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Electric polarizability of nuclei from a longitudinal sum rule

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Abstract

The nuclear electric polarizability is theoretically analyzed using a sum rule derived from the longitudinal part of the forward Compton amplitude. Beyond the leading dipole contribution, this approach leads to the presence of potential-dependent terms that do not show up in previous analyses. The significance of these new contributions is illustrated by performing an explicit calculation for a proton-neutron system interacting via a separable potential.

I. INTRODUCTION

For many years, the study of the electric and magnetic polarizabilities of hadronic systems has deserved a significant interest towards the understanding of strong interactions. The polarizabilities account for the distortion of a system (induced dipole moments) in the presence of external quasistatic electric and magnetic fields, so that they determine the lowest-order response of the system's internal degrees of freedom to the electromagnetic interactions [1]. In particular, in the case of a real Compton scattering (RCS) process, it is well known from low-energy theorems that the corresponding spin-averaged forward scattering amplitude is determined up to second order in the photon energy by the sum of both the electric and magnetic polarizabilities, $\alpha+\beta$ [2]. One has indeed

$$f(\omega^2) = -\frac{Z^2 e^2}{4\pi M} + (\alpha+\beta)\,\omega^2 + O(\omega^4)\,, \tag{1}$$

where ω is the energy of the (real) photon, and the first term on the right hand side corresponds to the Thomson limit, which depends only on global properties of the system: its mass and its electric charge.

The use of a forward dispersion relation allows to find a connection between the sum $\alpha+\beta$ in (1) and the total photo-absorption cross section for the system, $\sigma_{tot}(\omega)$. This relation can be expressed in the form of a sum rule [3],

$$\alpha+\beta = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty} d\omega\; \frac{\sigma_{tot}(\omega)}{\omega^2}\,, \tag{2}$$

where ω_{th} stands for the threshold inelastic excitation of the system. Thus the sum of the structure-dependent polarizabilities can be estimated by performing experimental measurements of $\sigma_{tot}(\omega)$ up to large enough photon energies.

Unfortunately, this approach cannot be used to determine both parameters α and β independently, as it is clear from the fact that only the sum $\alpha+\beta$ appears in (1). In order to get separate sum rules from RCS processes, it is necessary to take into account also nonforward amplitudes, which in general lead to the introduction of additional difficulties. This is e.g. the case if one considers the backward amplitude, which is related to the difference $\alpha-\beta$; it can be seen that the corresponding dispersion relation needs information not only on the s-channel photo-absorption cross sections, but also on the t-channel two-photon processes [4]. As a consequence, the analysis turns out to be more involved and includes some model dependence.

There is, however, an alternative possibility. As it was shown in Ref. [5] some time ago, it is possible to obtain a sum rule for α that involves the longitudinal part of the *virtual* photon cross section $\sigma_L(\omega,q^2)$ in the region of quasi-real photons. In fact, although the longitudinal cross section itself vanishes in the on-shell photon limit, it is found that the integral of the slope of $\sigma_L(\omega,q^2)$ at $q^2=0$ is directly related to the electric polarizability. The sum rule can be derived by considering an unsubtracted dispersion relation for the longitudinal forward virtual Compton amplitude $T_L(\omega,q^2)$, with the only ingredients of gauge invariance, analiticity and unitarity. The result in Ref. [5] explicitly reads

$$\alpha - (\frac{e^2}{4\pi})\frac{\mu^2}{4M^3} = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty} d\omega\;\lim_{q^2\to 0}\frac{\sigma_L(\omega,q^2)}{-q^2}\,, \tag{3}$$

where μ is the corresponding anomalous magnetic moment. We stress that the presence or not of a subtraction for T_L cannot be inferred from the Thomson limit, therefore Eq. (3) is valid under the assumption that the integrand in the right hand side has an adequate high-energy behaviour.

In this paper, we make use of the above longitudinal sum rule to derive a theoretical expression for the electric polarizability of nuclear systems. In order to deal with the nuclear photo-absorption cross section, we proceed by performing a conventional nonrelativistic treatment of the nuclear interactions, keeping only leading terms in powers of the inverse mass of the system. In this regard, however, there is an essential difference with respect to previous approaches: since our starting point for the evaluation of the electric polarizability is relation (3), the nonrelativistic approximations are introduced here only *after* having taken into account the properties of gauge invariance, analyticity and unitarity of the S matrix, which have already been invoked to obtain the sum rule. In contrast, these properties are not exactly satisfied from the very beginning when the nonrelativistic limit is taken directly on the real part of the Compton amplitude. This last procedure has been followed e.g. in Refs. [6,7]. We find that our treatment allows to improve the result for α obtained in these previous analyses, leading to the presence of additional contributions that depend on the nuclear interaction. To evaluate the significance of these contributions, we consider the simple case of a proton-neutron system interacting via a separable Yamaguchi potential.

The calculation of the electric polarizability from the sum rule (3) is performed in Section II. In Section III we present the application to the Yamaguchi-like interacting system, while Section IV contains our conclusions.

II. NUCLEAR ELECTRIC POLARIZABILITY

Starting from the sum rule in Eq. (3), we derive in this section the leading contributions to the nuclear electric polarizability, up to first order in the inverse nuclear mass M_A^{-1}. To this end, it is convenient to write the integrand on the right hand side of (3) in terms of the so-called longitudinal response, which is measured in many nuclei by means of inelastic electron scattering. This function is defined as

$$R_L(|\mathbf{q}|,\omega) = \sum_{n>0} |\langle n|\rho(\mathbf{q})|0\rangle|^2 \, \delta(\omega - \omega_{rec} - E_n + E_0)\,, \tag{4}$$

where $\rho(\mathbf{q})$ represents the charge operator, $\omega_{rec} = |\mathbf{q}|^2/(2M_A)$ is the recoil energy of the nuclear system, and E_0 ($|0\rangle$) and E_n ($|n\rangle$) are the eigenvalues (eigenstates) of the nuclear Hamiltonian corresponding to the ground and excited states, respectively. For a spinless nucleus, it is easy to see that Eq. (3) can be written as

$$\alpha = \frac{1}{2\pi}\int_{\omega_{th}}^{\infty} d\omega \;\; \frac{R_L(|\mathbf{q}|,\omega)}{\omega\,|\mathbf{q}|^2}\Bigg|_{|\mathbf{q}|\to\omega} . \tag{5}$$

From the analysis of electron scattering, it is seen that the longitudinal response displays a strong collectivity at small momenta, whereas for large momenta a single particle character is observed. In particular, it has been shown [8] that the proton-neutron dynamical correlations generate a high-energy tail that can be relevant for the Coulomb sum rule. We notice that, although this effect could also be significant in the quasi-real photon limit, the high-energy contributions to the longitudinal sum rule for α will be in general suppressed in view of the inverse powers of ω entering the integrand in Eq. (5).

Let us work out the right hand side of (5). We begin by considering the charge operator in (4), which at lowest relativistic order is given by

$$\rho(\mathbf{q}) = e \sum_{j=1}^{Z} \exp\left(i\mathbf{q}\cdot\mathbf{r}_j\right), \tag{6}$$

where Z is the number of protons and $\mathbf{r}_j$ are their coordinates with respect to the nuclear center of mass. Then, performing an expansion in powers of $|\mathbf{q}|$, one has

$$\begin{aligned}\alpha = \frac{e^2}{2\pi}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega}\lim_{|\mathbf{q}|\to\omega}\Bigg\{\delta\left(\omega - \frac{|\mathbf{q}|^2}{2M_A} - E_n + E_0\right)\sum_{n>0}\Bigg[|\langle 0|C_1|n\rangle|^2 \\ + \frac{|\mathbf{q}|^2}{4}\left(|\langle 0|C_2|n\rangle|^2 - \frac{4}{3}\langle 0|C_1|n\rangle\langle n|C_3|0\rangle\right) + O(|\mathbf{q}|^4)\Bigg]\Bigg\},\end{aligned} \tag{7}$$

where $C_n \equiv \sum_{i=1}^{Z} z_i^n$. In this expression, it is worth to notice the presence of the electric dipole operator C_1 at leading order in $|\mathbf{q}|$, while higher multipoles are also relevant to the next-to-leading order (their importance will be seen later). Taking explicitly the limit in the right hand side, and performing the integration in ω, the expression for α reads

$$\begin{aligned}\alpha = \frac{e^2}{2\pi}\sum_{n>0}\left(1-\frac{\epsilon_n}{M_A}\right)^{-1}\Bigg[\epsilon_n^{-1}|\langle 0|C_1|n\rangle|^2 \\ + \frac{\epsilon_n}{4}\left(|\langle 0|C_2|n\rangle|^2 - \frac{4}{3}\langle 0|C_1|n\rangle\langle n|C_3|0\rangle\right) + O(\epsilon_n^2)\Bigg],\end{aligned} \tag{8}$$

where $\epsilon_n = M_A\left(1-\sqrt{1-2\left(E_n-E_0\right)/M_A}\right)$. Finally, expanding up to first order in $(E_n - E_0)/M_A$, one gets

$$\begin{aligned}\alpha = \frac{e^2}{2\pi}\sum_{n>0}\Bigg[\frac{|\langle 0|C_1|n\rangle|^2}{E_n-E_0} + \frac{1}{2M_A}|\langle 0|C_1|n\rangle|^2 \\ + \frac{1}{4}(E_n-E_0)\left(|\langle 0|C_2|n\rangle|^2 - \frac{4}{3}\langle 0|C_1|n\rangle\langle n|C_3|0\rangle\right) \\ + \frac{3}{8M_A}(E_n-E_0)^2\left(|\langle 0|C_2|n\rangle|^2 - \frac{4}{3}\langle 0|C_1|n\rangle\langle n|C_3|0\rangle\right) + O(M_A^{-2})\Bigg].\end{aligned} \tag{9}$$

Eq. (9) gives α in terms of different energy-weighted sums involving nuclear matrix elements of the C_n operators ($n = 1, 2, 3$). It is also possible to express this result by means of sum rules, i.e. of average values of commutators and anticommutators of the C_n operators and the total nuclear Hamiltonian [9]. In order to do this, let us first rewrite the nuclear polarizability in terms of the moments m_p of the longitudinal response function, defined by

$$m_p(|\mathbf{q}|) \equiv \int_{0^+}^{\infty} d\tilde{\omega}\,(\tilde{\omega})^p\,R_L(|\mathbf{q}|,\omega), \tag{10}$$

where $\tilde{\omega} = \omega - |\mathbf{q}|^2/(2M_A)$. It can be easily verified that the following relations between the derivatives of the moments and the energy-weighted sums hold:

$$m'_{-1}(0) \equiv \left. \frac{dm_{-1}(|\mathbf{q}|)}{d|\mathbf{q}|^2} \right|_{|\mathbf{q}|^2=0} = \sum_{n>0} \frac{|\langle 0|C_1|n\rangle|^2}{E_n - E_0} \tag{11a}$$

$$m'_0(0) \equiv \left. \frac{dm_0(|\mathbf{q}|)}{d|\mathbf{q}|^2} \right|_{|\mathbf{q}|^2=0} = \sum_n |\langle 0|C_1|n\rangle|^2 \tag{11b}$$

$$m''_1(0) \equiv \left. \frac{d^2m_1(|\mathbf{q}|)}{(d|\mathbf{q}|^2)^2} \right|_{|\mathbf{q}|^2=0} = \frac{1}{2}\sum_n (E_n - E_0) \left(|\langle 0|C_2|n\rangle|^2 - \frac{4}{3}\langle 0|C_1|n\rangle\langle n|C_3|0\rangle \right) \tag{11c}$$

$$m''_2(0) \equiv \left. \frac{d^2m_2(|\mathbf{q}|)}{(d|\mathbf{q}|^2)^2} \right|_{|\mathbf{q}|^2=0} = \frac{1}{2}\sum_n (E_n - E_0)^2 \left(|\langle 0|C_2|n\rangle|^2 - \frac{4}{3}\langle 0|C_1|n\rangle\langle n|C_3|0\rangle \right) \tag{11d}$$

thus α can be written as

$$\alpha = \left(\frac{e^2}{4\pi}\right) \left[2\, m'_{-1}(0) + \frac{1}{M_A} m'_0(0) + m''_1(0) + \frac{3}{2M_A} m''_2(0) + O\left(M_A^{-2}\right) \right] . \tag{12}$$

Now, by using the closure property, it can be shown that the derivatives of the nonnegative moments in Eqs. (11) satisfy the following sum rules [9]:

$$m'_0(0) = \frac{1}{2}\langle 0|\{C_1, C_1\}|0\rangle \tag{13a}$$

$$m''_1(0) = \frac{1}{4}\langle 0|\, [C_2, [H, C_2]]\, |0\rangle - \frac{1}{3}\langle 0|\, [C_1, [H, C_3]]\, |0\rangle \tag{13b}$$

$$m''_2(0) = \frac{1}{4}\langle 0|\{[C_2, H], [H, C_2]\}|0\rangle - \frac{1}{3}\langle 0|\{[C_1, H], [H, C_3]\}|0\rangle\, . \tag{13c}$$

Moreover, some parts of the commutators and anticommutators can be explicitly evaluated; in particular, the double commutators containing the kinetic part T of the Hamiltonian ($H = T + V$) give rise to terms proportional to the proton radius, leading to

$$\langle 0|\, [C_2, [H, C_2]]\, |0\rangle = \frac{4}{m}\left(\frac{1}{3}\langle 0| \sum_{i=1}^{Z} \mathbf{r}_i^2 |0\rangle - \frac{1}{A}\langle 0|C_1\, C_1|0\rangle \right) + \langle 0|\, [C_2, [V, C_2]]\, |0\rangle \tag{14a}$$

$$\langle 0|\, [C_1, [H, C_3]]\, |0\rangle = \frac{1}{m}\left(1 - \frac{Z}{A}\right) \langle 0| \sum_{i=1}^{Z} \mathbf{r}_i^2 |0\rangle + \langle 0|\, [C_1, [V, C_3]]\, |0\rangle\, , \tag{14b}$$

where we have approximated $M_A \simeq m\, A$, being m the nucleon mass. By making use of relations (11-14), the nuclear electric polarizability can finally be written as

$$\alpha = \left(\frac{e^2}{4\pi}\right) \left[2\sum_{n>0} \frac{|\langle 0|C_1|n\rangle|^2}{E_n - E_0} + \frac{Z}{3M_A}\langle 0| \sum_{i=1}^{Z} \mathbf{r}_i^2 |0\rangle + \frac{1}{4}\langle 0|[C_2, [V, C_2]]|0\rangle - \frac{1}{3}\langle 0|[C_1, [V, C_3]]|0\rangle \right.$$
$$\left. + \frac{3}{8M_A}\langle 0|\{[C_2, V], [V, C_2]\}|0\rangle - \frac{1}{2M_A}\langle 0|\{[C_1, V], [V, C_3]\}|0\rangle + O(M_A^{-2}) \right] . \tag{15}$$

Eq. (15) represents the main result of this work. It is seen that, beyond the leading dipole term, there is a contribution to α proportional to $\langle \mathbf{r}^2 \rangle$, plus potential-dependent terms of order V and V^2/M_A. Let us remark that previous nonrelativistic analyses [6,7] lead only to the first two terms in (15). The significance of the potential-dependent contributions obtained from our calculation will be illustrated in the next section, where we consider the case of a simple solvable proton-neutron system. The study of Eq. (15) for realistic nuclear potentials leading to exchange-current contributions will be the subject of analysis in future work.

As a final comment, let us point out that the above result includes both the contributions coming from collective nuclear effects and those arising from the polarizability of the individual nucleons inside the nucleus. In fact, it can be seen that the photoabsorption cross section appears to be dominated by the nucleonic excitations for energies above the pion mass. Below this threshold, there is a region where the cross section still shows a volume character, which can be described in terms of the interaction of the photon with quasi-deuteron systems. Thus, for these energies, the contribution to the polarizability can also be understood as a nucleonic one, being the nucleons modified by the surrounding medium [10]. Within our formulation, the identification of the volume contributions is not trivial, since the result in Eq. (15) is obtained after performing an integration over the whole spectrum. We recall, however, that the sum rule for α considered here weights the longitudinal response inversely with ω, hence the effects coming from the high energy region are in general expected to be suppressed.

III. PROTON–NEUTRON SYSTEM WITH A YAMAGUCHI POTENTIAL

As an application of the above result, let us evaluate the contributions to α for a simple case, namely a proton-neutron system interacting via a separable potential

$$V = \lambda \, |g\rangle\langle g| \, , \qquad \lambda < 0 \, . \tag{16}$$

Denoting by $\psi_0(\mathbf{p})$ and $\psi_{\mathbf{k}}(\mathbf{p})$ the bound state wave function and the scattering solution for this potential respectively, the longitudinal structure function R_L will be given by

$$R_L(|\mathbf{q}|, \omega) = \int d^3k \; \delta(\omega - \frac{\mathbf{k}^2}{2M} - \epsilon_B) \left| \int d^3p \; \psi_{\mathbf{k}}(\mathbf{p}) \, \psi_0(\mathbf{p} - \mathbf{q}) \right|^2 \, , \tag{17}$$

where M is the total mass of the system and ϵ_B stands for the binding energy corresponding to the ψ_0 state. We consider for simplicity the particular case of a Yamaguchi potential [11],

$$\langle \mathbf{p}|g\rangle = \frac{\sqrt{\beta}}{\pi} \frac{1}{p^2 + \beta^2} \, , \qquad \epsilon_B = \frac{\beta^2}{2M} \, , \tag{18}$$

which allows to perform the integrals in (17) analytically. After some algebra, it is found [12] that the longitudinal structure function can be written in terms of the variables $x \equiv \sqrt{\omega/\epsilon_B - 1}$ and $y \equiv |\mathbf{q}|/\beta$ as

$$R_L(y, x) = \frac{16}{\pi \epsilon_B} \frac{x}{D^3} \left[(1 + x^2 + y^2)^2 + \frac{4}{3} x^2 y^2 - D \, F_0(y) \left(1 + \frac{y^2}{2} - \frac{y^4}{2(1 + x^2)} \right) \right] \, , \tag{19}$$

where $D \equiv (1 - x^2 + y^2)^2 + 4x^2$, and $F_0(y)$ is the elastic form factor,

$$F_0(y) = \frac{1}{Z}\langle\psi_0|\rho(\mathbf{q})|\psi_0\rangle = \left(1 + \frac{y^2}{4}\right)^{-2} . \tag{20}$$

In the case under consideration, the integrals corresponding to the moments in (11) are convergent, so that the terms in the right hand side of (12) can be evaluated using the expression for R_L in (19). Considering as before the terms in α up to order M^{-1}, we have

$$\alpha = \left(\frac{e^2}{4\pi}\right)\left[\frac{7}{12M\epsilon_B^2} + \frac{1}{2M^2\epsilon_B} - \frac{3}{16M^2\epsilon_B} + \frac{9}{8M^3}\right] , \tag{21}$$

where the four terms correspond to those in the right hand side of Eq. (12), respectively. Now, it is instructive to compare this expression for α with the main result in Eq. (15), in order to identify the contribution of the potential-dependent terms for this simple case. From (13c), we see that the V^2 part in (15) corresponds to the $m''_2(0)$ contribution, which yields the M^{-3} term in (21). This is consistent with the nonrelativistic approximation, which assumes implicitly that the nuclear interactions are relatively "weak", and both the potential and kinetic energies should be considered to be order $(1/M)$ [7]. Furthermore, it is found that the remaining potential contributions to (15) correspond to the $m''_1(0)$ term. In fact, in the $Z = 1$ case the kinetic contributions cancel out, and we end up with

$$\frac{1}{4}\langle 0|[C_2, [V, C_2]]|0\rangle - \frac{1}{3}\langle 0|[C_1, [V, C_3]]|0\rangle = m''_1(0) = -\frac{3}{16M^2\epsilon_B} \tag{22}$$

Finally, we see that the contributions of $m'_{-1}(0)$ and $m'_0(0)$ (first two terms in (21)) correspond to the first two terms in (15), respectively. Then we conclude, at least for this model, that *the lowest order potential-dependent contribution to α (i.e. that in (22)) has the same order of magnitude as the $\langle\mathbf{r}^2\rangle$ term.* This is once again consistent with treating the potential as order $1/M$. On the other hand, we recall that previous nonrelativistic calculations for α, such as those in Refs. [6,7] led only to the $\langle\mathbf{r}^2\rangle$ correction. In this simple example we find that the additional potential-dependent terms provided by the longitudinal sum rule approach contribute significantly to the electric polarizability and therefore should also be taken into account.

IV. CONCLUSIONS

We have presented here a novel approach to analyze the electric polarizability of nuclear systems. The distinctive feature of our analysis is the use of the sum rule displayed in Eq. (3), which arises from the assumption of an unsubtracted dispersion relation for the longitudinal forward virtual Compton amplitude. This sum rule involves the virtual photo-absorption cross section of the system in the region of quasi-real photons.

We have calculated the leading terms contributing to the electric polarizability for a spinless nucleus, up to first order in the inverse nuclear mass. To do this, we have assumed that the photo-absorption longitudinal cross section shows an adequate high-energy behaviour, so that the treatment of the nucleus as a nonrelativistic system is consistent with the use of the sum rule.

The main result of our calculation is shown in Eq. (15). As expected, it is found that the lowest order contribution to α is given by the inverse energy-weighted sum of the strengths of inelastic dipole excitations. Beyond this leading order, we obtain a contribution proportional to $\langle \mathbf{r}^2 \rangle$, plus potential-dependent terms that appear to be also significant. In particular, if the kinetic and potential energies are both considered to be order M^{-1} (which is consistent with the nonrelativistic approximation), the terms proportional to V in (15) are shown to be of the same order of magnitude as the $\langle \mathbf{r}^2 \rangle$ one. This is e.g. the case for a proton-neutron system interacting through a separable Yamaguchi potential. As stated above, the presence of the $\langle \mathbf{r}^2 \rangle$ contribution at the order M^{-1} had been derived many years ago by explicit nonrelativistic calculations of the Compton amplitude for a bound system. The origin of the new potential-dependent corrections in our analysis can be traced back to the properties of gauge invariance, analiticity and unitarity of the S matrix, which are implicitly taken into account once the sum rule for α has been invoked.

It is amazing that the requirement of completely general conditions on the virtual Compton scattering amplitude automatically leads to the presence of exchange current contributions to the nuclear polarizability. The analysis of such contributions for realistic nuclear potentials deserves a detailed study that will be the subject of future work.

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