

# Weak magnetic dipole moments in two-Higgs-doublet models

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## Abstract

We investigate the effects of the new scalars in a two-Higgs-doublet model on the weak magnetic dipole moments of the fermions at the  $Z$  peak. Proportionality of the Yukawa couplings to the fermion masses, and to  $\tan\beta$ , makes such effects more important for the third family, and potentially relevant. For the  $\tau$  lepton, the new diagrams are suppressed by  $v_\tau = 2\sin^2\theta_W - 1/2$ , or by powers of  $m_\tau/M_Z$ , but may still be comparable to the SM electroweak contributions. In contrast, we find that the new contributions for the bottom quark may be much larger than the SM electroweak contributions. These new effects may even compete with the gluonic contribution, if the extra scalars are light and  $\tan\beta$  is large. We also comment on the problem of the gauge dependence of the vertex, arising when the  $Z$  is off mass shell. We compute the contributions from the new scalars to the magnetic dipole moments for top-quark production at the NLC, and for bottom and  $\tau$  production at LEP2. In the case of the top, we find that the SM electroweak and gluonic contributions to the  $Zt\bar{t}$  vertex are comparable. The new contributions may be of the same order of magnitude as the standard-model ones, but not much larger.

# 1 Introduction

Thus far, the nature of the symmetry-breaking sector of the standard model (SM) remains largely untested, with a fundamental scalar yet to be found. In fact, the number of Higgs multiplets is not predicted and must be determined by experiment, just like the number of fermion families. Moreover, the success of supersymmetry coupling-constant unification suggests that one might have several Higgs doublets, since the minimal supersymmetric standard model (MSSM) requires two doublets. In this article we consider a two-Higgs-doublet model (THDM), which has physical scalars  $H^\pm$ ,  $A^0$ ,  $H^0$  and  $h^0$ .

While these particles remain undiscovered, we may look into the virtual effects that they induce in several phenomena. For instance, from the neutral-meson mixings one can set limits on the flavour-changing scalar vertices. These limits are so stringent that Glashow and Weinberg [1] and, independently, Paschos [2], introduced the concept of Natural Flavour Conservation, implemented through a discrete symmetry. We will assume that one doublet couples to the right-handed up-type quarks, while the other doublet couples to the right-handed down-type and charged-lepton fields. This is the so-called model II, which includes the scalar sector of the MSSM as a particular case. In model II, the Yukawa couplings are proportional to the fermion masses, so that one should look to the third family for noticeable effects.

Another interesting source of information comes from the radiative corrections to the gauge couplings of the fermions. On the one hand, the  $Z$  couples to a fermion  $f$  through a vector ( $v_f$ ) and an axial-vector ( $a_f$ ) coupling:

$$\frac{ie}{2s_W c_W} \bar{u}(p_-) \left[ \gamma^\mu \left( v_f(q^2) - a_f(q^2) \gamma_5 \right) \right] v(p_+). \quad (1)$$

These couplings occur already at tree level in the SM, with

$$\begin{aligned} v_f &= T_{3f} - 2Q_f s_W^2, \\ a_f &= T_{3f}. \end{aligned} \quad (2)$$

Here,  $s_W$  and  $c_W$  are the Weinberg angle's sine and cosine, and  $Q_f$  and  $T_{3f}$  are the fermion's charge and third component of the weak isospin, respectively. The momentum of the  $Z$  is  $q = p_- + p_+$ . Therefore, precise measurements are required to disentangle the loop effects. On the other hand, the anomalous weak magnetic dipole moments (WMDM)  $\mu_f$ , defined to be the couplings of the  $Z$  to  $f$  of the form

$$\frac{ie}{2m_f} \bar{u}(p_-) \left[ \mu_f(q^2) i\sigma^{\mu\nu} q_\nu \right] v(p_+), \quad (3)$$

arise only at loop level, making them preferred tools in the search for physics beyond the SM.

In this article, we compute the WMDMs of the  $\tau$  lepton and of the bottom quark in the model described above. We separate the SM contributions from the ones involving new scalar particles, and study the conditions under which the latter are numerically important. In particular, we reproduce the SM computation of the  $\tau$ -lepton's WMDM

in Ref. [3]. We give both analytic formulas for the WMDMs, and also their numerical values. The analytic formulas can also be applied to compute the usual magnetic dipole moments of the fermion with the photon<sup>1</sup>.

The article is organized as follows. In section 2, we discuss the WMDM of the  $\tau$  lepton in the THDM. In section 3 we present the results for the bottom quark. Section 4 is devoted to a discussion of the problem of the gauge non-invariance of the WMDM when the  $Z$  is off mass shell. This is crucial for the top quark, but also for the other fermions, when they are produced in colliders such as LEP2 or the NLC. We give some numerical values relevant for these colliders. We draw our conclusions in section 5. An appendix contains the analytic expressions for the  $Z$  and  $\gamma$  magnetic dipole moments (MDMs), induced at one-loop level in model II.

## 2 The WMDM of the $\tau$

One expects the  $\tau$  lepton to be a leading candidate in the search for new physics through WMDMs. This is due to the absence of gluon contributions, and to the fact that the energy and angular distribution of the decay products of the  $\tau$  can be used to extract information on the spin density matrix of the  $\tau$  pairs. Several groups have used this method to isolate the dispersive and absorptive parts of the weak electric dipole moment (WEDM) [5, 7] and of the WMDM [3, 6, 7]. An analysis, including all the form factors in terms of the spin density matrix of the  $\tau$ , is given in Ref. [8]. In particular, the transverse polarization of the  $\tau$  (within the collision plane) measures the real part of the WMDM and the imaginary part of the WEDM, while the normal polarization (perpendicular to the collision plane) measures the imaginary part of the WMDM and the real part of the WEDM. These are gauge-invariant, observable quantities, as long as the external particles are on mass shell:  $p_-^2 = p_+^2 = m_\tau^2$ , and  $q^2 = M_Z^2$ .

In the SM, as well as in model II, the WEDM is multiloop suppressed (since it must involve the quark mixing matrix), and therefore these polarizations measure directly the WMDM. The calculation of the WMDM of the  $\tau$  in the SM has been performed in Ref. [3], who found

$$\mu_\tau^{SM}(M_Z^2) = (-2.10 - i 0.61) \times 10^{-6}, \quad (5)$$

which is well below the expected experimental limit of  $10^{-4}$ .

A typical diagram contributing to the magnetic dipole moments is shown in Fig. 1. We will denote the different diagrams by the particles running in the loop, starting with the particle  $A$  in between the external fermions, and proceeding counterclockwise to particles  $B$  and  $C$ . We denote by  $\phi$  the SM Higgs scalar, and by  $\sigma$  ( $\chi$ ) the charged (neutral) Goldstone bosons.

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<sup>1</sup>In our notation, the tree-level coupling of the photon to  $f$  is  $ieQ_f\gamma^\mu$ , while the magnetic dipole moment is defined as in Eq. (3). The usual definition for the photon coupling [4],

$$ieQ_f \bar{u}(p_-) \left[ \gamma^\mu F_1(q^2) + \frac{i}{2m_f} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] v(p_+), \quad (4)$$

yields the following relation with our photon magnetic dipole moment:  $\mu_f = Q_f F_2$ .

The various contributions are suppressed by different powers of  $m_\tau$ . One power is always there due to the definition of  $\mu_\tau$  [See Eq. (3)]. Extra powers come from the Yukawa couplings (in the case of the scalars and of the Goldstone bosons) and, in the diagrams in which the  $Z$  couples in the loop to two particles with the same spin, from the mass insertion required to reproduce the chirality structure of the WMDM. The latter suppression is avoided when the  $B$  and  $C$  particles in Fig. 1 are a scalar and a vector (or vice-versa), that is, in the  $\tau\phi Z$  and  $\nu\sigma W$  diagrams. However, in the SM as in model II, the suppression thus avoided is offset by the proportionality of the Yukawa couplings to the fermion masses. That is not the case in the most general THDM, without Natural Flavour Conservation.

We have performed a complete computation of  $\mu_\tau$  in the SM, carefully checking its gauge invariance. Each of the four sets of diagrams in Fig. 2 is separately gauge invariant when the external particles are on mass shell. (One must also use the tree-level relation  $c_W = M_W/M_Z$  to explicitly verify gauge invariance.) This is also the case for the diagrams involving the SM Higgs particle  $\phi$  (the ones in Figs. 3a and 3b, with  $h^0 \rightarrow \phi$ ). The expressions for the MDM in the 't Hooft–Feynman gauge are listed in the appendix. We agree with the results in Ref. [3], except for a few signs, and for the small  $\tau\phi Z$  and  $\tau Z\phi$  contributions, which do not affect significantly the final numerical results.

For the numerical computations, we used the following values. At  $q^2 = M_Z^2$ ,  $\alpha_{em} = 1/128$  and  $s_W^2 = 0.232$ . (Notice that, due to the large cancellation in  $v_\tau = 2s_W^2 - 1/2$ , some of the numerical results are very sensitive to the input value of  $s_W^2$ .) We use  $M_Z = 91$  GeV,  $M_W = c_W M_Z$ , and  $m_\tau = 1.777$  GeV. We take the neutrino to be massless. Using this input, we get the values for the contributions to the  $\tau$  WMDM displayed in Table 1. We have grouped the diagrams into the gauge-invariant classes

$ABC$	$\mu_\tau^{ABC}$
$\gamma\tau\tau$	$(3.1853 - i 1.2713) \times 10^{-7}$
$Z\tau\tau + \chi\tau\tau$	$(4.1316 + i 1.9133) \times 10^{-8}$
$W\nu\nu + \sigma\nu\nu$	$(-9.8734 - i 5.0748) \times 10^{-7}$
$\nu WW + \nu\sigma W + \nu W\sigma + \nu\sigma\sigma$	$-1.4733 \times 10^{-6}$
Total	$-2.1008 \times 10^{-6} - i 6.1548 \times 10^{-7}$

Table 1: Standard-model contributions to the WMDM of the  $\tau$

shown in Fig. 2. Our final result agrees with that of Ref. [3].

We stress that all the results presented in this article are based on exact formulas, which we have evaluated using two completely different programs, as a cross-check. In many cases, the numerical results may be guessed at from the analytical expressions in the appendix by taking the relevant integrals to be dominated by the largest mass scale in the diagram. In the appendix, we show explicitly that this ‘guesstimate’ of the integrals is correct for the  $\gamma\tau\tau$  diagram. Experience shows that it also works for many other diagrams, making it a valuable tool in understanding the relative magnitudes of the different contributions.

We now consider the scalar-particle contributions, both in the SM and in the THDM. The relevant diagrams are the ones shown in Fig. 3, and their contributions

are given in the appendix. There are four types of scalar contributions: those with a neutral scalar and a  $Z$  (Fig. 3a), those with a neutral scalar (Fig. 3b), those with a neutral pseudo-scalar (Fig. 3c), and those with a charged scalar (Figs. 3d and 3e). In the SM, only the first two types of diagrams are present. They are functions of the Higgs mass which we display in Figs. 4 and 5, for the  $\tau\phi Z + \tau Z\phi$ , and  $\phi\tau\tau$  diagrams, respectively. We have taken all the scalar masses to be greater than 50 GeV. One can see that the SM Higgs contributions are never larger than about 2% of the non-Higgs contributions.

In model II, the contribution of diagram 3a to the WMDM is proportional to  $\cos^2\alpha + \tan\beta \sin\alpha \cos\alpha$ , for  $H^0$ , and to  $\sin^2\alpha - \tan\beta \sin\alpha \cos\alpha$ , for  $h^0$ , where  $\alpha$  is the mixing angle in the CP-even mass matrix, and  $\tan\beta$  is the ratio between the two vev's. In the following we shall take  $1 < \tan\beta < 70$  and let  $\alpha$  take any value. The proportionality factor is the function plotted in Fig. 4. We find that this contribution may increase or decrease the real part of the WMDM by about 80%, for a scalar of mass 50 GeV. Of course, this diagram does not originate an imaginary part since, at  $q^2 = M_Z^2$ , no cut can be made.

Similarly, the amplitude generated by diagram 3b is proportional to the function of the mass of the scalar shown in Fig. 5. The proportionality factors are  $(1 + \tan^2\beta)$  times  $\cos^2\alpha$  for  $H^0$ , or times  $\sin^2\alpha$  for  $h^0$ . The appearance of  $\tan^2\beta$  makes this contribution potentially large. It turns out that the contribution to the real part of the WMDM is negligible, but that the contribution to its imaginary part can make it to about one third of the SM value, for a scalar of mass 50 GeV. This happens despite the fact that this diagram has an extra  $m_\tau^2/M_Z^2$  suppression factor.

The contribution from the pseudo-scalar in diagram 3c is equal to  $\tan^2\beta$  times a function of the mass of  $A^0$  displayed in Fig. 6. We see that, if that mass is 50 GeV and  $\tan^2\beta$  is large, this may decrease the imaginary part of the WMDM by about 25%. The influence on the real part of the WMDM is negligible.

Finally we turn to the contribution from the charged scalar involved in diagrams 3d and 3e. We show in the appendix that such contributions are, in general, linear combinations of 1,  $\tan^2\beta$ , and  $\cot^2\beta$ . However, due to the zero mass of the neutrino, in this case only the term proportional to  $\tan^2\beta$  is non-vanishing. The factor of proportionality is displayed in Fig. 7 as a function of the mass of the charged scalar. We see that both the real and the imaginary parts of the charged-scalar contribution may make it to about  $1 \times 10^{-6}$ , if the mass of that particle is 50 GeV, and  $\tan\beta$  is maximal. This is a contribution of the same order of magnitude as the SM one, but of the opposite sign.

In conclusion, we find that all the new diagrams arising in model II may have an impact on the WMDM of the  $\tau$  lepton, if  $\tan\beta \sim 70$  and the mass of the scalar particles is  $\sim 50$  GeV. They may be comparable to the SM contributions, but never much larger than them. This is partly due to the masslessness of the neutrino, which eliminates some terms from the general expressions, and partly due to the proportionality to the vector coupling of the tau,  $v_\tau$ , which is approximately  $-0.036$ . This will not be the case for the quarks.

### 3 The WMDM of the bottom quark

The measurement of the WMDM of the bottom quark is more problematic since its polarization is affected by hadronization. Still, one may look for its influence on the  $Z \rightarrow b\bar{b}$  observables. Currently, this yields bounds of the order of  $10^{-2}$  [9].

Another possibility is brought about by the fact that the initial spin of the bottom quark is retained in the polarization of the  $\Lambda_b$  baryons that are produced directly [10, 11], although there might be substantial depolarization induced by the  $\Lambda_b$ 's produced from the decay of heavier b-baryons [10, 12]. This polarization can be studied by measuring the energy spectra of the charged-leptons [13], or the neutrinos [14], produced in the semileptonic decays of the  $\Lambda_b$ .

In the calculation of the WMDM of the bottom quark, we have used the values of  $M_Z$ ,  $M_W$ ,  $s_W$  and  $\alpha_{em}$  given in the previous section. In addition, we take  $m_b = 5$  GeV,  $m_t = 174$  GeV, and  $\alpha_S(M_Z) = 0.117$ . The values obtained are listed in Table 2. Contrary to what happened in the case of the  $\tau$ , for the case of the bottom, all the

$ABC$	$\mu_b^{ABC}$
$gbb$	$(3.5764 - i 1.9382) \times 10^{-4}$
$\gamma bb$	$(1.9900 - i 1.0785) \times 10^{-6}$
$Zbb + \chi bb$	$(3.0041 + i 1.3393) \times 10^{-6}$
$Wtt + \sigma tt$	$-2.2064 \times 10^{-6}$
$tWW + t\sigma W + tW\sigma + t\sigma\sigma$	$-6.4791 \times 10^{-6}$
Total	$(3.5394 - i 1.9356) \times 10^{-4}$

Table 2: Standard-model contributions to the WMDM of the bottom

gauge-invariant electroweak contributions are of the same order of magnitude, except for the Higgs contributions, which amount to less than 1% of the overall electroweak result. However, the WMDM of the bottom quark is dominated by the gluon diagram, shown in Fig. 8, which is about two orders of magnitude larger than the electroweak ones.

Still, the new scalars in model II may be important since the two factors limiting their impact on the WMDM of the  $\tau$  lepton are not present in the case of the bottom quark. In fact, the Yukawa-coupling factors in the vertices are now often factors of enhancement instead of suppression, because of the large coupling of the top, and also of the bottom in the large- $\tan\beta$  limit. In addition, the vector coupling  $v_b = 2/3 \sin^2 \theta_W - 1/2 \approx -0.345$  is an order of magnitude larger than  $v_\tau$ .

The shape of the contributions from diagrams 3a and 3b as functions of the scalar masses are similar to those of Figs. 4 and 5, although the numerical values are different. We find that the diagrams in Fig. 3a may increase or decrease the real part of the WMDM by some 35%, if one of the scalars of the THDM is very light while the other one is very heavy, and if  $\tan\beta$  is very large. Similarly, the  $h^0 bb$  or  $H^0 bb$  diagrams in Fig. 3b may be important for the imaginary part of the WMDM, increasing its SM value by about 70% if  $\tan\beta$  is large and the scalars are light.

As before, the pseudo-scalar contribution is proportional to  $\tan^2 \beta$  and, if  $\tan^2 \beta = 5000$  and  $m_{A^0} = 50$  GeV, that contribution is maximal, reaching  $(-3 + 9i) \times 10^{-5}$ .

Finally, we look at the contributions from the charged scalar  $H^\pm$ . These are real since we are assuming that the mass of the charged scalar is larger than  $M_Z/2$ . As we have mentioned above, these contributions are linear combinations of 1,  $\tan^2\beta$ , and  $\cot^2\beta$ . We find that these contributions are, at best, two orders of magnitude smaller than the QCD contribution, for  $\tan^2\beta = 5000$  or  $\tan^2\beta = 1$  (in which case it is dominated by the term proportional to 1).

So, contrary to what happened for the  $\tau$  lepton, for the bottom quark only the neutral-scalar and pseudoscalar contributions may change the SM value for the WMDM appreciably. One should note the remarkable fact that these new scalar contributions may compete with the strong gluonic corrections.

## 4 Magnetic dipole moments at $q^2 > M_Z^2$

The CDF and D0 Collaborations [15] have shown that the top quark is quite heavy. Because of this fact, problems arise in the definition of an appropriate WMDM for the top, since in order to produce a top pair the  $Z$  must be off mass shell. The root of the problem lies in the gauge dependence of any form factor arising in the Lorentz decomposition of the  $Z\bar{f}f$  and  $\gamma\bar{f}f$  vertices, when the gauge boson ( $Z$  or  $\gamma$ ) is off mass shell [16]. Indeed, the MDMs are unequivocally defined (gauge-invariant and observable) only when the incoming gauge boson is on mass shell ( $q^2 = 0$  for the  $\gamma$ , and  $q^2 = M_Z^2$  for the  $Z$ ), *i.e.*, when the physical process is dominated by the single-gauge-boson exchange.

Of course, this problem does not exist solely with the top quark. In particular, it occurs whenever one studies the radiative corrections to fermion-pair production at colliders with energy beyond the  $Z$  mass, such as LEP2 or the NLC.

Recently, the pinch technique [17] has been used to construct gauge-independent MDMs [18]. This technique consists in reorganizing the usual Feynman-diagram expressions into portions that are manifestly gauge independent. One extracts from the box diagrams those gauge-dependent pieces which are kinematically equivalent to the  $\gamma\bar{f}f$  or to the  $Z\bar{f}f$  vertex corrections. Those pieces offset the gauge-noninvariance of the vertex corrections.

One may now calculate these quantities in any gauge as long as the pinch contributions are consistently identified in the box and vertex graphs. However, in this particular case, the computation in the 't Hooft–Feynman gauge is most convenient since, in that gauge, the MDMs do not receive any contributions from the box diagrams. Indeed, such contributions from the box diagram arise from the longitudinal terms in the gauge-boson propagators, which are absent in this gauge. This means that, in the 't Hooft–Feynman gauge, the MDMs only receive contributions from the vertex corrections themselves. Thus, the fact that kinematically the top quark production requires  $q^2 > 4m_t^2$ , would be circumvented by a judicious definition of the form factors when the vector boson is off mass shell.

However, other authors [19] have claimed that the improved off-shell vertex functions are not uniquely defined. The basic problem is that one can still shift gauge-independent pieces between the various diagrams. This ambiguity means that the

gauge-invariant quantities obtained from the pinch technique are not observable by themselves. Still, they may be useful in determining which new physics effects may be important, and where to look for them.

Conscious of these problems, we have computed the contributions to the MDMs of the top quark with the  $\gamma$  and with the  $Z$  at  $\sqrt{q^2} = 500$  GeV (the center-of-mass energy expected for the next  $e^+ e^-$  linear collider, NLC) following the prescription of Ref. [18]. We have also computed the MDMs of the bottom and of the  $\tau$  at  $\sqrt{q^2} = 200$  GeV (relevant for LEP2). We do this in order to have an idea of the sensitivity of those MDMs to the new physics in the THDM. Our calculations are to be considered as preliminaries to a more general work in which the box diagrams should also be taken into account, in the computation of suitably defined physical observables.

## 4.1 The top quark at the NLC

For the NLC, we have taken  $\alpha_S = 0.096$ ,  $\alpha_{em} = 1/127$  and  $\sin^2 \theta_W = 0.240$ . These values are obtained by a SM renormalization-group running of these parameters from their measured values at  $q^2 = M_Z^2$ . In Table 3, we present the SM contributions to the WMDM in the  $Ztt$  vertex at  $\sqrt{q^2} = 500$  GeV. Notice that, contrary to the naive

$ABC$	$\mu_t^{ABC}$
$g_{tt}$	$(-2.6170 + i 4.5493) \times 10^{-3}$
$\gamma_{tt}$	$-7.1549 \times 10^{-5} + i 1.2438 \times 10^{-4}$
$Z_{tt} + \chi_{tt}$	$-2.4585 \times 10^{-4} - i 1.4502 \times 10^{-3}$
$W_{bb} + \sigma_{bb}$	$-4.2241 \times 10^{-4} + i 1.2600 \times 10^{-3}$
$bWW + b\sigma W + bW\sigma + b\sigma\sigma$	$-2.2436 \times 10^{-3} + i 8.7342 \times 10^{-4}$
Total	$(-5.6004 + i 5.3569) \times 10^{-3}$

Table 3: Standard-model contributions to the WMDM of the top, at  $\sqrt{q^2} = 500$  GeV

expectation, the electroweak sector gives contributions comparable to the gluonic one. Qualitatively, this difference with respect to the case of the bottom quark is due to the fact that here the largest mass scale that may dominate the integrals is never much larger than  $m_t^2$ , while the mass of the bottom satisfies  $m_b^2 \ll M_Z^2, m_t^2$ .

Now we can discuss the effect induced by the new scalars of the THDM. If the new scalar particles can be produced directly at  $\sqrt{q^2} = 500$  GeV, their effect on the MDMs will be to generate new non-vanishing imaginary parts due to unitarity. We are more interested in studying a possible scenario, in which they only have virtual effects. Therefore, we are looking for the possibility of sizeable effects induced by a charged scalar with mass larger than 250 GeV (meaning that no  $H^+ H^-$  event has been detected) and neutral scalars bounded by the following constraints:  $m_{h^0, H^0} \geq 409$  GeV (no scalar- $Z$  event); and  $m_{h^0, H^0} + m_{A^0} \geq 500$  GeV (no scalar-pseudoscalar event). The formulas for the various contributions are listed in the appendix. Notice that for the top quark there is a  $\tan \beta \leftrightarrow \cot \beta$  interchange with respect to the formulae for a fermion with  $T_{3f} = -1/2$ .



We find that the contribution from diagram 3a, which is real since we have taken the scalar masses to be greater than 409 GeV to avoid direct production, is now maximal for  $\tan\beta \sim 1$  but may only reach 10% of the SM value for scalar masses close to 409 GeV. This value decreases very rapidly as one goes away from 409 GeV into higher masses, as a consequence of the threshold effect that exists close to production.

Similarly, diagrams 3b and 3c take on their maximum values for  $\tan\beta \sim 1$ . This will limit their impact, although the extra  $m_t^2/M_Z^2$  prefactors enhance these contributions. We find that these contributions never go beyond 15% of the SM result, in their imaginary and real parts.

Finally, the charged-scalar contributions permit a test of both regimes:  $\tan\beta \gg 1$  and  $\tan\beta \sim 1$ . In the first regime the dominant piece is the one proportional to  $\tan\beta$ , which may reach two thirds of the SM value in its real part, but only 12% of the SM value in its imaginary part. For the  $\tan\beta \sim 1$  regime, one may reach around 20% of the SM real part and decrease the SM imaginary part by about 15%. These numbers are obtained for a charged scalar with mass 250 GeV, but decrease very slowly for higher values of the mass.

The SM contributions to the MDM coupling of the  $\gamma\bar{t}t$  vertex at  $\sqrt{q^2} = 500$  GeV are listed in Table 4. The gluonic contribution for this vertex is enhanced with respect

$ABC$	$\mu_t^{ABC}$
$g\bar{t}t$	$-8.2790 \times 10^{-3} + i 1.4392 \times 10^{-2}$
$\gamma\bar{t}t$	$(-2.2635 + i 3.9349) \times 10^{-4}$
$Z\bar{t}t + \chi\bar{t}t$	$-6.4295 \times 10^{-5} - i 3.5312 \times 10^{-3}$
$W\bar{b}b + \sigma\bar{b}b$	$(-1.4404 + i 4.2711) \times 10^{-4}$
$b\bar{W}W + b\sigma W + bW\sigma + b\sigma\sigma$	$-1.4978 \times 10^{-3} + i 7.0128 \times 10^{-4}$
Total	$(-1.0212 + i 1.2383) \times 10^{-2}$

Table 4: Standard-model contributions to the MDM of the top with the  $\gamma$ , at  $\sqrt{q^2} = 500$  GeV

to the previous one by  $Q_t/x_t \approx 3.16$ . This fact makes it the dominant contribution to this MDM.

Again the contributions from the new neutral scalars may, at most, reach 20% of the SM value, being more important for the imaginary part of the MDM. (Recall that the photon does not have a contribution from diagram 3a.) In contrast, the charged-scalar contributions may reach two thirds of the real part of the SM value, for  $\tan^2\beta = 5000$  and a charged-scalar mass near the threshold for  $H^+H^-$  production. These contributions are not as large for  $\tan^2\beta \sim 1$  where one can only reach 15% of the real part of the SM value.

## 4.2 The $\tau$ lepton and the bottom quark at LEP2

The calculations of the WMDM and the electromagnetic MDM for the bottom and  $\tau$  at LEP2 follow those in the previous section. At LEP2, with  $\sqrt{q^2} = 200$  GeV, we use  $\alpha_S = 0.106$ ,  $\alpha_{em} = 1/127.5$  and  $\sin^2\theta_W = 0.236$ . We remind the reader of the ambiguity in defining appropriate MDMs off the intermediate vector boson mass shell,

and report our results only qualitatively. We take these as a hint of where to look for important virtual effects of the new scalars. We shall take the charged-scalar masses to be greater than 100 GeV, together with the constraints  $m_{h^0, H^0} \geq 109$  GeV and  $m_{h^0, H^0} + m_{A^0} \geq 200$  GeV.

Phenomenologically, the  $\tau$  may be more interesting, due to the possibility of a clean measurement of its polarization. The SM values are:  $(-1.1142 - i 3.4060) \times 10^{-6}$  for the WMDM, and  $(1.3347 - i 2.6968) \times 10^{-6}$  for the MDM with the photon. These values are comparable with those obtained for the WMDM at  $M_Z^2$ .

Due to the arguments explained in section 2, the contributions from the new scalars in the THDM only affect the MDMs in the region of large  $\tan\beta$ . In the case of the WMDM, the contribution from diagram 3a may be very large. For a mass of the scalar near the threshold of 110 GeV, one gets two times the SM real part. All the other contributions are negligible, except for the charged-scalar ones which can reach 25% (8%) of the real (imaginary) part of the SM result, but with the opposite sign. This occurs for  $\tan^2\beta = 5000$  and charged scalar masses of 100 GeV.

For the photon vertex all the diagrams may give relevant contributions, for  $\tan^2\beta = 5000$  and light scalars. Diagram 3b may have an imaginary piece as large as 40% of the SM one. The pseudo-scalar correction may reduce the real part of the SM result by 60% and its imaginary part by 30%. Finally, the charged scalar may increase the real part by 25%.

For the bottom quark at LEP2, we have found the following results: the standard-model WMDM is  $(7.6143 - i 4.0685) \times 10^{-5}$ , while the standard-model MDM with the  $\gamma$  is  $(5.8639 - i 3.4638) \times 10^{-5}$ . These values are about a factor of five smaller than the WMDM at  $q^2 = M_Z^2$ . Both MDMs are dominated by the QCD correction to the vertex.

In the case of the  $Z$  vertex, there is a substantial contribution to the real part coming from diagram 3a, which may amount to three times the gluonic one, for a neutral scalar close to 109 GeV. On the other hand, the dominant new scalar contributions to the imaginary part come from diagram 3b, which may increase the gluonic value by 70%, and diagram 3c, which may decrease it by 50%. The pseudo-scalar diagram may decrease the real part of the WMDM by 25%. All these results are for  $\tan^2\beta = 5000$ .

For the photon vertex, the largest corrections to the imaginary part that one may obtain come from diagram 3b (increase of 70%) and diagram 3c (decrease of 50%), for large  $\tan\beta$ . Also, the pseudo-scalar diagram may decrease the real part by 25%.

## 5 Conclusions

We have performed a complete analysis of the magnetic dipole moments of the fermions in the SM and compared them to the corrections that may be induced by the virtual scalars in a THDM with a discrete symmetry, the so-called model II. For the  $\tau$  lepton, the masslessness of the neutrino and the smallness of  $v_\tau$  suppress the new contributions which may, nevertheless, be as large as the SM electroweak corrections. For the bottom quark, these suppressions disappear and the new physics yields MDMs much larger than the electroweak ones. They may even compete with the gluonic correction. We

have also presented the results for the MDMs defined by the pinch technique, which are relevant for the top quark produced at the NLC, and for the  $\tau$  lepton and for the bottom quark produced at LEP2. This definition is ambiguous, but it is useful in identifying candidate situations in which to look for new physics. In so doing we find some interesting results. The pure electroweak radiative corrections to the  $Zt\bar{t}$  vertex at NLC are as large as the gluonic corrections, although that is not the case for the  $\gamma t\bar{t}$  vertex. The diagrams in Fig. 3a can give contributions to the real part of the WMDM of the bottom and  $\tau$  two or three times larger than the SM values. In most other cases, precise measurements of the magnetic dipole moments would be required to disentangle the new effects. Those effects are maximal for light scalars, allowing for tests of the region of parameter space with large  $\tan\beta$ , both in the case of the  $\tau$  and of the bottom.

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## A Formulas for the magnetic moments

In this appendix we give the magnetic dipole moments of a fermion  $f$  induced at one-loop level in the THDM. We present the results in the 't Hooft–Feynman gauge. They hold for any fermion, and are written for both the  $Z$  and  $\gamma$  magnetic moments, for any value  $q^2$  of their squared momentum. However, some of the results are gauge-invariant only if the gauge bosons are on mass shell, as we have found by computing them in a general 't Hooft gauge.

For simplicity, we introduce the definition

$$\mu^{ABC} = \frac{\alpha_{em} m_f^2}{4\pi} b^{ABC}, \quad (6)$$

where  $A$ ,  $B$  and  $C$  are the particles in the loop, in the order displayed in Fig. 1. Moreover,  $x_f$  and  $y_f$  are defined as

$$x_f = \begin{pmatrix} v_f/(2s_W c_W) \\ Q_f \end{pmatrix}, \quad y_f = \begin{pmatrix} a_f/(2s_W c_W) \\ 0 \end{pmatrix}, \quad (7)$$

where the upper lines hold when the exterior gauge boson is the  $Z$ , while the lower lines hold for the  $\gamma$  MDMs. The quantities  $v_f$  and  $a_f$  are given in Eq. (2). The letter

$f$  refers to the fermion whose MDM is being calculated, and  $i$  to its  $SU(2)_L$ -doublet partner.

In order not to clutter our formulas, we will omit possible mixing-matrix elements, notably in the quark sector. The inclusion of those mixing-matrix elements is clearly trivial. Furthermore, our formulas are valid for any fermion. In the text, they have only been used to compute the MDMs of the fermions of the third family, because those are the ones for which the contributions from the extended scalar sector are larger.

We use for the integrals the notation of Ref. [3]:

$$\begin{aligned} & [I_{00}; I_\mu; I_{\mu\nu}](m_A, m_B, m_C, q) \\ &= \int \frac{d^4k}{i\pi^2} \frac{[1; k_\mu; k_\mu k_\nu]}{(k^2 - m_A^2)[(k - p_-)^2 - m_B^2][(k + p_+)^2 - m_C^2]}. \end{aligned} \quad (8)$$

They are decomposed as

$$\begin{aligned} I^\mu &= (p_- - p_+)^{\mu} I_{10} + (p_- + p_+)^{\mu} I_{11}, \\ I^{\mu\nu} &= (p_+^{\mu} p_+^{\nu} + p_-^{\mu} p_-^{\nu}) I_{21} + (p_+^{\mu} p_-^{\nu} + p_-^{\mu} p_+^{\nu}) I_{22} \\ &\quad + (p_+^{\mu} p_+^{\nu} - p_-^{\mu} p_-^{\nu}) I_{2,-1} + g^{\mu\nu} I_{20}. \end{aligned} \quad (9)$$

These functions are trivially related to the Passarino–Veltman functions [20]. If  $m_B = m_C$ ,  $I_{11}$  and  $I_{2,-1}$  vanish. Also, notice that, from  $I_{\mu\nu}$ , only the combination  $I_{21} - I_{22}$  appears in the expressions for the MDMs.

We have obtained the numerical values of these integrals in two independent ways. On the one hand, FeynCalc was used to reduce them to functions of the scalar integrals  $A_0$ ,  $B_0$  and  $C_0$ , for which there are explicit formulas. The results were checked against the ones obtained with the aid of the ff routines of van Oldenborgh [21].

## A.1 SM contributions, without the Higgs scalar

In this subsection we include the contributions from diagrams involving only SM particles, except the Higgs scalar. The latter contributions will be included in the next section, where we discuss the scalar sector of the THDM. We find

$$b^{\gamma ff} = 8 x_f Q_f^2 [I_{21} - I_{22} - I_{10}](0, m_f, m_f, q). \quad (10)$$

For quarks only, there is a similar contribution with a gluon  $g$  instead of the photon:

$$b^{gff} = 8 x_f C_F \frac{\alpha_S}{\alpha_{em}} [I_{21} - I_{22} - I_{10}](0, m_f, m_f, q), \quad (11)$$

where  $C_F = 4/3$ . For these contributions, the integrals can be computed analytically and one obtains the explicit formula

$$\begin{aligned} [I_{21} - I_{22} - I_{10}](0, m_f, m_f, q) &= \frac{1}{4q^2} \int_0^1 \frac{dy}{y(y-1) + m^2/q^2} \\ &= \frac{1}{2q^2\delta} \begin{cases} \log \frac{\delta-1}{\delta+1}, & \text{for } q^2 < 0; \\ \log \frac{1-\delta}{1+\delta} + i\pi, & \text{for } q^2 > 4m_f^2, \end{cases} \end{aligned} \quad (12)$$

where

$$\delta = \sqrt{1 - \frac{4m_f^2}{q^2}}. \quad (13)$$

The region  $0 < q^2 < 4m_f^2$  is unphysical. In the limit  $q^2 = 0$  one obtains  $1/(4m_f^2)$ , reproducing the Schwinger term. Thus, this integral is dominated by the largest mass scale: it is of order  $1/m_f^2$  when  $4m_f^2 \gg |q^2|$ , and of order  $1/q^2$  when  $q^2 \gg 4m_f^2$ . This is a common feature of these integrals, which is crucial to obtain the correct decoupling limits.

For the diagrams in Fig. 2b we find

$$\begin{aligned} b^{Zff} &= \frac{2}{s_W^2 c_W^2} \left[ x_f v_f^2 (I_{21} - I_{22} - I_{10}) + x_f a_f^2 (I_{21} - I_{22} - 5I_{10} + 2I_{00}) \right. \\ &\quad \left. + 2y_f v_f a_f (I_{21} - I_{22} - 3I_{10} + I_{00}) \right] (M_Z, m_f, m_f, q), \\ b^{\chi ff} &= \frac{x_f m_f^2}{s_W^2 M_W^2} [I_{21} - I_{22}] (M_Z, m_f, m_f, q). \end{aligned} \quad (14)$$

The sum of these two contributions is gauge-invariant.

Similarly, the sum of the two contributions

$$\begin{aligned} b^{Wii} &= 2 \frac{x_i + y_i}{s_W^2} [I_{21} - I_{22} - 3I_{10} + I_{00}] (M_W, m_i, m_i, q), \\ b^{\sigma ii} &= \frac{1}{s_W^2 M_W^2} \left\{ [m_f^2(x_i + y_i) + m_i^2(x_i - y_i)] (I_{21} - I_{22} - I_{10}) \right. \\ &\quad \left. + 2x_i m_i^2 I_{10} \right\} (M_W, m_i, m_i, q), \end{aligned} \quad (15)$$

is gauge-invariant.

Finally,

$$\begin{aligned} b^{iWW} &= \mathcal{S} \left( \begin{array}{c} \cot \theta_W \\ 1 \end{array} \right) \frac{1}{s_W^2} [2I_{21} - 2I_{22} + I_{10}] (m_i, M_W, M_W, q), \\ b^{i\sigma\sigma} &= \mathcal{S} \left( \begin{array}{c} \cot(2\theta_W) \\ 1 \end{array} \right) \frac{1}{s_W^2 M_W^2} \left[ (m_f^2 + m_i^2) (I_{21} - I_{22} - I_{10}) \right. \\ &\quad \left. + m_i^2 (I_{00} - 2I_{10}) \right] (m_i, M_W, M_W, q), \\ b^{i\sigma W} &= b^{iW\sigma} = \mathcal{S} \left( \begin{array}{c} -\tan(\theta_W) \\ 1 \end{array} \right) \frac{1}{2s_W^2} I_{10} (m_i, M_W, M_W, q). \end{aligned} \quad (16)$$

Once again, only the sum of these four contributions is gauge invariant. Moreover, gauge invariance in this case only occurs if, either  $q^2 = M_Z^2$  and one uses the upper line for the coefficients (for the case of a  $Z$  as external gauge boson), or  $q^2 = 0$  and one uses the lower line (for the case of a  $\gamma$  as external gauge boson). This is the reason behind the gauge noninvariance of the MDMs at arbitrary  $q^2$ . Gauge invariance of a physical

observable (such as the  $e^+e^- \rightarrow f\bar{f}$  cross section) is restored if we add to Eqs. (16) the result of the computation of the  $W$ -box diagram [18].

In Eqs. (16),  $\mathcal{S}$  is +1 if the fermion  $f$  has  $T_{3f} = -1/2$ , and  $-1$  if it has  $T_{3f} = 1/2$ . This we may write as  $\mathcal{S} = -2 T_{3f}$ . The factor  $\mathcal{S}$  arises from the anti-symmetry of the three-gauge-boson vertices under the  $W^+ \leftrightarrow W^-$  interchange.

## A.2 The scalar contributions

We now present the contributions to the MDMs that involve scalar particles in the loop. The letters  $h$ ,  $H$  and  $A$  refer to the neutral scalars  $h^0$ ,  $H^0$  and  $A^0$  of model II, respectively. The charged scalars  $H^\pm$  are denoted by the letter  $C$ . The corresponding Feynman rules may be found, for example, in Ref. [22].

The formulas we present are for a fermion with  $T_{3f} = -1/2$ , such as the bottom or the  $\tau$ . For a fermion with  $T_{3f} = 1/2$ , such as the top quark, one must make the following substitutions

$$\sin \beta \leftrightarrow \cos \beta \quad , \quad \sin \alpha \leftrightarrow \cos \alpha \quad . \quad (17)$$

We find

$$\begin{aligned} b^{iCC} &= \mathcal{S} \left( \begin{array}{c} \cot(2\theta_W) \\ 1 \end{array} \right) \frac{1}{s_W^2 M_W^2} \left\{ [(m_f \tan \beta)^2 + (m_i / \tan \beta)^2] (I_{21} - I_{22} - I_{10}) \right. \\ &\quad \left. + m_i^2 (2I_{10} - I_{00}) \right\} (m_i, M_C, M_C, q) \, , \\ b^{Cii} &= \frac{1}{s_W^2 M_W^2} \left\{ [(m_f \tan \beta)^2 (x_i + y_i) + (m_i / \tan \beta)^2 (x_i - y_i)] (I_{21} - I_{22} - I_{10}) \right. \\ &\quad \left. - 2x_i m_i^2 I_{10} \right\} (M_C, m_i, m_i, q) \, , \\ b^{Aff} &= \frac{x_f m_f^2}{s_W^2 M_W^2} \tan^2 \beta [I_{21} - I_{22}] (M_A, m_f, m_f, q) \, , \\ b^{Hff} &= \frac{x_f m_f^2}{s_W^2 M_W^2} \frac{\cos^2 \alpha}{\cos^2 \beta} [I_{21} - I_{22} - 2I_{10}] (M_H, m_f, m_f, q) \, , \\ b^{hff} &= \frac{x_f m_f^2}{s_W^2 M_W^2} \frac{\sin^2 \alpha}{\cos^2 \beta} [I_{21} - I_{22} - 2I_{10}] (M_h, m_f, m_f, q) \, . \end{aligned} \quad (18)$$

These diagrams contribute both to the photon and to the  $Z$  vertices.

In addition, there are a few diagrams contributing exclusively to the  $Z$  vertex. In some of them the external  $Z$  couples either to  $H^0$  and  $A^0$ , or to  $h^0$  and  $A^0$ , in the loop. Those diagrams do not yield any contribution to the WMDM. In other diagrams the external  $Z$  couples either to a  $Z$  and a  $H^0$ , or to a  $Z$  and a  $h^0$ , in the loop (see Fig. 3a). These diagrams have the same chiral properties as the MDM operator and, as a consequence, their contributions do not get  $(m_f/M_W)^2$  pre-factors. The results

are

$$\begin{aligned}
b^{fZH} + b^{fHZ} &= \frac{v_f}{s_W^3 c_W^3} (\cos^2 \alpha + \tan \beta \sin \alpha \cos \alpha) [I_{10} + I_{11}] (m_f, M_Z, M_H, q), \\
b^{fZh} + b^{fhZ} &= \frac{v_f}{s_W^3 c_W^3} (\sin^2 \alpha - \tan \beta \sin \alpha \cos \alpha) [I_{10} + I_{11}] (m_f, M_Z, M_h, q). \quad (19)
\end{aligned}$$

To obtain the SM Higgs scalar contributions from the expressions in this subsection, one deletes all the diagrams with  $H^\pm$ ,  $A^0$ , or  $H^0$ , and sets

$$\beta = \alpha + \pi/2, \quad (20)$$

in the diagrams with  $h^0$ .

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## FIGURE CAPTIONS

Figure 1: Typical Feynman diagram contributing to the magnetic dipole moments at one loop.

Figure 2: Electroweak one-loop vertex corrections to the MDMs, excluding the diagrams with the Higgs scalar.

Figure 3: One-loop vertex corrections due to the scalars in the THDM. The scalars  $H^0$ ,  $h^0$ ,  $A^0$  and  $H^\pm$  are denoted by the letters H, h, A and C, respectively. For the SM, only diagrams 3a and 3b exist, having the Higgs particle as the intermediate neutral scalar.

Figure 4: Plot of the contribution of the diagram in Fig. 3a to the WMDM of the  $\tau$ , as a function of the mass of the neutral scalar. This is to be multiplied by  $\cos^2 \alpha + \tan \beta \sin \alpha \cos \alpha$  for  $H^0$ , and by  $\sin^2 \alpha - \tan \beta \sin \alpha \cos \alpha$  for  $h^0$ , in the THDM.

Figure 5: Plot of the contribution of the diagram in Fig. 3b to the WMDM of the  $\tau$ , as a function of the mass of the neutral scalar. This is to be multiplied by  $(1 + \tan^2 \beta)$  times  $\cos^2 \alpha$  for  $H^0$ , and times  $\sin^2 \alpha$  for  $h^0$ , in the THDM.

Figure 6: Plot of the contribution of the diagram in Fig. 3c to the WMDM of the  $\tau$ , as a function of the mass of the neutral pseudo-scalar. This is to be multiplied by  $\tan^2 \beta$ .

Figure 7: Plot of the sum of the contributions of the diagrams in Figs. 3d and 3e to the WMDM of the  $\tau$ , as a function of the mass of the charged scalar. This is to be multiplied by  $\tan^2 \beta$ .

Figure 8: Gluonic one-loop vertex correction to the MDMs of the quarks.