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# UNIVERSALITY-BREAKING EFFECTS IN LEPTONIC $Z$ DECAYS

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## ABSTRACT

We analyze the possibility of universality violation in diagonal leptonic decays of the  $Z$  boson, in the context of interfamily "see-saw" models. In a minimal extension of the Standard Model with right-handed neutrino fields, we find that universality-breaking effects increase quadratically with the heavy Majorana neutrino mass and may be observed in the running  $LEP$  experiments.

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Heavy Majorana neutrinos with masses of few TeV have entered the domain of cosmology and astrophysics, as possible candidates to account for the net baryon number of the universe through lepton ( $L$ )-number violating processes [1]. On the other hand, if such heavy neutral leptons are realized in nature, their existence may be discovered through their production and the  $L$ -violating decay at present or future collider machines [2]. Another place of looking for new physics originating from heavy neutrinos is the rare leptonic decays of  $H^0$  [3] and  $Z$  particle [4]. Since such decays are forbidden in the minimal Standard Model ( $SM$ ), they constitute an interesting framework to constrain new physics beyond the  $SM$ . Such rare decays [3, 5] have recently been analyzed in a "see-saw"-type model [6] with intergenerational mixings [7]. An interesting aspect of this minimal scenario is that the Appelquist-Carazzone theorem [8] is not operative and vertex-correction diagrams with intermediate heavy Majorana neutrinos ( $N_i$ ) show a quadratic mass dependence, i.e.  $m_{N_i}^2/M_W^2$ . This mass dependence is a common feature for all theories based on the spontaneous-symmetry breaking mechanism. For example, in the Feynman-'t Hooft gauge this mass dependence can be seen to arise from the coupling of the would-be charged Goldstone bosons to heavy fermions. Similar non-decoupling effects in the quark sector have been extensively studied in the past for the flavor changing decays  $Z \rightarrow b\bar{s}$  [9] and the diagonal  $Z \rightarrow b\bar{b}$  [10].

In this note we study universality-breaking effects induced by heavy Majorana neutrinos in leptonic  $Z$  decays. Actually, we will analytically calculate the following physical observable:

$$U_{br} = \frac{\Gamma(Z \rightarrow \tau^+\tau^-) - \Gamma(Z \rightarrow l^+l^-, l = e \text{ or } \mu)}{\Gamma(Z \rightarrow \tau^+\tau^-) + \Gamma(Z \rightarrow l^+l^-, l = e \text{ or } \mu)}. \quad (1)$$

$U_{br}$  is a measure of universality violation in the leptonic sector provided lepton masses can be neglected and is experimentally constrained to be [11]  $|U_{br}| \leq 1.5 \cdot 10^{-2}$ . The model we are concerned with extends the  $SM$  by one right-handed neutrino field for each family. The renormalizable form of all relevant interactions and details of the notation we will

use here can be found in [7, 5].

In the *SM* the universality-violating parameter  $U_{br}$  has a value different from zero due to  $\tau$  lepton mass effects. However, pure phase-space (*PS*) corrections turn out to be rather small, i.e.  $|U_{br}^{(PS)}| \simeq 3 [1 + (1 - 4 \sin^2 \theta_W)^2]^{-1} m_\tau^2/M_Z^2 \simeq 1.1 \cdot 10^{-3}$ . This standard source for a non-zero value of  $U_{br}$  is beyond the sensitivity of the present *LEP* experiments. Since one can expect to analyze about  $10^5$  leptonic decays of the  $Z$  boson at *LEP* per year, one may reach an accuracy for  $|U_{br}|$  at the level of  $3 \cdot 10^{-3}$ . Since we are interested in values of  $|U_{br}|$  much larger than  $|U_{br}^{(PS)}|$ , we will neglect lepton-mass effects in the calculation of quantum corrections.

Decomposing now the transition matrix element  $\mathcal{T}(Z \rightarrow l\bar{l})$  into two parts (with the superscripts (0), (1) denoting the electroweak loop order), i.e.  $\mathcal{T}(Z \rightarrow l\bar{l}) = \mathcal{T}^{(0)}(Z \rightarrow l\bar{l}) + \mathcal{T}^{(1)}(Z \rightarrow l\bar{l})$ , with

$$\mathcal{T}^{(0)}(Z \rightarrow l\bar{l}) = \frac{ig_W}{4 \cos \theta_W} \varepsilon_Z^\mu \bar{u}_l \gamma_\mu (1 - 4 \sin^2 \theta_W - \gamma_5) v_{\bar{l}},$$

and defining  $\Delta\mathcal{T}^{(1)} = \mathcal{T}^{(1)}(Z \rightarrow \tau^+\tau^-) - \mathcal{T}^{(1)}(Z \rightarrow l^+l^-, l \neq \tau)$ , we find that

$$\Delta\mathcal{T}^{(1)} = \frac{ig_W \alpha_W}{8\pi \cos \theta_W} \varepsilon_Z^\mu \bar{u}_l \gamma_\mu (1 - \gamma_5) v_{\bar{l}} \Delta B_{ij} F_Z(\lambda_i, \lambda_j). \quad (2)$$

Then, the universality-breaking parameter  $U_{br}$  takes the simple form

$$U_{br} = \frac{\text{Re}(\mathcal{T}^{(0)} \Delta\mathcal{T}^{(1)})}{|\mathcal{T}^{(0)}|^2} = \frac{\alpha_W}{\pi} \frac{1 - 2 \sin^2 \theta_W}{1 + (1 - 4 \sin^2 \theta_W)^2} \Delta B_{ij} F_Z(\lambda_i, \lambda_j), \quad (3)$$

where

$$\lambda_i = \frac{m_{n_i}^2}{M_W^2}, \quad \Delta B_{ij} = B_{\tau i} B_{\tau j}^* - B_{li} B_{lj}^*, \quad l \neq \tau. \quad (4)$$

In Eq. (4),  $B_{li}$  is a Cabbibo-Kobayashi-Maskawa-type  $n_G \times 2n_G$  matrix appearing in the leptonic charged-current interaction, and  $m_{n_i}$  indicates the masses of all neutral leptons  $n_i$  in our minimal scenario (i.e.  $i = 1, 2, \dots, 2n_G$ , with  $n_G$  denoting the number of families). The function  $F_Z(\lambda_i, \lambda_j)$  originates from the one-loop graphs depicted in Fig. 1 and contains

all the non-decoupling physics mediated by heavy Majorana neutrinos. Its explicit form will be discussed below.

Before proceeding to give the analytical form of  $F_Z$ , we remark that the universality-violating parameter  $U_{br}$  does not involve any infra-red ( $IR$ ) singularities, as they do not depend on neutrino masses. Therefore, soft-photon emission graphs need not to be considered here. Also, the  $Z - l - l$  vertex should be renormalized at the one loop level. In fact, a large number of renormalization constants should not appear in the universality-breaking parameter  $U_{br}$ . If we adopt the on-shell renormalization scheme [12, 13], where the input renormalization parameters are the electric charge  $e$ ,  $M_W$ ,  $M_Z$ , the Higgs mass  $M_H$  and all fermion masses contained in the model, the counterterm Lagrangian  $\mathcal{L}_{int}^C$  relevant for the renormalization of the  $Z - l - \bar{l}$  vertex is then given by [14]

$$\begin{aligned} \mathcal{L}_{int}^C = & ie Z_{AZ}^{1/2} \bar{l} \gamma_\mu l Z^\mu + \frac{ie}{4s_W c_W} \left[ 1 + \frac{\delta e}{e} + \frac{1 - 2c_W^2}{2s_W^2} \delta\rho + \delta Z_{ZZ}^{1/2} \right. \\ & + \delta Z_L^l \left. \bar{l} \gamma_\mu (1 - \gamma_5) l Z^\mu - \frac{ie s_W}{c_W} \bar{l} \gamma_\mu \left[ 1 + \frac{\delta e}{e} + \frac{1}{2s_W^2} \delta\rho + \delta Z_{ZZ}^{1/2} \right. \right. \\ & \left. \left. + \delta Z_L^l \frac{1 - \gamma_5}{2} + \delta Z_R^l \frac{1 + \gamma_5}{2} \right] l Z^\mu, \right. \end{aligned} \quad (5)$$

where  $\delta\rho = \delta M_Z^2/M_Z^2 - \delta M_W^2/M_W^2$ ,  $c_W = \cos\theta_W = M_W/M_Z$  and  $s_W = \sin\theta_W$ . Due to  $GIM$ -type cancellation [15] the only non-vanishing contribution to the function  $F_Z$  comes from the wave-function renormalization constants of the left- and right-handed leptons, i.e.  $\delta Z_L^l$  and  $\delta Z_R^l$ . In fact, one has to calculate the difference of the self-energy derivatives given by  $\Delta Z_L^l = \delta Z_L^l - \delta Z_L^{l \neq \tau} = \Delta \partial \Sigma(\not{p}) / \partial \not{p} |_{\not{p}=m_l, m_\tau \rightarrow 0}$ . The corresponding constant for the right-handed leptons  $\Delta Z_R^l$  vanishes in the limit  $m_l, m_\tau \rightarrow 0$ . It is easy to see that only the neutrino-mass dependent self-energy graphs mediated by  $W^\pm$  and  $\chi^\pm$  are of interest here. The individual contributions to  $F_Z(\lambda_i, \lambda_j)$  arising from the diagrams 1(a)–1(f) and those from the wave-function renormalization constant  $\Delta Z_L^l$  are given by

$$F_Z^{(a)} = \frac{1}{2} \left[ C_{ij} \left( L_2(\lambda_i, \lambda_j) - \lambda_Z \left[ K_1(\lambda_i, \lambda_j) - K_2(\lambda_i, \lambda_j) + \tilde{K}(\lambda_i, \lambda_j) \right] \right) \right]$$

$$+ C_{ij}^* \sqrt{\lambda_i \lambda_j} K_1(\lambda_i, \lambda_j) \Big] , \quad (6)$$

$$F_Z^{(b)} = -\frac{1}{4} \left[ C_{ij} \lambda_i \lambda_j K_1(\lambda_i, \lambda_j) + C_{ij}^* \sqrt{\lambda_i \lambda_j} \left( \frac{1}{2} C_{UV} - \frac{1}{2} + \lambda_Z \tilde{K}(\lambda_i, \lambda_j) - L_2(\lambda_i, \lambda_j) \right) \right] , \quad (7)$$

$$F_Z^{(c)} = -\delta_{ij} \left[ \lambda_Z \tilde{I}(\lambda_i) + 3c_W^2 L_1(\lambda_i) \right] , \quad (8)$$

$$F_Z^{(d)} = \frac{1}{8} \delta_{ij} (1 - 2s_W^2) \lambda_i (C_{UV} - 2L_1(\lambda_i)) , \quad (9)$$

$$F_Z^{(e)} + F_Z^{(f)} = -\delta_{ij} \frac{s_W^2}{c_W} \lambda_i I(\lambda_i) , \quad (10)$$

$$F_Z^{(\Delta Z_L^l)} = -\frac{1}{8} \delta_{ij} (1 - 2s_W^2) \lambda_i \left( C_{UV} + \frac{3}{2} - \frac{3}{1 - \lambda_i} - \frac{(\lambda_i + 2)\lambda_i \ln \lambda_i}{(1 - \lambda_i)^2} \right) , \quad (11)$$

where

$$\begin{aligned} C_{ij} &= \sum_{k=1}^{n_G} B_{l_{ki}}^* B_{l_{kj}} , & \lambda_Z &= \frac{M_Z^2}{M_W^2} , \\ C_{UV} &= \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi - \ln \frac{M_W^2}{\mu^2} . \end{aligned} \quad (12)$$

The functions  $I$ ,  $\tilde{I}$ ,  $L_1$ ,  $K_1$ ,  $K_2$ ,  $\tilde{K}$  and  $L_2$  involved in Eqs. (6)-(11) are given in Appendix A. It is straightforward to see that the ultraviolet ( $UV$ ) divergences (i.e.  $C_{UV}$ ) cancel in the summation of all  $F_Z$  terms. To be precise, the  $UV$  pole in  $F_Z^{(d)}$  cancels against the  $UV$  one of the wave-function renormalization  $F_Z^{(\Delta Z_L^l)}$  and the  $UV$  constant in Eq. (7) vanishes due to the identity [3]:  $\sum_{i=1}^{2n_G} m_{n_i} B_{li} C_{ij}^* = 0$ .

For definiteness, we will consider a interfamily-mixing model with two families only. Employing relations between the mixing matrix  $B_{li}$  and heavy Majorana neutrino masses [5] together with Eqs. (3) and (6)–(11), we arrive at the simple result

$$|U_{br}| \simeq \frac{\alpha_W}{8\pi} \frac{(s_L^{\nu\tau})^4 - (s_L^{\nu l})^4}{(1 + x^{-1/2})^2} \lambda_{N_1} \left[ 1 + \frac{1}{2} \ln x - \frac{\ln x}{1 - x} (1 + 2x^{1/2}) \right] , \quad (13)$$

where  $x = m_{N_2}^2/m_{N_1}^2$ . We also assume  $N_2$  to be heavier than  $N_1$ , i.e.  $x \geq 1$ . In Eq. (13)  $s_L^{\nu l}$  is the usual neutrino-mixing angle between heavy Majorana neutrinos and the charged

lepton  $l$  and is generally defined as [16]  $(s_L^{\nu_l})^2 = \sum_{N_i} |B_{lN_i}|^2$ . The neutrino-mixing angle  $(s_L^{\nu_\tau})^2$  turns out to be severely constrained by the recent *LEP* data on  $\tau$  decays. In fact, a global analysis allowing mixing of exotic particles gives an upper bound of about  $7 \cdot 10^{-2}$  [17]. The  $e$ - and  $\mu$ -family is much more constrained, i.e.  $(s_L^{\nu_e(\mu)})^2 < 0.01$ . Due to this fact we have to deal with universality-breaking effects in the heaviest lepton family only. It is important to notice that the non-decoupling terms (i.e. proportional to  $m_N^2/M_W^2$ ) come from the "seemingly" suppressed  $(s_L^{\nu_\tau})^4$  terms. Table 1 shows the dramatic non-decoupling behaviour of the loop function  $F_Z$ . For comparison, we also show the corresponding values for a calculation where terms proportional to  $(s_L^{\nu_\tau})^4$  have been neglected. In our numerical estimates we have assumed that there is no large mass difference between the two heavy Majorana neutrinos, i.e.  $x \simeq 1$ . We have varied the heavy neutrino mass  $m_N$  ( $\sim m_{N_1} \sim m_{N_2}$ ) up to its perturbative unitarity bound. Such an upper bound may be imposed by requiring that the total width of  $N_1$  and  $N_2$ , denoted by  $\Gamma_{N_i}$ , satisfies the condition  $\Gamma_{N_i}/m_{N_i} \leq 1/2$  [5]. This leads to the constraint on the mass of the lightest heavy Majorana neutrino  $N_1$

$$m_{N_1}^2 \leq \frac{2M_W^2}{\alpha_W (s_L^{\nu_\tau})^2} \frac{1 + x^{-1/2}}{x^{1/2}}. \quad (14)$$

Taking the above upper bound into account, we find that the universality-violating parameter  $|U_{br}|$  can be up to 10 times larger than the naive value obtained by considering only terms proportional to the mixing  $(s_L^{\nu_\tau})^2$ . This enhancement factor (i.e. the crucial  $m_N^2/M_W^2$  dependence) results from the coupling of the would-be charged Goldstone boson  $\chi^+$  to the heavy Majorana neutrinos in the diagram 1(b). If we assume very large mass differences between the heavy neutrinos,  $|U_{br}|$  smoothly decreases to negligibly small values. The reason is that one effectively recovers the one generation "see-saw" model in such a case, i.e.  $m_{N_1} \rightarrow 0$  as  $x \rightarrow \infty$  in Eq. (14). In Fig. (2) we present exclusion plots for *LEP* experiments. We see, for example, that possible universality-breaking effects of the order of  $10^{-2}$  can easily be understood within our minimal model.

Non- $SM$  contributions, due to heavy neutrinos, could also be observed by comparing the prediction of the  $SM$  for the leptonic rates  $\Gamma(Z \rightarrow l^+l^-)$  with those expected when one includes heavy neutral lepton effects. Besides the universality-violating phenomena discussed in this paper, one could have non-decoupling physics introduced by these heavy neutral leptons which can be constrained by analyzing the oblique electroweak parameters  $S, T, U$  or  $\varepsilon_1, \varepsilon_2, \varepsilon_3$  as defined in [18]. These contributions occur both through vacuum polarization terms and in vertex corrections universal for all charged lepton flavors. However, a first estimate for the  $\delta\rho$  parameter [19] (i.e.  $\delta\rho = \alpha_{em}T$ ) provides a weaker bound than that obtained by Eq. (14). In particular, the dominant non- $SM$  contribution (denoted below as  $\delta\rho^*$ ) to  $\delta\rho$ , apart from heavy-top and Higgs-particle effects, comes from the  $\nu_i N_j, N_i N_j$  intermediate states of the  $Z$  self-energy graph. In this way, one gets

$$\delta\rho^*|_{q^2=0} \simeq \frac{\alpha_W}{16\pi} (s_L^{\nu\tau})^4 \frac{m_N^2}{M_W^2}, \quad \text{for } x = 1. \quad (15)$$

We can readily compare Eq. (15) with constraints on the masses of the heavy neutrinos that are derived on the basis of perturbative unitarity. For example, if  $(s_L^{\nu\tau})^2 = 0.1$ , the maximal value that  $\delta\rho^*$  can take is  $0.8 \cdot 10^{-2}$ , which is still in accordance with phenomenological constraints of a possible mass shift of the  $W$  boson [20]. However, to complete the analysis a global consideration of all electroweak oblique corrections is required.

In conclusion, we have explicitly shown that heavy neutral leptons introduce a quadratic mass dependence (i.e.  $\alpha_W m_N^2/M_W^2$ ) in the leptonic vertex function  $Z - l - \bar{l}$ . This situation is not peculiar for the minimal model considered in this work, but a general feature for all theories based on the spontaneous symmetry-breaking mechanism, e.g. similar effects will be present in the model described in [4]. In general, we have found that the mass of possible non-decoupling neutral particles can be constrained by the already existing or future  $LEP$  data, as a function of their mixing to the ordinary charged leptons.

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## A The one-loop integrals

It is useful first to define the functions  $B_1(\lambda_i)$  and  $B_2(\lambda_i, \lambda_j)$  which are given by the following expressions:

$$B_1(\lambda_i) = (1 - y)\lambda_i + y[1 - \lambda_Z yx(1 - x)] , \quad (\text{A1})$$

$$B_2(\lambda_i, \lambda_j) = 1 - y + y[x\lambda_i + (1 - x)\lambda_j - \lambda_Z yx(1 - x)] , \quad (\text{A2})$$

where  $x$  and  $y$  are Feynman parameters. The loop functions  $I$ ,  $\tilde{I}$ ,  $L_1$ ,  $K_1$ ,  $K_2$ ,  $\tilde{K}$  and  $L_2$  can then be written in terms of the following integrals:

$$I(\lambda_i) = \int \frac{dx dy y}{B_1(\lambda_i)} , \quad (\text{A3})$$

$$\tilde{I}(\lambda_i) = \int \frac{dx dy y^2}{B_1(\lambda_i)} [1 - yx(1 - x)] , \quad (\text{A4})$$

$$L_1(\lambda_i) = \int dx dy y \ln B_1(\lambda_i) , \quad (\text{A5})$$

$$K_1(\lambda_i, \lambda_j) = \int \frac{dx dy y}{B_2(\lambda_i, \lambda_j)} , \quad (\text{A6})$$

$$K_2(\lambda_i, \lambda_j) = \int \frac{dx dy y^2}{B_2(\lambda_i, \lambda_j)} , \quad (\text{A7})$$

$$\tilde{K}(\lambda_i, \lambda_j) , = \int \frac{dx dy y^3 x(1 - x)}{B_2(\lambda_i, \lambda_j)} , \quad (\text{A8})$$

$$L_2(\lambda_i, \lambda_j) = \int dx dy y \ln B_2(\lambda_i, \lambda_j) , \quad (\text{A9})$$

where the integration variables  $x$  and  $y$  are constrained to the interval  $[0, 1]$ . The above integrals can be best performed numerically.



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- [20] See for example, F. Halzen, B.A. Kniehl, M.L. Stong, *Z. Phys.* **C58** (1993) 119; who have recently performed a global analysis of all electroweak parameters in the *SM* at two-loop level.

## Figure and Table Captions

Fig. 1: One-loop irreducible vertex graphs contributing to the non-universality parameter  $U_{br}$  in the Feynman-'t Hooft gauge.

Fig. 2: *Exclusion plots for LEP experiments.* The areas lying to the right of the curves are excluded due to the following conditions: (i) The validity of perturbative unitarity (solid line) – see also text, (ii)  $|U_{br}| \leq 1.10^{-2}$  (dashed line), (iii)  $|U_{br}| \leq 7.10^{-3}$  (dash-dotted line), (iv)  $|U_{br}| \leq 3.10^{-3}$  (dotted line).

Tab. 1: Numerical estimates for the universality-violating quantity  $|U_{br}|$  within the perturbatively allowed parameter space. The values in parentheses are obtained by neglecting the "seemingly" suppressed terms proportional to  $(s_L^{\nu_\tau})^4$ .

Table 1

$m_N$ [TeV]	$(s_L^{\nu_\tau})^2$						
	0.06	0.05	0.04	0.03	0.02	0.01	0.005
0.3	$1.7 \cdot 10^{-4}$ ( $9.4 \cdot 10^{-5}$ )	$1.3 \cdot 10^{-4}$ ( $7.8 \cdot 10^{-5}$ )	$9.8 \cdot 10^{-5}$ ( $6.3 \cdot 10^{-5}$ )	$6.7 \cdot 10^{-5}$ ( $4.7 \cdot 10^{-5}$ )	$4.0 \cdot 10^{-5}$ ( $3.1 \cdot 10^{-5}$ )	$1.8 \cdot 10^{-5}$ ( $1.6 \cdot 10^{-5}$ )	$8.4 \cdot 10^{-6}$ ( $7.8 \cdot 10^{-6}$ )
0.5	$4.9 \cdot 10^{-4}$ ( $2.8 \cdot 10^{-4}$ )	$3.8 \cdot 10^{-4}$ ( $2.3 \cdot 10^{-4}$ )	$2.8 \cdot 10^{-4}$ ( $1.8 \cdot 10^{-4}$ )	$1.9 \cdot 10^{-4}$ ( $1.4 \cdot 10^{-4}$ )	$1.2 \cdot 10^{-4}$ ( $9.2 \cdot 10^{-5}$ )	$5.2 \cdot 10^{-5}$ ( $4.6 \cdot 10^{-5}$ )	$2.5 \cdot 10^{-5}$ ( $2.3 \cdot 10^{-5}$ )
0.7	$8.4 \cdot 10^{-4}$ ( $4.2 \cdot 10^{-4}$ )	$6.4 \cdot 10^{-4}$ ( $3.5 \cdot 10^{-4}$ )	$4.7 \cdot 10^{-4}$ ( $2.8 \cdot 10^{-4}$ )	$3.2 \cdot 10^{-4}$ ( $2.1 \cdot 10^{-4}$ )	$1.9 \cdot 10^{-4}$ ( $1.4 \cdot 10^{-4}$ )	$8.2 \cdot 10^{-5}$ ( $7.0 \cdot 10^{-5}$ )	$3.8 \cdot 10^{-5}$ ( $3.5 \cdot 10^{-5}$ )
1.0	$1.4 \cdot 10^{-3}$ ( $5.9 \cdot 10^{-4}$ )	$1.1 \cdot 10^{-3}$ ( $4.9 \cdot 10^{-4}$ )	$7.7 \cdot 10^{-4}$ ( $3.9 \cdot 10^{-4}$ )	$5.1 \cdot 10^{-4}$ ( $2.9 \cdot 10^{-4}$ )	$2.9 \cdot 10^{-4}$ ( $1.9 \cdot 10^{-4}$ )	$1.2 \cdot 10^{-4}$ ( $9.8 \cdot 10^{-5}$ )	$5.5 \cdot 10^{-5}$ ( $4.9 \cdot 10^{-5}$ )
2.0	$4.3 \cdot 10^{-3}$ ( $9.4 \cdot 10^{-4}$ )	$3.1 \cdot 10^{-3}$ ( $7.8 \cdot 10^{-4}$ )	$2.1 \cdot 10^{-3}$ ( $6.3 \cdot 10^{-4}$ )	$1.3 \cdot 10^{-3}$ ( $4.7 \cdot 10^{-4}$ )	$6.9 \cdot 10^{-4}$ ( $3.1 \cdot 10^{-4}$ )	$2.5 \cdot 10^{-4}$ ( $1.5 \cdot 10^{-4}$ )	$1.0 \cdot 10^{-4}$ ( $7.8 \cdot 10^{-5}$ )
3.0	$8.7 \cdot 10^{-3}$ ( $1.1 \cdot 10^{-3}$ )	$6.2 \cdot 10^{-3}$ ( $9.6 \cdot 10^{-4}$ )	$4.1 \cdot 10^{-3}$ ( $7.7 \cdot 10^{-4}$ )	$2.5 \cdot 10^{-3}$ ( $5.7 \cdot 10^{-4}$ )	$1.2 \cdot 10^{-3}$ ( $3.8 \cdot 10^{-4}$ )	$4.0 \cdot 10^{-4}$ ( $1.9 \cdot 10^{-4}$ )	$1.5 \cdot 10^{-4}$ ( $9.6 \cdot 10^{-5}$ )
4.0	$1.5 \cdot 10^{-2}$ ( $1.3 \cdot 10^{-3}$ )	$1.0 \cdot 10^{-2}$ ( $1.1 \cdot 10^{-3}$ )	$6.8 \cdot 10^{-3}$ ( $8.7 \cdot 10^{-4}$ )	$4.0 \cdot 10^{-3}$ ( $6.5 \cdot 10^{-4}$ )	$1.9 \cdot 10^{-3}$ ( $4.3 \cdot 10^{-4}$ )	$5.9 \cdot 10^{-4}$ ( $2.2 \cdot 10^{-4}$ )	$2.0 \cdot 10^{-4}$ ( $1.1 \cdot 10^{-4}$ )
6.0	—	—	—	$8.3 \cdot 10^{-3}$ ( $7.5 \cdot 10^{-4}$ )	$3.9 \cdot 10^{-3}$ ( $5.0 \cdot 10^{-4}$ )	$1.1 \cdot 10^{-3}$ ( $2.5 \cdot 10^{-4}$ )	$3.3 \cdot 10^{-4}$ ( $1.2 \cdot 10^{-4}$ )
8.0	—	—	—	—	—	$1.8 \cdot 10^{-3}$ ( $2.7 \cdot 10^{-4}$ )	$5.1 \cdot 10^{-4}$ ( $1.4 \cdot 10^{-4}$ )
10.0	—	—	—	—	—	—	$7.3 \cdot 10^{-4}$ ( $1.4 \cdot 10^{-4}$ )