell}'_R M^{(1)+}\ell'_L\right)$$
$$+ \frac{1}{2}\left(\bar{\nu}'^{c}_L M^{(2)}\nu'_L + \bar{\nu}'_L M^{(2)+}\nu^{c\prime}_L\right) \tag{12}$$

where

$$M^{(1)}_{ab} \equiv -f_{ab}\, u/\sqrt{2} \qquad \text{and} \qquad M^{(2)}_{ab} \equiv -\tilde{f}_{ab}\, v/\sqrt{2}$$

Following the method of Ref. 9), we use the polar reduction of a regular matrix, writing $M^{(i)} = H_i U_i$ ($i = 1,2$), where H_i is the unique square root of the self-adjoint positive matrix $M^{(i)}\cdot M^{(i)+}$, and U_i is the unitary matrix given by $U_i = H_i^{-1}\cdot M^{(i)}$. Due to the anticommuting character of the fermionic fields, we take $M^{(2)}$ as a complex symmetric matrix $M^{(2)} = M^{(2)T}$. This implies $U_2 = U_2{}^T$.

The positive definite matrices H_i have real and positive eigenvalues, and can be diagonalized by the similarity transformations $H_i = S_i{}^{+}\cdot D_i\cdot S_i$. After some algebra, we write the mass Lagrangian as

$$\mathcal{L}_{mass} = -\bar{\ell}\, D_1\, \ell - \frac{1}{2}\,\bar{\nu}\, D_2\, \nu \tag{13}$$

where the leptonic mass eigenstates are given by

$$\ell = S_1 (\ell'_L + U_1 \ell'_R)$$

$$\nu = S_2 (U_2 \nu'_L - \nu'^c_L) \tag{14}$$

It is easy to prove for the mass eigenstate neutrinos that

$$\nu = - A \nu^c \tag{15}$$

where $A \equiv S_2 \cdot U_2 \cdot S_2^T$ is a unitary diagonal matrix of phases. So Eq. (15) decouples into different relations for each kind of massive neutrino, and the ν_a fields are of Majorana type, as desired.

Substituting now the mass eigenstates in $\mathcal{L}_{F,\phi}$, and gauging away the non-physical scalar fields (the ones eaten by the gauge bosons to get masses), we obtain the Yukawa interactions among physical fields. They are:

$$\mathcal{L}^{\text{neutral}}_{\text{Yukawa}} = -\frac{1}{u} \bar{\ell} D_1 \ell (\cos\gamma\, \rho_H - \sin\gamma\, \rho_L) - \frac{i}{u} \bar{\ell} D_1 \gamma_5 \ell \sin\beta\, \theta$$
$$- \frac{1}{2v} \bar{\nu} D_2 \nu (\cos\gamma\, \rho_L + \sin\gamma\, \rho_H) - \frac{i}{2v} \bar{\nu} D_2 \gamma_5 \nu \cos\beta\, \theta$$

$$\mathcal{L}^{\text{charged}}_{\text{Yukawa}} = \left[\frac{\sqrt{2}}{u} \sin\alpha\, \bar{\nu} U D_1 \ell_R + \frac{1}{v} \cos\alpha\, \bar{\nu} D_2 U \ell_L \right] \omega^{(+)} \tag{16}$$
$$- \frac{1}{\sqrt{2}\, v} \phi^{(++)} \bar{\ell}^c_L V D_2 U \ell_L + \text{h. c.}$$

where we have written separately the contribution of the neutral and charged scalars. In the charged Yukawa sector, we have two different unitary mixing matrices which are given by

$$U = S_2 U_2 S_1^+$$

$$V = S_1^* S_2^+ \tag{17}$$

These two mixing matrices are related by means of the A matrix defined above

$$V = U^T A^* \tag{18}$$

Carrying out the same operation with the interaction Lagrangian among leptons and gauge bosons, we obtain

$$\mathcal{L}_{e.m.} = -e \, A_\mu \, \bar{\ell} \, \gamma^\mu \, \ell$$

$$\mathcal{L}_{n.c.} = \frac{e}{4 \sin\theta_W \cos\theta_W} \left\{ -\bar{\ell} \gamma^\mu [(1 - 4\sin^2\theta_W) + \gamma_5] \ell + \bar{\nu} \gamma^\mu \gamma_5 \nu \right\} Z_\mu$$

$$\mathcal{L}_{c.c.} = \frac{e}{\sqrt{2} \sin\theta_W} W_\mu^{(-)} \bar{\ell}_L \gamma^\mu U^+ \nu + h.c. \tag{19}$$

Some important conclusions can be extracted from these formulae:

1) The leptonic mixings appearing in the charged Higgs ω and gauge W_μ currents are exactly the same but for the mass factors characteristic of the Higgs sector. Therefore, both currents enjoy the same phase dependences. The reason for this identical behaviour stems from the fact that the two single charged boson pieces of the Lagrangian have the same $\bar{\nu}\ell$ field structure, and they are not characteristic of Majorana neutrinos.

2) The neutral scalar couplings have no mixings at all because, in this model, the diagonalization of the leptonic mass matrices implies the diagonalization of the neutral Yukawa couplings.

3) The doubly-charged Higgs current has a richer mixing structure of the form $\ell^T U^T A^* D_2 U \, \ell$. As seen, it only depends on the same U mixing matrix and on the Majorana phase matrix A, so there are no new phases. In fact, one can understand this peculiar mixing if one reinterprets the vertex as a product of two single charged currents $(\bar{\nu}U\ell)^T(\bar{\nu}U\ell)$, the D_2 and A^* matrices being the residual effect of the missing $(\overline{\nu v^T})^*$ neutrinos. This argument will be more apparent in Section 5, when we study a particular process. In any case, the phases present here are characteristic of Majorana behaviour.

4. - MAJORANA PHASES

Some care must be taken to carry out a correct counting of the observable phases. The unitary matrix U has in general $\frac{1}{2}n(n-1)$ moduli and $\frac{1}{2}n(n+1)$ phases but, as one easily realizes, it always appears acting over the charged lepton field, so, by redefinition of the unobservable e-phases, the number of U parameters is reduced to $\frac{1}{2}n(n-1)$ moduli and $\frac{1}{2}n(n-1)$ phases.

To handle the neutrino phases, one can take two different approaches: one can impose the $\nu = \nu^c$ condition fixing univocally the A matrix or, alternatively, one can absorb U-phases by means of the neutrino fields, leaving the A_{ii} elements as free parameters. The observable effects are the same in both parametrizations, as has been explicitly checked for two generations[14]. We will follow the second approach, which allows a clear distinction between the CP-invariant and CP-violating situations[15),16)]. In this scheme the different Majorana fields have different phase conditions, and the mixing matrix U is described in the Kobayashi-Maskawa manner. We have then the following parametrization:

$$\left\{ \begin{array}{l} U \rightarrow \left\{ \begin{array}{ll} 1/2\ n(n-1) & \text{moduli} \\ 1/2\ (n-1)(n-2) & \text{phases} \end{array} \right. \\ \\ (n-1) \text{ phases of the Majorana fields} \end{array} \right. \tag{20}$$

It is convenient to write the Fourier expansion of the free Majorana neutrinos in the form

$$\nu_i(x) = \frac{\phi_i}{(2\pi)^3} \int \frac{d^3p}{2E} \sum_\lambda \left\{ u_i(\bar{p},\lambda) a_i(\bar{p},\lambda) e^{-ipx} \right. \\ \left. + \varphi_i\, u_i^c(\bar{p},\lambda)\, a_i^+(\bar{p},\lambda)\, e^{ipx} \right\} \tag{21}$$

with $u_i^c(\bar{p},\lambda) \equiv C\bar{u}_i^T(\bar{p},\lambda)$ and arbitrary phases ϕ_i and φ_i. The diagonal mass matrix A which relates ν and ν^c is then given by $A_{ii} \equiv -\varphi_i\phi_i^2 \equiv \eta_i$, so the meaning of these phases is clear.

Let us discuss the phenomenological implications of all these new free parameters. Apart from the expected mixings in the charged-current leptonic vertex (as in the charged-quark current of the minimal standard model), and the similar ones appearing in the charged Yukawa couplings, we now have explicit phase dependences in the neutrino fields themselves. This implies several changes in the usual Feynman rules of these particles.

The "neutrino-neutrino" propagation is given by the same formula

$$S(p) \equiv -i\int d^4x\, e^{ipx}\langle 0|T(\nu(x)\bar{\nu}(0))|0\rangle = \frac{\not{p}+m}{p^2-m^2+i\epsilon} \tag{22}$$

as for Dirac particles. But Majorana fields have also "neutrino-antineutrino" propagation, given by

$$\tilde{S}(p) \equiv -i\int d^4x\, e^{ipx}\langle 0|T(\nu(x)\nu(0)^T)|0\rangle = \eta\,\frac{(\not{p}+m)C}{p^2-m^2+i\epsilon} \tag{23}$$

proportional to the product of phases $\eta = -(\varphi\phi^2)$. Thus we can have "CP-violating propagation" of the massive neutrinos.

On the other hand, the same phases appear when one makes the Wick contractions with the asymptotic neutrino fields. Besides, we have two different manners of doing these contractions because of the self-conjugate property of Majorana neutrinos. The rules for the external lines of the Feynman diagrams are given for these particles by

$$\left.\begin{aligned} \nu\ a^{+}(\bar{p},\lambda) &\rightarrow \phi\, u(\bar{p},\lambda) \\ \bar{\nu}\ a^{+}(\bar{p},\lambda) &\rightarrow (\phi\varphi)^{*}\, \bar{u}^{c}(\bar{p},\lambda) \end{aligned}\right\}\quad \text{incoming neutrino}$$

$$\left.\begin{aligned} a(\bar{p},\lambda)\ \nu &\rightarrow (\phi\varphi)\, u^{c}(\bar{p},\lambda) \\ a(\bar{p},\lambda)\ \bar{\nu} &\rightarrow \phi^{*}\, \bar{u}(\bar{p},\lambda) \end{aligned}\right\}\quad \text{outgoing neutrino} \tag{24}$$

Therefore, in order to get observable Majorana phases, we must look for processes in which there is a) neutrino-antineutrino propagation, and/or b) at least two different neutrino asymptotic states, and/or c) a vertex with a doubly-charged scalar.

5. - PHENOMENOLOGICAL IMPLICATIONS

In order to analyze specific examples in a compact form, we rewrite the interesting pieces of the Lagrangian as

$$\mathcal{L}^{(i)} = g_i B_i U_{ab} \bar{\nu}_a O^{(i)}_{ab} \ell_b + \text{h.c.} \quad , \qquad (i = 1, 2, 3)$$

$$\mathcal{L}^{(4)} = -\frac{1}{\sqrt{2}\, v} \phi^{(++)} \bar{\ell}^c_a \frac{1+\gamma_5}{2} \ell_b U_{ca} U_{cb} \eta^*_c m_{\nu_c} + \text{h.c.} \tag{25}$$

where

$$g_1 \equiv \frac{e}{\sqrt{2}\sin\theta_W} \quad ; \quad g_2 \equiv \frac{\sqrt{2}}{u} \sin\alpha \quad ; \quad g_3 \equiv \frac{\cos\alpha}{v}$$

$$O^{(1)}_{ab} \equiv \gamma^\mu \frac{1+\gamma_5}{2} \quad ; \quad O^{(2)}_{ab} \equiv m_{\ell_b} \frac{1-\gamma_5}{2} \quad ; \quad O^{(3)}_{ab} \equiv m_{\nu_a} \frac{1+\gamma_5}{2} \tag{26}$$

$$B_1 \equiv W^{(+)}_\mu \quad ; \quad B_2 \equiv B_3 \equiv \omega^{(+)}$$

and a sum over repeated indices is understood.

As an example of a situation in which the a) and c) phenomena are present, consider processes where a positive charged lepton ℓ^+_a converts into a negative charged lepton ℓ^-_b. This conversion is a second-order process in the $\mathcal{L}^{(i)}$ Lagrangian (i = 1,2,3) mediated by the neutrino-antineutrino propagator, as depicted in Fig. 1a, but it can also occur as a first-order process in $\mathcal{L}^{(4)}$ as indicated in Fig. 1b. The relevant part of the amplitudes for diagrams with the structure 1a is given by

$$T^{ij} = g_i g_j \bar{u}(p_b)\left(X_{ij} + X_{ji}\right) v^T(p_a)$$

$$X_{ij} \equiv \sum_c U^*_{cb}\left[\overline{O^{(j)}_{cb}}\right] \frac{\eta_c(\not{p}_a + \not{p}_i + m_{\nu_c})\, C}{(p_a+p_i)^2 - m^2_{\nu_c} + i\epsilon} \left[\overline{O^{(i)}_{ca}}\right]^T U^*_{ca} \tag{27}$$

where $i,j = (1,2,3)$ refers to the kind of interacting vertices considered, and $\bar{O} \equiv \gamma^\circ O^+ \gamma^\circ$.

Due to the $(1 \pm \gamma_5)$ factors appearing in the $O^{(i)}_{ab}$ operators, the piece $(\not{p}_a + \not{p}_i)$ of the neutrino-antineutrino propagator gives a non-vanishing contribution only to X_{13}, X_{31}, X_{23} and X_{32}, whereas the $m_{\nu c}$ piece only contributes to the other amplitudes. So, as $O^{(3)}_{ca} \propto m_{\nu c}$, the result is always proportional to $m_{\nu c}\eta_c$ (except for the piece $i=j=3$ where there is an additional $m^2_{\nu c}$ factor), and vanishes for zero neutrino masses, as it should.

The amplitude for the diagram 1b is given by

$$T^{(4)} = -\frac{\sqrt{2}}{v} \bar{u}(p_b) \frac{1-\gamma_5}{2} C \bar{v}^T(p_a) \sum_c U^*_{ca} U^*_{cb} \eta_c m_{\nu_c} \tag{28}$$

It is important to note that the $\phi^{(++)}$ vertex plays an equivalent role to the neutrino-antineutrino propagation. In fact, as is apparent in the formulae, the same leptonic mixings appear in both amplitudes, and one understands the role played by the Majorana A matrix in the doubly-charged Higgs coupling. It should not be surprising that the two kinds of diagrams have the same mixing structure $\mathcal{M}_{ab} \equiv \Sigma_c U^*_{ca} U^*_{cb} \eta_c m_{\nu c}$ because both correspond to the same $|\Delta L| = 2$ piece, and it is precisely the spontaneous breaking of leptonic number which lies at the origin of neutrino masses and consequently of Majorana phases.

Assuming that only two of the n generations were coupled, the matrix U may be parametrized with a Cabibbo angle θ, and the only observable phase would be the relative one between the Majorana neutrinos: $\eta_{12} \equiv \eta_1^* \eta_2$. The mixing factor would then be given by

$$\mathcal{M} = \eta_1 \begin{bmatrix} \cos^2\theta\, m_{\nu_1} + \sin^2\theta\, \eta_{12} m_{\nu_2} & \sin\theta\cos\theta(m_{\nu_1} - \eta_{12} m_{\nu_2}) \\ \sin\theta\cos\theta(m_{\nu_1} - \eta_{12} m_{\nu_2}) & \sin^2\theta\, m_{\nu_1} + \cos^2\theta\, \eta_{12} m_{\nu_2} \end{bmatrix} \qquad (29)$$

but for the i=j=3 piece where one must replace all the $m_{\nu i}$ factors by $m^3_{\nu i}$ [in writing this formula we have neglected the small neutrino masses in the denominators of Eq. (27)]. This is the same expression obtained in Ref. 9) for the particular i=j=1 case, where its relevance for μ^+-e^- conversion and neutrinoless double beta decay, induced by gauge couplings, is discussed. The limit[17] for a Majorana mass extracted from the laboratory experiment for the double beta decay of ^{76}Ge, $T_{\frac{1}{2}}(0\nu) > 3.7 \times 10^{22}$ y, quoted as $m_\nu \lesssim 10$ eV, corresponds to $\mathcal{M}_{ee}$. This is sensitive to the CP-violating phase and, for the case of mixing of two generations, we have

$$|\cos^2\theta\, m_{\nu_1} + \eta_{12} \sin^2\theta\, m_{\nu_2}| \lesssim 10 \text{ eV} \qquad (30)$$

The contributions of the charged scalars to these nuclear processes are negligible, because the charged Higgses are dominantly components of the triplet, not coupled to quarks.

The Yukawa coupling of the doubly-charged scalar to leptons is proportional to m_ν/v. The triplet vev has a bound[4] $v \lesssim 1$ MeV coming from Majoron emission by stellar objects. So, this coupling can be important even for small neutrino masses. The electromagnetic muon decay $\mu \to e\gamma$ is dominated[18] by diagrams with the charged Higgs scalars in the loop. To the leading order in neutrino masses, the rate does not depend on the Majorana phases in this particular process, due to unitarity arguments when summing over all internal charged leptons. The relevant mixing is given by

$$\sum_i \mathcal{M}_{ei} \mathcal{M}^*_{\mu i} = \sum_j U^*_{je}\, m^2_{\nu_j}\, U_{j\mu} \equiv \langle m^2_\nu \rangle_{e\mu} \qquad (31)$$

so this particular combination is the one constrained by the experimental bound on the $\mu \to e\gamma$ rate:

$$|\langle m_\nu^2 \rangle_{e\mu}|/v^2 < 7.3 \times 10^{-4} \tag{32}$$

The doubly-charged Higgs allows the existence of the $\mu^+ \to e^+e^+e^-$ decay channel at the tree level[19]. Its branching ratio to ordinary μ decay has been measured recently[20] with improved precision:

$$R \equiv \frac{\Gamma(\mu \to eee)}{\Gamma(\mu \to e\nu\nu)} < 2.4 \times 10^{-12} \tag{33}$$

The amplitude for this process contains the product of mixings associated with $\mathcal{M}^*_{ee}$ and $\mathcal{M}_{\mu e}$, the first factor being the same as the one seen in double beta decay. We get the following relation:

$$\frac{|\mathcal{M}_{ee}|\,|\mathcal{M}_{\mu e}|}{v^2} = 4\, G_F\, m_\phi^2\, R^{1/2} < 5.7 \times 10^{-6} \tag{34}$$

where the bound[19] on the mass of ϕ^{++}, coming from the neutral current to charged current ratio in neutrino scattering, has been used.

The result (34) contains an explicit dependence on the CP phases of the Majorana sector. When two generations are coupled, we have

$$|c_\theta s_\theta (m_{\nu_1} - \eta_{12} m_{\nu_2})|\,|c_\theta^2 m_{\nu_1} + \eta_{12} s_\theta^2 m_{\nu_2}| < 5.7 \times 10^{-6}\, v^2 \tag{35}$$

in a self-evident notation. From this result, it is apparent that even for very different physical situations such as $m_{\nu_1} \sim m_{\nu_2}$ or $m_{\nu_1} \sim m_{\nu_2} \tan^2\theta$, the interfering CP phase is relevant.

The pursuit of the experimental effort in neutrinoless double beta decay and rare decays of the muon, of the highest intrinsic importance, is of direct interest for obtaining information on the mixings and Majorana phases of the leptonic sector. In particular, double beta decay corresponds to a type a) situation in our classification, whereas the $\mu \to eee$ decay corresponds to a type c) case.

A type b) situation is provided by the $\nu_a \to \nu_b$ transition. There are now two kinds of contributions arising from the two different manners of doing contractions with the external neutrino fields. They are depicted schematically in Fig. 2, and explicitly written in Eq. (24). The relevant amplitude is given by

$$T^{ij} = g_i g_j \phi_a \phi_b^* \bar{u}(p_b) Y_{ij} u(p_a)$$

$$Y_{ij} = Y_{ij}^{(1)} - \eta_a^* \eta_b C Y_{ij}^{(2)T} C \tag{36}$$

where

$$Y_{ij}^{(1)} \equiv \sum_c U_{bc} O_{bc}^{(j)} \frac{\not{p} + m_{\ell_c}}{p^2 - m_{\ell_c}^2 + i\epsilon} [\overline{O_{ac}^{(i)}}] U_{ac}^*$$

$$Y_{ij}^{(2)} \equiv \sum_c U_{ac} O_{ac}^{(j)} \frac{\not{k} + m_{\ell_c}}{k^2 - m_{\ell_c}^2 + i\epsilon} [\overline{O_{bc}^{(i)}}] U_{bc}^* \tag{37}$$

The mixing structure of these expressions is a little more involved than in the previous example. However, the role played here by the $\eta_{ab} \equiv \eta_a^* \eta_b$ relative phase between the asymptotic neutrinos, which appears in the interference term between the $Y^{(1)}$ and $Y^{(2)}$ amplitudes, is apparent from Eq. (36). This relevant phase comes, in our choice of phase parametrization, from the different neutrino external legs due to their Majorana character.

The simplest example of this kind of process is the electromagnetic decay of Majorana neutrinos $\nu_a \to \nu_b \gamma$, which has been discussed in the past[21)] in a variety of astrophysical contexts. The amplitude, including mixings and the CP phases, is given by

$$T = -\frac{e G_F}{4\pi^2\sqrt{2}} \bar{u}(p_b) \sigma_{\mu\nu} q^\nu \epsilon^\mu(q) \Big\{ \sum_s U_{bs} U_{as}^* F(r_s, \rho_s)(m_{\nu_a} R + m_{\nu_b} L)$$
$$- \eta_{ab} \sum_s U_{bs}^* U_{as} F(r_s, \rho_s)(m_{\nu_a} L + m_{\nu_b} R) \Big\} u(p_a) \tag{38}$$

when the approximation $m_\ell \ll M_W, M_\omega$ is used. The form factor is

$$F(r,\rho) = -\frac{3}{4}(2-r) - \rho(1+\ln\rho) \; ; \; r_s \equiv \left(\frac{m_{\ell s}}{M_W}\right)^2, \; \rho_s \equiv \left(\frac{m_{\ell s}}{m_\omega}\right)^2 \tag{39}$$

The constant term -3/2 in the form factor does not contribute to the amplitude due to the unitarity of the U matrix. Unfortunately, the charged Higgs contribution also presents a GIM-like cancellation. The structure of the amplitude can be completely changed by the value of the phase η_{ab}. In fact, if one assumes CP invariance, U is a real matrix and η_{ab} can be +1 or -1, giving an amplitude of the electric dipole type $\sigma_{\mu\nu}\gamma_5$ or of the magnetic dipole type $\sigma_{\mu\nu}$ respectively. In the general case, the rate is given by

$$\begin{gathered}\Gamma = \frac{\alpha G_F^2}{64\pi^2}\left(\frac{m_{\nu_a}^2 - m^2_{\nu_b}}{m_{\nu_a}}\right)^3 \left| X_{ab} m_{\nu_b} - \eta_{ab} X_{ab}^* m_{\nu_a}\right|^2 \\ X_{ab} \equiv \sum_s F(r_s, \rho_s) U_{bs} U_{as}^* \end{gathered} \tag{40}$$

The result (40) shows a strong dependence on the Majorana phase for $m_{\nu_a} \sim m_{\nu_b}$. In this case, however, there is a big suppression of the rate due to the kinematical factor of phase space.

The same Majorana phases in a type b) situation appear in ordinary μ-decay[22], where we have two neutrinos in the final state. We have considered the diagrams with the intermediate gauge boson W, including the neutrino crossed diagrams associated with Majorana fields. We have analyzed the effect of neutrino masses, mixings and phases on the electron spectrum. We obtain the following result

$$\begin{aligned}\frac{d\Gamma}{dE} = \frac{m_\mu G_F^2}{12\pi^3} p_e E \Big\{ & 3W - 2E - \frac{m_e^2}{E} \\ & - \frac{3}{2m_\mu}\sum_j m_j^2\left(|U_{ej}|^2 + |U_{\mu j}|^2\right) - \frac{3}{m_\mu}\Big|\sum_j m_j \eta_j U_{ej} U_{\mu j}\Big|^2 \Big\}\end{aligned} \tag{41}$$

where $W = (m_\mu^2 + m_e^2)/2m_\mu$ is the maximum electron energy for massless neutrinos. Equation (41) is the result of summing incoherently all pairs of neutrinos in the final state, and we have assumed that all Majorana neutrinos are kinematically accessible, with $(m_j/m_\mu)^2 \ll 1$. We notice that the mass and mixing effects are of two different kinds. There is a term in the spectrum not specifically associated with Majorana neutrinos, but with massive neutrinos in general. The last term of Eq. (41), however, comes from the interference between the two different diagrams contributing to the process when neutrinos are Majorana particles. The structure of this interference is the same as the $|\Delta L| = 2$ amplitude with the mixing $\mathcal{M}_{e\mu}$, i.e., Eq. (29) for two generations. The detection of these terms would be difficult.

6. - CONCLUSIONS

The study of an explicit leptonic model, the Higgs triplet one, has allowed us to analyze the phase dependences of the whole Lagrangian, starting from its generating source: the spontaneous symmetry-breaking mechanism. We have shown that the Higgs sector contains the same leptonic mixing dependences as the gauge ones. These mixings have been parametrized in terms of a Kobayashi-Maskawa matrix U and the Majorana neutrino phases.

The single charged leptonic currents have always the mixing structure $\bar{\nu}U\ell$, but for the mass factors characteristic of the Higgs couplings. This $\Delta L = 0$ mixing is not related to the Majorana condition of massive neutrinos.

What is peculiar to Majorana behaviour is the existence of a doubly-charged leptonic current, corresponding to the $|\Delta L| = 2$ violation of lepton number. This current has the richer mixing structure $\ell^T U^T D_2 A^* U\ell$, where A is the diagonal matrix containing the Majorana phases and D_2 is the neutrino mass matrix. The $|\Delta L| = 2$ piece can be extracted directly from the $\phi^{(++)}$ vertex, but it also appears when two single charged currents are combined, making the $\nu\nu^T$ contraction of the neutrino fields. The D_2 and A^* matrix factors come in this later case from the neutrino-antineutrino propagator.

The A matrix appearing in the $|\Delta L| = 2$ amplitude leads to observable Majorana phase effects in the interference terms. We have analyzed these manifestations in several processes for which our conditions a) $|\Delta L| = 2$ neutrino propagation, or b) at least two different neutrino asymptotic states, or c) a vertex with a doubly-charged scalar, are met. For a type a) situation, neutrinoless double beta decay offers information on the mixing structure

$$\mathcal{M}_{ee} \equiv \Sigma_j \; U_{je}^* U_{je}^* \eta_j m_j,$$

which is experimentally bounded by about 10 eV in absolute value. The case b) is present in processes such as the electromagnetic decay of massive neutrinos and the ordinary muon decay. For neutrino decay, the corresponding rate depends on the relative CP phase of the external lines. For the two-neutrino μ-decay, the spectrum of outgoing electrons contains a Majorana effect associated with the non-diagonal mixing structure $\mathcal{M}_{e\mu}$, when the incoherent sum of neutrino pairs is made. These mixing amplitudes are also present in a process like the rare decay $\mu^+ \to e^+e^+e^-$, characteristic of a type c) situation. In fact, its rate is determined by the product $|\mathcal{M}_{ee}||\mathcal{M}_{\mu e}|$, but it also contains the triplet vev v, not present in the gauge contributions of the a) and b) cases. From the experimental limit to the rate, we get $|\mathcal{M}_{ee}||\mathcal{M}_{\mu e}| < 5.7 \cdot 10^{-6}\, v^2$, associated with the Yukawa couplings of the doubly-charged Higgs.

Some care must be taken in trying to generalize our results of the Majorana mixing effects to other theoretical models for the Higgs sector. Although it remains obvious that the mixings appearing in the Higgs and gauge sectors must be related, it is no longer true that they would be exactly the same. This nice property of the GR model is a consequence of the fact that each component of the lepton doublets gets its mass from only one vacuum expectation value. If two different Higgs multiplets were able to give mass to the same type of leptons, the diagonalization of the whole mass matrix would not imply that the leptonic currents of the different Yukawa pieces would have the same mixing structure.

On the other hand, in models with spontaneous CP violation, although our arguments are not spoiled because the vacuum expectation values are already enclosed in the fermionic matrices, its phases would appear in most of the Higgs coupling constants, leading to additional phase dependences which are not related with leptonic mixings.

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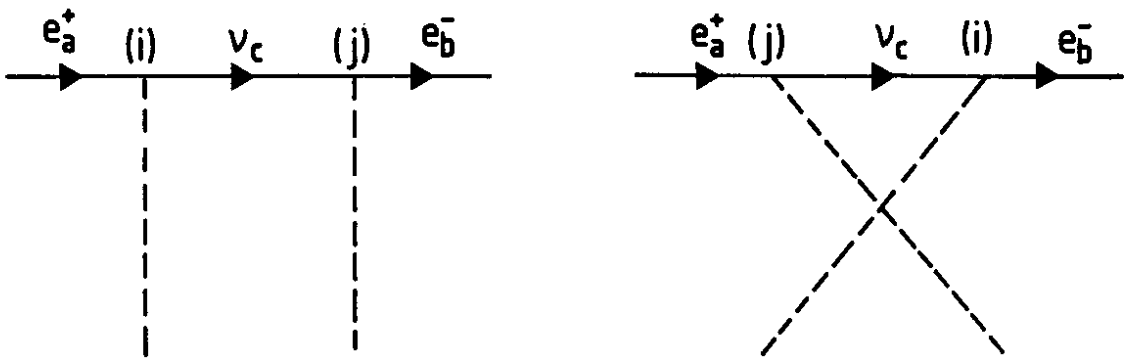
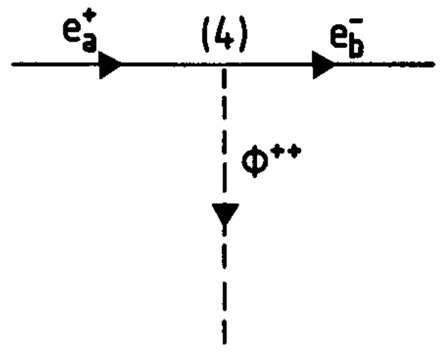
FIGURE CAPTIONS

Fig. 1 : Diagrams contributing to the conversion $\ell_a^+ \to \ell_b^-$.
a) Second-order W and ω emission mediated by Majorana neutrino propagation;
b) ϕ^{++} vertex.

Fig. 2 : Diagrams contributing to the conversion $\nu_a \to \nu_b$, where (i), (j) correspond to W and ω vertices.

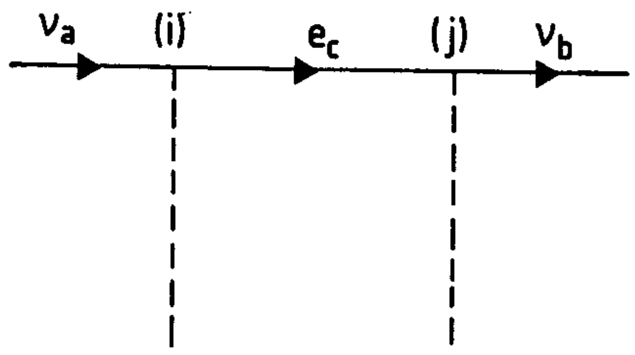


(a)

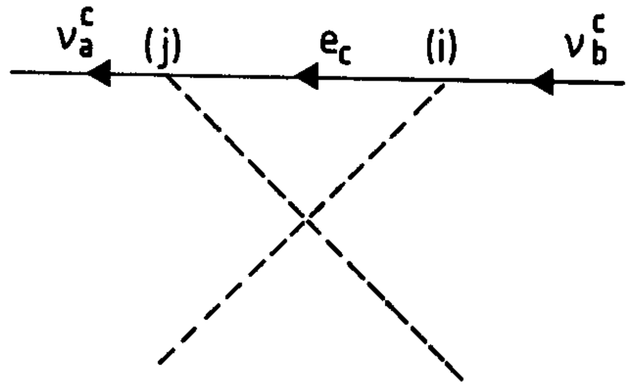


(b)

Fig. 1



(1)



(2)

Fig. 2