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NEUTRINOLESS DOUBLE BETA DECAY IN THE SU(4) SYMMETRY SCHEME

B. DESPLANQUES[1], S. NOGUERA[2,3], J. BERNABEU[2,3],

[1]Institut des Sciences Nucléaires, 38026 Grenoble-Cédex (France)

[2] Departamento de Fisica Teorica, Universidad de Valencia, C/ Dr Moliner, 50; E - 46100 Burjasot-Valencia (Spain)

[3]IFIC, Centro Mixto, CSIC - Universidad de Valencia (Spain)

Abstract. The SU(4) symmetry scheme, used in a previous paper for the study of the 2ν double β decay, is applied to the study of the 0ν double β decay. Differences and similarities between the 2 processes are emphasized. The sensitivity to short-range correlations is discussed.

I. Introduction

A great interest is presently devoted to double β decay and especially to the case where it occurs without ν emission [1-5]. The observation of this last process would be of the upmost importance for determining some of the neutrino properties. To get parameters relative to the neutrino however, some knowledge of nuclear matrix elements is necessary. As is well known, their estimate is not easy and it is highly desirable to check them, as far as possible, on other processses. An example of such a process is the double β decay with 2ν emission. It is fully allowed by the standard model and involves operators which have some similarity with those describing the 0ν process. Thus, the discussion of this important process cannot be separated from that for the other one.

The first comparison [6] of theory and experiment in 2ν double β decay has shown that corresponding predicted decay rates were too large by up to one or two orders of magnitude. Since then, it has been found that incorporating in QRPA approaches some particle-particle force was accounting for the observations [2,7]. How to explain the suppression of those decay rates has also been shown in quite different approaches based on the SU(4) symmetry [8,9]. In this paper, we extend to the 0ν process the SU(4) symmetry based analysis we did for the 2ν process [8]. With the help of this symmetry scheme, we shall emphasize in particular the similarities and differences that the two processes evidence. In this aim, we first remind a few results relative to the 2ν process, and then consider the 0ν process. The sensitivity of the results to short-range correlations is finally discussed.

2 - Two neutrinos double beta decay

It is well known that the SU(4) symmetry is not a fundamental one and its interest mainly resides in the fact that it easily allows to characterize the pieces of forces between nucleons. The dominant feature of the nucleon-nucleon force is the attraction in S states, which largely explains the binding of nuclei. It is essentially SU(4) symmetric, the relative strength of the interaction in $^1S_0$ and $^3S_1$ states often being quantified by a factor 0.6, corresponding to a relative strength of SU(4) non-symmetric and symmetric components of 0.25. In other channels this symmetry does not show up, but their contribution is generally

smaller. While it is not obvious, the relevance of the SU(4) symmetry in complex nuclei is nevertheless evidenced by several features [10] and in particular by the concentration in one state of the Gamow-Teller strength.

Due to the strong attractive interaction in S waves, the ground state of a nucleus will tend to have the lower SU(4) quantum numbers compatible with the number of protons and neutrons. Thus, for an even even nucleus, it will belong to the SU(4) supermultiplet [y,y,0] and will have $S = 0$, $T = y = (N-Z)/2$ [11]. Therefore a double β transition from ^AZ(g.s.) to $^A(Z+2)$(g.s.) involves initial and final states belonging to different supermultiplets [y,y,0] and [y-2, y-2,0] respectively.

It is important to notice that the difference in SU(4) assignments for initial and final states is independent of that one resulting from a different distribution of neutrons and protons among single particle orbits. It is rather similar to the difference between a nucleus with $N = Z = A/2$, for which ground state isospin may be equal to $T = 0$, and the nucleus with a proton replaced by a neutron ($N-1 = Z+1 = A/2$), whose isospin is at least equal to $T = 1$.

Assuming that the interaction is SU(4) invariant immediately implies the suppression of the 2ν double β decay transition since the corresponding matrix element :

$$M_{GT}^{2\nu} = \sum_n \frac{\langle f|\sigma\,\tau^+|\,n\rangle\langle n\,|\,\sigma\,\tau^+|\,i\rangle}{E_n - (E_i+E_f)/2} \qquad (1)$$

involves the SU(4) symmetry generators, which cannot connect different SU(4) multiplets. The corresponding result in the example implying the isospin given above is the suppression of the transition between $T = 0$ and $T = 1$ states induced by the isospin generator, $\sum_i \tau_i^+$. Acting on the $T = 0$ state, it gives zero.

In the simplest description of nuclei, most double β transitions of interest involve a transition between two neutrons in some orbit, ℓ_2, to two protons in a lower energy orbit, ℓ_1.Therefore, the vanishing of $M_{GT}^{2\nu}$ in the SU(4) symmetry limit may simply be trivial and would appear as a Pauli effect. Our statement about the suppression goes much beyond this case and holds as far as correlations between nucleons are produced by a SU(4) symmetric interaction. In such a case, the argument based on the Pauli principle does not work anymore (at least for nucleons as degrees of freedom).

Some finite value for 2ν double β decay can arise from the SU(4) symmetry breaking part of forces between nucleons. At the lowest order, it is due to the admixture into the final state, represented by [y-2,y-2,0] in the standard SU(4) notation, of some component represented by [y,y,0] . Calculations along these lines have been performed [8]. Assuming that initial and final states have seniority zero, (they are built by adding pairs of nucleons in a [11] Young tableau with L = 0) we have :

$$|i\rangle = |\ell_1^{n1}\, L_1 = 0\, [y_1,y_1]\,,\ \ell_2^{n2}\, L_2 = 0\, [y_2,y_2]\,;\, L = 0\, [y,y]\; S = 0\; T = y\rangle,$$

$$|f\rangle = |\ell_1^{n1+2}\, L_1 = 0\, [y_1-1,y_1-1]\,,\ \ell_2^{n2-2}\, L_2 = 0\, [y_2-1,y_2-1]\,;\, L = 0\, [y-2,y-2]\; S = 0\; T = y\rangle \quad (2)$$

where y_i is the half difference between the number of neutrons and protons in each shell in the initial state. We obtained for the amplitude the following expression :

$$M_{GT}^{2\nu} = (-)^{1+\ell_1+\ell_2}\,\frac{12}{\pi^2}\,\frac{C}{(E_f-E_s)\,(E_{GT}-(E_i+E_f)/2)}$$

$$\cdot \sum_\lambda (\ell_1\, \ell_2\, \lambda;00)^2 \int dq\, q^2\, |R_{12}^{\lambda}(q)|^2\, V(q), \quad (3)$$

where

$$C = \frac{1}{4}\left[\frac{(8\ell_1+2y_1-n_1+12)(n_1-2y_1+4)(8\ell_2-2y_2+8-n_2)(n_2+2y_2+8)y_1y_2}{(y_1+2)\,(y_2+2)}\right]^{1/2},$$

$$R_{12}^{\lambda}(q) = \int dr\, r^2\, j_\lambda(qr)\, R_{n_1\ell_1}(r)\, R_{n_2\ell_2}(r),$$

and E_s is the energy of some excited state of the final nucleus belonging to the supermultiplet of the initial nucleus [8].

Results are given in Table 1 for the effective force of Bertsch and Hamamoto [12] . In the first column we indicate the transition under consideration. For ^{82}Se, ^{128}Te and ^{130}Te we consider two different configurations. The actual status of the SU(4) algebra does not allow to consider the mixing between configurations. The theoretical results for $M_{GT}^{2\nu}$ are given in the second column. In the third column we give the

$M_{GT}^{2\nu}$ are given in the second column. In the third column we give the values of the amplitudes obtained from the experimental half lives taking $g_A/g_V = 1$. Numbers between brackets correspond to the experimental amplitudes obtained with $g_A/g_V = 1.25$.

The size of our results, compared to experiment, has been discussed elsewhere [8]. The important point that is worthwhile to consider here concerns the intrinsic sign, unobservable. By decreasing the strength of the force in triplet spin states ($^3S_1$), one tends to a situation where pairing dominates. Simultaneously, our results for $M_{GT}^{2\nu}$ first decrease in magnitude, go through zero and then increase, but with the opposite sign. By comparing with calculations using QRPA approaches, it seems that our results would correspond to a value of g^{pp}/g^{pair} significantly larger than 1.

3. Zero neutrino double beta decay

We here concentrate on the case where 0 ν double β decay would be due to a massive Majorana neutrino. The amplitudes relative to this process are given by :

$$M_F^{0\nu} = \langle f\,|\sum_{i,j}\tau_i^+\,\tau_j^+\, H_m(r_{ij})\,|\,i\rangle,$$

$$M_{GT}^{0\nu} = \langle f\,|\sum_{i,j}\tau_i^+\,\tau_j^+\,\sigma_i\cdot\sigma_j\, H_m(r_{ij})\,|\,i\rangle\,, \quad (4)$$

where $H_m(r_{ij})$ is the potential originated by the neutrino exchange [1]

$$H_m(r) = \frac{2R}{\pi r}\int^{\infty}\frac{q\,\sin qr}{\omega\,(\omega+\bar{E})}\,dq$$

where $R = 1.2\, A^{1/3}$ fm is the nuclear radius , $\omega = \sqrt{q^2+m_\nu^2}$ is the neutrino energy and $\bar{E}$ is some average energy of intermediate nuclear state which we have taken equal to 10 MeV.

The difference between (4) and the 2ν case is the appearance of some radial dependance in the transition operators, preventing to identify them with SU(4) generators. In the SU(4) symmetric case, the above amplitudes, in contrast to the 2ν case, are not a priori suppressed and can take finite values.

However, approximating $H_m(r)$ by some constant leads back to the 2ν case and therefore some suppression, with a different origin, might occur. Such a feature is characterized by the absence of contribution corresponding to the lower orbital multipole, with $L = 0$, in the expansion of the potential. Only multipoles with $L = 1, 2, \ldots$, whose contribution is somewhat smaller, make the value of the amplitudes, $M_F^{0\nu}$ and $M_{GT}^{0\nu}$, non-zero at this zeroth order of the calculation. The corresponding amplitudes are given by :

$$M_F^{0\nu,0} = \frac{1}{4\pi^2} (g_V/g_A)^2 (-)^{\ell_1+\ell_2} \, C \sum_\lambda (\ell_1\ell_2\lambda;00)^2 \int dq\, q^2 (R_{12}^\lambda(q))^2 \, H_m(q),$$

$$M_{GT}^{0\nu,0} = -\frac{3}{4\pi^2} (-)^{\ell_1+\ell_2} \, C \sum_\lambda (\ell_1\ell_2\lambda;00)^2 \int dq\, q^2 (R_{12}^\lambda(q))^2 \, H_m(q), \qquad (5)$$

where the C coefficient is the same as in eq. 3.

At the first order in the SU(4) symmetry breaking interaction, the above approximation consisting in taking a constant $H_m(r)$ gives now a non-zero contribution. Although this one involves some suppression, measured by the rate of symmetry breaking, it may not be quite negligible because it receives contributions from the dominant lower multipole, $L = 0$, in contrast to the 0th order contribution. It has been calculated, assuming that $H_m(r) = H_m$, thus allowing to use results for the 2ν case. The corresponding amplitudes are :

$$M_F^{0\nu,1} = 0,$$

$$M_{GT}^{0\nu,1} = \bar{H}_m \left(E_{GT} - \frac{(E_i+E_f)}{2}\right) M_{GT}^{2\nu}. \qquad (6)$$

For numerical results, we have taken $\bar{H}_m = 1.0$, which results from an average of $H_m(r)$ over dominant diagonal matrix elements of interest.

An important feature of this last contribution is its sign relatively to the 0th order contribution. It is such as to produce some cancellation. As seen from eq. (6), this is directly related to the sign of the 2ν amplitude discussed before, itself related to the relative strength of forces in $^3S_1$ and $^1S_0$ NN states. The rate of suppression is not large and varies from 20% for transitions where the lower allowed multipole has $L = 1$ (transition $g^2 \leftrightarrow f^2$, $g^2 \leftrightarrow h^2$) to 30% when $L = 3$ (transition $g^2 \leftrightarrow p^2$). We will come back to the consequences of this cancellation when discussing the effect of short-range correlations.

The total results, $M_{F(GT)}^{0\nu} = M_{F(GT)}^{0\nu,0} + M_{F(GT)}^{0\nu,1}$, are presented in Table 1. We also provide the corresponding limits on the combination of neutrino masses $\langle m_\nu \rangle$, which can be deduced from these results together with experimental limits on 0ν double β decay processes. Two sets of values are given, depending whether $\left(\frac{g_A}{g_V}\right)$ is taken to be 1.25 or 1.0. This last value may be appropriate for GT transitions and therefore for the 2ν process. It is not so obvious that it should be considered for the 0ν case, the answer being related to the nature of the mechanism responsible for the quenching of (g_A/g_V) in nuclei.

4. Effect of short-range correlations. Discussion

Motivated partly by the cancellation mentioned above, which may enhance further corrections, partly by the large effect of short-range correlations obtained by Engel et al [2] , we have looked at this effect in our approach. The same correlation function as theirs has been used. It has been incorporated in the calculation of matrix elements of H(r), but not in those of the Bertsch Hamamoto effective force which, in principle, already contains this effect. The essential feature that comes out from a detailed examination of the contributions corresponding to various multipoles is that the lower ones are reduced by a little amount (6% - 15%), while the higher ones, whose size is smaller however, can be reduced by a factor as large as 4 or 5 ($L = 7$). As a result, the zeroth order contribution to the 0ν amplitude is more reduced than the first order SU(4) symmetry breaking contribution of opposite sign, giving an overall reduction varying from 30 to 40% (for $\frac{g_A}{g_V} = 1$, see Table 1). The effect thus seems to be smaller than the one obtained by Engel et al. [2].

A closer comparison with the results of Engel et al. [2] however reveals that their 2ν amplitude is much larger than ours (by a factor 6 in ^{76}Ge). Rescaling accordingly the first order SU(4) symmetry breaking contribution to the 0ν amplitude would then lead to a dramatic cancellation with the 0th order contribution, the effect of short-range correlations reaching easily a factor 4 (and perhaps more), as obtained by these authors. Present results therefore support some of their results. It is not sure however that the effect is so dramatic. Comparison with experiment for the 2ν case in ^{76}Ge would only require a rescaling of the 1st order SU(4) symmetry breaking contribution by a factor 3. On the other hand, part of this factor, which is difficult to explain in our approach, may be due to the presence of a low energy 1^+ state in the intermediate nucleus, whose contribution would thus be enhanced in the 2ν case. Finally, one can imagine cases where the 0th order contribution should be rescaled at the same time as the 1st order contribution.

5. Conclusion

We described here an approach to the 0ν double β decay based on the SU(4) symmetry. Two types of contributions have been considered. The first one is fully allowed in the SU(4) symmetry limit, while the second one arises at the 1st order in the SU(4) symmetry breaking. In our calculations, this one has a sign opposite to the other one and it is the only contribution that may be related to the 2ν double β decay amplitude. There is in principle no way to scale the 0ν amplitude from the 2ν amplitude as once proposed.

Our approach is quite different from other ones and rather simple with many respects. It nevertheless accounts for some of the features which play an important role in them. Pairing is built in for a large part in the choice of our wave functions describing initial and final states. As to QRPA correlations, whose main role in the present case is to restore the dominant SU(4) symmetric character of the force, arbitrarily broken by the mean field BCS approximation [9], they don't need to be considered in the present approach where the breaking is limited to what is expected. In spite of differences, it is therefore not surprising that our results evidence several similarities with other ones, in particular at the level of the individual multipole contributions or of the $M_{GT}^{0\nu}/M_F^{0\nu}$ ratio. For this quantity, equal to $-3(g_A/g_V)^2$ in our calculations at the 0th order, the closeness to the result of Klapdor et al [5] further indicates that the effect of the single spin-orbit splitting, whose neglect may be a weak point of our approach, is not so important. The main difference with some works to be mentioned is the large contribution of the L = 0 ($J = 1^+$) multipole, strongly related to the 1st

order SU(4) symmetry breaking contribution. Due to the cancellation with the other contributions, it makes the results more sensitive to various effects, such as short-range correlations. This seems to confirm results obtained by Engel et al [2], but not those of Klapdor et al [5] or Faessler et al [3]. It is possible that the collapse of their QRPA approach when g^{pp}/g^{pair} becomes close to 1 has prevented them to consider values greater than 1, which rather corresponds to our case and to that of Engel et al.

We believe that the sign (and size) of this particular contribution, to which little attention has been given in the literature, probably represents one of the most important point to clarify in the near future. This is a necessary step if better limits on the ν mass from double β decay are to be obtained.

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TABLE CAPTION

Table 1. Results for 2ν double β decay and 0ν double β decay (without and with short-range correlations). In the first column are given the characteristics of the transition under consideration while in the last one are given the present limits on the 0ν double β decay life-times. Numbers between parentheses are obtained with $g_A/g_V = 1.25$. The experimental half-lives are taken from :

2ν : ^{76}Ge [13], ^{82}Se [14], ^{128}Te and ^{130}Te [15],

0ν : ^{76}Ge [16], ^{82}Se [17], ^{128}Te and ^{130}Te [15].

The relation between nuclear amplitudes and the half-life is given in ref. [1].

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	Transition	2 ν processes		0ν processes			0ν processes with SRC			$T^{0\nu}_{1/2}$ (exp) (y)
		$M^{2\nu}$ (MeV^{-1})	$M^{2\nu}_{exp}$ (MeV^{-1})	M_{GT}	M_F	limit on <mν> (eV)	M_{GT}	M_F	limit on <mν> (eV)	
^{76}Ge	$g^2 \rightarrow f^2$	-0.072	0.27 (0.17)	3.65	-1.50 (-.96)	2.8 (2.0)	2.67	-1.16 (-0.74)	3.8 (2.4)	> 4.7 10^{23}
^{82}Se	$g^2 \rightarrow p^2$	-0.028		0.98	-0.47 (-0.30)	31. (23.)	0.57	(-0.30) (-0.19)	52. (38.)	
			0.13							> 1.1 10^{22}
	$g^2 \rightarrow f^2$	-0.080	(0.082)	4.67	-1.91 (-1.22)	6.9 (4.9)	3.45	-1.47 (-0.94)	9.2 (5.9)	
^{128}Te	$h^2 \rightarrow g^2$	-0.082		7.86	-3.00 (-1.9)	0.77 (.55)	5.97	-2.33 (-1.49)	1.01 (0.72)	
			0.032 0.047							> 5 10^{24}
	$d^2 \rightarrow g^2$	0.016	(0.024-0.030)	-1.10	0.44 (0.28)	5.4 (3.9)	-0.65	0.29 (0.18)	8.9 (6.4)	
^{130}Te	$h^2 \rightarrow g^2$	-0.082		8.67	-3.28 (-2.10)	8.2 (5.8)	6.59	2.55 (-1.63)	11. (7.6)	
			.028-0.034							> 1.5 10^{21}
	$d^2 \rightarrow g^2$	0.016	(0.018-0.022)	-1.11	0.44 (0.28)	63. (45.)	-0.66	0.29 (0.18)	100.0 (75.)	