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SU(4) BREAKING, ALLOWED TRANSITIONS AND TOTAL MUON CAPTURE RATES IN NUCLEI

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A B S T R A C T

We study muon capture rates in light nuclei and show that one can evaluate them in a model independent way by correcting the Primakoff closure approximation to the first order in the neutrino energy dependence. In this framework the interplay between the existence of allowed transitions and SU(4) breaking is analyzed. In the correction to the first order in the neutrino energy dependence by a sum rule, Majorana and SU(4) breaking potentials are taken into account. The method is also applied to single multipolar transitions, in particular for the dipole transitions. General considerations are illustrated for ^{6}Li and ^{12}C by explicit calculations. The stability of the results varying the mean neutrino energy allows to use capture rates as a very sensitive tool to investigate the configuration mixing of the target because we connect the corrective term to spectroscopical information using directly the matrix elements of the effective potentials.

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1. INTRODUCTION

The problem of explaining the phenomenon of muon capture into the general framework of a universal theory of weak interactions has been the object of many theoretical works. Most of them study total and partial muon capture rates. One can consider total capture rates of selected nuclei, usually lighter nuclei, where a particular model may be employed. In general this use of model wave functions is the main source of trouble in the interpretation of muon capture. In order to minimize these effects, as far as total muon capture is concerned, a useful approximation is to use closure [1] which reduces the model dependence to the knowledge of the ground state of the nuclear target and has as a free parameter the mean neutrino energy ν. The choice of this parameter is critical because the dependence of the result on it is very strong.

A way out of this last problem has recently been suggested [2),3)] by a first order expansion of the capture rate around ν. One of the corrective terms involves a sum rule derived from the double commutator

$$\left[\sum_j e^{-i\vec{\nu}\cdot\vec{r}_j}\tau_j^+(\sigma_{zj}),\left[H,\sum_k e^{i\vec{\nu}\cdot\vec{r}_k}\tau_k^-(\sigma_{zk})\right]\right] \qquad (1)$$

which has been used in previous works in the particular case where no exchange potentials are present in the Hamiltonian H. We stress that this is a more restrictive requirement than that of SU(4) invariance. Indeed, Majorana exchange potentials break the theorem [4)]

$$\sum_b (E_b - E_a)\left|\langle b|\sum_i f_i|a\rangle\right|^2 = \frac{1}{2M}\langle a|\sum_i|\vec{\nabla}_i f_i|^2|a\rangle \qquad (2)$$

without violating SU(4) symmetry.

The results obtained in Ref. 3) allowed to conclude that the method is successful in obtaining independence on ν over a large zone of plausible values. In this paper we are interested in the extension of the study presented there in order to include the effect of the residual interactions on the different terms. In Section 2 the method is revised, and its application to single multipolar transitions is pointed out. This is illustrated in the case of ^{6}Li, for which we know [3)] that the LS coupling

works. The effects of the various potentials are present in the main term, usual closure approximation, through the complicated structure describing the ground state, such as configuration mixing. This is studied in Section 3, where the case of ^{12}C is worked out, using the wave function resulting from effective interactions in the 1p shell [5]. In the framework of SU(4) symmetry higher supermultiplets are admixed to the supermultiplet to which the ground state belongs. As a consequence, allowed axial transitions are present. Of completely different origin are the transitions which arise in non-scalar supermultiplets, like $^{6}Li \rightarrow {}^{6}He$, even neglecting SU(4) impurities. The corrections to the sum rule (2) are studied in Section 4, where central, single particle spin-orbit, two-body spin-orbit and tensor potentials are included. We show that in the limit of short range the expected value of the double commutator (1) can be written in terms of the effective potential matrix elements. The corresponding calculation in ^{6}Li and ^{12}C is presented. Finally, Section 5, gives some results and discussion. It turns out that the predicted capture rate is extremely sensitive to the configuration mixing in ^{12}C.

2. ZERO ORDER AND FIRST ORDER ENERGY WEIGHTED SUM RULES

The usual closure approximation [1] consists in reducing a complicated energy weighted sum rule to a zero order energy weighted sum rule in the hypothesis that the excited states may be approximated by a complete set. The tool for this simplification is the assumption that the neutrino energy ν_{ab} in each channel may be approximated by $\langle \nu_{ab} \rangle \equiv \nu$. Of course this procedure suffers of uncertainties due to the ambiguous physical meaning of the last parameter, so it has been suggested to expand in first order the energy dependence, i.e., take into acccunt zero order and first order energy weighted sum rules [2),3)]. Clearly this procedure may be pushed further to higher orders (converging therefore to the exact result), but since already in the first order independence on the specific value of the mean neutrino energy is obtained, within a large range of plausible values, we maintain the expansion up to the first order. In the notation of Ref. 3)

$$\Lambda_r(a \rightarrow b) \simeq \Lambda_r(a \rightarrow b)\Big|_{\nu_{ab}=\nu} + \frac{d\Lambda_r(a \rightarrow b)}{d\nu_{ab}}\Big|_{\nu_{ab}=\nu} (\nu_{ab} - \nu) \qquad (3)$$

Summing over final states $|b>$ assumed to form a complete set we get

$$\Lambda_r = \left\{ 1 + (m-\nu) \frac{d}{d\nu} \right\} \Lambda_r(\nu) - \frac{d}{d\nu} \left\{ \sum_b (E_b - E_a) \Lambda_r(a \to b) \Big|_{\nu_{ab}=\nu} \right\} \tag{4}$$

where the relation $\nu_{ab} = m-(E_b-E_a)$ is used for light nuclei (m is the muon mass).

As is clear from (4), studying $\Lambda_r(\nu)$ the properties of the ground state are involved whereas in the last term also the explicit dependence of the Hamiltonian on Majorana and SU(4) breaking potentials are important besides the kinetic energy contribution given by the theorem (2).

We note that the same procedure may be repeated for single multipolar transitions. Also in this case one can make a closure approximation and a correction of the first order to it. Clearly, if a single multipole is assumed to be resonant for some excitation energy $<E_b-E_a>= \omega$ and, therefore $<\nu_{ab}>=m-\omega$, the corrective term which is left after the application of the theorem (2) corresponds to shift the center of the resonant strength due to residual interactions [6]. We are going to apply the method to study the dipole ($\ell = 1$) contribution to muon capture by ^{6}Li, for which we assume a pure LS coupling. Then the vector and axial contributions are equal, and proceeding in the same way as in Ref. 3) we need to calculate for the main term

$$\Lambda_r(\nu)^{\ell=1} = \frac{1}{2} \left(\frac{\nu}{m}\right)^2 \left\{ 12\pi <a| \sum_i \frac{1+\tau_i^3}{2} |Y_1^0(\hat{x}_i)|^2 j_1^2(\nu r_i)|a> - D_S(\nu)^{\ell=1} \right\} \tag{5}$$

which in our case turns out to be

$$\Lambda_r(\nu)^{\ell=1} = \left(\frac{\nu}{m}\right)^2 \left\{ 2 J_1(s) + J_1(\mu) - \frac{2}{3} I_1^2(s,\mu) \right\} \tag{6}$$

where the radial integrals are given by

$$\mathcal{J}_\ell(x) = \int_0^\infty dr\ r^2\ R_x^2(r)\ j_\ell^2(\nu r)$$

$$I_\ell(x,y) = \int_0^\infty dr\ r^2\ R_x(r)\ R_y(r)\ j_\ell(\nu r) \qquad (7)$$

We use for these ones the harmonic oscillator wave functions with parameter $b = 1.98$ fm.

For the corrective term in Eq. (4) we take only the kinetic energy contribution, Eq. (2). By using the gradient formula we obtain from this theorem

$$\frac{\nu^2}{2M}\left\{2\,\mathcal{J}_0(s) + \mathcal{J}_0(p) + 4\,\mathcal{J}_2(s) + 2\,\mathcal{J}_2(p)\right\}$$

In Fig. 1 the dotted line gives $\Lambda_r(\nu)^{\ell=1}$, Eq. (6), as function of $x = \nu/m$, whereas the broken line is our prediction for the dipole contribution to the muon capture rate. We see that the result is quite independent of x, and we obtain $\Lambda_r^{\ell=1} = 0.208$. Comparing this with the total experimental result [7] $\Lambda_r(^6\mathrm{Li}) = 0.273 \pm 0.057$, we conclude that the dipole transition is dominant. The corresponding value of the average $\nu^{\ell=1}$ is about 90 MeV, which is not too high since the peak energy in photo-reactions is about 15 MeV [8]. Furthermore, in Section 4 we discuss some corrections which would enhance the mean excitation energy.

3. CONFIGURATION MIXING EFFECTS IN THE GROUND STATE

3.1 Foldy and Walecka [9] studied the properties of the correlation functions $D_{S,T,L}$ and found that for nuclei whose ground states ($L = 0$, $S = 0$, $T = 0$) belong to a scalar supermultiplet of SU(4)

$$D_S(\nu) = D_T(\nu) = D_L(\nu) \qquad (8)$$

We now investigate in some detail the mechanism through which the previous relation may be broken for scalar supermultiplets, which have SU(4) impurity admixtures, emphasizing the difference between this mechanism and

the allowed transitions present also in the SU(4) limit in other nuclei whose ground state is not a scalar supermultiplet, e.g., ^{6}Li with ground state quantum numbers ($L=0$, $S=1$, $T=0$) which belongs to a (100) multiplet. For nuclei whose ground state is a pure scalar supermultiplet there are only forbidden transitions, so that the presence of allowed transitions means inequivocally the existence of impurities in the ground state, e.g., (110) or (200) supermultiplets are admixed to (000). Due to the goodness of isospin we assume that only $T=0$ admixtures are present in the ground state, so that only axial or pseudoscalar allowed transitions are present as a consequence of SU(4) breaking. These transitions have as a counterpart M1 transitions in electromagnetic interactions between isospin rotated states (analogue states).

The most important difference between D_S and D_T, D_L is that the latter get contribution from allowed transitions, i.e., in closure approximation from

$$\langle a \left| \sum_{i,j} \tau_j^+ \tau_i^- \sigma_j \sigma_i j_0(\nu r_j) j_0(\nu r_i) \right| a \rangle$$

while D_S does not. Once these allowed transitions are subtracted other differences remain as e.g., the difference between D_T and D_L for values of $\nu \neq 0$. The combination $2D_T + D_L$ only depends on the scalar product $\vec{\sigma}_i \cdot \vec{\sigma}_j$, however, individually D_T and D_L can be written as proportional to

$$\sum_{i \neq j} \tau_j^+ \tau_i^- \left\{ Y_0(\hat{r}_{ij}) \otimes [\sigma_i \otimes \sigma_j]_0^0 \right\}_0^0 \qquad (9)$$

$$+ \alpha_{T,L} \sum_{i \neq j} \tau_j^+ \tau_i^- \left\{ Y_2(\hat{r}_{ij}) \otimes [\sigma_i \otimes \sigma_j]_2^0 \right\}_0^0$$

The second term in (9) will give a non-vanishing contribution only if the ground state has some component for which $S \neq 0$ and $L \neq 0$. In other words, $S=0$ or $L=0$ imply $D_T = D_L$ in closure approximation. This means that $S \neq 0$, $L \neq 0$ in the ground state acts itself as a SU(4) breaking spin dependent effect even if in the corrective term of (4) only the kinetic energy contribution is taken.

On the other hand, the ground state of ^{6}Li can be pictured as a core of ^{4}He plus two nucleons in $L=0$, $S=1$ state. The fact that $L=0$ forces the second term in (9) to be zero so that $D_T = D_L$. Furthermore, the dipole contribution becomes the same for D_S and D_T as was commented in Section 2, and for the allowed ($\ell=0$) contribution one can

demonstrate the validity of (8) provided that the transition to the ground state of ^{6}He is subtracted. This last point is immediately clear from the fact that the square of the matrix element of the partial transition

$$|\langle p^2(L=0, S=0, T=1)| \sum_{i=1}^{2} \tau_i^- \sigma_i j_0(\nu r_i) | p^2(L=0, S=1, T=0)\rangle|^2$$

is given by $2/3\ I_0^2(p)$. If this quantity is subtracted from $\Lambda_r(\nu)^{\ell=0}$ then the vector and axial contributions become equal [see Eq. (13) of Ref. 3)].

We remark that in general one should not identify allowed transitions with the ground state-ground state transitions [this is true in the SU(4) limit between states within a supermultiplet]. Indeed, let us take e.g., ^{12}C. The equality holds only if ^{12}C is described in jj coupling without configuration mixing and the same for ^{12}B ground state. Then effectively the latter is the only state which contributes to closure in the limit $\nu \to 0$. With configuration mixing the previous relation fails as we shall see below.

3.2 For the calculation of $D_{S,T,L}$ in ^{12}C we use the fact that only the scalar part, under $O(3) \times SU(2)_T$, of the operators contributes. Then, for the eight 1p nucleons the expected value of a two-body operator can be written as

$$\langle p^8(00)| \sum_{i<j} f_{ij} | p^8(00)\rangle = 28 \sum_{\alpha} \sum_{j_1 j_2 j_1' j_2'} [p^6(\alpha) j_1 j_2(\alpha) | p^8(00)]$$

$$\times [p^6(\alpha) j_1' j_2'(\alpha) | p^8(00)] \langle j_1 j_2(\alpha) | f_{12} | j_1' j_2'(\alpha)\rangle \qquad (10)$$

and, for $j_1 \neq j_2$ or/and $j_1' \neq j_2'$, the antisymmetric character of the states will be introduced by hand. The two-body coefficients of fractional parentage are the only nuclear model dependent quantities. We are going to use the wave function

$$|p^8(00)\rangle = C_1 |p_{3/2}^8\rangle + C_2 |p_{3/2}^6(10)\, p_{1/2}^2(10)\rangle + C_3 |p_{3/2}^6(01)\, p_{1/2}^2(01)\rangle \qquad (11)$$

$$+ C_4 |p_{3/2}^5(\tfrac{1}{2}\tfrac{1}{2})\, p_{1/2}^3(\tfrac{1}{2}\tfrac{1}{2})\rangle + C_5 |p_{3/2}^4(00)\, p_{1/2}^4(00)\rangle$$

where the amplitude values are obtained from effective interactions in the 1p shell [5].

In terms of the radial integrals $I_\ell(x,y)$ defined in Eq. (7) we obtain, after a tedious calculation, the results presented in Table I. We see that the coefficient of $I_0(p)I_2(p)$, which is a pure tensor under spatial rotations, is vanishing for D_S and $2D_T + D_L$, as shown before in general. Furthermore, the $\ell = 1$ contribution is the same for D_S, D_T and D_L.

If the harmonic oscillator wave functions, with parameter $b = 1.66$ fm, are used, the results of Table I for $D_{S,T,L}$ become of the form

$$D_{S,T,L}(\nu) = e^{-\frac{1}{2}b^2\nu^2} \left\{ C_0 + C_1\left(\tfrac{1}{2}b^2\nu^2\right) + C_2\left(\tfrac{1}{2}b^2\nu^2\right)^2 \right\} \qquad (12)$$

where the corresponding C' values are given in Table II.

The allowed contribution in the limit $\nu \to 0$ is then

$$\Lambda_r(\nu \to 0) = \left(\frac{\nu}{m}\right)^2 (B + C)\ 0.088 \qquad (13)$$

which is higher than the one deduced from $^{12}C \to {}^{12}B(g.s.)$ for which

$$\frac{\Lambda_r(^{12}C \to {}^{12}B(g.s.))}{I_0^2(p)} = \left(\frac{\nu}{m}\right)^2 (B + C)\ 0.064 \qquad (14)$$

This shows that the strength of the $0^+ \to 1^+$ transition is also contributing to other states in ^{12}B different from the ground state. In Fig. 2 the dotted line gives the value of $\Lambda_r(\nu)$ as obtained from Eq. (12) and Table II and then the $\nu \to 0$ behaviour is governed by (13). The broken line is instead $\Lambda_r(\nu)$ obtained when the value (14) is used for the $\nu \to 0$ limit. In the zone of ν- values in which we are interested the difference is only of the order of 6-7%.

The situation described above is in contrast with the simple jj coupling, where all the strength is given by the ground state transition, but its magnitude is too large by a factor 5: 0.064 becomes 0.297. In all cases the presence of this allowed transition indicates a SU(4) breaking effect, because in this scheme ^{12}C should be a scalar supermultiplet.

4. FIRST ORDER ENERGY WEIGHTED SUM RULES

4.1 The problem of studying the part of the capture rate that involves a first order energy weighted sum over excited states $|b>$ in Eq. (4), involves essentially two sum rules

$$\sum_b (E_b - E_a) \left| \langle b | \sum_j \tau_j^- e^{i\vec{\nu}\cdot\vec{r}_j} | a \rangle \right|^2 \qquad (15)$$

$$\sum_b (E_b - E_a) \left| \langle b | \sum_j \tau_j^- \vec{\sigma}_j e^{i\vec{\nu}\cdot\vec{r}_j} | a \rangle \right|^2$$

if we neglect here a possible difference between D_T and D_L about which we have commented in the last section. We can take advantage of the following sum rule

$$\frac{1}{2} \langle a | [O^+, [H, O]] | a \rangle = \sum_b (E_b - E_a) |\langle b | O | a \rangle|^2 \qquad (16)$$

which holds if

$$|\langle b | O^+ | a \rangle|^2 = |\langle b | O | a \rangle|^2 \qquad (17)$$

In our case

$$0 = \sum_j \tau_j^- e^{i\vec{\nu}\cdot\vec{r}_j} (\sigma_j)$$

The plane waves do not give trouble assuming $|a>$ and $|b>$ to be eigenstates of the parity operator, but we have to limit ourselves to $T = 0$ nuclei in order that (17) be satisfied. Indeed for $N > Z$ nuclei, $T = -M_T = (N-Z)/2$, τ^- excites only $T + 1$ levels while τ^+ excites $T-1$, T, $T + 1$ levels. So for $N > Z$ nuclei we cannot use the previous sum rule (16) but instead

$$\langle a | [O^+, H] O | a \rangle = \sum_b (E_b - E_a) |\langle b | O | a \rangle|^2 \qquad (18)$$

and we note that (16) contains two-body correlations while (18) contains also three-body correlations. One can avoid calculational problems by connecting muon capture to the $T+1$ fragment of the electromagnetic excitation, the relation is given by

$$|\langle T+1, M_T = -T-1 | \sum_i \tau_i^- O_i | T, M_T = -T \rangle|^2$$

$$= \frac{T+1}{2} |\langle T+1, M_T = -T | \sum_i \tau_i^3 O_i | T, M_T = -T \rangle|^2$$

We are going to study (16) for a Hamiltonian H of the form

$$H = \sum_i \frac{\vec{p}_i}{2M} + \sum_{i<j} V(r_{ij}) + \sum_{i<j} V^B(r_{ij}) P_{ij}^\sigma - \sum_{i<j} V^H(r_{ij}) P_{ij}^\tau + \sum_{i<j} V^M(r_{ij}) P_{ij}^x$$

$$+ \zeta \sum_i \vec{l}_i \cdot \vec{s}_i + \frac{1}{2} \sum_{i<j} (X + X_\tau P_{ij}^\tau)(\vec{r}_{ij} \times \vec{p}_{ij})(\vec{\sigma}_i + \vec{\sigma}_j) \qquad (19)$$

$$+ \sum_{i<j} (V_{T e}(r_{ij}) + V_{T e}^\tau(r_{ij}) P_{ij}^\tau) S_{ij}$$

where P_{ij}^x, P_{ij}^τ and P_{ij}^σ are the space, isospin and spin exchange operators respectively, and S_{ij} is given by

$$S_{ij} = 3 (\vec{\sigma}_i \cdot \vec{r}_{ij})(\vec{\sigma}_j \cdot \vec{r}_{ij}) / r_{ij}^2 - \vec{\sigma}_i \cdot \vec{\sigma}_j$$

For simplicity we give before only the results for the central potentials and single particle spin-orbit potential. Denoting by K^V the quantity

$$K^V = \frac{1}{2} \langle a | \left[\sum_j \tau_j^3 e^{-i \vec{\nu} \cdot \vec{r}_j}, [H, \sum_k \tau_k^3 e^{i \vec{\nu} \cdot \vec{r}_k}] \right] | a \rangle$$

we have that the contribution coming from central K_c^V and single particle spin orbit K_{ls}^V potentials is

$$K_c^V = -2 \langle a | \sum_{i<j} (1 - \cos \vec{\nu}.\vec{r}_{ij})(1 - \tau_i^3 \tau_j^3)[P_{ij}^x V^M(r_{ij}) - P_{ij}^\tau V^H(r_{ij})] | a \rangle$$

$$K_{ls}^V = 0 \qquad (20)$$

It should be noted that the sum effectively runs over neutron-proton couples.

Analogously, when the $\vec{\sigma}$'s are introduced in the operator, we have

$$\begin{aligned} K_c^A + K_{ls}^A = -\langle a | \tfrac{4}{3} \} \sum_i \vec{l}_i.\vec{s}_i + 2 \sum_{i<j} V^B(r_{ij}) \frac{1+\tau_i^3\tau_j^3}{2} (1-\cos\vec{\nu}.\vec{r}_{ij})\left(1 - \frac{\vec{\sigma}_i.\vec{\sigma}_j}{3}\right) P_{ij}^\sigma \\ + 2 \sum_{i<j} V^B(r_{ij}) \frac{1-\tau_i^3\tau_j^3}{2} \left(1 - \frac{\vec{\sigma}_i.\vec{\sigma}_j}{3}\right)(1+\cos\vec{\nu}.\vec{r}_{ij})\, P_{ij}^\sigma \\ - 2 \sum_{i<j} V^H(r_{ij}) \left(1 - \frac{\vec{\sigma}_i.\vec{\sigma}_j}{3} \cos\vec{\nu}.\vec{r}_{ij}\right)(1 - \tau_i^3\tau_j^3)\, P_{ij}^\tau \\ + 2 \sum_{i<j} V^M(r_{ij})(1-\cos\vec{\nu}.\vec{r}_{ij})\left(1 - \tau_i^3\tau_j^3 \frac{\vec{\sigma}_i.\vec{\sigma}_j}{3}\right) P_{ij}^x \, | a \rangle \end{aligned} \qquad (21)$$

In the limit of short range $\vec{\nu}\cdot\vec{r}_{ij} \to 0$

$$K_c^V + K_{ls}^V = 0$$

$$K_c^A + K_{ls}^A = -\langle a | \tfrac{4}{3} \} \sum_i \vec{l}_i.\vec{s}_i - 2 \sum_{i<j} (1 - \tau_i^3\tau_j^3)\left(1 - \frac{\vec{\sigma}_i.\vec{\sigma}_j}{3}\right)[V^H(r_{ij}) P_{ij}^\tau - V^B(r_{ij}) P_{ij}^\sigma] | a \rangle \qquad (22)$$

We note that the limit $\vec{\nu}\cdot\vec{r}_{ij} \to 0$ for (21) is equivalent to take $e^{i\vec{\nu}\cdot\vec{r}_i} \to 1$, so that this approximation connects the corrections to closure approximation to allowed Gamow-Teller transitions. From the above formulae one deduces that Wigner + Majorana potentials, which are the most important ones, do not give any contribution if the last one has zero range, i.e., of the form $\delta(r_{ij})$. This property holds both for $T = 0$ and $T \neq 0$ nuclei since it is

related to the single $[O,H]$ commutator. This depends on the fact that in the zero range case Wigner+Majorana potentials behave effectively as a Wigner local potential. Since the effective internucleonic potential is not so well known in its explicit radial dependence it does not seem reasonable at the present level of our calculation to perform detailed evaluations which would be model dependent anyway. On the other hand, the short range limit can be related directly to spectroscopical data (see Section 4.2). But the limit $\vec{\nu}\cdot\vec{r}_{ij}\to 0$ would not be a good estimate because it would give zero for (20) while we know from photoreactions that the sum rule is really enhanced relative to the kinetic energy contribution. This, as we know, is mainly a dipole effect and therefore would involve an expansion to $(\vec{\nu}\cdot\vec{r}_{ij})^2$ order. We shall assume that due to approximate SU(4) invariance these corrections are the same for D_S, D_T, D_L. Possible differences would come essentially from taking into account the fact that the axial dipole excitations have a different mean energy than the vector ones. We will here neglect systematically this effect which for ^6Li, ^{12}C should be small according to estimates in Ref. 6). Our main difference between vector and axial excitations will be obtained taking only the short range limit $\vec{\nu}\cdot\vec{r}_{ij}\to 0$, i.e., we use (22). This is in agreement with the fact that the main effect distinguishing D_S from D_T and D_L is due to allowed transitions. The commutators with the other potentials K_{S-O}, K_{Te} where S-O and Te refer to the two-body spin orbit and tensor potentials respectively, in the limit $\vec{\nu}\cdot\vec{r}_{ij}\to 0$ are

$$K^{V}_{S-O} = 0$$
$$K^{V}_{Te} = 0 \qquad (23)$$

$$K^{A}_{S-O} = -2\langle a|\frac{2}{3}X\sum_{i<j}(\vec{r}_{ij}\times\vec{p}_{ij})(\vec{\sigma}_i+\vec{\sigma}_j) - \frac{1}{3}X_\tau\sum_{i<j}(1-\tau_i^3\tau_j^3)(\vec{r}_{ij}\times\vec{p}_{ij})(\vec{\sigma}_i+\vec{\sigma}_j)P_{ij}^{\tau}$$
$$+ X_\tau\sum_{i<j}(\vec{r}_{ij}\times\vec{p}_{ij})(\vec{\sigma}_i+\vec{\sigma}_j)P_{ij}^{\tau}\left(1-\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{3}\right)(1-\tau_i^3\tau_j^3)|a\rangle \qquad (24)$$

$$K^{A}_{Te} = -2\langle a|\frac{2}{3}\sum_{i<j}V_{Te}(r_{ij})S_{ij}(2+\tau_i^3\tau_j^3) + \frac{2}{3}\sum_{i<j}V^{\tau}_{Te}(r_{ij})S_{ij}P_{ij}^{\tau}(1+2\tau_i^3\tau_j^3)$$
$$+\sum_{i<j}V^{\tau}_{Te}(r_{ij})S_{ij}P_{ij}^{\tau}\left(1-\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{3}\right)(1-\tau_i^3\tau_j^3)|a\rangle$$

As far as the term (20) is concerned we calculate it very simply by a Van Vleck potential $V = V_o + ap^2$, where the coefficient a, which is a function of the density, is adjusted to fit photoreaction data. We note that in our case the mass renormalization $(M \to M^*)$ due to the momentum dependent potential is not subject to the criticism of being spurious as for example would be true for electrons in an atom for which we know that the potentials are Coulombic which are perfectly local and would not give any correction to the theorem (2). In our case instead this is a true physical effect which is observed e.g., in photoreactions. Moreover, the value provided by photoreactions is a good estimate also for muon capture because in both reactions dipole transitions are prominent so that the weight given by these reactions to the spatial distribution is the same. We illustrate this point with the calculations of the sum rules for ^6Li and ^{12}C for which we take the ratio $M/M^* \begin{cases} 1.3 & \text{for } ^6Li \\ 1.6 & \text{for } ^{12}C \end{cases}$ consistently with photoreactions [8].

For the calculation of K^A it is interesting to remark that in our approximation we can connect very easily the spectroscopical information contained in Cohen-Kurath matrix elements [5] expressed in LS coupling with the corrective term since the commutator with a certain potential contains again the same potential in the final expression. So we can obtain our result in terms of matrix elements of effective potentials. Phenomenologically it is noticeable that we correct closure approximation with the effective kinetic energy term and K^A arising from the analog of M1 transition operators which are SU(4) generators. This shows clearly the importance of the SU(4) structure of the ground state; indeed for a double closed shell nucleus the last correction is exactly zero.

4.2 We now give the explicit relations between expected values of the commutator and matrix elements of the effective potential. If in the LS scheme we define

$$\langle p^n(\alpha) | \sum_{i<j} f_{ij} | p^n(\alpha) \rangle = \sum_{LS, L'S'} \sum_{JT} C(LS, L'S', JT) \langle p^2, LS(JT) | f_{12} | p^2, L'S'(JT) \rangle \qquad (25)$$

where f_{12} is a scalar operator, we obtain for (22)

$$K_c^A = -4 \sum_{LJ} \left\{ \left[C(L0, L0, J0) - \frac{1}{9} C(L1, L1, J1) \right] \langle L | {}^{00}V_c - {}^{11}V_c | L \rangle \right.$$
$$\left. + \frac{1}{3} \left[C(L1, L1, J0) - C(L0, L0, J1) \right] \langle L | {}^{10}V_c - {}^{01}V_c | L \rangle \right\} \tag{26}$$

where ${}^{ST}V_c$ refers to the central potential for these values of S, T. For the single particle spin-orbit contribution we have

$$K_{\ell 1}^A = \frac{8}{9} (\epsilon_{1/2} - \epsilon_{3/2}) \left[\frac{1}{2} N_{3/2} - N_{1/2} \right] \tag{27}$$

where $\epsilon_{\frac{3}{2},\frac{1}{2}}$ are the single particle energies in the corresponding sub-shell, and $N_{\frac{3}{2},\frac{1}{2}}$ are the effective number of nucleons defined by

$$\langle a | \sum_i f_i | a \rangle = \sum_j N_j \langle j | f_1 | j \rangle$$

for a scalar single particle operator.

Analogously, the two-body spin-orbit and tensor contributions are given by

$$K_{S-O}^A = -\frac{4}{3} \sum_{LJT} C(L1, L1, JT) \frac{T+1}{2T+1} \langle L1(J) | {}^T V_{S-O} | L1(J) \rangle$$
$$K_{Te}^A = -\frac{4}{3} \sum_{LL'JT} C(L1, L'1, JT) \frac{6T+1}{2T+1} \langle L1(J) | {}^T V_{Te} | L'1(J) \rangle \tag{28}$$

It is straightforward to show that all these expressions for $K^A = K_c^A + K_{\ell s}^A + K_{S-O}^A + K_{Te}^A$ vanish for a double closed shell nucleus, like ^{16}O.

In the case of ^6Li, with $L = 0$, $S = 1$ coupling, only K_c^A is different from zero.

For ^{12}C the coefficients $C(LS, L'S', JT)$ are related to the two-body fractional parentage coefficients introduced in Eq. (10) by doing

the change of scheme from jj to LS coupling. The matrix elements of the effective potentials are given in Ref. 5), obtained from spectroscopy in the 1p shell.

Our results are $K^A = K_c^A \simeq 2$ MeV for ^{6}Li and $K_c^A \simeq 6$ MeV, $K_{ls}^A \simeq 2$ MeV, $K_{S-O}^A \simeq 5$ MeV, $K_{Te}^A \simeq -1.6$ MeV for ^{12}C. It is interesting to point out that all these corrections tend to enhance the mean excitation energy of the axial transitions relative to the vector ones, with the exception of the tensor interaction.

5. RESULTS AND DISCUSSION

Taking into account the corrective terms in Eq. (4), as explained in Section 4, we can write our answer for the capture rate as

$$\Lambda_r = \left\{ 1 + (m-\nu) \frac{d}{d\nu} \right\} \Lambda_r(\nu) - 2 \frac{m}{M^*} \left(\frac{\nu}{m} \right)^3 - \frac{1}{2} \frac{K^A}{m} \frac{\nu}{m} \qquad (29)$$

In Fig. 1 the dotted-dashed line is the answer for the dipole contribution in ^{6}Li when the effective mass correction is included. We obtain now $\nu^{\ell=1} \simeq 88$ MeV, which is consistent with the peak energy observed in photoreactions [8]. The solid line is our total result for the capture rate; we obtain $\Lambda_r = 0.280$ to be compared with the experimental result [7] $\Lambda_r(^6\text{Li}) = 0.273 \pm 0.057$. The dipole part is predicted to be 0.186, whereas the allowed contribution (which is exhausted by the ground state transition) is of the order of 0.076.

In Fig. 2 our results for ^{12}C are presented. The solid line gives a result $\Lambda_r = 0.124$ to be compared with experiment [7] $\Lambda_r(^{12}\text{C}) = 0.125 \pm 0.005$. If the broken line is used for the main term $\Lambda_r(\nu)$, the result we obtain from Eq. (29) is that indicated in the dotted-dashed line, which is somewhat smaller than experiment. In Ref. 10) it was noticed that the capture rate is extremely sensitive to the configuration mixing of ^{12}C so that there are wild discrepancies between experimental data and theoretical predictions if the extreme jj coupling or the LS coupling are used. This is seen in Table III, where the results of Λ_r are given in terms of the parameter ν.

As one can notice from the Figures the kinetic energy correction to closure approximation is sufficient to achieve stability of the value Λ_r, over a large range of ν. The effective mass correction gives as a result that the stability region is shifted in ν and is consistent with what we know about the reaction mechanism, i.e., excitation of giant resonance analogue states. More quantitatively, we have the result that in the mean the transition strength is in the region of about 17 MeV in ^{6}Li and 25 MeV in ^{12}C. The values of the effective masses we use are obtained fitting the integrated cross-section for photoabsorption from threshold to 140 MeV. A dependence on ν of M^* could be expected if the momentum dependent potential has terms of higher order than the second, in this sense our formulation is a truncation to the second order. Since quadrupole and higher multipoles do not seem to play an important role, the approximation $M^* = cte$, and, taken from photoreactions, does not seem to be dangerous. It can only affect the values for very high ν where the curves in Fig. 2 decrease very rapidly.

From Table III it is evident that total muon capture rates may be used as a sensitive tool to investigate the detailed structure of the configuration mixing in the ground state of the target. The information coming from muon capture rates can complement very well that of M1 transitions and allowed β decays because muon capture induces transitions to states which are the same which β decay, e.g., $^{12}C \rightarrow {}^{12}B(g.s)$ but also to states which cannot β decay or decay by M1 transitions because they are unstable for particle emission. In the results for the total Λ_r there is a variation of 10% if one takes all the strength of the allowed axial transition to be saturated in the ground state transition, as one can see in Fig. 2. The main ingredient in the discrepancy between the prediction for the capture rate using Cohen-Kurath wave function and an extreme jj coupling is the different strength of this allowed transition. In general this means that the dynamics of allowed transitions, analogous to M1 electromagnetic transitions, is very important and therefore the SU(4) group, even if broken, is very useful in analyzing them since the transition operators are generators of the group.

In conclusion what we have done is a consistent treatment of muon capture which seems to be in agreement with all we know about weak interactions and nuclear structure and being free from uncertainties associated to the mean neutrino energy in the Primakoff closure approximation.

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	$I_0^2(s)$	$I_0^2(p)$	$I_0(p)\ I_2(p)$	$I_2^2(p)$	$I_1^2(s,p)$
D_S	2.0	4.0	0.	5.83	8.0
D_T	2.0	3.47	-0.12	6.10	8.0
D_L	2.0	3.47	0.24	6.84	8.0

Table I : The functions $D_{S,T,L}(\nu)$ for ^{12}C in terms of the radial integrals.

	C_0	C_1	C_2
D_S	6.0	0.0	1.09
D_T	5.47	0.31	1.08
D_L	5.47	0.43	1.12

Table II : Values of the C's defined in Eq. (12) for the three functions $D_{S,T,L}$.

ν (MeV)	61	66	71	77	82	87
jj	0.220	0.227	0.229	0.226	0.217	0.200
L-S	0.080	0.087	0.091	0.089	0.082	0.068
C-K	0.116	0.122	0.125	0.123	0.114	0.098

Table III : Total capture rate Λ_r in the extreme jj or L-S couplings and using the Cohen-Kurath wave function. The stability of the results in a zone around 75 MeV is apparent. The experimental result is $\Lambda_r = 0.125 \pm 0.005$.

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FIGURE CAPTIONS

Figure 1 :

Reduced capture rate Λ_r for ^{6}Li. Our prediction for the total capture rate (solid line), in agreement with experiment, is compared to the dipole contribution with the effective mass correction to the kinetic energy term (dotted-dashed line) and without it (broken line). The dotted line is the dipole contribution in the Primakoff closure approximation.

Figure 2 :

Reduced capture rate Λ_r for ^{12}C. The total capture rate (solid line), in agreement with experiment, is compared to the ν-dependence in the Primakoff closure approximation (dotted line). The dotted-dashed and broken lines are, respectively, the corresponding predictions when only the ground state transition for the allowed contribution is included.

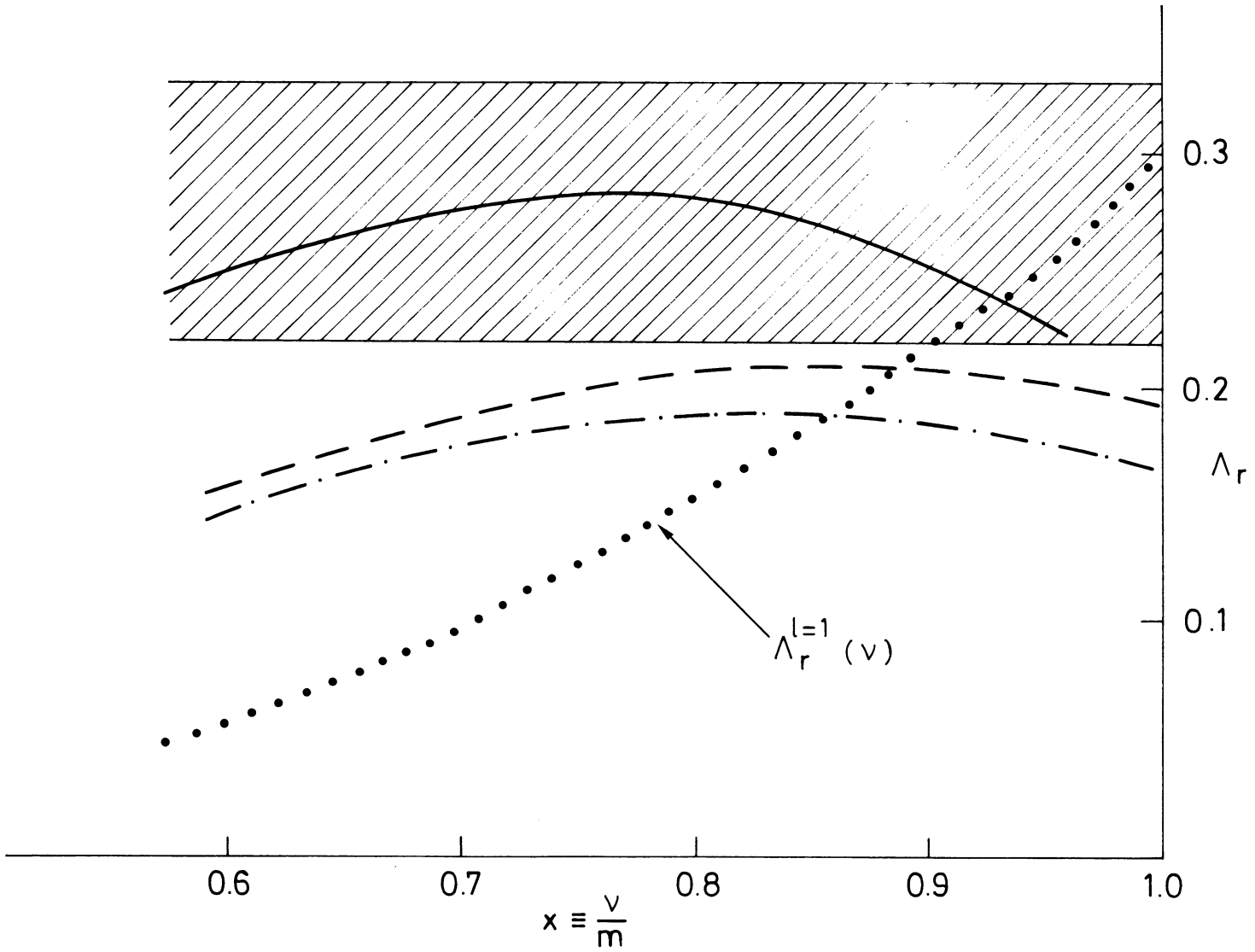

Fig. 1

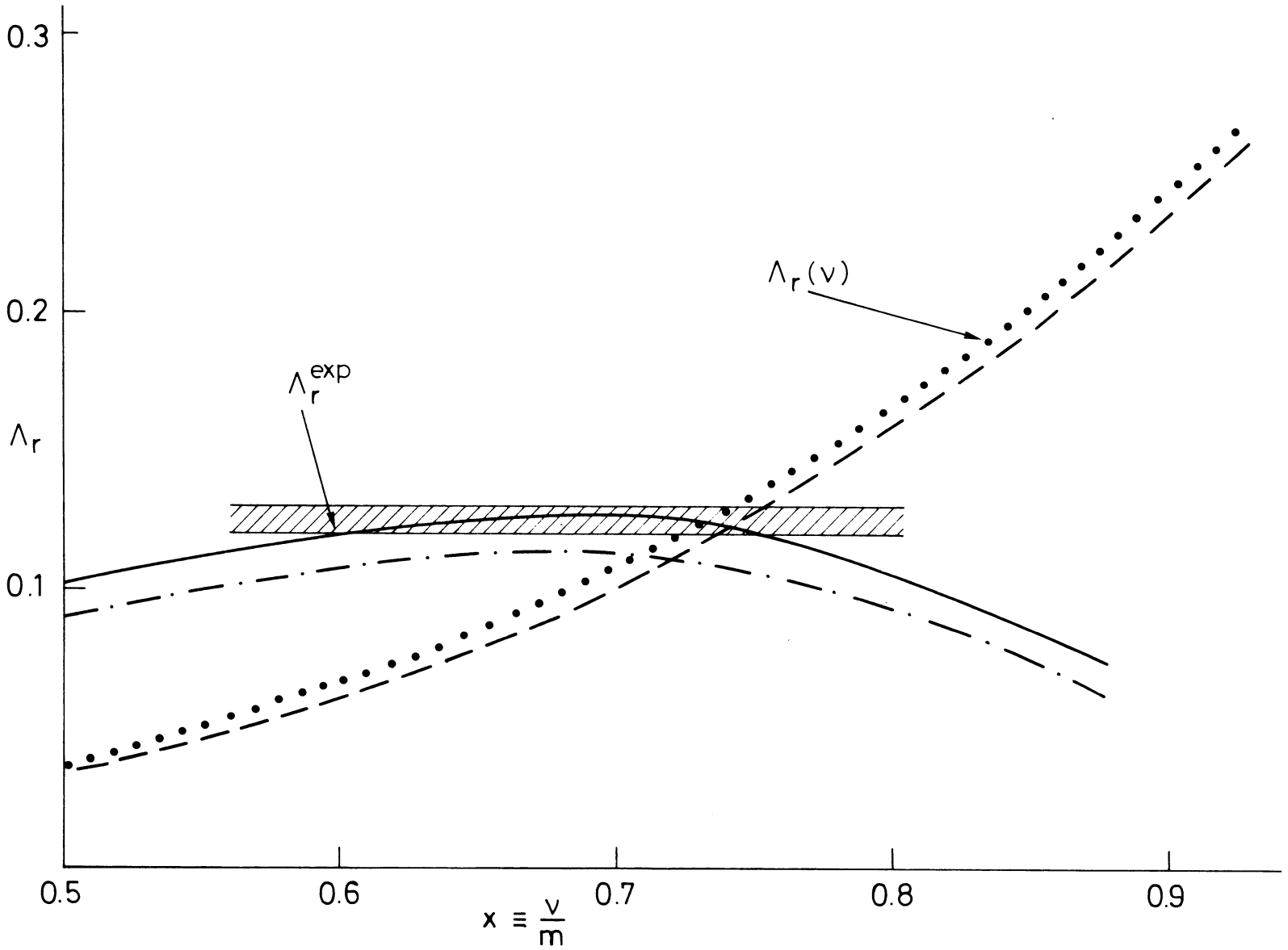
0.3
0.2
0.1
Λ_r
Λ_r^{exp}
$\Lambda_r(\nu)$
0.5
0.6
0.7
0.8
0.9
$x \equiv \frac{\nu}{m}$

Fig. 2