An effective field theory approach to the electroweak corrections at LEP energies

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Abstract

In the framework of the effective field theory (EFT) we discuss the electroweak (EW) corrections at LEP energies. We obtain the effective Lagrangian in the large m_t limit, and reproduce analytically the dominant EW corrections to the LEP2 processes $e^+e^- \to \gamma Z$ and $e^+e^- \to ZZ$. To include effects of finite top–quark and Higgs masses, we use the effective Lagrangian at tree level and fit LEP1/SLD observables with four arbitrary parameters, plus $\alpha_s(m_Z)$. The EFT approach works remarkably well. Using the effective couplings determined from the fit, and tree–level EFT formulae, we predict the cross sections for $e^+e^- \to ZZ, \gamma Z$ at a level better than 1%.

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I. INTRODUCTION

The radiative corrections play a very important role in the analysis of the Standard Model (SM) predictions [1,2]. The knowledge of radiative corrections up to definite order for different processes is necessary to perform accurate tests of the SM, allowing to probe the quantum structure of the theory and also to search for possible effects of new physics [3]. The precision achieved in these tests has been significantly increased in the last years, in view of the experimental information provided by the e^+e^- colliders LEP and SLC and the $\bar{p}p$ collider Tevatron [4], and the theoretical computation of the SM predictions including radiative corrections beyond the level of one loop [3,5,6]. In fact, the calculation of higher–order radiative corrections has reached an extremely complicated level and heavily relies

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now on computer facilities [7]. For the main processes measured at LEP1, the final results are presented through computer programs [8].

The main goal of this paper is to show how the effective field theory (EFT) [9–12] can help in the estimation of the electroweak (EW) corrections at LEP2 energies by using the precise measurements of LEP1 and SLD. Indeed, the standard approach to EW radiative corrections in the SM requires firstly the evaluation of those corrections for LEP1/SLD observables, in order to extract the relevant SM parameters from the available experimental data. It is seen that the extracted values depend strongly on the top–quark mass, m_t, and (to a lesser extent) on the Higgs mass, m_H. Then, in terms of these parameters, one can calculate the radiative corrections to LEP2 observables. As expected, the results also show a strong dependence on m_t and m_H. However, this is cancelled almost completely by the m_t and m_H dependences of the input parameters extracted from LEP1/SLD. This is not surprising, since for both LEP1 and LEP2 energies top quarks and Higgs bosons are always virtual. It is then conceivable that a description in terms of an effective theory without explicit top quarks or Higgs bosons is good enough for both LEP1 and LEP2. All top quark and Higgs–boson mass dependences will be absorbed in the effective Lagrangian parameters, which can be determined at LEP1/SLD and then used to make predictions for LEP2 that will be trivially independent on m_t and m_H. We will show that this program can be carried out basically at tree level, achieving precisions for LEP2 predictions at the % level, which should be enough for most purposes.

We will focus on the neutral gauge boson production at LEP2,

$$e^+e^- \to \gamma Z \tag{1}$$

and

$$e^+e^- \to ZZ \ . \tag{2}$$

The process (1), so–called "Z radiative return", has already been observed at LEP2 with a hard photon and an on–shell Z in the final state [13] and it is expected that about 10000 γZ events will be collected until the end of LEP2. This process is the main source of the well–known Initial State Radiation in e^+e^- collisions [14]. The process (2) is similar to (1), with a lower cross section [15]. Both processes (1) and (2) are interesting for seeking for possible nonstandard effects in LEP2, such as the presence of anomalous three–gauge boson couplings [16].

The complete one–loop EW corrections in the SM for (1) and (2) have been calculated some years ago by Böhm and Sack [17] and by Denner and Sack [18] respectively, using an on–shell renormalization scheme [19] in the 't Hooft–Feynman gauge. For both processes (1) and (2), the differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left(1 + \delta^{\mathrm{QED}} + \delta^{\mathrm{EW}}\right), \tag{3}$$

where $(d\sigma/d\Omega)_0$ is the corresponding cross section in the Born approximation. For convenience, the one–loop corrections have been split in two: δ^{QED} contains the "pure" QED —or photonic— contributions, while δ^{EW} includes the remaining, non–QED EW corrections.

The pure QED correction δ^{QED} for neutral gauge boson production is gauge invariant and depends on the experimental conditions. To get an analytical result, it is possible to use the soft–photon approximation for the Bremsstrahlung of an additional photon with energy $\omega \lesssim \Delta E$, being ΔE some energy cutoff. In this way δ^{QED} has been calculated for the processes (1) and (2) in [20] and [18] respectively. It is seen that, with a cutoff $\Delta E = 0.1\sqrt{s}$, δ^{QED} can reach up to $\sim -10\%$ for process (1) and $\sim -17\%$ for process (2) at LEP2 energies. In this paper we will concentrate in the analysis of the non–QED corrections, δ^{EW}. For a full calculation of the pure one–loop QED contributions we refer the reader to the articles in Refs. [20] and [18].

The analytical results for δ^{EW} obtained in [17] and [18] involve huge formulae. We will show that within the framework of our EFT, the analysis of the EW corrections for these processes can be carried out with good accuracy in a very simple way. Our approach just requires the knowledge of tree–level expressions for the corresponding cross sections, plus the introduction of a few input parameters, which can be fitted from existing experimental data for LEP1 and SLD observables. The quality of the approach can be tested by comparing our results with the full one–loop calculations for δ^{EW} mentioned above.

The paper is organized as follows: in section II we briefly discuss how to integrate the top quark to obtain an effective Lagrangian that can be used at LEP energies, and give explicit formulae for the effective couplings in the limit of large m_t. In section III we use the previously obtained effective Lagrangian to calculate, in the large m_t limit, the dominant radiative corrections to $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$. We show that this simple tree–level analytical calculation in EFT is able to reproduce, in this limit, the results obtained through a full one–loop calculation in the SM. In sections IV and V we go beyond the large m_t limit, assuming that our effective Lagrangian is valid for both LEP1 and LEP2 energies. In section IV we consider different Z–pole observables measured at LEP1 and SLD, and use tree–level formulae (plus standard QCD and QED corrections) expressed in terms of the effective Lagrangian couplings to get the corresponding theoretical predictions. Then we use the experimental results of LEP1 and SLD to fit the parameters of the Lagrangian. Finally the same Lagrangian is used in section V to give predictions for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2 energies using again tree–level formulae. We expect to include in this way all the leading EW corrections to the observables studied. This is checked by comparing our results with full one–loop calculations. To conclude, in section VI we collect the main results of this paper.

II. AN EFFECTIVE EW LAGRANGIAN FOR LEP ENERGIES

The effective Lagrangian for $\mu \leq m_t$ is obtained by integrating the top quark at $\mu = m_t$. At one–loop level this is done by computing all diagrams containing at least one top quark. In the case of the kinetic terms of the gauge bosons it is enough to compute a few gauge boson self–energies. After a trivial field redefinition one obtains (for details see [11])

$$\mathcal{L}_{\text{eff}} = W_\mu^+ \partial^2 W^{-\mu} + \frac{g_+^2(\mu)}{4}\left(v^2 + \delta v_+^2(\mu)\right)W_\mu^+ W^{-\mu} + \frac{1}{2}W_\mu^3 \partial^2 W^{3\mu} + \frac{1}{2}B_\mu \partial^2 B^\mu$$

$$+ \frac{1}{2}\left(g_3(\mu)W_3^\mu - g'(\mu)B^\mu\right)\left[\frac{1}{4}\left(v^2 + \delta v_3^2(\mu)\right) - \delta Z_{3Y}(\mu)\partial^2\right]\left(g_3(\mu)W_{3\mu} - g'(\mu)B_\mu\right)$$

$$+ \ \bar{\psi} \ i\not{D}(\ g_+(\mu)W^+, \ g_3(\mu)W_3, \ g'(\mu)B) \ \psi$$

$$+ i\bar{b}\not{\partial}b + \frac{1}{2} \left(g_3(\mu)W_3^\mu - g'(\mu)B^\mu \right) (1 + \epsilon_b(\mu))\overline{b_L}\gamma_\mu b_L + \frac{1}{3}g'(\mu)B^\mu\bar{b}\gamma_\mu b \ . \tag{4}$$

Here quark mixing has been neglected and ψ stands for all the fermions but the bottom and top quarks. Since the top quark has been integrated out, there are no charged current couplings for the bottom. In addition, the standard neutral couplings of the bottom quark get further modified due to both vertex and wave function corrections to b_L. The contributions to the b_L self–energy have been absorbed in (4) by a redefinition of the b_L field, whereas the remaining corrections are collected in $\epsilon_b(\mu)$. The mixing between the W_3 and B wave functions has been treated by including a ∂^2 operator in the form of a "mass term" to make simpler the subsequent diagonalization. The covariant derivative $\not{D}$ in Eq. (4) is just a simplified notation to refer to the standard gauge interactions to the fermions, but with the renormalized couplings $g_+(\mu)$, $g_3(\mu)$ and $g'(\mu)$.

Higher dimensional operators suppressed by the corresponding inverse powers of the top–quark mass, as well as other operators not relevant for the discussion in this paper —e.g. four fermion operators involving the bottom quark— have not been included. Triangle diagram contributions to gauge boson interactions depending on the top–quark mass are small and have also been neglected. In addition, in (4) we have not included trilinear couplings of gauge bosons; these will be important for some processes such as W boson production at LEP2.

The redefinition of fields also leads to a redefinition of coupling constants. The initially unique coupling constant g splits into g_+ and g_3 below the top–quark mass scale [10]. At one loop one obtains $g_+^2(\mu) \simeq g^2 \left(1 - g^2\delta Z_+(\mu)\right)$, $g_3^2(\mu) \simeq g^2 \left(1 - g^2\delta Z_3(\mu) - g^2\delta Z_{3Y}(\mu)\right)$ and $g'^2(\mu) \simeq g'^2 \left(1 - g'^2\delta Z_Y(\mu) - g'^2\delta Z_{3Y}(\mu)\right)$, where $g^2\delta Z_+(\mu)$, $g^2\delta Z_3(\mu)$, $g'^2\delta Z_Y(\mu)$ are the top–quark induced wave function renormalizations of the W^+, W_3 and B gauge bosons respectively while $g'g\delta Z_{3Y}(\mu)$ is the top–quark induced $W_3 - B$ wave–function mixing. Similarly $v_+^2(\mu) \equiv v^2 + \delta v_+^2(\mu)$ and $v_3^2(\mu) \equiv v^2 + \delta v_3^2(\mu)$ are also different.

In order to obtain the effective Lagrangian at LEP scales $\mu \simeq m_Z$ it is necessary to perform the matching of the effective theory to the full theory at scales $\mu = m_t$, and then to scale down the effective Lagrangian, using the renormalization group equations, for each of the "couplings" $g_+(\mu), g_3(\mu), g'(\mu), \delta v_+^2(\mu), \delta v_3^2(\mu), \delta Z_{3Y}(\mu)$ and $\epsilon_b(\mu)$. In the limit of large m_t one obtains [11]

$$\frac{v_+^2(m_Z)}{v_3^2(m_Z)} \simeq \frac{v_+^2(m_t)}{v_3^2(m_t)} \simeq 1 + \frac{3}{(4\pi)^2}\frac{m_t^2}{v^2} \tag{5a}$$

$$\frac{g_+^2(m_Z)}{g_3^2(m_Z)} \simeq 1 + \frac{2g^2}{3(4\pi)^2} \log\left(\frac{m_t}{m_Z}\right) \tag{5b}$$

$$\delta Z_{3Y}(m_Z) \simeq -\frac{1}{3(4\pi)^2} \log\left(\frac{m_t}{m_Z}\right) \tag{5c}$$

$$\epsilon_b(m_Z) \simeq -2\frac{m_t^2}{(4\pi)^2 v^2} - \frac{1}{(4\pi)^2}\left(\frac{17}{6}g^2 + \frac{1}{6}g'^2\right) \log\left(\frac{m_t}{m_Z}\right) \ . \tag{5d}$$

One observes here the leading non–decoupling top mass effects, appearing both in the universal self–energy coupling [21] and in the specific vertex to b quarks [22]. QCD correc-

tions to the parameters in (5) can be easily included, if needed [11,12] (for QCD corrections to electroweak parameters in the large m_t limit see also [23] and references therein). Finally, to get the effective Lagrangian at the m_Z scale one still has to diagonalize the neutral gauge boson sector, including a further wave function renormalization of the Z field to absorb the δZ_{3Y} term (i.e. the mixing between the W_3 and B wave functions). The effective Lagrangian reads

$$
\mathcal{L}_{\text{eff}} = W^+_\mu \partial^2 W^{-\mu} + m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} A_\mu \partial^2 A^\mu + \frac{1}{2} Z_\mu \partial^2 Z^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu
$$
$$
+ \ \bar{\psi} \ i \slashed{D} \left(\frac{e_W(m_Z)}{s_Z} W^+, \ \frac{e_Z(m_Z)}{s_Z c_Z} Z, \ e(m_Z) A \right) \psi
$$
$$
+ i \bar{b} \slashed{\partial} b - \frac{e_Z(m_Z)}{2 s_Z c_Z} \bar{b} \gamma_\mu \left(g_V^b - g_A^b \gamma_5 \right) b Z^\mu + \frac{1}{3} e(m_Z) \bar{b} \gamma_\mu b A^\mu \ , \tag{6}
$$

where $c_Z \equiv \cos\theta_W(m_Z)$, $s_Z \equiv \sin\theta_W(m_Z)$, are the cosine and the sine, respectively, of the effective weak mixing angle at the scale m_Z. As usual, $\theta_W(m_Z)$ is determined by the diagonalization of the mass matrix for the neutral gauge bosons. It is trivially related to the gauge couplings by $\tan\theta_W(m_Z) \equiv g'(m_Z)/g_3(m_Z)$. In the same way, $e(m_Z) = g_3(m_Z) s_Z$ is the electromagnetic coupling at the scale m_Z. As commented above, the Z field needs a further rescaling owing to the δZ_{3Y} term in (4). This leads us to define $e_Z(m_Z)$, which appears in all Z couplings and which is related to $e(m_Z)$ through

$$
e_Z^2(m_Z) = e^2(m_Z) \left(1 - \frac{g^2}{c_Z^2} \delta Z_{3Y}(m_Z) \right) \ . \tag{7}
$$

Similarly, we found it convenient to express the coupling of the W^+ gauge bosons, $g_+(m_Z)$, in terms of an effective coupling $e_W(m_Z) \equiv g_+(m_Z) s_Z$. From this definition and Eq. (5b) we get

$$
e_W^2(m_Z) = e^2(m_Z) \frac{g_+^2(m_Z)}{g_3^2(m_Z)} \ . \tag{8}
$$

As it is usually done for the electromagnetic coupling, we can define

$$
\alpha(m_Z) \equiv \frac{e^2(m_Z)}{4\pi} \equiv \frac{\alpha}{1 - \Delta\alpha} \ , \tag{9}
$$
$$
\alpha_Z(m_Z) \equiv \frac{e_Z^2(m_Z)}{4\pi} \equiv \alpha(m_Z) \left(1 + \delta\alpha_Z \right) \ , \tag{10}
$$
$$
\alpha_W(m_Z) \equiv \frac{e_W^2(m_Z)}{4\pi} \equiv \alpha(m_Z) \left(1 + \delta\alpha_W \right) \ . \tag{11}
$$

Here $\alpha = 1/137.036$ is the fine structure constant and $\Delta\alpha$ is the QED shift produced by the running from its on–shell value to $\mu = m_Z$. It can be obtained from $e^+e^- \to hadrons$ data. If $\alpha(m_Z)$ is given in the $\overline{\text{MS}}$ scheme one obtains $\Delta\alpha = 0.067$ [24,1]. $\delta\alpha_Z$ and $\delta\alpha_W$ represent the additional shifts in α_Z and α_W due, in part, to the heavy top. From (5b), (5c), (7) and (8) we obtain

$$\delta\alpha_Z \simeq \frac{\alpha}{12\pi s_Z^2 c_Z^2} \log\left(\frac{m_t}{m_Z}\right) , \tag{12}$$

$$\delta\alpha_W \simeq \frac{\alpha}{12\pi s_Z^2} \log\left(\frac{m_t}{m_Z}\right) . \tag{13}$$

Given the size of these corrections, we expect the three couplings $\alpha(m_Z)$, $\alpha_Z(m_Z)$, and $\alpha_W(m_Z)$ to be almost equal, at least, at the percent level. Their values can be extracted directly from experiment and we will see that, indeed, that is the case. Therefore, if only precisions at the percent level are needed, one can safely assume $\alpha(m_Z) = \alpha_Z(m_Z) = \alpha_W(m_Z)$.

The physical W and Z masses are given, in the large m_t limit, by the equations

$$m_W^2 = \frac{e_W^2(m_Z)}{4s_Z^2} v_+^2(m_Z)$$

$$m_Z^2 = \frac{e_Z^2(m_Z)}{4c_Z^2 s_Z^2} v_3^2(m_Z) . \tag{14}$$

Then, we can obtain the cosine of the Sirlin weak mixing angle [25] in terms of the cosine of the effective mixing angle at the scale m_Z:

$$c_W^2 \equiv \frac{m_W^2}{m_Z^2} = c_Z^2 \frac{e_W^2(m_Z)}{e_Z^2(m_Z)} \frac{v_+^2(m_Z)}{v_3^2(m_Z)} . \tag{15}$$

If we write now the relation between these two mixing angles as $s_Z^2 = s_W^2 + \delta s_W^2$, from equations (5a–5c), (7) and (8) we immediately obtain

$$\delta s_W^2 = \frac{\alpha}{\pi} \left[\frac{3}{16s_Z^2} \frac{m_t^2}{m_Z^2} + \frac{(3 - 2s_Z^2)}{12s_Z^2} \log\left(\frac{m_t}{m_Z}\right) \right] . \tag{16}$$

On the other hand, the coupling of the bottom quark to the Z boson gets extra contributions due to the vertex diagrams involving the top quark. These contributions can be taken into account by parametrizing the effective g_V^b and g_A^b couplings as

$$g_V^b = -\frac{1}{2}\left(1 + \epsilon_b(m_Z)\right) + \frac{2}{3}s_Z^2 , \qquad g_A^b = -\frac{1}{2}\left(1 + \epsilon_b(m_Z)\right) , \tag{17}$$

where $\epsilon_b(m_Z)$ is given in Eq. (5d). Finally, although it is not relevant for the neutral current processes we want to study, one can also relate m_W and s_Z with the Fermi coupling constant, G_F, measured in the muon decay [6]. This relation reads

$$\frac{G_F}{\sqrt{2}} \simeq \frac{e_W^2(m_Z)}{8m_W^2 s_Z^2} , \tag{18}$$

and allows to estimate $\alpha_W(m_Z)$ from s_Z, m_W and G_F.

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III. $e^+e^- \to ZZ$ AND $e^+e^- \to \gamma Z$ IN THE LARGE m_t LIMIT

We can use now the Lagrangian (6) at tree level, together with the results in the previous section, to estimate the dominant electroweak corrections to $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2 in the large m_t limit.

From the Lagrangian (6) and the diagrams in Fig. 1 we easily obtain the cross section for $e^+e^- \to ZZ$. It is the usual tree–level result obtained in the SM, but expressed in terms of the effective couplings $\alpha_Z(m_Z)$ and s_Z:

$$
\left(\frac{d\sigma^{ZZ}}{d\Omega}\right)_{\text{eff}} = \frac{\alpha_Z^2(m_Z)}{32 s_Z^4 c_Z^4} \left(g_V^4 + 6 g_V^2 g_A^2 + g_A^4\right)
$$

$$
\times \frac{1}{s}\left(1 - \frac{4m_Z^2}{s}\right)^{\frac{1}{2}} \left[\frac{s^2 + 6m_Z^4}{ut} - \frac{m_Z^4(s - 2m_Z^2)^2}{(ut)^2} - 2\right], \tag{19}
$$

where $g_V = -1/2 + 2s_Z^2$, $g_A = -1/2$ are, respectively, the vector and the axial–vector couplings of the Z boson to the electrons, and s, t and u are the usual Mandelstam variables

$$
s = (p_+ + p_-)^2, \qquad t = (p_+ - p_1)^2, \qquad u = (p_+ - p_2)^2, \tag{20}
$$

being p_+, p_- and p_1, p_2 the lepton and gauge boson four–momenta respectively. In the center of mass frame, the dependence on the scattering angle θ is carried by t and u through the relations

$$
t = -\frac{1}{2}\left[s - m_1^2 - m_2^2 - \lambda^{1/2}(s, m_1^2, m_2^2)\cos\theta\right],
$$

$$
u = -\frac{1}{2}\left[s - m_1^2 - m_2^2 + \lambda^{1/2}(s, m_1^2, m_2^2)\cos\theta\right], \tag{21}
$$

where $\lambda(s, m_1^2, m_2^2) \equiv (s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2$ (here $m_1 = m_2 = m_Z$).

The accuracy of the effective cross section (19) can be tested by comparing with full explicit EW calculations in the SM. As commented in the introduction, the EW corrections to $e^+e^- \to ZZ$ were calculated in [18]. There, the Born cross section is defined in terms of the fine structure constant α and the Sirlin weak mixing angle s_W. That is, our expression (19), but changing $\alpha_Z(m_Z) \to \alpha$ and $s_Z \to s_W$. With respect to this reference value, the effective cross section (19) shows a correction

$$
\delta_{\text{eff}}^{\text{EW (ZZ)}} \equiv \frac{\left(\frac{d\sigma^{ZZ}}{d\Omega}\right)_{\text{eff}} - \left(\frac{d\sigma^{ZZ}}{d\Omega}\right)_0}{\left(\frac{d\sigma^{ZZ}}{d\Omega}\right)_0}. \tag{22}
$$

To compare the results from our effective Lagrangian with those in [18] we rewrite Eq. (19) in terms of α and s_W by using (10) and $s_Z^2 = s_W^2 + \delta s_W^2$. The leading electroweak corrections can be easily obtained in the large m_t limit:

$$
\delta^{\text{EW (ZZ)}} \simeq 2\,\Delta\alpha + 2\,\delta\alpha_Z + \frac{s_Z^4 c_Z^4}{(g_V^4 + 6 g_V^2 g_A^2 + g_A^4)} \frac{d}{ds_Z^2}\left[\frac{(g_V^4 + 6 g_V^2 g_A^2 + g_A^4)}{s_Z^4 c_Z^4}\right]\delta s_W^2, \tag{23}
$$

with $\delta\alpha_Z$ and δs_W^2 given by (12) and (16) respectively. Using these equations we find

$$\delta^{\text{EW (ZZ)}} \simeq 2\,\Delta\alpha - \frac{\alpha}{\pi s_Z^4 c_Z^2}\left[\frac{3\,(1 - 6\,s_Z^2 + 12\,s_Z^4 - 8\,s_Z^6 - 16\,s_Z^8)}{8\,(1 - 8\,s_Z^2 + 24\,s_Z^4 - 32\,s_Z^6 + 32\,s_Z^8)}\frac{m_t^2}{m_Z^2}+\right.$$
$$\left.+ \frac{(3 - 21\,s_Z^2 + 56\,s_Z^4 - 72\,s_Z^6)}{6\,(1 - 8\,s_Z^2 + 24\,s_Z^4 - 32\,s_Z^6 + 32\,s_Z^8)}\log\left(\frac{m_t}{m_Z}\right)\right].\tag{24}$$

We have checked explicitly this result against the one obtained in [18] by taking there the large m_t limit, and found complete agreement.

Let us discuss now the case of γZ production at LEP2. As in the previous case, we begin by writing the SM lowest–order differential cross section for the process, which is obtained from the diagrams in Fig. 1 after replacing one of the Z bosons by a photon. Here we find

$$\left(\frac{d\sigma^{\gamma Z}}{d\Omega}\right)_{\text{eff}} = \frac{\alpha\alpha_Z(m_Z)}{4s_Z^2 c_Z^2}(g_V^2 + g_A^2)\frac{(s - m_Z^2)}{s^2}\left(\frac{s^2 + m_Z^4}{2ut} - 1\right).\tag{25}$$

Although in principle the situation is similar to the case of ZZ production, there is a crucial difference if the photon is a real one, that is with $q^2 = 0$. In that case, in the $\overline{\text{MS}}$ scheme we are using, and choosing a renormalization scale $\mu = m_Z$, one finds that the photon self–energy diagrams of Fig. 2 contain large logarithms $\sum_f Q_f^2 \log(m_f/m_Z)$, which effectively produce the "running back" of the electromagnetic coupling constant from $\alpha(m_Z)$ to $\alpha(m_e) \simeq \alpha$. This agrees with common wisdom that says that real, on–shell photons (no matter whether they are soft or hard) couple with α strength. Thus, the use of our effective Lagrangian at the scale $\mu = m_Z$ has to be supplemented with the rule that a real photon couples with its on–shell coupling.

As in the previous case, we can easily obtain the electroweak correction

$$\delta^{\text{EW (}\gamma Z\text{)}} = \Delta\alpha + \delta\alpha_Z + \frac{s_Z^2 c_Z^2}{(g_V^2 + g_A^2)}\frac{d}{ds_Z^2}\left[\frac{(g_V^2 + g_A^2)}{s_Z^2 c_Z^2}\right]\delta s_W^2,\tag{26}$$

and using (12) and (16) we get, in the large m_t limit,

$$\delta^{\text{EW (}\gamma Z\text{)}} = \Delta\alpha - \frac{\alpha}{\pi s_Z^4 c_Z^2}\left[\frac{3\,(1 - 2\,s_Z^2 - 4\,s_Z^4)}{16\,(1 - 4\,s_Z^2 + 8\,s_Z^4)}\frac{m_t^2}{m_Z^2} + \frac{(3 - 9\,s_Z^2 - 4\,s_Z^4)}{12\,(1 - 4\,s_Z^2 + 8\,s_Z^4)}\log\left(\frac{m_t}{m_Z}\right)\right].\tag{27}$$

As in the ZZ production, we should be able to test our EFT approach by comparing our results with those arising from explicit calculations of the EW effects in the limit of large m_t. For this process, an explicit one–loop calculation of $\delta^{\text{EW (}\gamma Z\text{)}}$ in the SM has been carried out by Böhm and Sack [17]. However, we could not reproduce their results in the large m_t limit. We believe they missed a factor $1/2$ entering the Z boson renormalized self–energy, therefore the bulk of the EW corrections, namely $\Delta\alpha$, has been overestimated. This is confirmed by the study of the crossed reaction $e^-\gamma \to e^- Z$, which has been extensively analysed in the literature (see for instance [26]). Here the EW corrections are reproduced completely, in the large m_t limit, with Eq. (27)[1].

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[1]Notice that in our approach the size of the electroweak corrections is exactly the same in the case of $e^+e^- \to \gamma Z$ and $e^-\gamma \to e^- Z$.

IV. GLOBAL FIT FOR LEP1/SLD OBSERVABLES

The analytical results of the previous section, obtained in the large m_t limit, are very interesting and very useful to test the overall approach and contain the bulk of EW radiative corrections. However, the top–quark mass is not so large and there could be other corrections at least comparable to those considered. In addition, Higgs mass corrections, though in principle can also be included, have not been taken into account in the previous analysis. This makes it difficult to achieve precisions better than the 2–3% with the above analytical approach. As an alternative procedure, we can use our effective Lagrangian at tree level with arbitrary couplings, and fit those couplings with LEP1/SLD observables. In this way, the effective couplings will contain not only the leading top–quark and Higgs mass dependences but also other universal non–leading corrections. This includes, a priori, also possible effects of new physics in the effective couplings. In this sense, the procedure is related to the "S, T, U" [27] or "epsilon" [28] analyses proposed already in the literature. The excellent agreement between the SM predictions and LEP1/SLD observables suggests, however, that our EFT couplings do not include any significant effect arising from non–standard physics.

Once we have fitted the parameters entering the effective Lagrangian for the processes observed at LEP1/SLD, we can use the result to give predictions for $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow \gamma Z$ cross sections at LEP2 energies, considering once again tree–level formulae. With this procedure, we expect to achieve precisions better than 1%, which should be enough for most LEP2 observables. This can be checked by comparing our results with known one–loop calculations.

The list of LEP1 and SLD observables that we consider for our fit is presented in Table I. These include: the Z mass (m_Z), the total Z width (Γ_Z), the hadronic cross section (σ_{had}), the ratio of the widths $Z \rightarrow hadrons$ to $Z \rightarrow l^+l^-$, $l = e, \mu, \tau$ (R_l), the ratios of the widths $Z \rightarrow \bar{b}b$ (R_b) and $Z \rightarrow \bar{c}c$ (R_c) to $Z \rightarrow hadrons$, the leptonic ($A_{FB}^{(0,l)}$), b–quark ($A_{FB}^{(0,b)}$) and c–quark ($A_{FB}^{(0,c)}$) C–odd forward–backward asymmetries, and the P–odd leptonic (A_l), b–quark (A_b) and c–quark (A_c) asymmetries. The quoted experimental value for the leptonic asymmetry A_l is the average of LEP1 and SLD results, assuming lepton universality.

From the Lagrangian (6), it is immediate to see that the lowest–order formulae in the EFT scheme are basically the same as in the SM, just taking $e_Z(m_Z)/(s_Z c_Z)$ and s_Z as the weak $Z\bar{f}f$ coupling constant and the sine of the Weinberg angle respectively. Only special care has to be taken in the case of the $Z\bar{b}b$ coupling, which requires the inclusion of the additional parameter $\epsilon_b(m_Z)$ defined in the previous section. The tree–level expressions for the LEP1/SLD observables, as well as the leading QCD and QED corrections, are well–known and will not be reproduced here.

The parameters to be fitted are five, namely m_Z, $\alpha_Z(m_Z)$, s_Z^2, $\epsilon_b(m_Z)$ and $\alpha_s(m_Z)$, although the value of $\alpha_s(m_Z)$ could be obtained independently from other processes [1]. In any case, the result of the fit is found to be rather stable with respect to the value of $\alpha_s(m_Z)$.

We obtain from the fit the following values for the parameters:

$$m_Z = 91.1867 \pm 0.0020$$
$$\alpha_Z(m_Z) = 0.007788 \pm 0.000012$$
$$s_Z^2 = 0.23103 \pm 0.00021$$

$$\epsilon_b(m_Z) = -0.0053 \pm 0.0023$$
$$\alpha_s(m_Z) = 0.1215 \pm 0.0052 \tag{28}$$

with

$$\chi^2/\text{ndf} = 2.6/7. \tag{29}$$

If instead we fix $\alpha_s(m_Z)$ to its world average $\alpha_s(m_Z) = 0.119$ [1] we obtain $m_Z = 91.1867 \pm 0.0020$, $\alpha_Z(m_Z) = 0.007790 \pm 0.000011$, $s_Z^2 = 0.23102 \pm 0.00021$, and $\epsilon_b(m_Z) = -0.0045 \pm 0.0017$ with $\chi^2/\text{ndf} = 2.9/8$.

Notice that in these fits we neglect the correlations in the input data. We have checked that the impact of these correlations in the results of the fit is negligible. We did so by performing a fit including the main correlations (we have considered only the correlations which are larger than 10% [29]: Γ_Z–$\sigma_{\text{had}} \approx -0.19$, R_l–$\sigma_{\text{had}} \approx 0.13$,$R_b$–$R_c \approx -0.17$, $A_{FB}^{(0,b)}$–$A_{FB}^{(0,c)} \approx 0.13$). The errors remain unchanged and the central values are shifted at most by 10% of one standard deviation.

The predictions for the LEP1/SLD observables obtained with these values can be read from the third column in Table I. We also quote in the fourth column the deviations of the different observables from the measured central values in units of experimental standard deviations (the pull). It can be seen that all the predictions deviate less than 1.5σ from the measured values. This is reflected in the very low χ^2 in (29), and shows that for LEP1 and SLD data the EFT approach works remarkably well.

From Eq. (18), using the value of s_Z^2 in (28), the W boson mass, $m_W = 80.42 \pm 0.08$ GeV, and the Fermi constant, we can also estimate the value of $\alpha_W(m_Z)$. We obtain $1/\alpha_W(m_Z) = 127.2 \pm 0.3$, to be compared with $1/\alpha_Z(m_Z) = 128.4 \pm 0.2$ from our fit (28), and to $1/\alpha(m_Z) = 127.88 \pm 0.09$ obtained by running from the Thomson limit. We see that, as expected from (12) and (13), the differences are really small and the three couplings can be taken as equal if only precisions at the 1% level are needed.

V. PREDICTIONS FOR $e^+e^- \rightarrow ZZ$ AND $e^+e^- \rightarrow \gamma Z$ AT LEP2

Now, once the effective couplings at the scale of m_Z have been determined, our goal is to use the same approach to predict the magnitude of the electroweak corrections for processes to be measured at LEP2. To estimate the corresponding cross sections, we will use the values of $\alpha_Z(m_Z)$, m_Z and s_Z^2 from the result of the fit (28). Although the relevant scale at LEP2 could reach 190 GeV, the running of the parameters from m_Z to 190 GeV will give at most corrections of the order of $\alpha/\pi \log 2$ which are small[2]. In addition, in the processes we are interested in this paper, $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow \gamma Z$, the gauge bosons are on–shell, therefore the relevant scale is fixed by their masses.

Let us take the EFT tree–level expressions (19) and (25) for $e^+e^- \rightarrow ZZ$ and $e^+e^- \rightarrow \gamma Z$ respectively, with α_Z, m_Z and s_Z^2 from (28), and compute the size of the deviations from

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[2]Note, however, that if needed, these corrections can be easily included in our approach.

the Born cross sections expressed in the on–shell scheme as in Eq. (22) (or its equivalent for γZ production). For the process $e^+e^- \to ZZ$ we obtain

$$\delta_{\text{eff}}^{\text{EW (ZZ)}} \simeq 5.4 \pm 0.4 \ \% \,. \tag{30}$$

Since for our fit we have used LEP1/SLD values, we have also taken the last SM fit for Z–pole data to evaluate the Born cross section written in terms of on–shell parameters. Thus, we have used for the Sirlin weak mixing angle the value $s_W^2 = 0.2236 \pm 0.0008$ [1].

On the other hand, the full one–loop EW correction to $(d\sigma/d\Omega)_0$ can be obtained from the analysis carried out by Denner and Sack [18], after updating the values for the masses of the gauge bosons and —especially— the top quark. It can be seen that, for LEP2 energies, the shift is strongly dominated by the one–loop correction to the Z self–energies, which contain the top quark dependence. To obtain the EW corrections from [18] we use the value of s_W^2 quoted above, together with $m_t = 168 \pm 8$ GeV (arising from Z–pole analysis [1]) and $m_H = m_Z$. We find

$$\delta_{\text{SM}-1 \ \text{loop}}^{\text{EW (ZZ)}} \simeq 5.3 \pm 1.0 \ \% \,, \tag{31}$$

where the error is mainly due to the uncertainty in m_t. As can be seen, the agreement between the values in (30) and (31) is remarkably good. Notice that, in general, one would expect $\delta_{\text{SM}-1 \ \text{loop}}^{\text{EW (ZZ)}}$ to depend on the scattering angle θ. However, it can be seen [18] that for these energies the distribution is almost flat, so that the constant value in (31) represents a good approximation. This is also consistent with our approach. As can be seen from Eqs. (19) and (21), the dependence of the differential Born cross section with the scattering angle is contained in the Mandelstam variables t and u, which only enter the factor in square brackets in (19). The shift of α and $\sin\theta_W$ from the on–shell to the effective values leads only to a global correction that does not affect the θ–dependence.

It is important to remark that the value in (30) has been found in a quite straightforward way, whereas that in (31) can be obtained only after a very lengthy calculation. In addition, the one–loop result, though in principle more precise, depends not only on m_t but also on other uncertain parameters, such as the Higgs mass and the running of the electromagnetic coupling from the Thomson limit to the m_Z scale.

For $e^+e^- \to \gamma Z$ we use Eq. (25), taking once again α_Z, m_Z and s_Z^2 from (28). Notice that, as discussed in the previous section, the value of the electromagnetic coupling to be used in (25) is the on–shell fine structure constant α. This is because the photon is on shell, i.e. with $q^2 = 0$. For the rest of the parameters the same considerations as for $e^+e^- \to ZZ$ apply. We obtain

$$\delta_{\text{eff}}^{\text{EW }(\gamma Z)} \simeq 3.7 \pm 0.2 \ \% \,. \tag{32}$$

As commented in section III we cannot use the expressions in [17] to check this last result. Still, we can take into account the known calculations [26] for the crossed reaction $e^-\gamma \to e^-Z$. Within the EFT approach, the cross section for this process shows exactly the same dependence on the parameters $\alpha_Z(m_Z)$ and s_Z as in (25), therefore for both $e^-\gamma \to e^-Z$ and $e^+e^- \to \gamma Z$ the correction $\delta_{\text{eff}}^{\text{EW}}$ will be exactly the same.

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For a center–of–mass energy of 100 GeV, and using a top–quark mass of 140 GeV, the analysis in Ref. [26] shows that the EW corrections to $e^-\gamma \to e^- Z$ are $\simeq 4.2\%$. Once again, it is found that this result is almost independent from the scattering angle (see Table 3 of [26]). Now increasing m_t up to 168 GeV, and taking into account the errors in s_W and m_t as in the $e^+ e^- \to ZZ$ case, we find

$$\delta^{\mathrm{EW}\, e^-\gamma\to e^- Z}_{\mathrm{SM-1\ loop}} \simeq 3.1 \pm 0.4\% \,. \tag{33}$$

That means, our result (32) lies within the expected level of accuracy. Moreover, our approach succeeds in predicting the flat behaviour of δ^{EW} with respect to the scattering angle.

Notice that in the definition of δ^{EW} we refer to the Born cross section. The latter is defined in terms of the Sirlin weak mixing angle (or equivalently, the W mass), which is not measured with sufficiently high precision in LEP1/SLD and introduces some error. In order to estimate the accuracy of our approach, it is better to compare directly the values for the cross sections obtained from both the EFT and SM one–loop analyses. In this way the comparison is much less sensitive to the top–quark mass, which does not appear explicitly in σ_{eff}. Thus for both $e^+ e^- \to ZZ$ and $e^-\gamma \to e^- Z$ we compute the ratio

$$\Delta \equiv \frac{\sigma_{\mathrm{eff}} - \sigma_{\mathrm{SM-1\ loop}}}{\sigma_{\mathrm{SM-1\ loop}}} \,, \tag{34}$$

obtaining

$$\Delta^{(ZZ)} = 0.0012 \pm 0.0038 \tag{35}$$

and

$$\Delta^{(\gamma Z)} = 0.0065 \pm 0.0017 \,. \tag{36}$$

In both cases, the agreement between EFT and one–loop SM values is found to be better than 1%. For the crossed reaction $e^+ e^- \to \gamma Z$, the result is expected to be similar to that obtained for $e^-\gamma \to e^- Z$.

It is also worth to mention that in our effective Lagrangian approach we trivially find that pure Compton processes (only containing real photons and electrons), such as $e^-\gamma \to e^-\gamma$ and $\gamma\gamma \to e^- e^+$, have zero EW radiative corrections, since the tree–level cross sections are independent of s_Z and $\alpha_Z(m_Z)$. It can be seen that this result is also obtained from full one–loop SM calculations [30], for a range of center of mass energies of 100–200 GeV. Our approach is again successful in this case.

VI. CONCLUSIONS

In this paper we elaborate an effective field theory approach to the analysis of the electroweak corrections at LEP energies.

We review how to obtain the effective EW Lagrangian that arises when the top quark is integrated out. At the leading order in the top–quark mass, we obtain the effective couplings that are relevant for LEP energies.

Using this Lagrangian at tree level we obtain analytical formulae for the differential cross sections for the LEP2 processes $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$, in the large m_t limit. The results agree completely with full one–loop EW calculations.

This approach allows us to compute LEP2 observables in the large m_t limit. However, this cannot be used in general to achieve precisions better than 2–3%. To go beyond that, we consider an effective Lagrangian similar to that arising from the EFT, but leaving the couplings as free parameters. Then, using the effective Lagrangian at tree level, we fit the parameters from present LEP1 and SLD data. In this way, the effective couplings should take into account the effects arising from virtual top quarks and Higgs, as well as other possible universal contributions. The fit is performed for 12 LEP1/SLD observables, and the parameters to be determined are five, including m_Z and $\alpha_s(m_Z)$. The results are amazingly good: as shown in Table I, in all cases the difference between fitted and experimental values is less than 1.5σ. Finally, taking the effective couplings from the LEP1/SLD fit, we compute the differential cross sections for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2 energies, using once again the effective Lagrangian at tree level. The predictions are in this way completely independent on the masses of the top quark and the Higgs boson. Our results are compared with the values of the corresponding Born cross sections written in terms of on–shell parameters: at LEP2 energies, the EW corrections for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ amount to 5.4 % and 3.7 % respectively for a fixed value of the on–shell weak mixing angle. In addition, our EFT cross sections are compared with those arising from one–loop analyses in the SM. The agreement is found to be better than 1 % for both $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$.

It is worth to point out that our effective Lagrangian can be used to estimate the size of EW corrections in other LEP2 processes that involve the subprocesses $e^+e^- \to VV$, $Ve \to Ve$ and $VV \to \bar{f}f$ ($V = \gamma, Z$). This is $e.g.$ the case of the scattering $e^+e^- \to e^+e^-\bar{b}b$, which is a very important background process in searches for new particles, and where the full one–loop EW calculation turns out to be very hard . Another important example is the neutrino counting process $e^+e^- \to \nu\bar{\nu}\gamma$, for which the full one–loop EW calculation is also missing. The extension of the effective Lagrangian to include triple gauge boson couplings is presently under study.

ACKNOWLEDGMENTS

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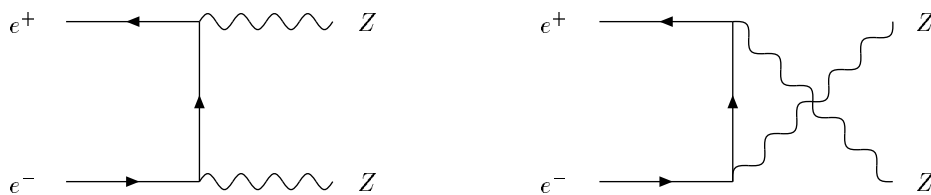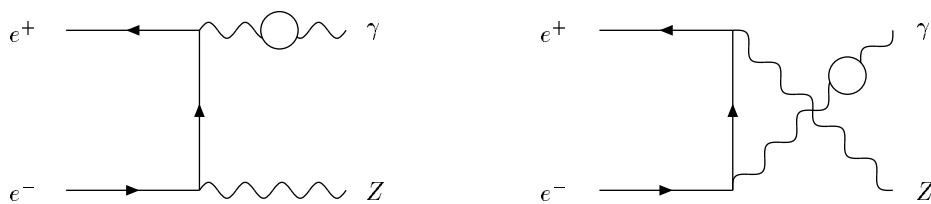
FIG. 1. Tree–level contributions to $e^+e^- \to ZZ$.

FIG. 2. Photon self–energy diagrams contributing to $e^+e^- \to \gamma Z$.

TABLES

TABLE I. Results for combined LEP1 and SLD observables obtained within the EFT scheme using the fitted values for $\alpha_Z(m_Z)$, m_Z, s_Z, $\epsilon_b(m_Z)$ and $\alpha_s(m_Z)$. We have used EFT tree–level formulae plus standard QCD and QED corrections.

Observable	Experimental value	Fitted value	Pull
m_Z [GeV]	91.1867 ± 0.0020	91.1867	0.00
Γ_Z [GeV]	2.4948 ± 0.0025	2.4949	0.04
σ_{had} [nb]	41.486 ± 0.053	41.499	0.24
R_l	20.775 ± 0.027	20.779	0.15
R_b	0.2170 ± 0.0009	0.2169	-0.09
R_c	0.1734 ± 0.0048	0.1711	-0.47
$A_{FB}^{(0,l)}$	0.0171 ± 0.0010	0.0170	-0.05
$A_{FB}^{(0,b)}$	0.0984 ± 0.0024	0.1015	1.30
$A_{FB}^{(0,c)}$	0.0741 ± 0.0048	0.0726	-0.31
A_l	0.1521 ± 0.0021	0.1506	-0.71
A_b	0.900 ± 0.050	0.899	-0.02
A_c	0.650 ± 0.058	0.643	-0.12