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CONFIGURATION MIXING AND TOTAL MUON CAPTURE RATES

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A B S T R A C T

We modify the Primakoff closure approximation to get independence on the mean neutrino energy and energy weighted sum rules are used for the corrective terms. A near model independent discussion is then possible, and the total rates are shown to be a very sensitive tool to investigate configuration mixing of the target. Wild discrepancies with experiment would arise if the limits of pure jj or LS couplings are used for ^{12}C, whereas Cohen-Kurath wave function gives a very good result.

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Total muon captures of muons can be written in terms of matrix elements of the form

$$\sum_b \left(\frac{\nu_{ab}}{m}\right)^2 \left| \langle b | \sum_j e^{i \vec{\nu}_{ab} \cdot \vec{r}_j} \tau_j^- \, (\vec{\sigma})_j | a \rangle \right|^2 \quad (1)$$

where the momentum transfer ν_{ab} for the partial transition $a \to b$ is given by $\nu_{ab} = m-(E_b-E_a)$ (m is the muon mass). If the sum over final states b is done explicitly the result is very sensitive to the model used, because both the right excitation energies and wave functions are necessary in the calculation.

In order to minimize these effects closure approximation was first suggested by Primakoff [1]. It is obtained by summing over the final states, assumed to form a complete set, replacing ν_{ab} by an average value ν. This approximation reduces the model dependence to the knowledge of the ground state of the nuclear target, but it has the drawback that the result depends on the parameter ν as ν^3 to ν^4, and clearly its choice is critical.

A way out to this last problem, as suggested in Ref. 2), is to expand the expression for the capture rate $\Lambda_r(a \to b)$ in a power series around ν. Limiting the expansion to the first order (this truncation will be justified a posteriori by the stability of the result)

$$\Lambda_r(a \to b) \approx \Lambda_r(a \to b)\Big|_{\nu_{ab}=\nu} + \frac{d\Lambda_r(a \to b)}{d\nu_{ab}}\Big|_{\nu_{ab}=\nu} (\nu_{ab}-\nu) \quad (2)$$

and doing the sum over the final states b we obtain

$$\Lambda_r = \left\{ 1 + (m-\nu)\frac{d}{d\nu} \right\} \Lambda_r(\nu) - \frac{d}{d\nu} \left\{ \sum_b (E_b - E_a) \Lambda_r(a \to b)\Big|_\nu \right\} \quad (3)$$

where $\Lambda_r(\nu)$ is the result in the Primakoff closure approximation. Energy weighted sum rules can be used for the last term in (3), and then in the result only the properties of the ground state, through $\Lambda_r(\nu)$, are involved. This opens the way for a near model independent discussion, including SU(4) symmetry breaking. We will illustrate this point for the case of ^{12}C using the wave function resulting from effective interactions in the 1p shell [3].

The energy weighted sum rule in Eq. (3) involves the expected value of the double commutator

$$\left[\sum_j e^{-i\vec{\nu}.\vec{r}_j} \tau_j^+ (\sigma)_j \, , \left[H , \sum_k e^{i\vec{\nu}.\vec{r}_k} \tau_k^- (\sigma)_k \right] \right] \tag{4}$$

where the dependence of the Hamiltonian H on Majorana and SU(4) breaking potentials is important besides the kinetic energy contribution given by

$$\sum_b (E_b - E_a) |\langle b | \sum_i f_i | a \rangle|^2 = \frac{1}{2M} \langle a | \sum_i | \vec{\nabla}_i f_i |^2 | a \rangle \tag{5}$$

We study (4) for a Hamiltonian of the form

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2M} + \sum_{i<j} V(r_{ij}) + \sum_{i<j} V^B(r_{ij}) P_{ij}^\sigma - \sum_{i<j} V^H(r_{ij}) P_{ij}^\tau + \sum_{i<j} V^M(r_{ij}) P_{ij}^x$$

$$+ \zeta \sum_i \vec{\ell}_i \cdot \vec{s}_i + \frac{1}{2} \sum_{i<j} (X + X_\tau P_{ij}^\tau)(\vec{r}_{ij} \times \vec{p}_{ij})(\vec{\sigma}_i + \vec{\sigma}_j) \tag{6}$$

$$+ \sum_{i<j} (V_{Te}(r_{ij}) + V_{Te}^\tau(r_{ij}) P_{ij}^\tau) S_{ij}$$

where P_{ij}^x, P_{ij}^σ, P_{ij}^τ are the space, spin and isospin exchange operators respectively and S_{ij} is given by

$$S_{ij} \equiv 3 (\vec{\sigma}_i . \vec{r}_{ij})(\vec{\sigma}_j . \vec{r}_{ij}) / r_{ij}^2 - \vec{\sigma}_i . \vec{\sigma}_j$$

Working out the commutator (4) for the potentials (6) one has that the limit $\vec{\nu} \cdot \vec{r}_{ij} \to 0$ is equivalent to $e^{i\vec{\nu}.\vec{r}_i} \to 1$, so that this approximation connects the corrections to closure approximation directly to allowed Gamow Teller transitions. Since the effective internucleonic potential is not so well known in its explicit radial dependence, at the present level of our calculation it does not seem reasonable to perform detailed evaluations which would be model dependent anyway. On the other hand, as we show in Ref. 4), the short range limit can be directly related to spectroscopical data. This limit has a drawback of giving zero for the sum rule (4) of the vector operator evaluated with the potentials while we know from photo-reactions that the sum rule is really enhanced relative to the kinetic energy contribution. This as we know is mainly a dipole effect and therefore would involve an expansion to $(\vec{\nu}.\vec{r}_{ij})^2$. We shall assume that due to approximate SU(4) invariance these corrections are the same for axial and vector matrix elements. Possible differences would come essentially from taking into account the fact that the axial dipole excitations are higher in energy than the vector ones. We neglect systematically this effect which is not too important according to estimates in Ref. 5). As far as the explicit evaluation is concerned we take a Van Vleck potential $V = V_o + ap^2$ where the coefficient a, which depends on the density, is adjusted to fit photoreaction data. The dipole being the prominent contribution both in photoreactions as well as in muon capture, the weight given by these two reactions is the same so that effectively M^* is well evaluated (for ^{12}C, $M/M^* = 1.6$). Our main difference between the vector and axial matrix elements is obtained taking only the short range limit $\vec{\nu} \cdot \vec{r}_{ij} \to 0$ in (4). This is in agreement with the fact that the main difference between axial and vector transitions is due to allowed transitions. It is interesting to remark that performing the commutator in our approximation we can connect very easily the spectroscopical information contained in Cohen Kurath matrix elements [3] expressed in L-S coupling, since the commutator with a certain potential contains again the same potential in the final expression. So our results are given in terms of matrix elements of effective potentials. Phenomenologically it is noticeable that we correct closure approximation with the effective kinetic energy term and the term in the commutator analogous to M1 transitions. We can write the final expression for the capture rate as

$$\Lambda_r = \left\{ 1 + (m-\nu)\frac{d}{d\nu} \right\} \Lambda_r(\nu) - 2\,\frac{m}{M^*}\left(\frac{\nu}{m}\right)^3 - \frac{1}{Z}\,\frac{K^A}{m}\,\frac{\nu}{m} \qquad (6)$$

where K^A is the result of the commutator (4) for the various potentials in the short range limit.

In the Figure we give the results for ^{12}C. The value $M/M^* = 1.6$ is used and the capture rate is calculated in the extreme cases of jj coupling (broken line) and LS coupling (dotted line). Our result using the wave function of Ref. 3) is given in the solid line and it is in agreement with the experimental result [6] $\Lambda_r^{exp} = 0.125 \pm 0.005$. For comparison we give also $\Lambda_r(\nu)$ (Primakoff closure approximation) with this wave function, and we see that the value of ν which reproduces our result for Λ_r is $\nu = 78$ MeV.

The strong sensitivity of the capture rate to the particular coupling is immediately apparent from the Figure. From the fact that small variations in M^* do not change the capture rate appreciably (we have seen that a variation of 10% in M^* gives a variation of 4-5% in Λ_r) it is evident that total muon capture rates may be used as a sensitive tool to investigate the detailed structure of the configuration mixing. The information coming from muon capture rates can complement very well that of M1 transitions and allowed β decays because muon capture induces transitions to states which are the same as in β decay, e.g., $^{12}C \rightarrow {}^{12}B$(g.s.), but also to states which cannot β decay or decay by M1 transitions because they are particle emission unstable. Indeed there is an appreciable variation in the total capture rate if one assumes all the allowed strength to be saturated in the ground state transition (dashed-dotted line in the Figure). Our result indicates that higher lying 1^+ levels contribute to the allowed strength about 25%. On the other side in the jj coupling the axial allowed strength is exhausted by the ground state transition, but the main discrepancy arises from the fact that its magnitude is too large by a factor 4.5. This is the indication of the departure of the ground state of ^{12}C from a pure jj configuration.

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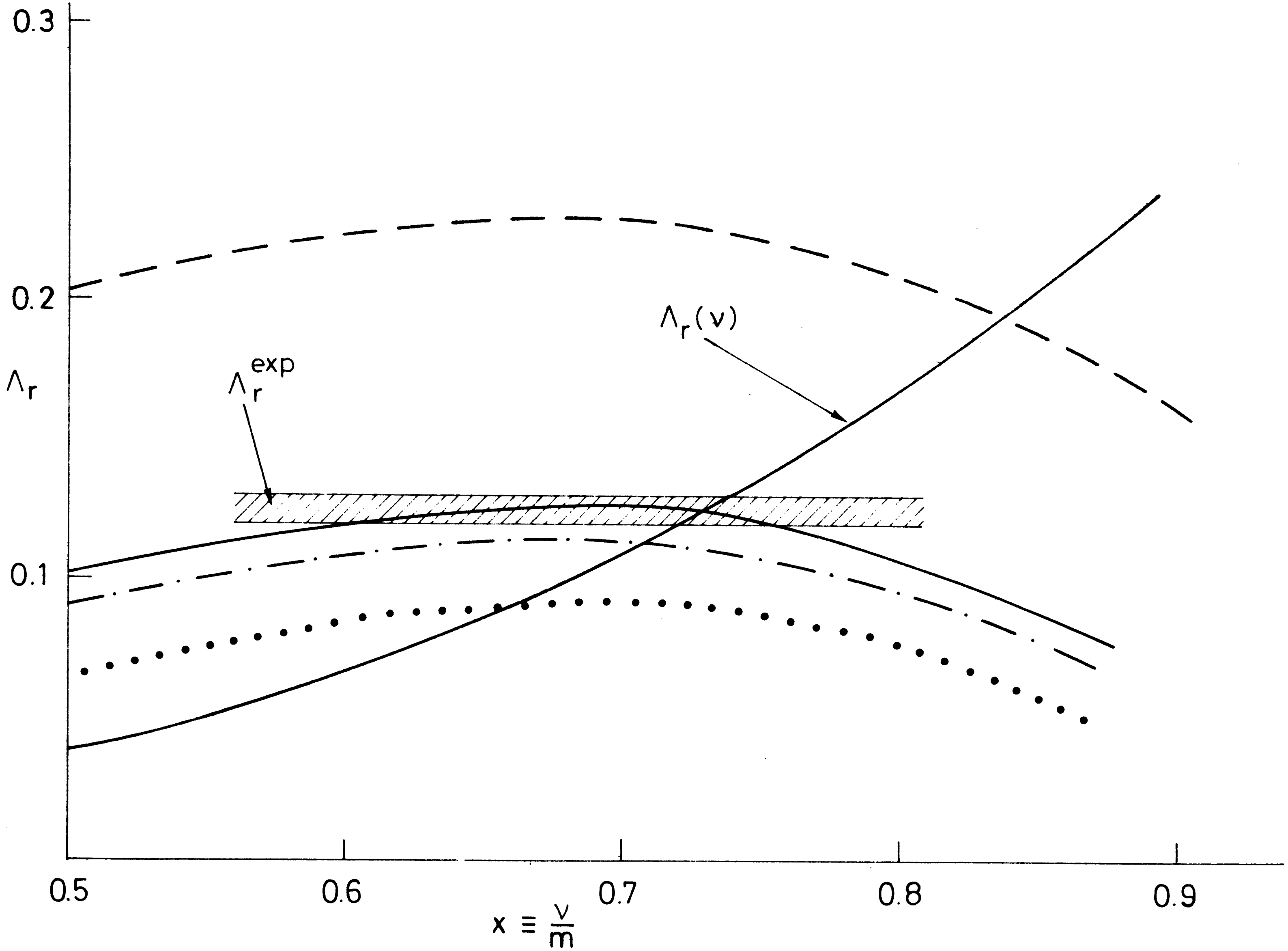
FIGURE CAPTION

Reduced capture rate Λ_r for ^{12}C. Our result (solid lines) is compared to the extreme cases of jj coupling (broken line) and LS coupling (dotted line). The dashed-dotted line would be the prediction when only the ground state transition for the allowed contribution is included. The line $\Lambda_r(\nu)$ is the ν- dependence in the Primakoff closure approximation, using the wave function of Ref. 3).



0.3
0.2
Λ_r
0.1
Λ_r^{exp}
$\Lambda_r(\nu)$
0.5
0.6
0.7
0.8
0.9
$x \equiv \frac{\nu}{m}$