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SPONTANEOUS BREAKDOWN OF CP IN LEFT RIGHT SYMMETRIC MODELS

G. Barenboim[1,2] and J. Bernabéu[1]

[1] *Departament de Física Teòrica, Universitat de València*
[2] *IFIC, Centre Mixte Universitat de València - CSIC*
E-46100 Burjassot, Valencia, Spain

Abstract
We show that it is possible to obtain spontaneous CP violation in the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, i.e. in a left right symmetric model containing a bidoublet and two triplets in the scalar sector. For this to be a natural scenario, the non-diagonal quartic couplings between the two scalar triplets and the bidoublet play a fundamental role. We analyze the corresponding Higgs spectrum, the suppression of FCNC's and the manifestation of the spontaneous CP phase in the electric dipole moment of the electron.

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1 Introduction

Understanding the origin of CP violation is one of the outstanding open questions in particle physics [1]. Although one can incorporate CP violation in the three generation standard model through the CKM mechanism, there is no deep understanding on the origin of it. The indication that the amount of CP violation one has in the standard model through the CKM mechanism is probably not enough to generate the baryon asymmetry [2] suggests to look for other sources of CP violation beyond the standard model [3, 4].

One of the most attractive extensions of the standard electroweak model uses $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ as a gauge group [5]. This model is formulated so that parity is a spontaneously broken symmetry. In these theories the observed V-A structure of the weak interactions is only a low energy phenomenon which should disappear when one reaches the energies of order $\mathbf{v_R}$ or higer, where $\mathbf{v_R}$ is the vacuum expectation value (vev) for the right handed scalar. In such a picture, all interactions above these energies are supposed to be parity conserving.

The enlargement of the gauge group and the increase in the number of Higgs scalars seems to be the necessary price to be paid in orden to bring parity violation on the same footing as other, continuous symmetries. Therefore, we are dealing with a theory which predicts the doubled number of charged gauge bosons (4 $W_L^\pm$ and $W_R^\pm$ against the 2 $W^\pm$ of the standard model) and also the doubled number of massive neutral gauge bosons.

Regarding the Higgs sector of the left right symmetric models, there are two distinct alternatives. All models contain a bidoublet field ϕ, the masses of the W_L and Z derive primarily from the vev k_1 and k_2 of the two neutral members of this doublet. Since experimental constraints from $K_L - K_S$ mixing force W_R to be very heavy [6], an additional Higgs representation, with large vev (v_R) for its neutral member is required that couples primarily to the W_R. To preserve the left right symmetry, there must be a corresponding Higgs representation coupling to the W_L, but the vev of its neutral member (v_L) must be smaller in order to preserve the standard model relation between the W_L and Z masses. If the additional Higgs fields are members of doublets, then the above criteria can be met, but the theory then fails to incorporate a natural explanation of the smallness of neutrino masses [7]. In contrast, if the extra neutral Higgs fields are members of triplets, all requirements are well satisfied. Because of that, we choose to investigate models containing extra triplet Higgs fields Δ_L and Δ_R. The resulting Higgs sector has many exotic features, and our ability to experimentally probe these features is an important issue.

In this paper we analize in detail one of the most interesting features of such a theory, namely, the possibility that spontaneous CP violation could occur with the described Higgs structure. Whether or not a significant number of the Higgs bosons of a left right symmetric model can be sufficiently light to be detectable is, in fact, a serious issue [8]. We also comment on it.

The work is organized as follows : we begin by reviewing the Higgs sector of the minimal left right symmetric model. The most general left right symmetric Higgs potential is also presented, while its minimization is carried out in section 3. We analize there its phase degrees of freedom and in section 4 we show that, for a Higgs potential without explicit CP violation, spontaneous CP violation does occur. For the model to be consistent with the observed phenomena we study the Higgs spectrum (section 5) and the FCNCs constraints (section 6). To do this we write the Higgs bosons coupling in a manner such that the flavour diagonal and flavour changing couplings are explicitly displayed. In section 7 we present an example where this CP violation could be seen. Finally, we draw our conclusions.

2 The Higgs Sector

This is the main sector of our work. Here we analize in detail the symmetry breaking in left right symmetric models with special emphasis on the spontaneous violation of parity. The theory we have in mind is the minimal theory in terms of its Higgs sector, which manifestly preserves parity prior to symmetry breaking.

2.1 The Higgs content

The scalar fields of the minimal model are [10]

$$\phi\;(\frac{1}{2},\frac{1}{2}^*,0) \quad \Delta_L\;(1,0,2) \quad \Delta_R\;(0,1,2) \tag{1}$$

where the $SU(2)_L$, $SU(2)_R$ and $B-L$ quantum numbers are indicated in parentheses. A convenient representation of the fields is given by the 2×2 matrices

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{2}$$

$$\Delta_L = \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & \frac{-\delta_L^+}{\sqrt{2}} \end{pmatrix} \tag{3}$$

$$\Delta_R = \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & \frac{-\delta_R^+}{\sqrt{2}} \end{pmatrix} \tag{4}$$

Following some previous conventions [9], the neutral Higgs field ϕ^0 is written in terms of correctly normalized real and imaginary components as

$$\phi^0 = \frac{1}{\sqrt{2}}\left(\phi_0^r + i\phi_0^i\right) \tag{5}$$

These fields transform according to the relation

$$\begin{array}{ccccccc} \phi & \longrightarrow & U_L\phi U_R^\dagger & , & \tilde{\phi} & \longrightarrow & U_L\tilde{\phi}U_R^\dagger, \\ \Delta_L & \longrightarrow & U_L\Delta_L U_L^\dagger & , & \Delta_L^\dagger & \longrightarrow & U_L\Delta_L^\dagger U_L^\dagger, \\ \Delta_R & \longrightarrow & U_R\Delta_R U_R^\dagger & , & \Delta_R^\dagger & \longrightarrow & U_R\Delta_R^\dagger U_R^\dagger, \end{array} \tag{6}$$

where $U_{L,R}$ are the general $SU(2)_L$ and $SU(2)_R$ unitary transformations, and $\tilde{\phi} \equiv \tau_2\phi^*\tau_2$

The gauge symmetry breaking proceeds in two stages. In the first stage, the electrically neutral component of Δ_R, denoted by δ_R^0, acquires a vev v_R, and breaks the gauge symmetry down to $SU(2)_L \otimes U(1)_Y$ where

$$\frac{Y}{2} = I_{3R} + \frac{B-L}{2} \tag{7}$$

The parity symmetry breaks down at this stage. In the second stage, the vevs of the electrically neutral components of ϕ, (k_1 and k_2) break the symmetry down to $U(1)_Q$. At the first stage, the charged right handed gauge bosons denoted by $W_R^{\pm}$ and the neutral gauge boson called Z' acquire masses proportional to v_R and become much heavier than the usual left handed $W_L^{\pm}$ and the Z bosons, which pick up masses proportional to k_1 and k_2 only at the second stage.

Experimental constraints force the relation that $k_1, k_2 \ll v_R$, as we will see later. Making two of them complex leads to an interesting model of CP violation.

2.2 The Higgs potential

Let now discuss the form of the scalar field potential [9, 11, 12]. For our theory to be left right symmetric, it is necessary that the lagrangian be invariant under the discrete left right symmetry defined by:

$$\Psi_L \longleftrightarrow \Psi_R \quad \Delta_L \longleftrightarrow \Delta_R \quad \phi \longleftrightarrow \phi^\dagger \tag{8}$$

where $\Psi_{L,R}$ are column vectors containing the left-handed and right-handed fermionic fields of the theory. Our theory is a left right symmetric one: the lagrangian should be invariant under the exchange of the fields ϕ_1 and ϕ_2 [13] :

$$\phi_1 \;\longleftrightarrow\; \phi_2 \tag{9}$$

Furthermore, the most general scalar field potential cannot have trilinear terms: because of the nonzero $B-L$ quantum numbers of the Δ_L and Δ_R triplets, these must always appear in the quadratic combinations $\Delta_L^\dagger \Delta_L$, $\Delta_R^\dagger \Delta_R$, $\Delta_L^\dagger \Delta_R$ or $\Delta_R^\dagger \Delta_L$. These combinations can never be combined with a single bidoublet ϕ in such a way as to form $SU(2)_L$ and $SU(2)_R$ singlets. Nor can three bidoublets be combined so as to yield a singlet. However, quartic combinations of the form $Tr(\Delta_L^\dagger \phi \Delta_R \phi^\dagger)$ are in general allowed by the left right symmetry. Following these strict conditions, the most general form of the Higgs potential is

$$\mathbf{V} = \mathbf{V}_\phi + \mathbf{V}_\Delta + \mathbf{V}_{\phi\Delta} \tag{10}$$

where

$$\begin{aligned}\mathbf{V}_\phi = & -\mu_1^2 Tr(\phi^\dagger\phi) \,-\, \mu_2^2 \left[Tr(\tilde{\phi}\phi^\dagger) + Tr(\tilde{\phi}^\dagger\phi)\right] \,+\, \lambda_1 \left[Tr(\phi\phi^\dagger)\right]^2 + \\ & \lambda_2 \left\{\left[Tr(\tilde{\phi}\phi^\dagger)\right]^2 + \left[Tr(\tilde{\phi}^\dagger\phi)\right]^2\right\} \,+\, \lambda_3 \left[Tr(\tilde{\phi}\phi^\dagger) Tr(\tilde{\phi}^\dagger\phi)\right] + \\ & \lambda_4 \left\{Tr(\phi^\dagger\phi)\left[Tr(\tilde{\phi}\phi^\dagger) + Tr(\tilde{\phi}^\dagger\phi)\right]\right\}\end{aligned}$$

$$
\begin{aligned}
\mathbf{V}_{\Delta} = & -\mu_3^2 \left[Tr(\Delta_L\Delta_L^\dagger) + Tr(\Delta_R\Delta_R^\dagger)\right] + \rho_1 \left\{\left[Tr(\Delta_L\Delta_L^\dagger)\right]^2 + \left[Tr(\Delta_R\Delta_R^\dagger)\right]^2\right\} + \\
& \rho_2 \left[Tr(\Delta_L\Delta_L)Tr(\Delta_L^\dagger\Delta_L^\dagger) + Tr(\Delta_R\Delta_R)Tr(\Delta_R^\dagger\Delta_R^\dagger)\right] + \\
& \rho_3 \left[Tr(\Delta_L\Delta_L^\dagger)Tr(\Delta_R\Delta_R^\dagger)\right] + \\
& \rho_4 \left[Tr(\Delta_L\Delta_L)Tr(\Delta_R^\dagger\Delta_R^\dagger) + Tr(\Delta_L^\dagger\Delta_L^\dagger)Tr(\Delta_R\Delta_R)\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{V}_{\phi\Delta} = & \alpha_1 \left\{Tr(\phi^\dagger\phi)\left[Tr(\Delta_L\Delta_L^\dagger) + Tr(\Delta_R\Delta_R^\dagger)\right]\right\} + \alpha_2 \left[Tr(\tilde{\phi}^\dagger\phi)Tr(\Delta_R\Delta_R^\dagger) + \right. \\
& \left. Tr(\tilde{\phi}\phi^\dagger)Tr(\Delta_L\Delta_L^\dagger) + Tr(\tilde{\phi}\phi^\dagger)Tr(\Delta_R\Delta_R^\dagger) + Tr(\tilde{\phi}^\dagger\phi)Tr(\Delta_L\Delta_L^\dagger)\right] + \\
& \beta_1 \left[Tr(\phi\Delta_R\phi^\dagger\Delta_L^\dagger) + Tr(\phi^\dagger\Delta_L\phi\Delta_R^\dagger)\right] + \beta_2 \left[Tr(\tilde{\phi}\Delta_R\phi^\dagger\Delta_L^\dagger) + \right. \\
& \left. Tr(\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger) + Tr(\phi\Delta_R\tilde{\phi}^\dagger\Delta_L^\dagger) + Tr(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger)\right]
\end{aligned}
$$

where we have written out each term completely to display the full parity symmetry. Note that all terms in the potential are self conjugate as a consequence of the discrete left right symmetry, so that all the parameters have to be real in order to preserve hermiticity. In this way our potential is CP conserving.

The neutral Higgs fields δ_R^0, δ_L^0, ϕ_1^0 and ϕ_2^0 can potentially acquire vevs, v_R, V_L, k_1 and k_2, respectively. Explicitly, we have

$$
\langle\phi\rangle = \begin{pmatrix} \frac{k_1}{\sqrt{2}} & 0 \\ 0 & \frac{k_2}{\sqrt{2}} \end{pmatrix} \quad , \quad \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ \frac{v_{L,R}}{\sqrt{2}} & 0 \end{pmatrix} \tag{11}
$$

3 The symmetry breaking

Let us now discuss the phases of the vevs that are acquired by the neutral components of Δ_R, Δ_L and ϕ. A priori, it is possible that one could allow for the possibility of phases in the left right transformation defined in Eq. (8), for example $\Delta_L \longleftrightarrow e^{i\varphi_L}\Delta_R$ or $\phi \longleftrightarrow e^{i\varphi_\phi}\phi^\dagger$. However, one may always absorb these phases by appropiate global phase rotations of the fields. We will assume that this has been done.

Since we have employed our global phase degrees of freedom in eliminating phases from the left right transformation symmetry; our only remaining freedom in choosing vevs is that allowed by the underlying U_L and U_R transformations. Of these, only the T_L^3 and T_R^3 components are useful for the vevs of the neutral Higgs fields. Using

$$
U_L = \begin{pmatrix} e^{i\theta_L} & 0 \\ 0 & e^{-i\theta_L} \end{pmatrix} \tag{12}
$$

and the corresponding form for U_R, one finds,

$$
\begin{aligned}
k_1 &\longrightarrow k_1 e^{i(\theta_L-\theta_R)} \\
k_2 &\longrightarrow k_2 e^{-i(\theta_L-\theta_R)} \\
v_L &\longrightarrow v_L e^{-2i\theta_L} \\
v_R &\longrightarrow v_R e^{-2i\theta_R}
\end{aligned} \tag{13}
$$

Clearly, we have the choice of two phases at will. We use them to fix θ_L and $(\theta_L - \theta_R)$ so that v_L and k_2 are real.

We can now consider the minimization of the potential. There are six minimization conditions:

$$\frac{\partial \mathbf{V}}{\partial Re(k_1)} = \frac{\partial \mathbf{V}}{\partial Im(k_1)} = \frac{\partial \mathbf{V}}{\partial k_2} = \frac{\partial \mathbf{V}}{\partial Re(v_R)} = \frac{\partial \mathbf{V}}{\partial Im(v_R)} = \frac{\partial \mathbf{V}}{\partial v_L} = 0 \tag{14}$$

This is due to the complex character of v_R and k_1 ($v_R = |v_R|e^{i\theta}$ and $k_1 = |k_1|e^{i\alpha}$) They are:

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial Re(k_1)} = {} & 2k_1^2 k_2 \lambda_4 + k_2^3 \lambda_4 - 2k_2 \mu_2^2 + \alpha_2 k_2 (v_L^2 + v_R^2) + k_1 \lambda_1 \cos(\alpha)(k_1^2 + k_2^2) + \\ & 4k_1 k_2^2 \lambda_2 \cos(\alpha) + 2k_1 k_2^2 \lambda_3 \cos(\alpha) - k_1 \mu_1^2 \cos(\alpha) + k_1^2 k_2 \lambda_4 \cos(2\alpha) + \\ & \frac{1}{2}\alpha_1 k_1 (v_L^2 + v_R^2) \cos(\alpha) + \beta_2 k_1 v_L v_R \cos(\alpha - \theta) + \frac{1}{2}\beta_1 k_2 v_L v_R \cos(\theta)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial Im(k_1)} = {} & k_1 \lambda_1 \sin(\alpha)(k_1^2 + k_2^2) - 4k_1 k_2^2 \lambda_2 \sin(\alpha) + 2k_1 k_2^2 \lambda_3 \sin(\alpha) - k_1 \mu_1^2 \sin(\alpha) + \\ & \frac{1}{2}\alpha_1 k_1 (v_L^2 + v_R^2) \sin(\alpha) + k_1^2 k_2 \lambda_4 \sin(2\alpha) - \beta_2 k_1 v_L v_R \sin(\alpha - \theta) + \\ & \frac{1}{2}\beta_1 k_2 v_L v_R \sin(\theta)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial k_2} = {} & k_2 \lambda_1 (k_1^2 + k_2^2) + 2k_1^2 k_2 \lambda_3 - k_2 \mu_1^2 + \frac{1}{2}\alpha_1 k_2 (v_L^2 + v_R^2) \sin(\alpha) + 2k_1 k_2^2 \lambda_4 \cos(\alpha) + \\ & k_1 (k_1^2 + k_2^2) \lambda_4 \cos(\alpha) - 2k_1 \mu_2^2 \cos(\alpha) + \alpha_2 k_1 (v_L^2 + v_R^2) \cos(\alpha) + \\ & 4k_1^2 k_2 \lambda_2 \cos(2\alpha) + \beta_2 k_2 v_L v_R \cos(\theta) + \frac{1}{2}\beta_1 k_1 v_L v_R \cos(\alpha - \theta)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial v_L} = {} & \Big\{\alpha_1 v_L (k_1^2 + k_2^2) - 2\mu_3^2 v_L + 2\rho_1 v_L (v_L^2 + v_R^2) + 4\alpha_2 k_1 k_2 v_L \cos(\alpha) + \\ & \beta_1 k_1 k_2 v_R \cos(\alpha - \theta) + \beta_2 v_R \cos(2\alpha - \theta)(k_1^2 + k_2^2)\Big\} \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial Re(v_R)} = {} & \Big\{\beta_2 v_L (k_2^2 + k_1^2 \cos(2\alpha)) + \beta_1 k_1 k_2 v_L \cos(\alpha) + 2\alpha_2 k_1 k_2 v_R \cos(\alpha - \theta) + \\ & \alpha_1 v_R \cos(\theta)(k_1^2 + k_2^2) + \alpha_3 k_2^2 v_R \cos(\theta) - 2\mu_3^2 v_R \cos(\theta) + \\ & \rho_3 v_L^2 v_R \cos(\theta) + 2\rho_1 v_R^3 \cos(\theta) + 2\alpha_2 k_1 k_2 v_R \cos(\alpha + \theta)\Big\} \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{V}}{\partial Im(v_R)} = {} & \Big\{\beta_2 v_L k_1^2 \sin(2\alpha) + \alpha_3 k_2^2 v_R \sin(\theta) - 2\alpha_2 k_1 k_2 v_R \sin(\alpha - \theta) + -2\mu_3^2 v_R \sin(\theta) \\ & \alpha_1 v_R \sin(\theta)(k_1^2 + k_2^2) + \beta_1 k_1 k_2 v_L \sin(\alpha) + \rho_3 v_L^2 v_R \sin(\theta) + 2\rho_1 v_R^3 \sin(\theta) + \\ & 2\alpha_2 k_1 k_2 v_R \sin(\alpha + \theta)\} \frac{1}{2}\end{aligned}$$

In these equations and the ensuing discussion, v_R refers to the magnitude $|v_R|$ and similarly for k_1.

Some of these first derivative equations can be used to determine μ_1^2, μ_2^2, and μ_3^2, the remaining first derivative equations impose strong constraints on the quartic couplings appearing in the Higgs potential, and on the relative phases of the vevs. In addition, at a true local minimum all the physical Higgs bosons must have positive square masses for a solution of (14). This implies that various combinations of the potential parameters must be positive. Of the twenty real degrees of freedom contained in this Higgs sector, six are absorbed in giving masses to the left and right handed gauge bosons $W_L^\pm$, $W_R^\pm$ Z and Z'.

In previous works [9, 14, 15] three minimization conditions are used to determine the mass terms μ_1^2, μ_2^2, and μ_3^2, while the other equations are used to find (or better saying to not find) the phase degrees of freedom such as to have CP violation. This procedure is well adapted to find the solutions in the absence of β quartic terms in $V_{\phi\Delta}$, Eq. (10). This is the desired situation under the existence of a symmetry which avoids the presence of FCNC's. That analysis, which leads to the absence of spontaneous CP phases, is not the appropiate one to account for **all** the solutions when the non diagonal quartic terms are present.

We are going to proceed in a different way. These six first derivative equations are going to be used to determine not only μ_1^2, μ_2^2, and μ_3^2 but also other three parameters of our choice. In this way we are going to have a minimum for any choice of the CP violating phases, α and θ. These new relations must be satisfied in order to generate a minimum of the Higgs potential, but they can have the unnatural property of relating parameters across widely differing scales, what we usually call fine tuning. Only if this is not the case, our analysis would be valid.

4 Vacuum expectation value scenarios

In fact, it is worth emphasizing what has occurred in our analysis up to this point. We have required our Higgs potential to have a minimum which allows spontaneous CP violation; in order to have a phenomenologically accepted minimum we have to analize our potential parameters to avoid fine tuning.

Analyzing it, we notice that we have two possible scenarios: (a) $k_1 = k_2 = k$ (b) $k_1 \neq k_2$.

First scenario : $k_1 = k_2 = k$

In this case, from our six minimization equations, we can take four to obtain μ_1^2, μ_2^2, μ_3^2 and ρ_1. The remaining two equations are:

$$2\beta_2 k v_L v_R \sin(\alpha)\sin(\alpha-\theta) = 0$$

$$\frac{k^2 v_L \sec(\theta)\sin(\alpha-\theta)}{2}\left(\beta_1 + 2\beta_2\cos(\alpha)\right) = 0$$

So we again have at this point two possibilities, namely $\alpha = \theta$ or $\beta_1 = \beta_2 = 0$. They yield for the $\alpha = \theta$ case

$$
\begin{aligned}
\mu_1^2 &= 2k^2\left(\lambda_1 - 2\lambda_2 + \lambda_3 + \lambda_4\cos(\alpha)\right) + \frac{\alpha_1}{2}(v_L^2 + v_R^2) + \frac{\beta_1}{2}v_L v_R \\
\mu_2^2 &= k^2\left(\lambda_4 + 2\lambda_2\cos(\alpha)\right) + \frac{\alpha_2}{2}(v_L^2 + v_R^2) + \frac{\beta_2}{2}v_L v_R \\
\mu_3^2 &= k^2\left(\alpha_1 + 2\alpha_2\cos(\alpha)\right) + \frac{k^2(v_L^2 + v_R^2)}{2v_L v_R}(\frac{\beta_1}{2} + \beta_2) + \frac{\rho_3}{2}v_L \\
\rho_1 &= \frac{\beta_1 k^2 + \rho_3 v_L v_R + 2\beta_2 k^2\cos(\alpha)}{2v_L v_R}
\end{aligned} \tag{15}
$$

and for the $\beta_1 = \beta_2 = 0$ case

$$
\begin{aligned}
\mu_1^2 &= 2k^2\left(\lambda_1 - 2\lambda_2 + \lambda_3 + \lambda_4\cos(\alpha)\right) + \frac{\alpha_1}{2}(v_L^2 + v_R^2) \\
\mu_2^2 &= k^2\left(\lambda_4 + 2\lambda_2\cos(\alpha)\right) + \frac{\alpha_2}{2}(v_L^2 + v_R^2) \\
\mu_3^2 &= k^2\left(\alpha_1 + 2\alpha_2\cos(\alpha)\right) + \frac{\rho_3}{2}(v_L^2 + v_R^2) \\
\rho_1 &= \frac{\rho_3}{2}
\end{aligned} \tag{16}
$$

Second scenario : $k_1 \neq k_2$

In this case we have

$$
\begin{aligned}
\mu_1^2 &= \lambda_1(k_1^2 + k_2^2) + \frac{1}{2}\alpha_1(v_L^2 + v_R^2) + 2k_1k_2\lambda_4\cos(\alpha) + \beta_1 v_L v_R k_1 k_2\csc(\alpha)\sin(\alpha - \theta) \\
&\quad \left[\left(k_1^2\sin(2\alpha - \theta) - k_2^2\sin(\theta)\right)(k_2^2 - k_1^2)\right]^{-1}\left(k_1^2\sin(3\alpha - \theta) - \right. \\
&\quad \left. k_2^2\left(2\sin(\alpha - \theta) + \sin(\alpha)\right)\right) \\
\mu_2^2 &= 2\lambda_3 k_1 k_2\cos(\alpha) + \frac{1}{2}\lambda_4(k_1^2 + k_2^2) + \frac{1}{2}\alpha_2(v_L^2 + v_R^2) + \frac{1}{4}\beta_1 v_L v_R\sec(\alpha)\left(\cos(\alpha - \theta) + \right. \\
&\quad \left.\csc(\alpha)\sin(\theta)\cos(2\alpha)\right) - \frac{1}{4}\beta_1\frac{v_L v_R k_1^2}{k_2^2 - k_1^2}\sec(\alpha)\csc(\alpha)\sin(\alpha - \theta)\left(\cos(2\alpha) + \frac{k_1^2}{k_2^2}\right) \\
&\quad \left[k_1^2\left(\sin(\alpha - \theta) + \sin(3\alpha + \theta)\right) - k_2^2\left(3\sin(\alpha - \theta) + \sin(\alpha + \theta)\right)\right] \\
\mu_3^2 &= \frac{1}{2}\alpha_1(k_1^2 + k_2^2) + \frac{1}{2}\rho_3(v_L^2 + v_R^2) - 2\alpha_2 k_1 k_2\cos(\alpha) + \beta_1 k_1 k_2(k_1^2 - k_2^2)\sin(\alpha) \\
&\quad \left[2v_L v_R(v_R^2 - v_L^2)\left(k_2^2\sin(\theta) - k_1^2\sin(2\alpha - \theta)\right)\right]^{-1} \\
\lambda_2 &= \frac{1}{2}\lambda_3 + \beta_1 v_L v_R\csc(\alpha)\sin(\theta)\left\{(8k_1k_2)^{-1} + k_1\left[k_1^2\left(\sin(\alpha - \theta) + \sin(3\alpha + \theta)\right) - \right.\right. \\
&\quad \left.\left. k_2^2\left(3\sin(\alpha - \theta) + \sin(\alpha + \theta)\right)\right]\left[\left(k_1^2\sin(2\alpha - \theta) - k_2^2\sin(\theta)\right)(k_1^2 - k_2^2)\right]^{-1}\right\} \\
\rho_1 &= \frac{\rho_3}{2} + \frac{\beta_1 k_1 k_2\sin(\alpha)\,(k_2^2 - k_1^2)}{2v_L v_R\left(k_2^2\sin(\theta) - k_1^2\sin(2\alpha - \theta)\right)} \\
\beta_2 &= \frac{\beta_1 k_1 k_2\sin(\alpha - \theta)}{\left(k_2^2\sin(\theta) - k_1^2\sin(2\alpha - \theta)\right)}
\end{aligned} \tag{17}
$$

As the reader can see, all the parameters (except for the μ_i^2) are of the same order, so that no special fine tuning is needed. Certainly, to demostrate that our different models are free of phenomenological disaster requires further analysis. The phenomenology of this class of models will be examined in the following; however, a complete analytical analysis of these models is far too complex to be exhausted in this work. We shall ilustrate only some aspects of this class of models here. We will turn our discussion toward the Higgs spectrum.

5 The Higgs spectrum

The complete form of the Higgs mass matrices for the general case are given in the Appendix. Let us now examine them to see if they are able to generate the proper masses for the physical particles, in the scenarios presented above.

We first consider the scenario with both, $k_1 = k_2 = k$ and $\beta_1 = \beta_2 = 0$. It is enough to inspect the doubly charged Higgs mass matrix to discard this model on a phenomenological basis. In fact, we have in the $\{\delta_R^{++}\ ,\ \delta_L^{++}\}$ basis

$$\mathcal{M}_{++}^{\in} = \begin{pmatrix} 2\rho_2 v_R^2 & 2\rho_4 v_L v_R \cos(\theta) \\ 2\rho_4 v_L v_R \cos(\theta) & 2\rho_2 v_L^2 \end{pmatrix} \tag{18}$$

with eigenvalues proportional to v_R^2 and $v_L v_R$ which are phenomenologically unacceptable. To escape from this bound is completely impossible, even with a severe fine tuning of the ρ parameters.

But this cannot be surprising, because in this model, in which the β-type Higgs potential terms are absent, the first derivative conditions become homogeneous [9], i.e.

$$\frac{\partial \mathbf{V}}{\partial Re(k_i)} = k_i f_{k_i}(...)$$

where $f_{k_i}(...)$ is a general quadratic function of the vevs, and k_i represents any of the four vevs. Therefore, we can satisfy the first derivative conditions by setting either $f_{k_i}(...) = 0$ or $k_i = 0$. As was shown in previous works [12], the latter solution is the only one phenomenologically acceptable for two of the vevs (k_2 and v_L), in such a way that no phase degrees of freedom remain. In this case spontaneous CP violation cannot occur. Thus, we can conclude that the β-type Higgs potential terms (the quartic ones)must be present in order to have the desired spontaneous CP violation.

Following the order of increasing complexity, we will focus now on our second case, where k_1 is still equal to k_2 but now $\alpha = \theta$. Here the doubly charged Higgs mass matrix is specially easy to analize. Let us recall the reader first, that for left right symmetric models to be consistent with the observed phenomena, the symmetry breaking pattern that should arise is $v_R \gg k_1, k_2 \gg v_L$. For this reason, we can safely neglect terms of order (v_L/v_R) . Additionally, we will assume that $k^2 \sim v_L v_R$. We make this assumption because this choice allows us to solve the model easily. If this was not the case, we can take the largest contributing term between them and the features of the model remain the same. The schematic form of the mass matrix for the doubly charged Higgs sector in the $\{\delta_R^{++}\,,\,\delta_L^{++}\}$ basis is

$$\mathcal{M}^{\in}_{++} = \begin{pmatrix} \rho v_R^2 & (\rho+\beta)v_L v_R\cos(\theta) \\ (\rho+\beta)v_L v_R\cos(\theta) & \beta v_R^2 \end{pmatrix} \tag{19}$$

Here, we have introduced a shorthand notation where the parametres $\{\rho\,,\,\beta\}$ without subscripts stand for a generic parameter of their class, and we have indicated for each entry only the largest contributing terms. The exact entries are presented in the Appendix. (The same generic notation will be used for the other mass matrices that follow). The eigenstates will have masses of order v_R, with mixing of order (v_L/v_R).

For the singly charged Higgs sector, we will exhibit the result in the $\{\phi_1^+\,,\,\phi_2^+\,,\,\delta_R^+\,,\,\delta_L^+\}$ basis

$$\mathcal{M}^{\in}_{+} = \begin{pmatrix} (\lambda-\beta)k^2 & \beta k^2 & 0 & \beta k v_R \\ \beta k^2 & (\lambda-\beta)k^2 & 0 & \beta k v_R \\ 0 & 0 & 0 & \beta k^2 \\ \beta k v_R & \beta k v_R & \beta k^2 & \beta v_R^2 \end{pmatrix} \tag{20}$$

The mass scales of the various Higgs bosons are as expected. The singly charged Higgs mass matrix has the two required zero eigenvalues (which eventually become longitudinal components of W_L^+ and W_R^+), and the other two masses will be set by k and v_R, respectively.

For the neutral sector, we will work with an 8×8 square matrix since, because of our CP violating phases, the real and imaginary components of the neutral Higgs scalars couple to each other in the mass matrix (one cannot avoid this and still achieve spontaneous CP violation). This huge mass matrix in the $\{\phi_1^r\,,\,\phi_2^r\,,\,\delta_R^r\,,\,\delta_L^r\,,\,\phi_1^i\,,\,\phi_2^i\,,\,\delta_R^i\,,\,\delta_L^i\}$ basis has the following form

$$\mathcal{M}^{\in}_{\prime} = \begin{pmatrix} \mathcal{M}^{\in}_{\nabla\nabla} & \mathcal{M}^{\in}_{\nabla\rangle} \\ \mathcal{M}^{\dagger\in}_{\nabla\rangle} & \mathcal{M}^{\in}_{\rangle\rangle} \end{pmatrix} \tag{21}$$

where

$$\mathcal{M}^{\in}_{\nabla\nabla} = \begin{pmatrix} (\lambda-\beta)k^2 & (\lambda-\beta)k^2 & \alpha k v_R\cos(\theta) & \beta k v_R\cos(\theta) \\ (\lambda-\beta)k^2 & (\lambda-\beta)k^2 & \alpha k v_R\cos(\theta) & \beta k v_R \\ \alpha k v_R\cos(\theta) & \alpha k v_R\cos(\theta) & \beta v_R^2 & (\beta+\rho)k^2\cos(\theta) \\ \beta k v_R\cos(\theta) & \beta k v_R & (\beta+\rho)k^2\cos(\theta) & \beta v_R^2\cos(\theta) \end{pmatrix}$$

$$\mathcal{M}^{\in}_{\rangle\rangle} = \begin{pmatrix} (\lambda-\beta)k^2 & -(\lambda-\beta)k^2 & \alpha k v_R\sin(\theta)^2 & \beta k v_R \\ -(\lambda-\beta)k^2 & (\lambda-\beta)k^2 & \alpha k v_R\sin(\theta)^2 & \beta k v_R \\ \alpha k v_R\sin(\theta)^2 & \alpha k v_R\sin(\theta)^2 & \beta v_R^2 & \beta k^2\cos(\theta) \\ \beta k v_R & \beta k v_R & \beta k^2\cos(\theta) & \beta v_R^2 \end{pmatrix}$$

$$\mathcal{M}^{\in}_{\nabla\rangle} = \sin(\theta)\begin{pmatrix} (\lambda+\beta)k^2 & -(\lambda+\beta)k^2 & \alpha k v_R & \beta k v_R \\ (\lambda+\beta)k^2 & -(\lambda+\beta)k^2 & \alpha k v_R & \beta k v_R \\ \alpha k v_R\cos(\theta) & \alpha k v_R\cos(\theta) & \beta v_R^2 & \beta k^2 \\ \beta k v_R & \beta k v_R & (\beta+\rho)k^2 & 0 \end{pmatrix}$$

One can observe that (keeping only the leading terms), as required in order to give Z and Z' mass, there are two zero mass Goldstone boson eigenstates. Four of the remaining ones have mass of order v_R and the last two of order k. Thus, if v_R is very large, all the non-standard-model Higgs bosons in the neutral Higgs sector will be heavy except for one. As such, it could happen that the only signature of an underlying left right symmetric theory that will be accesible at present and foreseable machines, will be these light Higgs (one charged and one neutral) in addition to one of the neutral Higgs bosons that plays the role of the standard model Higgs boson in the left right model.

Last but not least, our $k_1 \neq k_2$ case. This case amounts to a complicated version of the previous one, but with similar results. (Exact mass matrices could be found in the Appendix).

Thus, we have arrived at two models which potentially yield a reasonable phenomenology, for a relatively constrained set of Higgs boson couplings and vevs. We find this result to be particularly interesting, given the relatively large number of free parameters in the models to adjust the remaining phenomenology [16].

6 The FCNCs

We are going to analyze now, the requirements that must be fullfilled in order to suppress the FCNC. For this purpose we have to analyze the quarks-Higgs boson couplings. The most general Yukawa interaction invariant separately under $SU(2)_L$ and $SU(2)_R$ transformations is [9, 12, 17]

$$\mathcal{L}_{\mathcal{Y}} = \bar{\Psi}^i{}_L \left(f_{ij}\phi + g_{ij}\tilde{\phi} \right) \Psi^j_R \ + \ h.c. \tag{22}$$

where $\Psi = \begin{pmatrix} \breve{u}_i \\ \breve{d}_i \end{pmatrix}$. These states are weak eigenstates. f and g are the Yukawa coupling matrices, and the i, j indices are family indices. Due to the left right symmetry requirement on the lagrangian , we require that $f = f^\dagger$ and $g = g^\dagger$. We can rotate the weak eigenstates into mass eigenstates with unitary matrices $\mathbf{V}$ in this way

$$\breve{u}_\alpha = \mathbf{V}^u_\alpha u_\alpha$$
$$\breve{d}_\alpha = \mathbf{V}^d_\alpha d_\alpha$$

where $\breve{u}$ and $\breve{d}$ are vectors representing the up and down type quarks and the index $\alpha = L, R$.

In terms of these matrices, the usual Cabibbo-Kobayashi-Maskawa matrix (CKM) in the left and right sectors is given by

$$\mathbf{V}^{CKM}_\alpha = \mathbf{V}^{u\dagger}_\alpha \mathbf{V}^d_\alpha$$

We want now to build the quark mass matrices, so that we have to worry only about the (u and d) diagonal terms. Taking the vevs of the ϕ fields, we can determine the u and d type quark mass matrices

$$\frac{1}{\sqrt{2}} \bar{u}_L \mathbf{V}^{u\dagger}_L \left(f k_1 + g k_2^* \right) \mathbf{V}^u_R u_R \equiv \bar{u}_L \mathbf{M}^u u_R$$

$$\frac{1}{\sqrt{2}}\bar{d}_L \mathbf{V}_L^{d\dagger}\left(fk_2 + gk_1^*\right)\mathbf{V}_R^d d_R \equiv \bar{d}_L \mathbf{M}^d d_R \tag{23}$$

where $\mathbf{M}^u$ and $\mathbf{M}^d$ represent the diagonal matrix of physical quark masses . For $|k_1|^2 \neq |k_2|^2$ and $k_\pm^2 \equiv |k_1|^2 \pm |k_2|^2$ we can invert these equations, to solve f and g in terms of the physical masses of the up and down quarks and the diagonalizing matrices

$$\begin{aligned} f &= \frac{\sqrt{2}}{k_-^2}\left(k_1^* \mathbf{V}_L^u \mathbf{M}^u \mathbf{V}_R^{u\dagger} - k_2^* \mathbf{V}_L^d \mathbf{M}^d \mathbf{V}_R^{d\dagger}\right) \\ g &= \frac{\sqrt{2}}{k_-^2}\left(-k_2 \mathbf{V}_L^u \mathbf{M}^u \mathbf{V}_R^{u\dagger} + k_1 \mathbf{V}_L^d \mathbf{M}^d \mathbf{V}_R^{d\dagger}\right) \end{aligned} \tag{24}$$

We can now write the general interaction term for the quark mass eigenstates with the neutral ϕ-type Higgs fields

$$\begin{aligned} &\frac{\sqrt{2}}{k_-^2}\bar{u}_L\left[\mathbf{M}^u\left(k_1^*\phi_1^0 - k_2\phi_2^{0*}\right) + \mathbf{V}_L^{CKM}\mathbf{M}^d\mathbf{V}_R^{CKM\dagger}\left(-k_2^*\phi_1^0 + k_1\phi_2^{0*}\right)\right]u_R \ , \\ &\frac{\sqrt{2}}{k_-^2}\bar{d}_L\left[\mathbf{M}^d\left(k_1\phi_1^{0*} - k_2^*\phi_2^0\right) + \mathbf{V}_L^{CKM\dagger}\mathbf{M}^u\mathbf{V}_R^{CKM}\left(-k_2\phi_1^{0*} + k_1^*\phi_2^0\right)\right]d_R \end{aligned} \tag{25}$$

To identify the flavour changing and flavour conserving combinations, we define the new reciprocally orthogonal neutral fields

$$\begin{aligned} \phi_+^0 &= \frac{1}{k_+^2}\left(-k_2^*\phi_1^0 + k_1\phi_2^{0*}\right) \\ \phi_-^0 &= \frac{1}{k_+^2}\left(k_1^*\phi_1^0 + k_2\phi_2^{0*}\right) \end{aligned} \tag{26}$$

where the inverse transformations are

$$\begin{aligned} \phi_1^0 &= \frac{1}{k_+^2}\left(-k_2\phi_+^0 + k_1\phi_-^0\right) \\ \phi_2^0 &= \frac{1}{k_+^2}\left(k_1\phi_+^{0*} + k_2\phi_-^{0*}\right) \end{aligned} \tag{27}$$

In terms of these new fields, the coupling to the quarks are

$$\begin{aligned} &\frac{\sqrt{2}}{k_-^2}\bar{u}_L\left[\phi_-^0\frac{k_-^2}{k_+}\mathbf{M}^u + \phi_+^0\left(\frac{-2k_1^*k_2}{k_+}\mathbf{M}^u + k_+\mathbf{V}_L^{CKM}\mathbf{M}^d\mathbf{V}_R^{CKM\dagger}\right)\right]u_R \\ &\frac{\sqrt{2}}{k_-^2}\bar{d}_L\left[\phi_-^{0*}\frac{k_-^2}{k_+}\mathbf{M}^d + \phi_+^{0*}\left(\frac{-2k_1^*k_2}{k_+}\mathbf{M}^d + k_+\mathbf{V}_L^{CKM\dagger}\mathbf{M}^d\mathbf{V}_R^{CKM}\right)\right]d_R \end{aligned} \tag{28}$$

It is easy to see that these couplings are not diagonal since the CKM matrices are not diagonal. This non-diagonality always yields powerful constraints. It is obvious from Eq. (28) that only the two components of the complex field ϕ_-^0 can have flavour diagonal coupling. Thus, the real component of the ϕ_-^0 must be the analogue to the standard model Higgs boson, and the imaginary component must correspond to the massless Goldstone field absorbed by the Z. So both of them are flavour conserving. In order that the flavour changing couplings of the ϕ_+^0 in (28) not to enter in conflict with experiment we can follow two approaches : i) the mass eigenstates containing significant mixtures of ϕ_+^0 can have a large mass, the exact requirements will be presented in an example below. ii) Similarly to Ref. [18] one can invoke the assumption of global U(1) family symmetries with saying that the off diagonal elements of f and g, and consequently those of (28), have small values; sufficient for a natural suppression of family changing currents.

6.1 A "flavour diagonal" basis

As it was stated before, for the FCNC analysis, we find it useful to rotate the neutral fields into what we call the "flavour diagonal" basis. That is, we go from the original $\{\phi_1^r, \phi_2^r, \delta_R^r, \delta_L^r, \phi_1^i, \phi_2^i, \delta_R^i, \delta_L^i\}$ basis to the $\{\phi_-^r, \phi_+^r, \delta_R^r, \delta_L^r, \phi_-^i, \phi_+^i, \delta_R^i, \delta_L^i\}$ basis with a flavour diagonal ϕ_-. Recall that in our model, using Eq. (26) the "flavour diagonal" fields are

$$\phi_-^0 = \left[\left(-k_2\phi_1^r + k_1\cos(\alpha)\phi_2^r + k_1\sin(\alpha)\phi_2^i\right) + i\left(-k_2\phi_1^i - k_1\cos(\alpha)\phi_2^i + k_1\sin(\alpha)\phi_2^r\right)\right]\frac{1}{k_+}$$

$$\phi_+^0 = \left[\left(k_1\cos(\alpha)\phi_1^r + k_1\sin(\alpha)\phi_1^i + k_2\phi_2^r\right) + i\left(k_1\cos(\alpha)\phi_1^i - k_1\sin(\alpha)\phi_1^r - k_2\phi_2^i\right)\right]\frac{1}{k_+} \quad (29)$$

We define the rotation matrix $\mathbf{R}$ as

$$\mathbf{R} = \frac{1}{k_+}\begin{pmatrix} -k_2 & k_1\cos(\alpha) & 0 & 0 & 0 & k_1\sin(\alpha) & 0 & 0 \\ k_1\cos(\alpha) & k_2 & 0 & 0 & k_1\sin(\alpha) & 0 & 0 & 0 \\ 0 & 0 & k_+ & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_+ & 0 & 0 & 0 & 0 \\ 0 & k_1\sin(\alpha) & 0 & 0 & -k_2 & -k_1\cos(\alpha) & 0 & 0 \\ -k_1\sin(\alpha) & 0 & 0 & 0 & k_1\cos(\alpha) & -k_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_+ & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_+ \end{pmatrix} \quad (30)$$

which will accomplish our change of basis.

We can now examine the components of the mass matrix in this "flavour diagonal" basis, although its complete analytical study is far too complex to be considered. We shall ilustrate the viability of this class of models by examining a toy model in which $\lambda_3 = \lambda_4 = 0$ and $\alpha = \theta = \frac{\pi}{2}$. We make this choice, which allows us to solve the model exactly, because the cancelation of these parameters (λ_3 and λ_4) can be justified by the imposition of a discrete symmetry in the Higgs potential [9].

To analyze the mass matrix, let us assume for simplicity that the differences amongst the vevs of the bidoublet are smaller than their common scale, k, namely $\mid k_i^2 - k_j^2 \mid / \left(k_i^2 + k_j^2\right) \ll 1$. This together with the above mentioned conditions and neglecting terms of relative order v_L/v_R, has the effect of decoupling the 8×8 neutral Higgs mass matrix in three separated pieces.

The first is a 1×1 matrix containing only the ϕ_-^r, the second one is a 4×4 matrix which couples the ϕ_+^r, Δ_R^r, ϕ_-^i and Δ_L^i fields and the last one, which couples the remaining fields, i.e. Δ_L^r, ϕ_+^i and Δ_R^i. Thus ϕ_-^r is an unmixed mass eigenstate with mass $m^2_{\phi_-^r} \approx \beta k_>^2$, where $k_>$ is the biggest of k_1 and k_2.

The second set of eigenstates is that arising from diagonalizing the 4×4 submatrix which couples the ϕ^r_+ , Δ^r_R , ϕ^i_- and Δ^i_L fields, which yields

$$\begin{aligned}
h^0_1 &= -\varepsilon\phi^r_+ - \varepsilon\Delta^r_R + \phi^i_- \\
h^0_2 &= \phi^r_+ - 2\varepsilon\Delta^r_R + \varepsilon\phi^i_- - \Delta^i_L \\
h^0_3 &= -\Delta^r_R + \Delta^i_L \\
h^0_4 &= \varepsilon\phi^r_+ + \Delta^r_R + \Delta^i_L
\end{aligned} \tag{31}$$

where $\varepsilon = k_>/v_R$. The masses of these four states are given to first order in ε by

$$\begin{aligned}
m^2_{h^0_1} &\approx 0 \\
m^2_{h^0_2} &\approx kv_R \\
m^2_{h^0_3} &\approx v^2_R \\
m^2_{h^0_4} &\approx v^2_R
\end{aligned} \tag{32}$$

In the last set , we have

$$\begin{aligned}
h^0_5 &= \Delta^r_L \\
h^0_6 &= \phi^i_+ + \varepsilon\Delta^i_R \\
h^0_7 &= -\varepsilon\phi^i_+ + \Delta^i_R
\end{aligned} \tag{33}$$

with masses

$$\begin{aligned}
m^2_{h^0_5} &\approx v^2_R \\
m^2_{h^0_6} &\approx 0 \\
m^2_{h^0_7} &\approx v^2_R
\end{aligned} \tag{34}$$

From this, we can see that the real part of ϕ^0_- is the standard model Higgs boson with diagonal couplings to quarks (see Eq. (28)) , while its imaginary part is (approximately) the massless Goldstone mode which will be eaten by the Z. As desired, the mass eigenstate h^0_2 containing a significant mixture of ϕ^0_+ (real) has large mass, while its imaginary part, seen in h^0_6, will be eaten by the Z'.

What this model ilustrates is that, despite the great danger to lose the possibility of decoupling the mass scale of the FCNC Higgs bosons from the mass scale of the standard model one, there is at least some instances where this can be done and still have spontaneous CP violation. A detailed numerical analysis of the general case, shows that it is in fact possible to decouple the mass scales without further restrictions on the model.

7 CP violation in the leptonic sector : an example

As we have shown up to now, it is possible to have spontaneous CP violation in left right symmentric models. The question is now: where can we see it ?. In the quark sector, the charged current involves a unitary mixing matrix. The elements of this matrix are complex, and this fact gives rise to CKM CP violation in the standard model. The possibility of spontaneous CP phases induces physics beyond the standard model.

In models with massive neutrinos, we have mixing matrix in the leptonic sector as well. Complex numbers in this matrix would imply CP violation in the leptonic sector. It is well known that if CP is conserved, an elementary fermion cannot have an electric dipole moment. Now, we want to examine the electric dipole moment of charged leptons introduced by CP violation in the leptonic sector induced by complex vevs.

In left right symmetric theories, the charged current interaction of the lepton is given by

$$\mathcal{L}_{\rfloor\rfloor} = \frac{g}{\sqrt{2}} \sum_{a=1}^{3} \left(W_L^{\mu} \bar{l_{aL}} \gamma_\mu \nu_{aL} + W_R^{\mu} \bar{l_{aR}} \gamma_\mu N_{aR} \right) + \text{h.c.} \tag{35}$$

We can always choose a representation in which the mass matrix of the charged leptons l_a is diagonal. The gauge bosons mass matrix will be given by

$$\begin{pmatrix} \frac{g^2}{2}\left(2v_L^2 + k_1^2 + k_2^2\right) & -g^2 k_1 k_2 e^{i\alpha} \\ -g^2 k_1 k_2 e^{-i\alpha} & \frac{g^2}{2}\left(2v_R^2 + k_1^2 + k_2^2\right) \end{pmatrix} \tag{36}$$

and in that basis the neutrino mass matrix by

$$\begin{pmatrix} \mu_\nu & \mu_D \\ \mu_D^T & \mu_N \end{pmatrix} \tag{37}$$

In Eq. (37) μ_ν, μ_D and μ_N are 3×3 matrices and they are given by

$$\begin{aligned} \mu_\nu &= f v_L \\ \mu_D &= h k_1 e^{i\alpha} + \hat{h} k_2 \\ \mu_N &= f v_R e^{i\theta} \end{aligned}$$

where f,h and $\hat{h}$ are the corresponding Yukawa couplings matrices.

As the reader can see, the mass parameters in Eq. (36) as well as in Eq. (37) are complex. However, by an appropiate choice of the phases of the various fields, some of them can be chosen to be real. For example, note that the charged current interaction in Eq. (35) is invariant under the phase redefinitions $W_R \longrightarrow e^{i\alpha} W_R$, $N_R \longrightarrow e^{-i\alpha} N_R$. Using this freedom, we can arrange (36) to be real, while the phase α, will appear in μ_D and μ_N in Eq. (37). Thus, by this phase convention, the CP violating effects arise through the neutrino mass matrix.

The neutrino mass matrix, though complex, is symmetric, so it can be diagonalized by using a 6×6 unitary matrix V which gives

$$\begin{pmatrix} \nu_L \\ N_L \end{pmatrix} = V \chi_L$$

where χ is a column matrix of mass eigenstates. Equivalently, this equation can be written as

$$\begin{aligned} \nu_{aL} &= \sum_{i=1}^{6} P_{ai} \chi_{iL} \\ N_{aL} &= \sum_{i=1}^{6} Q_{ai} \chi_{iL} \end{aligned} \tag{38}$$

where P and Q are 3×6 matrices, and are both complex. We can also diagonalize the gauge bosons mass matrix. This leads to

$$\begin{pmatrix} W_L \\ W_R \end{pmatrix} = U \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \tag{39}$$

where W_1 and W_2 are the mass eigenstates. With these notations, we can rewrite Eq. (35) in terms of physical fields

$$\mathcal{L}_{\rfloor\rfloor} = \frac{g}{\sqrt{2}} \sum_{a=1}^{3} \sum_{i=1}^{3} \sum_{j=1}^{2} W_j^\mu \left(U_{Lj} P_{ai} \bar{l}_{aL} \gamma_\mu \chi_{iL} + U_{Ri} Q_{ai} \bar{l}_{aR} \gamma_\mu \chi_{iR} \right) + \text{h.c.} \tag{40}$$

The one loop graph involving weak gauge bosons that contribute to the electric dipole moment is shown if Fig 1. However, if we choose to calculate the form factor in a general ξ gauge, extra diagrams appear where one or both of the W_i lines are replaced by the unphysical gauge bosons wich are absorbed by the longitudinal component of W_i in the unitary gauge. A straightforward calculation gives

$$d_a^{(ij)} = -\frac{eg^2 m_i}{64\pi^2 M_j^2} U_{Lj} U_{Rj} \text{Im} \left(P_{ai} Q_{ai} \right) \left[\frac{r_{ij}^2 - 11 r_{ij} + 4}{(r_{ij} - 1)^2} + \frac{6 r_{ij}^2 \ln r_{ij}}{(r_{ij} - 1)^3} \right] \tag{41}$$

where

$$r_{ij} \equiv m_i^2 / M_j^2$$

The result for d_a can be found by summing over the indices i of the heavy neutrinos and j of the W bosons.

At this piont, a few comments are in order. First, if the neutrino mass matrix in Eq. (37) was real, the diagonalizing matrices P and Q can be chosen to be real, so that we would have no electric dipole moment. Second, for d_a to be nonzero , we need $U_{Lj} U_{Rj} \neq 0$ that is, we need the $W_L - W_R$ mixing in the mass matrix. Otherwise the mass eigenstates W_1 and W_2 become the same as the gauge eigenstates, and the diagrams with W_L (W_R) running in the loop do not produce any CP violation at the one loop level. Third, if all the neutrino masses are small compared to M_j, the expression in square brackets in Eq. (41) becomes a constant, viz.,4. On the other hand, if all the neutrino masses are big compared to M_j, the expression in square brackets in Eq. (41) becomes a constant again, viz. 1.

Following the diagonalization procedure the reader can see that in any case we have

$$d_a \propto \sin(\alpha) \tag{42}$$

as $\sum_i m_i P_{ai} Q_{ai} = (\mu_D)_{aa}$. This is illustrated in Figure 1b.

Let us now see what magnitude we can expect for the electric dipole moment of the electron. If $M_2 \gg M_1$, the dominant term in Eq. (41) is the term with $j = 1$. Using

$$\frac{g^2}{8M_1} = \frac{G_F}{\sqrt{2}}$$

then,

$$d_e = 10^{-24}\text{e-cm}\ \sin(2\zeta)\sum_i \frac{m_i}{1\text{MeV}}\text{Im}(P_{ei}Q_{ei})S \tag{43}$$

where S is the expression in square brakets in Eq. (41) and ζ is the W mixing.

Two limits on S are interesting:
i) $m_i \ll M_1$, then $S \simeq 4 - 3r_{i1}$
ii) $m_i \gg M_1$, then $S \simeq 1 + \frac{6\ln r_{i1}}{r_{i1}}$
In the second case, it appears that d_e does not vanish even if we take the right handed scale v_R to infinity. However, this is not the case , decoupling is recovered because ζ is proportional to $1/v_R^2$. Taking the limit $\zeta \leq .001$ [19] d_e becomes

$$d_e \leq 2\ 10^{-27}\text{e-cm}\frac{\text{Im}(\mu_D)_{ee}}{1\text{MeV}}S \tag{44}$$

nearly close to the experimental bound [20] $\mid d_e \mid\leq 10^{-26}$e-cm.

It is interesting to see that the value of d_e is essentially determined by ζ, the $W_L - W_R$ mixing parameter and by $(\mu_D)_{ee}$; although we have found two CP violating phases, only α appears in this process. The reader may then wonder if the same phenomenology could be achieved by eliminating the triplet phase (or even the triplet), i.e. still have spontaneous CP violation with only one complex vev, but this is not the case. As can be seen from the first derivative equations of section 3, if one sets $\theta = 0$, the only viable solution is $\alpha = 0$. That is, to get spontaneous CP violation in our case, two of the vevs must be complex; although only one is visible in our example.

8 Conclusions

In this paper we have presented a detailed analysis of the spontaneous symmetry breaking and Higgs sector of the conventional $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ left right symmetric model, containing one bidoublet Higgs field, one left handed triplet field and one right handed triplet field. Specifically, we have performed a critical assessment of the phenomenological viability of having spontaneous CP violation.

We have shown that it is possible to obtain a minimum of the Higgs potential which yields that spontaneous CP violation; this task is further complicated by relations among the parameters which may have the (unnatural) property of relating parameters across widely differing scales.

There are many attractive aspects to a left right symmetric gauge theory including (i) a mechanism for neutrino mass generation, (ii) the identification of the $U(1)$ quantum number with $(B - L)$, and (iii) a collection of potentially observable Higgs and gauge bosons including doubly charged Higgs bosons.

We have analyzed in this class of models the phase degrees of freedom and we have found that, for a Higgs potential without explicit CP violation, spontaneous CP violation might occur, that is, the vacuum expectation values of the Higgs fields can be chosen to be complex. For this to happen, the β-type Higgs potential terms, which quartically mix all the Higgs fields, should be present.

The increasing of physical Higgs fields in the left right theory introduces new features, in particular we have now FCNC. This feature can be analized by examining the quarks-Higgs bosons Yukawa terms in the lagrangian. Such analysis has shown that additional constraints have to be imposed on the theory (which are not present in the standard model) but they do not spoil it. The flavour changing neutral Higgs bosons in the left right models of this type, can be made heavy in order to avoid the significant contribution to FCNC at tree level they could give rise to. Besides that, we have shown that the spontaneous CP violating phase of the left right symmetric theory can manifest itself in the electric dipole moment of an elementary fermion.

Certainly, there are many exciting features and potential signatures for this kind of left right symmetric models which violate CP spontaneously. We hope that our presentation would have proved sufficiently transparent to allow the reader to judge himself the degree of skepticism that is appropiate when considering the phenomenology of these theories with extended (and complicated) Higgs sectors.

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A General Higgs mass matrices

In this appendix we give a variety of useful results for the mass-squared matrices of the various Higgs sectors. We will present here the result before the first derivative constraints have been substituted, so that the expresions will be useful for the different scenarios

A.1 Components of the neutral Higgs mass matrix

We first compute the components of the neutral Higgs mass matrix in the
$\{\phi_1^r\,,\,\phi_2^r\,,\,\delta_R^r\,,\,\delta_L^r\,,\,\phi_1^i\,,\,\phi_2^i\,,\,\delta_R^i\,,\,\delta_L^i\}$ basis. The mass matrices are symmetric matrices which we require to have positive eigenvalues.

$$
\begin{aligned}
\mathcal{M}^{\in}_{\infty\infty} &= -\mu_1^2 + \lambda_1 k_1^2\left(2\cos(\alpha)^2+1\right) + 4\lambda_2 k2^2 + 2\lambda_3 k2^2 + 6\lambda_4 k_1 k_2 \cos(\alpha) + \\
&\quad \frac{1}{2}\alpha_1(v_L^2+v_R^2) + \beta_2 v_L v_R \cos(\theta) + \lambda_1 k_2^2 \\
\mathcal{M}^{\in}_{\infty\in} &= -2\mu_2^2 + k_1 k_2 \cos(\alpha)\left(\lambda_1 + 4\lambda_2 + 2\lambda_3\right) + 3\lambda_4 k2^2 + \lambda_4 k_1^2\left(1+2\cos(\alpha)^2\right) \\
&\quad \alpha_2(v_L^2+v_R^2) + \frac{1}{2}\beta_1 v_L v_R \cos(\theta) \\
\mathcal{M}^{\in}_{\infty\ni} &= \alpha_1 k_1 v_R \cos(\alpha)\cos(\theta) + 2\alpha_2 k_2 v_R \cos(\theta) + \frac{1}{2} v_L\left(\beta_1 k_2 + 2k_1\beta_2\cos(\alpha)\right) \\
\mathcal{M}^{\in}_{\infty\triangle} &= v_L\left(\alpha_1 k_1 + 2\alpha_2 k_2\right) + \frac{1}{2} v_R \cos(\theta)\left(\beta_1 k_2 + 2\beta_2 k_1 \cos(\alpha)\right) \\
\mathcal{M}^{\in}_{\infty\bigtriangledown} &= \beta_2 v_L v_R \sin(\theta) + \lambda_1 k_1^2 \sin(2\alpha) + 2\lambda_4 k_1 k_2 \sin(\alpha) \\
\mathcal{M}^{\in}_{\infty/} &= -\frac{1}{2}\beta_1 v_L v_R \sin(\theta) + \lambda_4 k_1^2 \sin(2\alpha) - 8\lambda_2 k_1 k_2 \sin(\alpha) \\
\mathcal{M}^{\in}_{\infty\prime} &= \beta_2 v_L k_1 \sin(\alpha) + \alpha_1 v_R k_1 \sin(\theta)\cos(\alpha) + 2\alpha_2 v_R k_2 \sin(\theta) \\
\mathcal{M}^{\in}_{\infty\forall} &= -\beta_2 v_R k_1 \cos(\theta)\sin(\alpha) + \beta_2 v_R k_1 \cos(\alpha)\sin(\theta) + \frac{1}{2}\beta_1 v_R k_2 \sin(\theta) \\
\mathcal{M}^{\in}_{\in\in} &= -\mu_1^2 + \lambda_1\left(3k_2^2 + k_1^2\right) + 4\lambda_2 k_1^2\left(\cos(\alpha)^2 - \sin(\alpha)^2\right) + 2\lambda_3 k_1^2 + 6\lambda_4 k_1 k_2 \cos(\alpha) \\
&\quad \frac{1}{2}\alpha_1(v_L^2+v_R^2) + \beta_3 v_L v_R \cos(\theta) \\
\mathcal{M}^{\in}_{\in\ni} &= \frac{1}{2} v_L\left(\beta_1 k_1 \cos(\alpha) + 2\beta_2 k_2\right) + v_R \cos(\theta)\left(\alpha_1 k_2 + 2\alpha_2 k_1 \cos(\alpha)\right) \\
\mathcal{M}^{\in}_{\in\triangle} &= v_L\left(\alpha_1 k_2 + 2\alpha_2 k_1 \cos(\alpha)\right) + v_R \cos(\theta)\left(\beta_1 k_1 \cos(\alpha) + 2\beta_2 k_2\right) + \\
&\quad \frac{1}{2}\beta_1 v_R k_1 \sin(\theta)\sin(\alpha) \\
\mathcal{M}^{\in}_{\in\bigtriangledown} &= \frac{1}{2}\beta_1 v_L v_R \sin(\theta) + 2\lambda_4 k_1^2 \sin(\alpha)\cos(\theta) + 2k_1 k_2 \sin(\alpha)\left(\lambda_1 + 4\lambda_2 + 2\lambda_3\right) \\
\mathcal{M}^{\in}_{\in/} &= -\beta_2 v_L v_R \sin(\theta) - 8\lambda_2 k_1^2 \sin(\alpha)\cos(\alpha) - 2\lambda_4 k_1 k_2 \sin(\alpha) \\
\mathcal{M}^{\in}_{\in\prime} &= \frac{1}{2}\beta_1 v_L k_1 \sin(\alpha) + 2\alpha_2 v_R k_1 \sin(\theta)\cos(\alpha) + \alpha_1 k_2 v_R \sin(\theta) \\
\mathcal{M}^{\in}_{\in\forall} &= \frac{1}{2}\beta_1 v_R k_1 \sin(\theta-\alpha) + \beta_2 k_2 v_R \sin(\theta) \\
\mathcal{M}^{\in}_{\ni\ni} &= \rho_1 v_R^2\left(1+2\cos(\theta)^2\right) + \frac{1}{2}\rho_3 v_L^2 - \mu_3^2 + 2\alpha_2 k_1 k_2 \cos(\alpha) + \frac{1}{2}\alpha_1(k_1^2+k_2^2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^{\in}_{\ni\triangle} &= \rho_3 v_L v_R \cos(\theta) + \frac{1}{2}\beta_1 k_1 k_2 \cos(\alpha) + \frac{1}{2}\beta_2 k_1^2 \left(\cos(\theta)^2 - \sin(\theta)^2\right) + \frac{1}{2}\beta_2 k_2^2 \\
\mathcal{M}^{\in}_{\ni\triangledown} &= -\beta_2 v_L k_1 \sin(\alpha) + \alpha_1 v_R k_1 \cos(\theta)\sin(\alpha) \\
\mathcal{M}^{\in}_{\ni/} &= \frac{1}{2}\beta_1 v_L k_1 \sin(\alpha) - 2\alpha_2 v_R k_1 \cos(\theta)\sin(\alpha) \\
\mathcal{M}^{\in}_{\ni'} &= 2\rho_1 v_R^2 \sin(\theta)\cos(\theta) \\
\mathcal{M}^{\in}_{\ni\forall} &= -\frac{1}{2}\beta_1 k_1 k_2 \sin(\alpha) \\
\mathcal{M}^{\in}_{\triangle\triangle} &= -\mu_3^2 + 3\rho_1 v_L^2 + \frac{1}{2}\rho_3 v_R^2 + \frac{1}{2}\alpha_1(k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 \cos(\alpha) \\
\mathcal{M}^{\in}_{\triangle\triangledown} &= \beta_2 v_R k_1 \sin(\theta - \alpha) + \alpha_1 v_L k_1 \sin(\alpha) + \frac{1}{2}\beta_1 k_2 v_R \sin(\theta) \\
\mathcal{M}^{\in}_{\triangle/} &= -2\alpha_2 v_L k_1 \sin(\alpha) - \frac{1}{2}\beta_1 v_R k_1 \sin(\theta - \alpha) - \beta_3 k_2 v_R \sin(\theta) \\
\mathcal{M}^{\in}_{\triangle'} &= \beta_2 k_1^2 \sin(\alpha)\cos(\alpha) + \frac{1}{2}\beta_1 k_1 k_2 \sin(\alpha) - \beta_2 k_2 v_R \sin(\theta) \\
\mathcal{M}^{\in}_{\triangle\forall} &= 0 \\
\mathcal{M}^{\in}_{\triangledown\triangledown} &= -\mu_1^2 + \lambda_1 k_1^2 \left(2\sin(\alpha)^2 + 1\right) + k_2^2\left(\lambda_1 + 2\lambda_3 - 4\lambda_4\right) + 2\lambda_4 k_1 k_2 \cos(\alpha) + \\
&\quad \frac{1}{2}\alpha_1(v_L^2 + v_R^2) - \beta_2 v_L v_R \cos(\theta) \\
\mathcal{M}^{\in}_{\triangledown/} &= 2\mu_2^2 - 8\lambda_2 k_2 k_2 \cos(\alpha) - \lambda_4\left(k_1^2\left(1 + 2\sin(\alpha)^2\right) + k_2^2\right) - \alpha_2(v_L^2 + v_R^2) + \\
&\quad \frac{1}{2}\beta_1 v_L v_R \cos(\theta) \\
\mathcal{M}^{\in}_{\triangledown'} &= \alpha_1 k_1 v_R \sin(\alpha)\sin(\theta) + \frac{1}{2}v_L\left(\beta_1 k_2 + 2k_1\beta_2\cos(\alpha)\right) \\
\mathcal{M}^{\in}_{\triangledown\forall} &= -\frac{1}{2}\beta_1 k_2 v_R \cos(\theta) - \beta_2 k_1 v_R \cos(\theta - \alpha) \\
\mathcal{M}^{\in}_{/} &= -\mu_1^2 + \lambda_1\left(k_2^2 + k_1^2\right) - 4\lambda_2 k_1^2\left(\cos(\alpha)^2 - \sin(\alpha)^2\right) + 2\lambda_3 k_1^2 + 2\lambda_4 k_1 k_2 \cos(\alpha) \\
&\quad \frac{1}{2}\alpha_1(v_L^2 + v_R^2) - \beta_3 v_L v_R \cos(\theta) \\
\mathcal{M}^{\in}_{'/} &= -\frac{1}{2}v_L\left(\beta_1 k_1 \cos(\alpha) + 2\beta_2 k_2\right) - 2\alpha_2 k_1 v_R \sin(\theta)\sin(\alpha) \\
\mathcal{M}^{\in}_{\forall/} &= \frac{1}{2}\beta_1 k_1 v_R \cos(\theta - \alpha) + \beta_3 k_2 v_R \cos(\theta) \\
\mathcal{M}^{\in}_{'} &= \rho_1 v_R^2\left(1 + 2\sin(\theta)^2\right) + \frac{1}{2}\rho_3 v_L^2 - \mu_3^2 + 2\alpha_2 k_1 k_2 \cos(\alpha) + \frac{1}{2}\alpha_1(k_1^2 + k_2^2) \\
\mathcal{M}^{\in}_{\forall'} &= \frac{1}{2}\beta_1 k_1 k_2 \cos(\alpha) + \frac{1}{2}\beta_2 k_1^2\left(\cos(\theta)^2 - \sin(\theta)^2\right) + \frac{1}{2}\beta_2 k_2^2 \\
\mathcal{M}^{\in}_{\forall\forall} &= -\mu_3^2 + \rho_1 v_L^2 + \frac{1}{2}\rho_3 v_R^2 + \frac{1}{2}\alpha_1(k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 \cos(\alpha)
\end{aligned}
\tag{A.1}
$$

A.2 Singly charged Higgs mass matrix

In a manner similar to the previous section, we compute the components of the singly charged Higgs mass matrix, in the $\{\phi_1^+\,,\ \phi_2^+\,,\ \delta_R^+\,,\ \delta_L^+\}$ basis

$$
\begin{aligned}
\mathcal{M}^{+\in}_{\infty\infty} &= -\mu_1^2 + \lambda_1(k_1^2+k_2^2) + 2\lambda_4 k_1 k_2 \cos(\alpha) + \frac{1}{2}\alpha_1(v_L^2+v_R^2) \\
\mathcal{M}^{+\in}_{\infty\in} &= -\alpha_2(v_L^2+v_R^2) + 2\mu_2^2 - \lambda_4(k_1^2+k_2^2) - 2k_1k_2\cos(\theta)\,(\lambda_3+2\lambda_2) \\
\mathcal{M}^{+\in}_{\infty\ni} &= -\left(\beta_2 v_L k_1 \cos(\alpha) + \frac{1}{2}\beta_1 v_L k_2\right)\frac{1}{\sqrt{2}} \\
\mathcal{M}^{+\in}_{\infty\triangle} &= \left(\frac{1}{2}\beta_1 v_R k_1 \cos(\theta-\alpha) + \beta_2 v_R k_2 \cos(\theta)\right)\frac{1}{\sqrt{2}} \\
\mathcal{M}^{+\in}_{\in\in} &= \frac{1}{2}\alpha_1(v_L^2+v_R^2) - \mu_1^2 + \lambda_1(k_1^2+k_2^2) + 2\lambda_4 k_1 k_2 \cos(\alpha) \\
\mathcal{M}^{+\in}_{\in\ni} &= \left(\frac{1}{2}\beta_1 v_L k_1 \cos(\alpha) + \beta_3 v_L k_2\right)\frac{1}{\sqrt{2}} \\
\mathcal{M}^{+\in}_{\in\triangle} &= -\left(\beta_2 v_R k_1 \cos(\theta-\alpha) + \frac{1}{2}\beta_1 v_R k_2 \cos(\theta)\right)\frac{1}{\sqrt{2}} \\
\mathcal{M}^{+\in}_{\ni\ni} &= -\mu_3^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 \cos(\alpha) + \rho_1 v_R^2 + \frac{1}{2}\rho_3 v_L^2 \\
\mathcal{M}^{+\in}_{\ni\triangle} &= \frac{1}{4}\beta_1(k_1^2+k_2^2) + \beta_2 k_1 k_2 \cos(\alpha) \\
\mathcal{M}^{+\in}_{\triangle\triangle} &= -\mu_3^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 \cos(\alpha) + \rho_1 v_L^3 + \frac{1}{2}\rho_3 v_R^2
\end{aligned} \tag{A.2}
$$

A.3 Doubly charged Higgs mass matrix

We now present the doubly charged Higgs mass matrix components in the $\{\delta_R^{++}, \delta_L^{++}\}$ basis .

$$
\begin{aligned}
\mathcal{M}^{++\in}_{\infty\infty} &= -\mu_3^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 \cos(\alpha) + \rho_1 v_R^2 + 2\rho_2 v_R^2 + \frac{1}{2}\rho_3 v_L^2 \\
\mathcal{M}^{++\in}_{\infty\in} &= 2\rho_4 v_L v_R \cos(\theta) + \frac{1}{2}\left(\beta_1 k_1 k_2 \cos(\alpha) + \beta_2 k_1^2\left(\cos(\alpha)^2 - \sin(\alpha)^2\right) + \beta_2 k_2^2\right) \\
\mathcal{M}^{++\in}_{\in\in} &= -\mu_3^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 \cos(\alpha) + \rho_1 v_L^2 + 2\rho_2 v_L^2 + \frac{1}{2}\rho_3 v_R^2
\end{aligned} \tag{A.3}
$$

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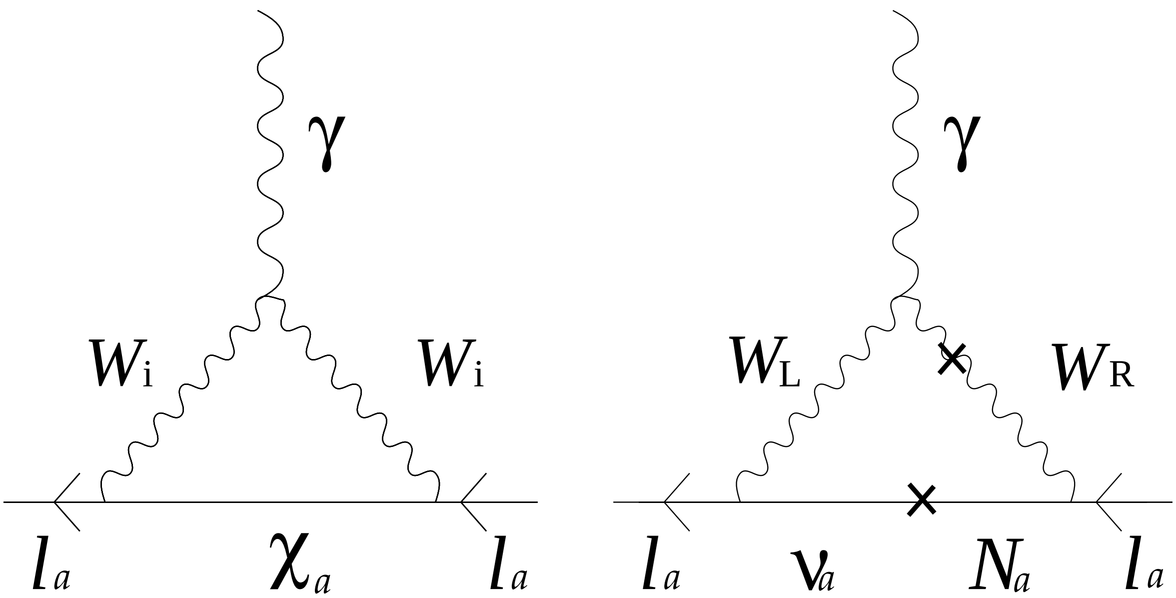
Figure captions:

Figure 1: (a) diagrams in the mass eigenstate basis for the calculation of d_e; (b) same diagrams in terms of the gauge eigenstates, showing the different mass insertions.



(a) (b)